Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.2-d-x-^m-a+b-x^n+c-x^-2-n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [664]. This is test number [46].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (664)	% 0.00 (0)
Mathematica	% 99.70 (662)	% 0.30 (2)
Maple	% 74.70 (496)	% 25.30 (168)
Maxima	% 45.63 (303)	% 54.37 (361)
Fricas	% 80.57 (535)	% 19.43 (129)
Sympy	% 42.47 (282)	% 57.53 (382)
Giac	% 62.05 (412)	% 37.95 (252)
Mupad	% 54.22 (360)	% 45.78 (304)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

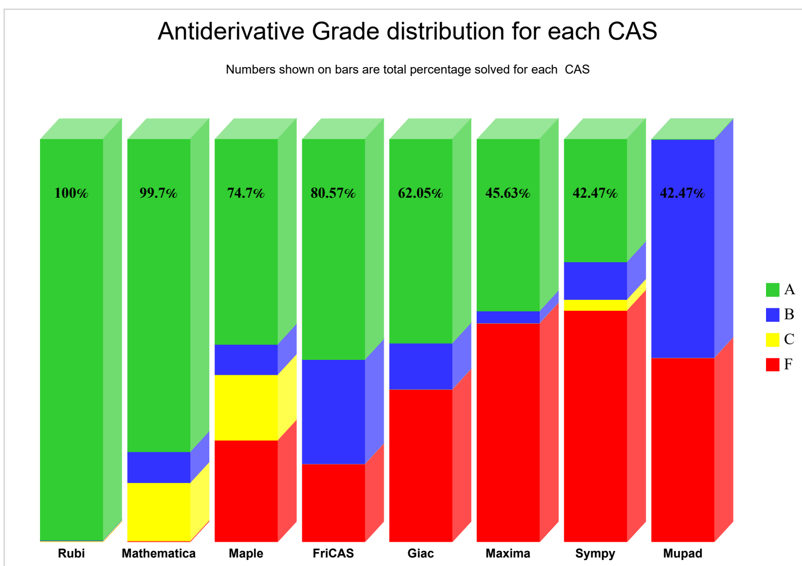
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

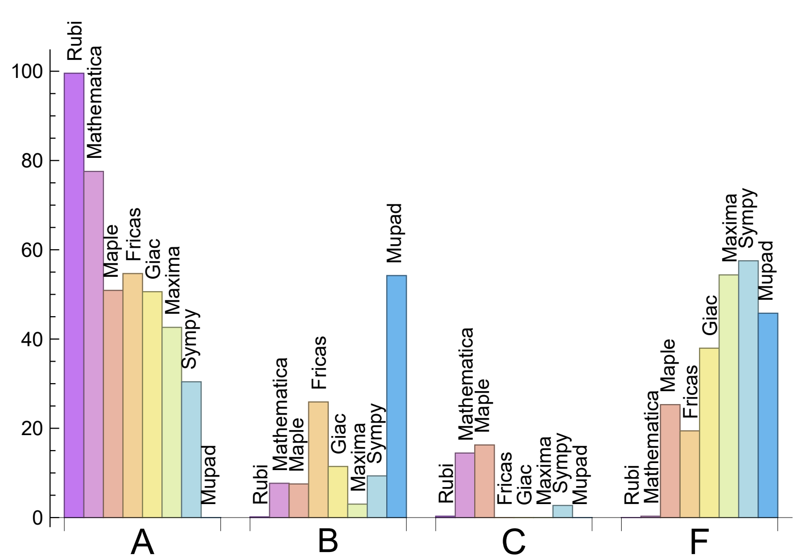
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.55	0.15	0.30	0.00
Mathematica	77.56	7.68	14.46	0.30
Maple	50.90	7.53	16.27	25.30
Maxima	42.62	3.01	0.00	54.37
Fricas	54.67	25.90	0.00	19.43
Sympy	30.42	9.34	2.71	57.53
Giac	50.60	11.45	0.00	37.95
Mupad	0.00	54.22	0.00	45.78

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	168	100.00 %	0.00 %	0.00 %
Maxima	361	67.59 %	9.97 %	22.44 %
Fricas	129	66.67 %	10.85 %	22.48 %
Sympy	382	75.92 %	23.30 %	0.79 %
Giac	252	93.65 %	1.59 %	4.76 %
Mupad	304	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

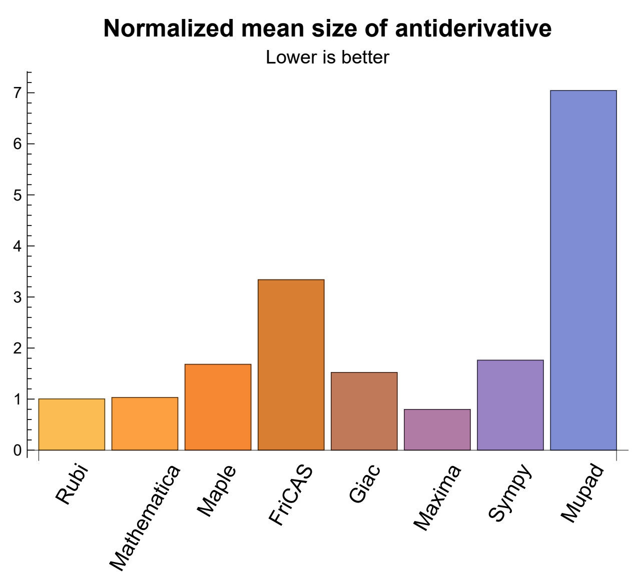
1.3 Performance

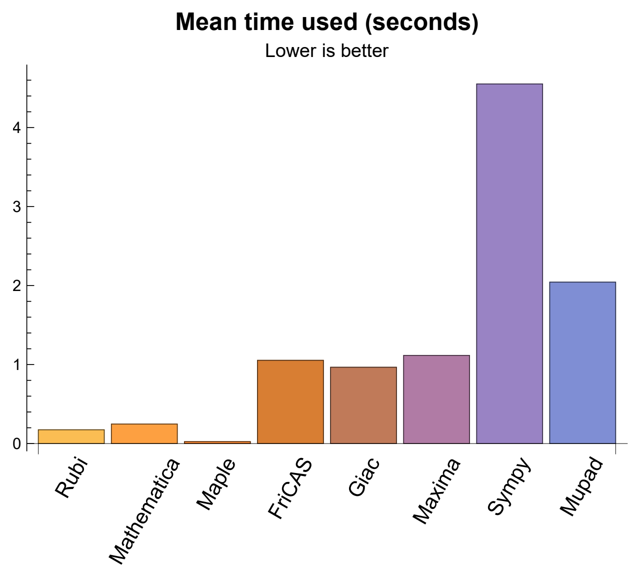
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	157.95	1.00	137.00	1.00
Mathematica	0.25	149.32	1.03	83.00	0.92
Maple	0.02	283.22	1.68	78.00	0.79
Maxima	1.12	80.96	0.80	55.00	0.76
Fricas	1.05	774.34	3.34	143.00	1.50
Sympy	4.55	183.60	1.76	59.00	0.94
Giac	0.97	256.17	1.52	91.00	0.85
Mupad	2.04	1652.88	7.04	119.00	0.99

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {132, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 309, 327, 329, 331, 333, 338, 340, 341, 343, 344, 346, 347, 351, 353, 367, 385, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 602, 603, 604, 605, 606}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

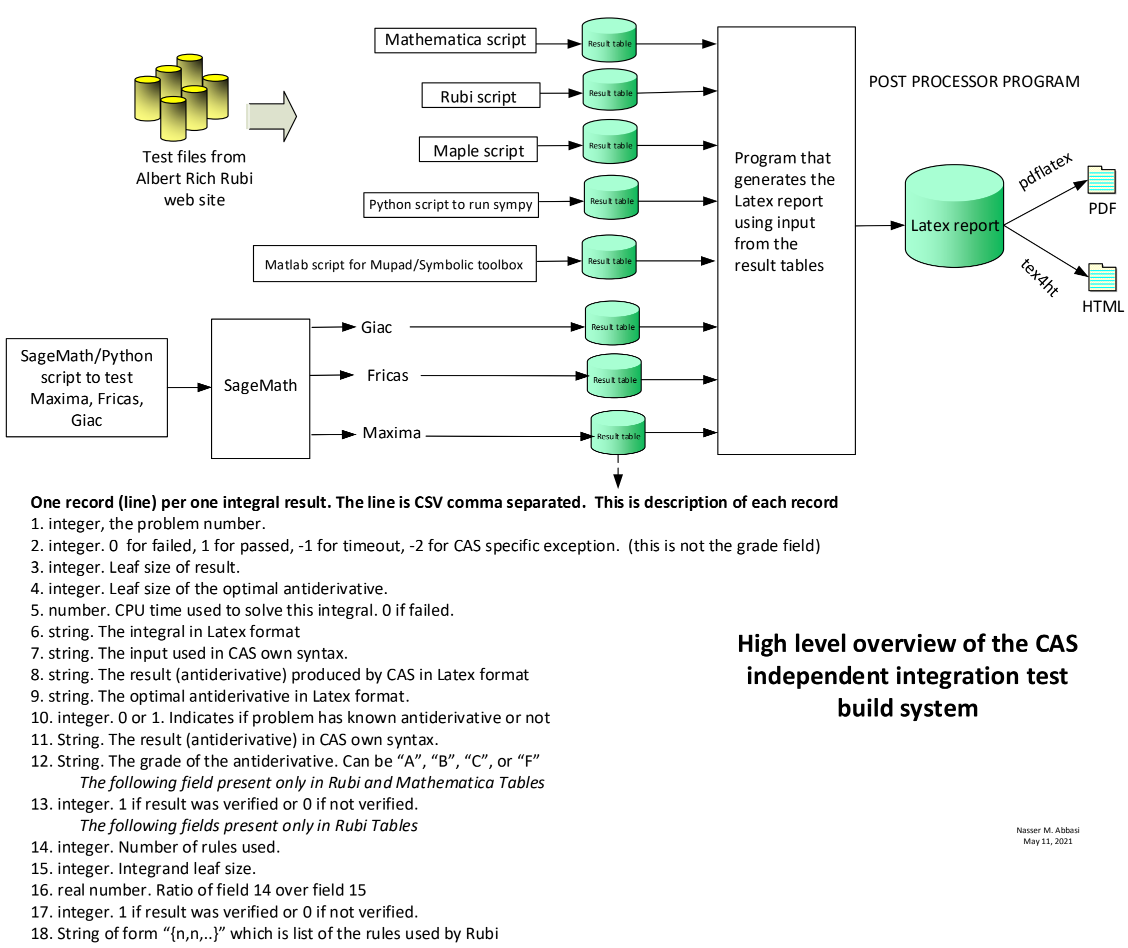
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664 }

B grade: { 154 }

C grade: { 176, 478 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 328, 330, 332, 339, 342, 345, 348, 349, 350, 352, 355, 368, 369, 370, 371, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 562, 563, 565, 566, 567, 572, 579, 582, 583, 584, 585, 586, 587, 588, 593, 596, 597, 598, 599, 604, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663 }

B grade: { 61, 154, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 232, 233, 243, 244, 245, 246, 247, 252, 564, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 589, 590, 591, 592, 594, 595, 600, 601, 602, 603, 605, 607, 608, 609, 610, 611, 612, 664 }

C grade: { 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 184, 250, 251, 257, 258, 309, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 333, 334, 335, 336, 337, 338, 340, 341, 343, 344, 346, 347, 351, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 409, 410, 411, 457, 458, 478, 500, 501, 502, 503, 504, 505, 546, 547, 559, 560, 561 }

F grade: { 661, 662 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 111, 113, 114, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 237, 238, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 313, 314, 315, 316, 317, 318, 328, 329, 330, 331, 332, 333, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 368, 370, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 427, 436, 437, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 523, 524, 525, 526, 527, 528, 533, 540, 544, 572, 579 }

B grade: { 61, 82, 107, 109, 110, 112, 115, 116, 154, 248, 311, 334, 336, 369, 371, 375, 377, 409, 411, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 438, 439, 449, 450, 456, 549, 550, 551, 552, 553, 554, 555, 565, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade: { 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 319, 320, 321, 322, 323, 324, 325, 326, 358, 359, 360, 361, 362, 363, 364, 365, 366, 378, 379, 380, 381, 382, 383, 384, 457, 458, 500, 501, 504, 505, 515, 522, 548, 556, 557, 558, 559, 560, 561, 596, 597, 598, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 472, 473, 474, 475, 476, 477, 478, 506, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 545, 546, 547, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 348, 350, 352, 354, 356, 368, 370, 372, 374, 376, 386, 387, 388, 390, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 548, 596, 597, 598 }

B grade: { 9, 21, 42, 45, 48, 79, 82, 85, 88, 154, 389, 391, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade: { }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 353, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 472, 476, 477, 478, 481, 500, 501, 502, 503, 504, 505, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 240, 241, 242, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 296, 298, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 348, 349, 350, 352, 354, 355, 356, 359, 362, 365, 368, 370, 374, 376, 386, 388, 392, 394, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 460, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 478, 479, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 565, 572, 579, 586, 616, 617, 639, 641, 642 }

B grade: { 82, 143, 144, 145, 146, 147, 148, 149, 150, 154, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 239, 248, 293, 295, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 344, 346, 351, 353, 357, 358, 360, 361, 363, 364, 366, 369, 371, 372, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 387, 389, 390, 391, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 454, 456, 457, 458, 470, 471, 552, 556, 557, 558, 559, 560, 561, 593, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade: { }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 459, 461, 472, 476, 477, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.1.6 Sympy

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 89, 90, 91, 92, 93, 94, 95, 96, 97, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 321, 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 388, 390, 392, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 440, 441, 442, 443, 444, 445, 446, 447, 448, 456, 457, 460, 493, 494, 495, 496, 498, 499, 502, 503, 598, 613, 615, 618, 620, 638, 640, 643, 645 }

B grade: { 138, 139, 140, 141, 142, 310, 312, 314, 316, 387, 389, 391, 393, 395, 412, 413, 414, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 607, 608, 609, 610, 611, 612, 614, 616, 617, 619, 621, 622, 623, 624, 631, 633, 639, 641, 642, 644, 646, 647, 648, 649, 655, 657, 663, 664 }

C grade: { 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 338, 340, 341, 343, 344, 346 }

F grade: { 1, 2, 3, 4, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 318, 319, 320, 325, 326, 327, 347, 367, 385, 419, 420, 421, 428, 429, 430, 438, 439, 449, 450, 451, 452, 453, 454, 455, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 500, 501, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 625, 626, 627, 628, 629, 630, 632, 634, 635, 636, 637, 650, 651, 652, 653, 654, 656, 658, 659, 660, 661, 662 }

2.1.7 Giac

A grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 120, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 188, 189, 204, 205, 222, 223, 237, 238, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 475, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 496, 500, 501, 502, 515, 516, 517, 518, 523, 524, 525, 552, 614, 616, 619, 622, 624, 628, 637, 641, 663, 664 }

B grade: { 45, 61, 82, 118, 119, 125, 154, 170, 172, 173, 175, 176, 178, 179, 181, 248, 249, 311, 313, 315, 317, 349, 355, 389, 391, 456, 473, 474, 522, 558, 597, 598, 607, 608, 609, 610, 611, 612, 613, 615, 617, 620, 621, 623, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

C grade: { }

F grade: { 1, 2, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 182, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 347, 367, 385, 449, 450, 451, 452, 453, 454, 457, 458, 459, 472, 476, 477, 478, 479, 493, 494, 495, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604, 605, 606, 618, 643, 661, 662 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 30, 43, 44, 45, 46, 47, 48, 49, 61, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 94, 97, 100, 108, 111, 125, 126, 127, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 204, 205, 222, 223, 224, 225, 236, 237, 238, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 455, 456, 457, 458, 460, 465, 468, 469, 470, 471, 473, 474, 475, 478, 485, 496, 552, 565, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade: { }

F grade: { 7, 8, 10, 11, 13, 14, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 89, 90, 92, 93, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 449, 450, 453, 454, 459, 461, 462, 463, 464, 466, 467, 472, 476, 477, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	46	53	0	0	40
normalized size	1	1.00	0.81	0.75	0.88	1.02	0.00	0.00	0.77
time (sec)	N/A	0.048	0.024	0.006	0.443	1.162	0.000	0.000	1.226

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	0	29
normalized size	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	1.16
time (sec)	N/A	0.005	0.008	0.004	0.458	1.237	0.000	0.000	1.152

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	17	21	0	14	21
normalized size	1	1.00	1.00	1.17	0.74	0.91	0.00	0.61	0.91
time (sec)	N/A	0.005	0.008	0.004	0.449	1.191	0.000	24.996	1.151

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	46	46	38	54	0	52	51
normalized size	1	1.00	0.60	0.60	0.49	0.70	0.00	0.68	0.66
time (sec)	N/A	0.057	0.010	0.004	0.465	0.838	0.000	23.789	1.282

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	37	46	48	38	51
normalized size	1	1.00	1.00	0.79	0.77	0.96	1.00	0.79	1.06
time (sec)	N/A	0.023	0.017	0.007	0.946	0.841	0.147	0.316	0.095

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	83	13	12	23	59
normalized size	1	1.00	0.49	0.46	1.05	0.16	0.15	0.29	0.75
time (sec)	N/A	0.023	0.012	0.003	0.490	0.848	0.107	0.395	1.259

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
normalized size	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.023	0.008	0.001	0.466	0.496	0.109	0.371	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
normalized size	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.024	0.007	0.003	0.463	0.771	0.106	0.388	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	52	13	12	22	33
normalized size	1	1.00	1.06	0.97	1.44	0.36	0.33	0.61	0.92
time (sec)	N/A	0.028	0.009	0.003	0.462	0.567	0.105	0.362	1.229

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
normalized size	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.018	0.009	0.007	0.451	0.826	0.105	0.326	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	10	10	8	20	-1
normalized size	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01
time (sec)	N/A	0.012	0.007	0.001	0.440	0.788	0.103	0.379	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	96	11	10	28	109
normalized size	1	1.00	0.49	0.45	1.28	0.15	0.13	0.37	1.45
time (sec)	N/A	0.020	0.009	0.010	0.462	0.840	0.129	0.289	1.377

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	38	36	14	14	8	29	-1
normalized size	1	1.00	0.49	0.47	0.18	0.18	0.10	0.38	-0.01
time (sec)	N/A	0.021	0.009	0.001	0.463	0.767	0.125	0.323	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	37	34	15	15	8	26	-1
normalized size	1	1.00	0.50	0.46	0.20	0.20	0.11	0.35	-0.01
time (sec)	N/A	0.020	0.007	0.003	0.456	0.863	0.134	0.403	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	99	17	10	43	112
normalized size	1	1.00	0.52	0.51	1.32	0.23	0.13	0.57	1.49
time (sec)	N/A	0.021	0.009	0.009	0.465	0.641	0.162	0.346	1.384

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	13	13	14	30	33
normalized size	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43
time (sec)	N/A	0.021	0.008	0.003	0.454	0.785	0.168	0.354	1.208

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.022	0.008	0.003	0.456	0.873	0.179	0.348	1.176

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	37	34	86	13	14	30	33
normalized size	1	1.00	0.47	0.43	1.09	0.16	0.18	0.38	0.42
time (sec)	N/A	0.022	0.008	0.003	0.488	0.870	0.185	0.336	1.184

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.022	0.008	0.003	0.457	0.934	0.193	0.339	1.267

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.021	0.008	0.003	0.453	0.872	0.197	0.379	1.164

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	117	15	15	31	35
normalized size	1	1.00	0.49	0.46	1.48	0.19	0.19	0.39	0.44
time (sec)	N/A	0.022	0.008	0.004	0.473	0.873	0.209	0.357	1.151

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.022	0.009	0.004	0.452	0.788	0.217	0.307	1.162

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.043	0.020	0.007	0.472	0.755	0.000	0.385	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	167	61	58	114	35	0	67	-1
normalized size	1	1.40	0.51	0.49	0.96	0.29	0.00	0.56	-0.01
time (sec)	N/A	0.053	0.015	0.008	0.463	0.926	0.000	0.303	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.043	0.016	0.007	0.448	0.825	0.000	0.295	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.041	0.015	0.008	0.452	0.640	0.000	0.359	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	58	83	35	0	45	46
normalized size	1	1.00	0.78	0.74	1.06	0.45	0.00	0.58	0.59
time (sec)	N/A	0.050	0.015	0.007	0.487	0.622	0.000	0.437	1.249

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.041	0.016	0.007	0.469	0.779	0.000	0.368	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.041	0.015	0.007	0.480	0.804	0.000	0.447	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	60	57	52	35	0	44	36
normalized size	1	1.00	1.67	1.58	1.44	0.97	0.00	1.22	1.00
time (sec)	N/A	0.030	0.016	0.007	0.453	0.871	0.000	0.413	1.220

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.038	0.015	0.005	0.443	0.807	0.000	0.374	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	59	56	32	32	0	64	-1
normalized size	1	1.00	0.36	0.35	0.20	0.20	0.00	0.40	-0.01
time (sec)	N/A	0.033	0.014	0.003	0.715	0.724	0.000	0.360	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	60	57	152	32	0	65	-1
normalized size	1	1.00	0.38	0.36	0.95	0.20	0.00	0.41	-0.01
time (sec)	N/A	0.048	0.020	0.010	0.517	0.735	0.000	0.426	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	67	-1
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	-0.01
time (sec)	N/A	0.042	0.016	0.007	0.496	0.829	0.000	0.337	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	37	37	0	65	-1
normalized size	1	1.00	0.37	0.36	0.23	0.23	0.00	0.40	-0.01
time (sec)	N/A	0.042	0.018	0.006	0.498	0.876	0.000	0.384	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	62	59	156	38	0	85	-1
normalized size	1	1.00	0.39	0.37	0.97	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.048	0.020	0.010	0.528	0.877	0.000	0.365	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	69	-1
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01
time (sec)	N/A	0.042	0.015	0.006	0.525	0.882	0.000	0.362	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	37	37	0	68	-1
normalized size	1	1.00	0.37	0.36	0.23	0.23	0.00	0.42	-0.01
time (sec)	N/A	0.041	0.017	0.006	0.497	0.776	0.000	0.293	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	61	60	220	39	0	86	-1
normalized size	1	1.00	0.38	0.37	1.36	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.047	0.015	0.012	0.700	0.860	0.000	0.340	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	70	-1
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01
time (sec)	N/A	0.041	0.014	0.006	0.738	0.684	0.000	0.470	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	61	58	37	37	0	67	-1
normalized size	1	1.00	0.38	0.36	0.23	0.23	0.00	0.41	-0.01
time (sec)	N/A	0.041	0.014	0.007	0.677	0.714	0.000	0.391	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	63	60	253	39	0	85	-1
normalized size	1	1.00	0.39	0.37	1.57	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.046	0.021	0.013	0.724	0.851	0.000	0.377	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.92
time (sec)	N/A	0.040	0.013	0.007	0.665	0.939	0.000	0.361	1.214

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.041	0.016	0.007	0.718	0.860	0.000	0.419	1.224

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	148	35	0	68	151
normalized size	1	1.00	1.44	1.37	3.61	0.85	0.00	1.66	3.68
time (sec)	N/A	0.018	0.013	0.007	0.546	0.820	0.000	0.357	1.208

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.041	0.013	0.010	0.689	0.817	0.000	0.358	1.191

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.041	0.016	0.007	0.686	0.838	0.000	0.360	1.207

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	58	179	37	0	69	151
normalized size	1	1.00	0.73	0.69	2.13	0.44	0.00	0.82	1.80
time (sec)	N/A	0.040	0.014	0.012	0.522	0.969	0.000	0.414	1.209

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.040	0.014	0.007	0.766	0.883	0.000	0.304	1.215

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.064	0.025	0.013	0.830	0.775	0.000	0.326	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.058	0.020	0.010	0.603	0.890	0.000	0.322	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	145	57	0	105	-1
normalized size	1	1.00	0.52	0.50	0.91	0.36	0.00	0.66	-0.01
time (sec)	N/A	0.120	0.021	0.012	0.725	0.855	0.000	0.351	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.061	0.024	0.010	0.805	0.806	0.000	0.443	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.057	0.020	0.008	0.674	0.887	0.000	0.401	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	114	57	0	105	-1
normalized size	1	1.00	0.70	0.67	0.96	0.48	0.00	0.88	-0.01
time (sec)	N/A	0.090	0.021	0.009	0.666	0.851	0.000	0.377	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.060	0.021	0.008	0.774	0.776	0.000	0.339	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.059	0.020	0.009	0.714	0.871	0.000	0.345	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	80	83	57	0	67	-1
normalized size	1	1.00	1.06	1.03	1.06	0.73	0.00	0.86	-0.01
time (sec)	N/A	0.056	0.021	0.009	0.625	0.822	0.000	0.291	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.059	0.021	0.009	0.691	0.774	0.000	0.416	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	-1
normalized size	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.057	0.020	0.008	0.511	0.797	0.000	0.317	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	79	52	57	0	66	36
normalized size	1	1.00	2.28	2.19	1.44	1.58	0.00	1.83	1.00
time (sec)	N/A	0.029	0.021	0.009	0.660	0.894	0.000	0.373	1.244

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	-1
normalized size	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.054	0.020	0.005	1.101	0.907	0.000	0.368	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	81	78	53	53	0	101	-1
normalized size	1	1.00	0.33	0.32	0.21	0.21	0.00	0.41	-0.00
time (sec)	N/A	0.051	0.019	0.004	1.016	0.863	0.000	0.376	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	206	55	0	104	-1
normalized size	1	1.00	0.33	0.31	0.82	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.069	0.025	0.008	1.156	0.864	0.000	0.352	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	105	-1
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.061	0.021	0.008	0.820	0.863	0.000	0.334	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	103	-1
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.41	-0.00
time (sec)	N/A	0.059	0.021	0.008	1.231	0.625	0.000	0.362	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	214	61	0	124	-1
normalized size	1	1.00	0.34	0.33	0.85	0.24	0.00	0.49	-0.00
time (sec)	N/A	0.074	0.026	0.011	1.196	0.646	0.000	0.308	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	59	59	0	107	-1
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.059	0.023	0.007	0.972	0.858	0.000	0.351	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	106	-1
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.058	0.020	0.007	0.672	0.605	0.000	0.341	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	282	61	0	126	-1
normalized size	1	1.00	0.34	0.33	1.12	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.073	0.024	0.013	1.245	0.661	0.000	0.367	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	59	59	0	107	-1
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.058	0.021	0.008	1.099	0.898	0.000	0.326	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	59	59	0	105	-1
normalized size	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.059	0.021	0.008	1.326	0.867	0.000	0.336	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	313	61	0	127	-1
normalized size	1	1.00	0.34	0.33	1.24	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.071	0.028	0.012	0.797	0.777	0.000	0.397	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	108	-1
normalized size	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00
time (sec)	N/A	0.062	0.017	0.006	1.013	0.642	0.000	0.397	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	59	59	0	106	-1
normalized size	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.060	0.016	0.006	0.974	0.750	0.000	0.350	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	342	61	0	125	-1
normalized size	1	1.00	0.34	0.33	1.36	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.071	0.019	0.013	1.097	0.861	0.000	0.378	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	108	-1
normalized size	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00
time (sec)	N/A	0.059	0.017	0.006	0.986	0.604	0.000	0.335	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	59	59	0	105	-1
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.060	0.018	0.007	0.990	0.911	0.000	0.346	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	374	61	0	123	-1
normalized size	1	1.00	0.34	0.33	1.49	0.24	0.00	0.49	-0.00
time (sec)	N/A	0.069	0.028	0.014	1.097	0.742	0.000	0.370	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	107	231
normalized size	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	0.92
time (sec)	N/A	0.058	0.017	0.009	0.985	0.582	0.000	0.340	1.260

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	107	231
normalized size	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.058	0.018	0.008	1.163	0.839	0.000	0.448	1.323

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	210	57	0	106	231
normalized size	1	1.00	1.98	1.90	5.12	1.39	0.00	2.59	5.63
time (sec)	N/A	0.018	0.017	0.007	1.097	0.921	0.000	0.291	1.222

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	107	231
normalized size	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.061	0.020	0.008	1.045	0.868	0.000	0.361	1.305

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	59	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.056	0.017	0.009	0.983	0.936	0.000	0.318	1.241

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	80	241	59	0	107	231
normalized size	1	1.00	0.99	0.95	2.87	0.70	0.00	1.27	2.75
time (sec)	N/A	0.040	0.018	0.007	1.159	0.795	0.000	0.322	1.217

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	59	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.058	0.018	0.010	0.984	0.899	0.000	0.330	1.220

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	59	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.056	0.018	0.008	0.832	0.924	0.000	0.340	1.230

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	272	59	0	107	231
normalized size	1	1.00	0.65	0.62	2.12	0.46	0.00	0.84	1.80
time (sec)	N/A	0.057	0.020	0.010	1.021	0.870	0.000	0.395	1.222

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	131	113	109	123	32	146	-1
normalized size	1	1.00	0.55	0.47	0.45	0.51	0.13	0.61	-0.00
time (sec)	N/A	0.124	0.056	0.010	2.178	0.647	0.212	0.371	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	128	110	106	106	22	143	-1
normalized size	1	1.00	0.54	0.47	0.45	0.45	0.09	0.61	-0.00
time (sec)	N/A	0.114	0.032	0.007	1.365	0.855	0.203	0.409	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	15	13	10	22	33
normalized size	1	1.00	0.80	0.73	0.34	0.30	0.23	0.50	0.75
time (sec)	N/A	0.035	0.008	0.007	0.932	0.539	0.173	0.340	1.392

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	98	304	24	124	-1
normalized size	1	1.00	0.54	0.48	0.49	1.50	0.12	0.61	-0.00
time (sec)	N/A	0.085	0.025	0.004	1.277	0.654	0.179	0.417	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	98	299	20	122	-1
normalized size	1	1.00	0.54	0.48	0.49	1.48	0.10	0.60	-0.00
time (sec)	N/A	0.117	0.023	0.004	2.733	0.795	0.190	0.365	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	39	43	18	15	32	48
normalized size	1	1.00	0.52	0.49	0.54	0.22	0.19	0.40	0.60
time (sec)	N/A	0.034	0.012	0.008	1.146	0.686	0.276	0.343	1.387

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	133	111	106	103	29	131	-1
normalized size	1	1.00	0.56	0.47	0.45	0.43	0.12	0.55	-0.00
time (sec)	N/A	0.108	0.031	0.007	1.742	0.628	0.234	0.371	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	240	140	117	106	143	32	125	-1
normalized size	1	0.99	0.58	0.48	0.44	0.59	0.13	0.51	-0.00
time (sec)	N/A	0.107	0.034	0.013	2.211	0.680	0.261	0.334	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	122	54	51	73	33	31	50	75
normalized size	1	0.98	0.43	0.41	0.58	0.26	0.25	0.40	0.60
time (sec)	N/A	0.051	0.016	0.014	1.195	0.720	0.362	0.367	1.399

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	235	299	149	512	0	0	-1
normalized size	1	1.00	0.84	1.07	0.53	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.075	0.025	1.810	0.697	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	235	299	146	503	0	0	-1
normalized size	1	1.00	0.85	1.08	0.53	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.066	0.014	1.700	0.748	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	16	26	0	0	34
normalized size	1	1.00	0.71	0.63	0.42	0.68	0.00	0.00	0.89
time (sec)	N/A	0.029	0.011	0.006	0.888	0.608	0.000	0.000	1.193

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	237	301	147	514	0	0	-1
normalized size	1	1.00	0.86	1.09	0.53	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.069	0.010	1.361	0.697	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	235	299	145	499	0	0	-1
normalized size	1	1.00	0.82	1.05	0.51	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.068	0.010	1.916	0.594	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	88	90	0	0	-1
normalized size	1	1.00	0.50	0.73	0.60	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.030	0.017	1.153	0.701	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	260	316	148	201	0	0	-1
normalized size	1	1.00	0.82	1.00	0.47	0.64	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.084	0.020	1.949	0.636	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	266	322	150	242	0	0	-1
normalized size	1	1.00	0.84	1.02	0.47	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.088	0.018	1.801	0.624	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	97	133	117	119	0	0	-1
normalized size	1	1.00	0.52	0.71	0.62	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.033	0.024	0.994	0.602	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	218	519	195	723	0	0	-1
normalized size	1	1.00	0.61	1.45	0.54	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.128	0.023	1.348	0.658	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	39	32	43	58	0	0	42
normalized size	1	1.00	0.50	0.41	0.55	0.74	0.00	0.00	0.54
time (sec)	N/A	0.053	0.016	0.009	0.857	0.720	0.000	0.000	1.284

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	229	521	195	734	0	0	-1
normalized size	1	1.00	0.62	1.42	0.53	1.99	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.135	0.022	1.766	0.828	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	221	519	193	723	0	0	-1
normalized size	1	1.00	0.61	1.44	0.54	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.180	0.132	0.019	2.284	0.581	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	16	48	0	0	34
normalized size	1	1.00	0.71	0.63	0.42	1.26	0.00	0.00	0.89
time (sec)	N/A	0.030	0.010	0.009	0.512	0.561	0.000	0.000	1.259

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	219	521	191	734	0	0	-1
normalized size	1	1.00	0.61	1.45	0.53	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.120	0.011	2.184	0.799	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	211	519	189	719	0	0	-1
normalized size	1	1.00	0.58	1.43	0.52	1.98	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.111	0.008	2.152	0.733	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	132	178	0	0	-1
normalized size	1	1.00	0.43	0.87	0.59	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.044	0.024	0.875	0.619	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	242	536	192	311	0	0	-1
normalized size	1	1.00	0.61	1.35	0.48	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.129	0.025	2.333	1.043	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	234	542	194	352	0	0	-1
normalized size	1	1.00	0.59	1.36	0.49	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.135	0.025	2.059	0.912	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	119	219	163	207	0	0	-1
normalized size	1	1.00	0.44	0.81	0.61	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.051	0.021	0.851	0.919	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	243	369	0	900	-1
normalized size	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00
time (sec)	N/A	0.138	0.091	0.008	0.952	0.830	0.000	0.673	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	119	159	0	384	-1
normalized size	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00
time (sec)	N/A	0.085	0.065	0.015	1.037	0.901	0.000	0.524	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	35	35	0	83	-1
normalized size	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01
time (sec)	N/A	0.038	0.023	0.003	1.034	0.861	0.000	0.452	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.018	0.033	0.000	0.988	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.020	0.023	0.000	0.852	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.018	0.030	0.000	0.788	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.019	0.135	0.000	1.023	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	110	150	115	163	0	375	207
normalized size	1	1.00	0.64	0.87	0.67	0.95	0.00	2.18	1.20
time (sec)	N/A	0.113	0.056	0.014	0.808	0.943	0.000	0.530	1.310

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	79	108	0	235	137
normalized size	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05
time (sec)	N/A	0.079	0.033	0.007	1.222	0.879	0.000	0.439	1.219

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	60	54	70	0	132	85
normalized size	1	1.00	0.61	0.71	0.64	0.83	0.00	1.57	1.01
time (sec)	N/A	0.055	0.020	0.007	0.628	0.877	0.000	0.479	1.193

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.006	0.057	0.000	0.616	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.006	0.043	0.000	0.893	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	32	40	30	37	0	58	46
normalized size	1	1.00	0.78	0.98	0.73	0.90	0.00	1.41	1.12
time (sec)	N/A	0.028	0.006	0.007	0.947	0.751	0.000	0.482	1.155

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0	-1
normalized size	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.007	0.030	0.000	0.711	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	204	0	0	0	0	0	-1
normalized size	1	1.04	3.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.173	0.014	0.000	1.102	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.011	0.023	0.000	0.890	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.007	0.038	0.000	0.807	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.006	0.046	0.000	0.711	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.012	0.047	0.000	0.785	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.006	0.056	0.000	1.188	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	254	316	75	1758
normalized size	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	21.70
time (sec)	N/A	0.083	0.052	0.006	0.000	0.935	2.695	1.138	1.981

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	1199
normalized size	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	19.03
time (sec)	N/A	0.058	0.023	0.003	0.000	1.117	1.369	1.186	1.800

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	174
normalized size	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	4.58
time (sec)	N/A	0.036	0.010	0.002	0.000	1.028	0.638	0.983	1.232

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	223	253	66	1362
normalized size	1	1.00	0.96	0.96	0.00	3.23	3.67	0.96	19.74
time (sec)	N/A	0.070	0.023	0.007	0.000	0.906	6.731	1.003	1.922

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	119	0	293	345	93	4281
normalized size	1	1.00	1.03	1.34	0.00	3.29	3.88	1.04	48.10
time (sec)	N/A	0.125	0.029	0.009	0.000	1.255	113.787	1.137	2.026

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	70	61	0	5601	279	0	4069
normalized size	1	1.00	0.11	0.10	0.00	8.81	0.44	0.00	6.40
time (sec)	N/A	1.249	0.030	0.141	0.000	3.288	14.581	0.000	12.146

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	5260	196	0	2280
normalized size	1	1.00	0.11	0.09	0.00	8.34	0.31	0.00	3.61
time (sec)	N/A	1.024	0.030	0.005	0.000	2.536	6.903	0.000	3.397

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	44	43	0	3799	175	0	2695
normalized size	1	1.00	0.08	0.08	0.00	6.81	0.31	0.00	4.83
time (sec)	N/A	0.520	0.017	0.004	0.000	1.613	2.182	0.000	8.111

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	42	43	0	2551	122	0	2129
normalized size	1	1.00	0.08	0.08	0.00	4.57	0.22	0.00	3.82
time (sec)	N/A	0.574	0.016	0.003	0.000	1.239	1.800	0.000	7.712

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	43	41	0	2875	158	0	1543
normalized size	1	1.00	0.08	0.07	0.00	5.15	0.28	0.00	2.77
time (sec)	N/A	0.472	0.017	0.004	0.000	1.337	1.528	0.000	5.388

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	45	40	0	3978	155	0	2597
normalized size	1	1.00	0.08	0.07	0.00	7.13	0.28	0.00	4.65
time (sec)	N/A	0.595	0.017	0.002	0.000	1.380	4.353	0.000	8.495

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	71	61	0	5266	252	0	2978
normalized size	1	1.00	0.12	0.10	0.00	8.63	0.41	0.00	4.88
time (sec)	N/A	0.818	0.031	0.006	0.000	2.366	3.188	0.000	6.889

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	75	62	0	5771	241	0	4063
normalized size	1	1.00	0.12	0.10	0.00	9.43	0.39	0.00	6.64
time (sec)	N/A	0.815	0.034	0.005	0.000	2.628	52.501	0.000	10.654

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	29	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.83	0.83	0.77
time (sec)	N/A	0.024	0.006	0.005	0.489	1.086	0.126	0.394	1.253

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	24	22
normalized size	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	0.79
time (sec)	N/A	0.019	0.005	0.006	0.487	1.065	0.124	0.343	0.046

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	19	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.81
time (sec)	N/A	0.014	0.004	0.004	0.519	0.916	0.116	0.364	0.055

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	21	21	18	17	17	15	19	16
normalized size	1	2.10	2.10	1.80	1.70	1.70	1.50	1.90	1.60
time (sec)	N/A	0.014	0.004	0.003	0.465	1.126	0.108	0.328	0.383

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	31	23	21	20	24	21
normalized size	1	1.00	1.00	1.15	0.85	0.78	0.74	0.89	0.78
time (sec)	N/A	0.019	0.006	0.010	0.451	1.050	0.141	0.353	1.261

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	28	35	29	36	26
normalized size	1	1.00	1.00	1.06	0.82	1.03	0.85	1.06	0.76
time (sec)	N/A	0.032	0.006	0.010	0.682	0.903	0.170	0.301	1.233

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	41	35	40	34	41	32
normalized size	1	1.00	1.00	1.00	0.85	0.98	0.83	1.00	0.78
time (sec)	N/A	0.036	0.005	0.008	0.588	0.953	0.188	0.365	0.043

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	118	94	94	102	144	96	124
normalized size	1	1.00	0.95	0.76	0.76	0.82	1.16	0.77	1.00
time (sec)	N/A	0.114	0.054	0.010	1.230	0.819	0.619	0.365	0.244

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	92	92	90	129	94	119
normalized size	1	1.00	0.93	0.75	0.75	0.74	1.06	0.77	0.98
time (sec)	N/A	0.094	0.028	0.007	1.331	0.991	0.611	0.432	1.420

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	89	89	99	134	91	118
normalized size	1	1.00	0.93	0.75	0.75	0.83	1.13	0.76	0.99
time (sec)	N/A	0.082	0.027	0.007	1.305	1.120	0.607	0.336	0.186

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	85	85	88	126	87	104
normalized size	1	1.00	0.98	0.75	0.75	0.78	1.12	0.77	0.92
time (sec)	N/A	0.074	0.026	0.007	1.227	1.131	0.615	0.454	0.160

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	84	106	134	86	114
normalized size	1	1.00	0.96	0.75	0.75	0.95	1.20	0.77	1.02
time (sec)	N/A	0.068	0.024	0.006	1.607	0.938	0.600	0.303	1.371

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	106	84	84	102	110	86	113
normalized size	1	1.00	0.95	0.75	0.75	0.91	0.98	0.77	1.01
time (sec)	N/A	0.069	0.023	0.007	1.706	1.122	0.589	0.335	1.363

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	84	84	84	119	86	113
normalized size	1	1.00	0.96	0.75	0.75	0.75	1.06	0.77	1.01
time (sec)	N/A	0.069	0.025	0.005	1.597	1.023	1.857	0.363	1.364

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	84	124	124	86	110
normalized size	1	1.00	0.96	0.75	0.75	1.11	1.11	0.77	0.98
time (sec)	N/A	0.065	0.023	0.007	1.283	0.996	1.819	0.339	0.225

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	89	89	117	139	91	119
normalized size	1	1.00	0.99	0.75	0.75	0.98	1.17	0.76	1.00
time (sec)	N/A	0.082	0.041	0.007	1.085	0.851	1.830	0.414	1.380

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	89	89	126	128	91	118
normalized size	1	1.00	0.95	0.75	0.75	1.06	1.08	0.76	0.99
time (sec)	N/A	0.078	0.049	0.008	1.275	1.085	1.742	0.411	1.362

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	96	112	141	98	124
normalized size	1	1.00	0.94	0.75	0.76	0.89	1.12	0.78	0.98
time (sec)	N/A	0.103	0.048	0.011	1.198	1.063	1.836	0.363	0.189

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	96	153	136	98	121
normalized size	1	1.00	0.94	0.75	0.76	1.21	1.08	0.78	0.96
time (sec)	N/A	0.099	0.059	0.014	1.658	1.067	1.776	0.358	1.399

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	59	44	0	1028	26	638	320
normalized size	1	1.00	0.14	0.11	0.00	2.50	0.06	1.55	0.78
time (sec)	N/A	0.428	0.013	0.013	0.000	0.929	0.180	0.525	1.821

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.035	0.011	0.005	1.041	0.755	0.128	0.433	1.214

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	41	40	0	1583	26	824	304
normalized size	1	1.00	0.10	0.10	0.00	3.85	0.06	2.00	0.74
time (sec)	N/A	0.281	0.009	0.007	0.000	1.347	0.181	0.688	1.717

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	39	40	0	1031	24	637	327
normalized size	1	1.00	0.09	0.10	0.00	2.51	0.06	1.55	0.80
time (sec)	N/A	0.278	0.009	0.007	0.000	1.272	0.179	0.574	1.840

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.023	0.006	0.001	1.135	0.890	0.116	0.495	1.217

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	40	38	0	1583	26	812	304
normalized size	1	1.00	0.11	0.10	0.00	4.22	0.07	2.17	0.81
time (sec)	N/A	0.250	0.010	0.007	0.000	1.320	0.180	0.570	0.451

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	375	42	37	0	1027	20	629	327
normalized size	1	2.02	0.23	0.20	0.00	5.52	0.11	3.38	1.76
time (sec)	N/A	0.242	0.008	0.007	0.000	1.291	0.184	0.499	1.788

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	38	34	41	35	36
normalized size	1	1.00	1.34	0.85	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.039	0.012	0.006	1.447	1.051	0.148	0.352	1.231

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	61	50	0	1598	24	826	286
normalized size	1	1.00	0.15	0.12	0.00	3.84	0.06	1.99	0.69
time (sec)	N/A	0.300	0.013	0.009	0.000	1.314	0.198	0.550	1.656

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	65	50	0	1066	31	642	324
normalized size	1	1.00	0.16	0.12	0.00	2.55	0.07	1.54	0.78
time (sec)	N/A	0.342	0.013	0.009	0.000	1.427	0.206	0.551	1.717

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	43	51	48	45	41
normalized size	1	1.00	1.06	0.83	0.90	1.06	1.00	0.94	0.85
time (sec)	N/A	0.052	0.013	0.007	1.130	0.890	0.172	0.417	0.064

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	54	51	0	1623	39	836	318
normalized size	1	1.00	0.13	0.12	0.00	3.84	0.09	1.98	0.75
time (sec)	N/A	0.369	0.014	0.010	0.000	1.331	0.218	0.549	1.592

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	38	33	0	1996	24	0	513
normalized size	1	1.00	0.10	0.09	0.00	5.24	0.06	0.00	1.35
time (sec)	N/A	0.403	0.010	0.007	0.000	2.569	0.154	0.000	2.615

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.024	0.008	0.003	1.360	1.005	0.113	0.504	0.046

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	37	36	0	1435	24	0	351
normalized size	1	1.00	0.09	0.09	0.00	3.60	0.06	0.00	0.88
time (sec)	N/A	0.305	0.009	0.006	0.000	0.979	0.148	0.000	2.614

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	208	0	0	451	0	0	543
normalized size	1	1.00	0.90	0.00	0.00	1.95	0.00	0.00	2.35
time (sec)	N/A	0.300	0.160	0.053	0.000	1.056	0.000	0.000	2.939

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	164	0	0	367	0	0	315
normalized size	1	1.00	0.96	0.00	0.00	2.15	0.00	0.00	1.84
time (sec)	N/A	0.152	0.134	0.046	0.000	0.815	0.000	0.000	1.866

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	136	0	0	303	0	0	193
normalized size	1	1.00	0.89	0.00	0.00	1.98	0.00	0.00	1.26
time (sec)	N/A	0.136	0.069	0.032	0.000	1.039	0.000	0.000	1.591

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	99	0	0	237	0	98	87
normalized size	1	1.00	0.92	0.00	0.00	2.19	0.00	0.91	0.81
time (sec)	N/A	0.085	0.074	0.028	0.000	1.181	0.000	0.491	1.390

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	87	0	0	197	0	76	72
normalized size	1	1.00	1.05	0.00	0.00	2.37	0.00	0.92	0.87
time (sec)	N/A	0.060	0.010	0.026	0.000	1.034	0.000	0.556	1.390

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	106	0	0	566	0	0	88
normalized size	1	1.00	0.97	0.00	0.00	5.19	0.00	0.00	0.81
time (sec)	N/A	0.111	0.042	0.036	0.000	1.242	0.000	0.000	1.362

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	0	0	601	0	0	91
normalized size	1	1.00	1.00	0.00	0.00	5.37	0.00	0.00	0.81
time (sec)	N/A	0.114	0.048	0.043	0.000	0.990	0.000	0.000	1.553

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	215	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.043	0.039	0.000	1.141	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	0	0	259	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.074	0.050	0.000	1.153	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	139	0	0	325	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.088	0.060	0.000	1.185	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	173	0	0	389	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.116	0.061	0.000	1.438	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	358	0	0	0	0	0	-1
normalized size	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.561	0.045	0.000	1.016	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	337	0	0	0	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.411	0.043	0.000	1.066	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	335	0	0	0	0	0	-1
normalized size	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.338	0.033	0.000	0.817	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	340	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.355	0.046	0.000	1.057	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	340	0	0	0	0	0	-1
normalized size	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.318	0.039	0.000	0.828	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	241	0	0	641	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.402	0.349	0.049	0.000	1.092	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	192	0	0	535	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.192	0.032	0.000	0.992	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	175	0	0	451	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.159	0.031	0.000	1.057	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	0	0	361	0	172	223
normalized size	1	1.00	0.99	0.00	0.00	2.41	0.00	1.15	1.49
time (sec)	N/A	0.116	0.146	0.036	0.000	1.111	0.000	0.561	1.575

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	126	0	0	297	0	135	115
normalized size	1	1.00	1.02	0.00	0.00	2.40	0.00	1.09	0.93
time (sec)	N/A	0.086	0.085	0.033	0.000	1.135	0.000	0.611	1.444

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	0	0	727	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	4.69	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.143	0.042	0.000	1.503	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	134	0	0	713	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	4.75	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.111	0.065	0.000	1.139	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	134	0	0	713	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	4.72	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.172	0.056	0.000	1.095	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	149	0	0	771	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.73	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.206	0.056	0.000	1.457	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	138	0	0	319	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.164	0.051	0.000	1.112	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	167	0	0	383	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.140	0.065	0.000	1.081	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	206	0	0	473	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.210	0.078	0.000	1.835	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	243	0	0	557	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.486	0.093	0.000	2.033	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	453	0	0	0	0	0	-1
normalized size	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.872	0.045	0.000	1.032	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	410	0	0	0	0	0	-1
normalized size	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.696	0.043	0.000	1.041	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0	-1
normalized size	1	1.00	3.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.670	0.038	0.000	0.817	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	379	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.483	0.046	0.000	1.032	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	379	0	0	0	0	0	-1
normalized size	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.482	0.042	0.000	1.105	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	137	0	0	303	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.111	0.036	0.000	1.027	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	104	0	0	241	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.050	0.040	0.000	1.204	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	0	0	203	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.033	0.027	0.000	0.643	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	161	0	61	55
normalized size	1	1.00	1.00	0.00	0.00	2.37	0.00	0.90	0.81
time (sec)	N/A	0.057	0.016	0.024	0.000	1.003	0.000	0.674	1.492

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	118	0	40	34
normalized size	1	1.00	1.00	0.00	0.00	2.74	0.00	0.93	0.79
time (sec)	N/A	0.034	0.006	0.020	0.000	1.071	0.000	0.676	1.575

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	124	0	0	36
normalized size	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	0.82
time (sec)	N/A	0.040	0.006	0.020	0.000	0.899	0.000	0.000	1.571

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	179	0	0	56
normalized size	1	1.00	1.00	0.00	0.00	2.49	0.00	0.00	0.78
time (sec)	N/A	0.060	0.024	0.033	0.000	1.166	0.000	0.000	1.560

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	0	0	221	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.059	0.037	0.000	1.072	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	112	0	0	263	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.078	0.042	0.000	1.224	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	141	0	0	327	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.098	0.046	0.000	1.292	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.078	0.023	0.000	1.313	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.073	0.020	0.000	1.306	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	163	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.073	0.024	0.000	1.291	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	343	0	0	0	0	0	-1
normalized size	1	1.00	2.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.345	0.048	0.000	1.252	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	342	0	0	0	0	0	-1
normalized size	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.299	0.048	0.000	1.120	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	181	0	0	591	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.181	0.065	0.000	1.414	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	137	0	0	459	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.119	0.055	0.000	0.905	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	107	0	0	387	0	0	84
normalized size	1	1.00	0.89	0.00	0.00	3.22	0.00	0.00	0.70
time (sec)	N/A	0.090	0.098	0.028	0.000	1.051	0.000	0.000	1.658

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	38	0	68	0	45	38
normalized size	1	1.00	1.05	0.97	0.00	1.74	0.00	1.15	0.97
time (sec)	N/A	0.029	0.099	0.006	0.000	0.973	0.000	1.321	1.429

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	67	0	45	37
normalized size	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	0.97
time (sec)	N/A	0.025	0.024	0.006	0.000	0.867	0.000	1.407	1.370

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	0	0	389	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	4.23	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.123	0.062	0.000	0.848	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	137	0	0	485	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.083	0.087	0.000	1.092	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	179	0	0	615	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.126	0.098	0.000	1.271	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	223	0	0	705	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	2.75	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.177	0.133	0.000	1.501	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	340	0	0	0	0	0	-1
normalized size	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.318	0.031	0.000	0.794	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	362	0	0	0	0	0	-1
normalized size	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.481	0.030	0.000	0.840	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	359	0	0	0	0	0	-1
normalized size	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.465	0.031	0.000	0.874	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	407	0	0	0	0	0	-1
normalized size	1	1.00	2.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.715	0.063	0.000	0.584	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	405	0	0	0	0	0	-1
normalized size	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.642	0.081	0.000	0.662	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1510	449	260
normalized size	1	1.00	0.69	2.98	1.09	2.39	14.95	4.45	2.57
time (sec)	N/A	0.061	0.071	0.008	1.102	0.842	5.824	0.465	1.524

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	78	50	71	314	119	89
normalized size	1	1.00	0.67	1.50	0.96	1.37	6.04	2.29	1.71
time (sec)	N/A	0.021	0.029	0.003	1.163	0.846	1.468	0.346	1.358

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	84	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.061	0.045	0.000	1.079	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	0.081	0.026	0.000	0.982	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	-1
normalized size	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.271	0.017	0.000	1.154	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.094	0.013	0.000	1.268	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.105	0.013	0.000	1.031	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.170	0.011	0.000	1.106	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.240	0.068	0.000	1.032	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	162	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.203	0.026	0.000	0.874	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	162	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.210	0.039	0.000	0.738	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.106	0.029	0.000	0.882	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.158	0.033	0.000	0.818	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.178	0.032	0.000	0.920	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.183	0.026	0.000	0.817	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.147	0.016	0.000	0.907	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	157	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.200	0.017	0.000	0.924	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.160	0.030	0.000	0.864	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.177	0.036	0.000	0.925	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.201	0.033	0.000	0.918	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.198	0.038	0.000	0.909	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.163	0.043	0.000	0.913	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	164	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.221	0.045	0.000	0.797	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.008	0.025	0.000	0.818	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	25	24	31	22	24	25
normalized size	1	1.00	0.80	0.83	0.80	1.03	0.73	0.80	0.83
time (sec)	N/A	0.012	0.014	0.009	2.156	0.860	0.126	0.340	0.047

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	18	23	15	18	18
normalized size	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82
time (sec)	N/A	0.010	0.006	0.007	0.984	0.792	0.106	0.279	1.315

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	24	15	19	21
normalized size	1	1.00	1.00	0.87	0.83	1.04	0.65	0.83	0.91
time (sec)	N/A	0.009	0.009	0.006	2.015	0.788	0.118	0.372	1.365

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	11
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00
time (sec)	N/A	0.003	0.002	0.003	0.887	0.791	0.094	0.355	0.018

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	23	15	19	20
normalized size	1	1.00	0.87	0.87	0.83	1.00	0.65	0.83	0.87
time (sec)	N/A	0.007	0.005	0.004	2.173	0.838	0.116	0.392	0.026

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
normalized size	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.012	0.010	0.013	0.974	0.812	0.125	0.361	0.044

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	25	31	26	25	25
normalized size	1	1.00	1.00	0.83	0.83	1.03	0.87	0.83	0.83
time (sec)	N/A	0.012	0.011	0.010	1.976	0.820	0.147	0.335	0.040

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	33	44	31	33	31
normalized size	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	0.94
time (sec)	N/A	0.016	0.012	0.015	0.877	0.816	0.153	0.355	0.054

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	30	36	29	31	30
normalized size	1	1.00	0.89	0.76	0.81	0.97	0.78	0.84	0.81
time (sec)	N/A	0.016	0.011	0.012	2.487	0.973	0.170	0.296	0.046

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	69	83	132	90	83	45
normalized size	1	1.00	0.90	0.66	0.80	1.27	0.87	0.80	0.43
time (sec)	N/A	0.055	0.068	0.010	2.022	0.795	0.176	0.339	1.369

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	70	84	129	90	84	47
normalized size	1	1.00	0.94	0.71	0.85	1.30	0.91	0.85	0.47
time (sec)	N/A	0.051	0.061	0.008	2.075	0.903	0.179	0.293	1.326

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	68	82	126	82	82	45
normalized size	1	1.00	0.93	0.70	0.85	1.30	0.85	0.85	0.46
time (sec)	N/A	0.051	0.065	0.006	1.986	0.958	0.170	0.357	0.082

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	70	84	128	83	84	46
normalized size	1	1.00	0.93	0.71	0.85	1.29	0.84	0.85	0.46
time (sec)	N/A	0.050	0.049	0.007	2.065	0.919	0.174	0.345	0.045

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	68	82	127	88	82	44
normalized size	1	1.00	0.94	0.70	0.85	1.31	0.91	0.85	0.45
time (sec)	N/A	0.047	0.046	0.004	2.002	0.851	0.200	0.336	1.305

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	98	75	88	130	97	88	49
normalized size	1	1.00	0.92	0.71	0.83	1.23	0.92	0.83	0.46
time (sec)	N/A	0.052	0.069	0.009	2.025	0.886	0.207	0.358	1.314

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	73	90	140	99	87	51
normalized size	1	1.00	0.91	0.69	0.85	1.32	0.93	0.82	0.48
time (sec)	N/A	0.052	0.071	0.010	2.198	0.617	0.224	0.443	1.365

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	103	80	95	145	102	96	55
normalized size	1	1.00	0.91	0.71	0.84	1.28	0.90	0.85	0.49
time (sec)	N/A	0.054	0.076	0.012	2.012	0.938	0.232	0.297	0.093

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	78	95	145	102	94	55
normalized size	1	1.00	0.89	0.69	0.84	1.28	0.90	0.83	0.49
time (sec)	N/A	0.055	0.076	0.010	1.932	0.948	0.234	0.364	0.104

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.006	0.020	0.000	0.709	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	39	41	34	46	34	35	26
normalized size	1	1.00	1.22	1.28	1.06	1.44	1.06	1.09	0.81
time (sec)	N/A	0.013	0.022	0.009	1.032	0.764	0.122	0.326	0.047

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	18	23	15	19	20
normalized size	1	1.00	0.85	0.73	0.69	0.88	0.58	0.73	0.77
time (sec)	N/A	0.013	0.007	0.006	1.128	0.768	0.104	0.410	0.048

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	29	40	26	30	21
normalized size	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.010	0.011	0.009	0.914	0.799	0.118	0.496	1.269

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	11
normalized size	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85
time (sec)	N/A	0.003	0.002	0.003	0.884	0.854	0.095	0.499	0.022

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	29	40	26	30	21
normalized size	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.008	0.007	0.010	0.918	0.873	0.119	0.471	0.035

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	47	24	32	19	30	22
normalized size	1	1.00	0.93	1.68	0.86	1.14	0.68	1.07	0.79
time (sec)	N/A	0.015	0.010	0.015	0.943	0.807	0.126	0.381	0.058

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	50	37	54	36	38	26
normalized size	1	1.00	1.28	1.56	1.16	1.69	1.12	1.19	0.81
time (sec)	N/A	0.013	0.017	0.017	0.953	0.671	0.150	0.414	0.044

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	54	35	50	29	36	32
normalized size	1	1.00	0.95	1.46	0.95	1.35	0.78	0.97	0.86
time (sec)	N/A	0.019	0.012	0.018	0.828	0.663	0.152	0.453	0.052

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	49	55	42	59	41	42	32
normalized size	1	1.00	1.26	1.41	1.08	1.51	1.05	1.08	0.82
time (sec)	N/A	0.017	0.014	0.020	0.937	0.912	0.176	0.328	0.051

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	43	28	49	32	30	26
normalized size	1	1.00	1.12	1.26	0.82	1.44	0.94	0.88	0.76
time (sec)	N/A	0.009	0.016	0.013	2.127	0.797	0.151	0.280	1.285

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	42	29	46	32	31	23
normalized size	1	1.00	1.21	1.45	1.00	1.59	1.10	1.07	0.79
time (sec)	N/A	0.008	0.014	0.015	2.027	0.822	0.153	0.400	0.034

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	27	43	26	29	21
normalized size	1	1.00	1.15	1.56	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.007	0.013	0.013	1.918	0.856	0.147	0.512	0.034

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	33	42	29	45	27	31	23
normalized size	1	1.00	1.14	1.45	1.00	1.55	0.93	1.07	0.79
time (sec)	N/A	0.008	0.012	0.010	1.711	0.833	0.149	0.335	0.032

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	42	27	44	31	29	21
normalized size	1	1.00	1.22	1.56	1.00	1.63	1.15	1.07	0.78
time (sec)	N/A	0.005	0.009	0.013	2.132	0.855	0.156	0.366	0.029

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	47	35	55	37	37	26
normalized size	1	1.00	1.11	1.31	0.97	1.53	1.03	1.03	0.72
time (sec)	N/A	0.010	0.016	0.014	2.086	0.522	0.179	0.375	0.042

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	47	37	63	39	34	28
normalized size	1	1.00	1.06	1.31	1.03	1.75	1.08	0.94	0.78
time (sec)	N/A	0.009	0.018	0.013	1.983	0.730	0.184	0.457	1.292

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	52	42	68	44	43	34
normalized size	1	1.00	1.19	1.21	0.98	1.58	1.02	1.00	0.79
time (sec)	N/A	0.012	0.020	0.020	2.434	0.850	0.201	0.451	0.044

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	42	68	44	41	34
normalized size	1	1.00	1.00	1.21	0.98	1.58	1.02	0.95	0.79
time (sec)	N/A	0.012	0.019	0.016	1.940	0.896	0.211	0.452	0.047

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	82	0	0	0	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.059	0.016	0.000	0.736	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	254	316	75	3916
normalized size	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	48.35
time (sec)	N/A	0.084	0.051	0.006	0.000	0.987	4.113	17.069	2.687

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	210	360	0	1071	134	2043	5659
normalized size	1	1.00	1.09	1.88	0.00	5.58	0.70	10.64	29.47
time (sec)	N/A	0.339	0.123	0.030	0.000	0.639	4.419	18.166	3.071

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	2654
normalized size	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	42.13
time (sec)	N/A	0.058	0.024	0.004	0.000	1.068	2.180	17.121	2.608

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	171	216	0	567	76	1036	1220
normalized size	1	1.00	1.08	1.36	0.00	3.57	0.48	6.52	7.67
time (sec)	N/A	0.126	0.086	0.018	0.000	0.919	2.509	18.741	2.810

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	260
normalized size	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	6.84
time (sec)	N/A	0.034	0.009	0.003	0.000	0.919	0.770	17.345	1.370

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	133	120	0	619	88	1030	1105
normalized size	1	1.00	0.86	0.78	0.00	4.02	0.57	6.69	7.18
time (sec)	N/A	0.094	0.081	0.013	0.000	0.677	3.359	19.496	2.271

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	223	253	68	1690
normalized size	1	1.00	0.96	0.96	0.00	3.23	3.67	0.99	24.49
time (sec)	N/A	0.068	0.023	0.008	0.000	1.038	14.469	16.281	2.185

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	75	240	0	1134	153	2055	5451
normalized size	1	1.00	0.41	1.30	0.00	6.16	0.83	11.17	29.62
time (sec)	N/A	0.224	0.030	0.025	0.000	0.967	15.344	15.891	2.416

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	119	0	293	0	94	8817
normalized size	1	1.00	1.03	1.34	0.00	3.29	0.00	1.06	99.07
time (sec)	N/A	0.124	0.032	0.010	0.000	0.996	0.000	14.501	2.788

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	70	63	0	6296	0	0	12709
normalized size	1	1.00	0.18	0.17	0.00	16.52	0.00	0.00	33.36
time (sec)	N/A	0.647	0.040	0.038	0.000	3.735	0.000	0.000	3.491

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	5082	0	0	10382
normalized size	1	1.00	0.19	0.16	0.00	13.52	0.00	0.00	27.61
time (sec)	N/A	0.574	0.037	0.007	0.000	1.796	0.000	0.000	3.969

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	44	43	0	3912	230	0	8033
normalized size	1	1.00	0.14	0.13	0.00	12.04	0.71	0.00	24.72
time (sec)	N/A	0.307	0.024	0.003	0.000	1.322	61.577	0.000	3.511

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	42	43	0	2479	126	0	8169
normalized size	1	1.00	0.13	0.13	0.00	7.63	0.39	0.00	25.14
time (sec)	N/A	0.296	0.021	0.003	0.000	1.133	2.917	0.000	3.633

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	43	43	0	2746	172	0	6067
normalized size	1	1.00	0.14	0.14	0.00	8.72	0.55	0.00	19.26
time (sec)	N/A	0.290	0.023	0.003	0.000	1.077	4.099	0.000	2.336

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	45	40	0	3929	177	0	10337
normalized size	1	1.00	0.14	0.13	0.00	12.47	0.56	0.00	32.82
time (sec)	N/A	0.304	0.025	0.003	0.000	1.313	19.792	0.000	3.416

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	71	63	0	5125	0	0	10509
normalized size	1	1.00	0.20	0.17	0.00	14.12	0.00	0.00	28.95
time (sec)	N/A	0.412	0.037	0.009	0.000	2.418	0.000	0.000	2.809

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	75	62	0	6324	0	0	16497
normalized size	1	1.00	0.21	0.17	0.00	17.33	0.00	0.00	45.20
time (sec)	N/A	0.399	0.042	0.012	0.000	2.699	0.000	0.000	5.568

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	488	0	0	0	0	0	-1
normalized size	1	1.00	3.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	1.130	0.026	0.000	0.892	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	35	42	35	37
normalized size	1	1.00	1.00	0.82	0.80	0.80	0.95	0.80	0.84
time (sec)	N/A	0.037	0.011	0.004	2.423	0.797	0.139	0.342	0.046

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	98	43	42	40	51	42	43
normalized size	1	1.00	1.81	0.80	0.78	0.74	0.94	0.78	0.80
time (sec)	N/A	0.058	0.173	0.006	2.394	1.050	0.140	0.315	0.042

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.031	0.008	0.003	2.435	0.921	0.133	0.390	0.040

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	94	62	61	61	76	61	51
normalized size	1	1.00	1.25	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.077	0.119	0.004	2.490	0.873	0.206	0.389	0.093

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.021	0.006	0.001	2.470	0.775	0.118	0.361	1.302

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	62	61	61	76	61	51
normalized size	1	1.00	1.05	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.064	0.048	0.004	2.589	0.910	0.202	0.309	1.275

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	138	87	36	32	41	36	34
normalized size	1	1.00	3.54	2.23	0.92	0.82	1.05	0.92	0.87
time (sec)	N/A	0.035	0.084	0.008	3.047	0.786	0.155	0.383	1.295

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	100	57	42	45	53	42	43
normalized size	1	1.00	1.85	1.06	0.78	0.83	0.98	0.78	0.80
time (sec)	N/A	0.052	0.049	0.006	2.392	0.797	0.164	0.378	0.036

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	141	94	41	49	48	46	41
normalized size	1	1.00	2.94	1.96	0.85	1.02	1.00	0.96	0.85
time (sec)	N/A	0.051	0.102	0.010	2.791	0.765	0.184	0.306	0.063

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	142	95	73	84	88	73	62
normalized size	1	1.00	1.60	1.07	0.82	0.94	0.99	0.82	0.70
time (sec)	N/A	0.096	0.110	0.009	2.372	0.790	0.249	0.266	0.039

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	139	110	0	212	192	109	100
normalized size	1	1.00	0.99	0.78	0.00	1.50	1.36	0.77	0.71
time (sec)	N/A	0.097	0.275	0.045	0.000	0.868	0.709	0.395	0.103

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	70	82	66	38
normalized size	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.43
time (sec)	N/A	0.058	0.020	0.013	0.000	0.824	0.179	0.326	1.310

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	211	197	108	99
normalized size	1	1.00	0.96	0.78	0.00	1.51	1.41	0.77	0.71
time (sec)	N/A	0.107	0.165	0.015	0.000	0.880	0.725	0.404	0.067

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	211	214	108	97
normalized size	1	1.00	0.96	0.78	0.00	1.51	1.53	0.77	0.69
time (sec)	N/A	0.083	0.151	0.010	0.000	0.950	0.713	0.335	1.311

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	70	82	66	40
normalized size	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.45
time (sec)	N/A	0.052	0.017	0.010	0.000	0.942	0.182	0.387	0.037

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	140	114	0	224	218	113	102
normalized size	1	1.00	0.97	0.79	0.00	1.54	1.50	0.78	0.70
time (sec)	N/A	0.112	0.208	0.013	0.000	0.983	0.738	0.330	0.048

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	148	114	0	240	197	113	104
normalized size	1	1.00	1.01	0.78	0.00	1.63	1.34	0.77	0.71
time (sec)	N/A	0.099	0.294	0.013	0.000	0.935	0.748	0.377	0.030

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	75	0	90	94	100	52
normalized size	1	1.00	0.97	0.77	0.00	0.92	0.96	1.02	0.53
time (sec)	N/A	0.084	0.034	0.013	0.000	1.121	0.217	0.404	0.038

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	171	119	0	246	209	120	110
normalized size	1	1.00	1.11	0.77	0.00	1.60	1.36	0.78	0.71
time (sec)	N/A	0.142	0.344	0.013	0.000	0.888	0.764	0.426	0.032

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.056	0.020	0.000	0.772	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.040	0.012	0.006	2.112	0.823	0.140	0.342	0.047

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	47	48	99	29
normalized size	1	1.00	0.96	0.77	0.00	0.82	0.84	1.74	0.51
time (sec)	N/A	0.043	0.015	0.010	0.000	0.875	0.127	0.321	1.305

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.033	0.008	0.004	1.968	0.858	0.137	0.423	1.281

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	98	65	0	171	70	76	53
normalized size	1	1.00	1.20	0.79	0.00	2.09	0.85	0.93	0.65
time (sec)	N/A	0.071	0.132	0.007	0.000	0.914	0.209	0.348	0.050

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.022	0.007	0.001	1.937	0.927	0.122	0.410	1.285

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	65	0	171	70	64	53
normalized size	1	1.00	1.01	0.79	0.00	2.09	0.85	0.78	0.65
time (sec)	N/A	0.057	0.052	0.008	0.000	1.001	0.208	0.407	0.045

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	38	34	41	38	36
normalized size	1	1.00	1.34	0.85	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.038	0.013	0.006	1.947	0.880	0.157	0.346	1.292

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	50	49	99	29
normalized size	1	1.00	0.96	0.77	0.00	0.88	0.86	1.74	0.51
time (sec)	N/A	0.042	0.015	0.010	0.000	0.799	0.153	0.314	1.273

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	43	51	48	48	41
normalized size	1	1.00	1.06	0.83	0.90	1.06	1.00	1.00	0.85
time (sec)	N/A	0.054	0.014	0.007	1.968	1.000	0.190	0.404	0.066

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	56	75	0	193	83	56	63
normalized size	1	1.00	0.58	0.78	0.00	2.01	0.86	0.58	0.66
time (sec)	N/A	0.094	0.018	0.011	0.000	0.896	0.261	0.415	0.056

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	59	44	0	716	26	254	209
normalized size	1	1.00	0.17	0.12	0.00	2.01	0.07	0.71	0.59
time (sec)	N/A	0.333	0.015	0.016	0.000	0.942	3.202	0.347	0.157

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	41	32	0	215	165	205	53
normalized size	1	1.00	0.15	0.12	0.00	0.78	0.60	0.75	0.19
time (sec)	N/A	0.243	0.011	0.013	0.000	0.840	0.219	0.398	0.096

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	39	40	0	567	24	253	474
normalized size	1	1.00	0.11	0.12	0.00	1.63	0.07	0.73	1.37
time (sec)	N/A	0.209	0.011	0.012	0.000	0.954	3.187	0.480	1.328

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	40	40	0	567	26	253	286
normalized size	1	1.00	0.11	0.11	0.00	1.60	0.07	0.71	0.81
time (sec)	N/A	0.200	0.011	0.013	0.000	1.015	3.281	0.457	0.079

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	215	165	205	53
normalized size	1	1.00	0.15	0.11	0.00	0.78	0.60	0.75	0.19
time (sec)	N/A	0.211	0.011	0.006	0.000	0.863	0.220	0.400	0.037

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	61	52	0	732	29	258	253
normalized size	1	1.00	0.17	0.14	0.00	2.03	0.08	0.72	0.70
time (sec)	N/A	0.239	0.016	0.016	0.000	0.952	3.293	0.462	1.292

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	65	50	0	756	31	258	213
normalized size	1	1.00	0.18	0.14	0.00	2.04	0.08	0.70	0.58
time (sec)	N/A	0.236	0.014	0.014	0.000	0.739	3.252	0.353	1.288

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	54	43	0	238	182	217	63
normalized size	1	1.00	0.19	0.15	0.00	0.83	0.63	0.76	0.22
time (sec)	N/A	0.241	0.017	0.011	0.000	0.991	0.273	0.383	1.296

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	54	51	0	614	37	265	486
normalized size	1	1.00	0.14	0.14	0.00	1.63	0.10	0.70	1.29
time (sec)	N/A	0.286	0.015	0.019	0.000	0.970	3.366	0.381	0.064

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.058	0.019	0.000	1.006	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	50	62	60	50	64
normalized size	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	1.03
time (sec)	N/A	0.056	0.033	0.004	1.281	0.836	0.145	0.497	0.134

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	117	0	154	54	66	130
normalized size	1	1.00	1.08	1.30	0.00	1.71	0.60	0.73	1.44
time (sec)	N/A	0.142	0.152	0.050	0.000	0.886	0.207	0.561	1.343

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	45	56	53	45	59
normalized size	1	1.00	0.96	0.60	0.82	1.02	0.96	0.82	1.07
time (sec)	N/A	0.033	0.023	0.003	1.169	0.904	0.135	0.497	1.358

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	110	0	165	49	47	117
normalized size	1	1.00	0.93	1.36	0.00	2.04	0.60	0.58	1.44
time (sec)	N/A	0.084	0.048	0.024	0.000	1.057	0.201	0.551	0.119

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	31	43	42	31	30
normalized size	1	1.00	1.65	0.83	1.35	1.87	1.83	1.35	1.30
time (sec)	N/A	0.026	0.010	0.001	1.482	0.839	0.118	0.601	1.333

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	60	0	128	49	41	125
normalized size	1	1.00	0.99	0.80	0.00	1.71	0.65	0.55	1.67
time (sec)	N/A	0.058	0.037	0.016	0.000	0.947	0.204	0.419	0.054

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	35	51	58	58	51	42
normalized size	1	1.00	0.96	0.61	0.89	1.02	1.02	0.89	0.74
time (sec)	N/A	0.034	0.030	0.009	1.505	0.699	0.158	0.484	1.412

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	65	117	0	158	56	68	130
normalized size	1	1.00	0.73	1.31	0.00	1.78	0.63	0.76	1.46
time (sec)	N/A	0.072	0.017	0.028	0.000	0.926	0.238	0.456	1.304

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	42	56	76	65	63	49
normalized size	1	1.00	0.91	0.64	0.85	1.15	0.98	0.95	0.74
time (sec)	N/A	0.067	0.035	0.010	1.292	0.822	0.188	0.551	1.358

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	122	0	180	65	77	136
normalized size	1	1.00	0.75	1.26	0.00	1.86	0.67	0.79	1.40
time (sec)	N/A	0.134	0.018	0.027	0.000	0.885	0.271	0.509	0.124

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	440	58	46	0	1012	29	240	216
normalized size	1	0.96	0.13	0.10	0.00	2.20	0.06	0.52	0.47
time (sec)	N/A	0.416	0.015	0.013	0.000	1.050	1.591	0.725	1.438

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	41	40	0	725	26	239	149
normalized size	1	1.00	0.10	0.09	0.00	1.68	0.06	0.55	0.35
time (sec)	N/A	0.295	0.012	0.013	0.000	0.975	1.530	0.663	1.465

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	39	40	0	843	24	239	454
normalized size	1	1.00	0.09	0.09	0.00	1.87	0.05	0.53	1.01
time (sec)	N/A	0.280	0.011	0.016	0.000	0.840	1.482	0.765	0.196

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	431	40	40	0	955	26	239	275
normalized size	1	1.01	0.09	0.09	0.00	2.24	0.06	0.56	0.64
time (sec)	N/A	0.259	0.010	0.010	0.000	0.973	1.510	0.593	0.086

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	42	37	0	733	26	239	403
normalized size	1	1.00	0.10	0.09	0.00	1.77	0.06	0.58	0.97
time (sec)	N/A	0.257	0.010	0.010	0.000	0.989	1.519	0.528	0.083

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	61	52	0	1017	32	244	292
normalized size	1	1.00	0.15	0.12	0.00	2.44	0.08	0.59	0.70
time (sec)	N/A	0.285	0.016	0.013	0.000	1.022	1.609	0.603	1.291

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	65	50	0	1057	34	244	492
normalized size	1	1.00	0.14	0.11	0.00	2.27	0.07	0.52	1.06
time (sec)	N/A	0.367	0.016	0.012	0.000	0.864	1.624	0.702	0.184

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	575	0	0	0	0	0	-1
normalized size	1	1.00	4.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.572	0.018	0.000	0.922	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	50	62	58	53	64
normalized size	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.03
time (sec)	N/A	0.046	0.032	0.004	1.393	0.714	0.143	0.416	0.120

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	103	67	92	114	170	97	90
normalized size	1	1.00	1.14	0.74	1.02	1.27	1.89	1.08	1.00
time (sec)	N/A	0.072	0.054	0.006	1.359	0.875	0.380	0.463	1.328

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	45	57	53	48	59
normalized size	1	1.00	0.96	0.60	0.82	1.04	0.96	0.87	1.07
time (sec)	N/A	0.032	0.022	0.003	1.479	0.789	0.135	0.425	0.102

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	91	62	87	109	165	92	77
normalized size	1	1.00	1.12	0.77	1.07	1.35	2.04	1.14	0.95
time (sec)	N/A	0.053	0.034	0.003	1.917	0.621	0.371	0.445	1.381

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	31	43	42	33	30
normalized size	1	1.00	1.65	0.83	1.35	1.87	1.83	1.43	1.30
time (sec)	N/A	0.027	0.010	0.003	1.413	0.582	0.118	0.537	1.570

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	91	62	87	107	165	92	83
normalized size	1	1.00	1.21	0.83	1.16	1.43	2.20	1.23	1.11
time (sec)	N/A	0.038	0.029	0.004	1.450	0.871	0.366	0.433	1.302

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	64	51	59	58	54	42
normalized size	1	1.00	0.96	1.12	0.89	1.04	1.02	0.95	0.74
time (sec)	N/A	0.030	0.031	0.010	1.445	0.861	0.158	0.477	0.429

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	67	92	125	172	97	88
normalized size	1	1.00	1.16	0.75	1.03	1.40	1.93	1.09	0.99
time (sec)	N/A	0.058	0.057	0.007	1.462	0.875	0.406	0.384	0.064

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	71	56	76	66	66	49
normalized size	1	1.00	0.92	1.08	0.85	1.15	1.00	1.00	0.74
time (sec)	N/A	0.063	0.036	0.012	1.395	0.823	0.190	0.505	1.348

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	72	99	130	199	104	95
normalized size	1	1.00	1.14	0.74	1.02	1.34	2.05	1.07	0.98
time (sec)	N/A	0.090	0.071	0.009	1.325	0.849	0.439	0.409	1.385

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	160	205	0	304	58	148	246
normalized size	1	1.00	0.94	1.21	0.00	1.79	0.34	0.87	1.45
time (sec)	N/A	0.114	0.266	0.061	0.000	0.868	1.245	0.676	1.445

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	160	206	0	255	53	147	147
normalized size	1	1.00	0.96	1.23	0.00	1.53	0.32	0.88	0.88
time (sec)	N/A	0.078	0.150	0.030	0.000	0.753	1.222	0.748	0.194

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	132	206	0	271	49	147	269
normalized size	1	1.00	0.76	1.19	0.00	1.57	0.28	0.85	1.55
time (sec)	N/A	0.075	0.191	0.040	0.000	0.889	1.203	0.615	1.472

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	166	131	110	0	255	53	147	269
normalized size	1	1.14	0.90	0.76	0.00	1.76	0.37	1.01	1.86
time (sec)	N/A	0.060	0.044	0.029	0.000	0.958	1.185	0.624	0.081

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	160	206	0	251	53	147	245
normalized size	1	1.00	0.95	1.22	0.00	1.49	0.31	0.87	1.45
time (sec)	N/A	0.060	0.156	0.029	0.000	0.900	1.212	0.482	0.079

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	174	211	0	313	63	152	250
normalized size	1	1.00	1.01	1.23	0.00	1.82	0.37	0.88	1.45
time (sec)	N/A	0.089	0.267	0.029	0.000	0.927	1.250	0.542	1.339

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	166	209	0	327	63	152	268
normalized size	1	1.00	0.91	1.15	0.00	1.80	0.35	0.84	1.47
time (sec)	N/A	0.118	0.262	0.030	0.000	0.928	1.281	0.669	0.204

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	189	216	0	300	73	159	257
normalized size	1	1.00	1.09	1.25	0.00	1.73	0.42	0.92	1.49
time (sec)	N/A	0.140	0.274	0.039	0.000	1.200	1.290	0.539	1.487

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	189	216	0	332	70	159	291
normalized size	1	1.00	1.00	1.14	0.00	1.76	0.37	0.84	1.54
time (sec)	N/A	0.163	0.268	0.043	0.000	1.038	1.298	0.556	0.210

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	16
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.76
time (sec)	N/A	0.013	0.004	0.005	0.587	0.877	0.116	0.282	0.063

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	19	22	22
normalized size	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85
time (sec)	N/A	0.019	0.005	0.006	0.861	0.925	0.130	0.364	1.315

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.034	0.013	0.003	2.046	1.083	0.143	2.721	1.346

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.022	0.006	0.001	2.092	0.955	0.131	2.962	1.333

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	197	66	36	32	41	33	34
normalized size	1	1.00	5.05	1.69	0.92	0.82	1.05	0.85	0.87
time (sec)	N/A	0.035	0.037	0.039	2.033	0.873	0.165	0.408	0.060

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	208	73	41	49	48	45	41
normalized size	1	1.00	4.33	1.52	0.85	1.02	1.00	0.94	0.85
time (sec)	N/A	0.051	0.042	0.024	2.070	0.803	0.202	0.251	1.369

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	197	66	0	32	41	33	34
normalized size	1	1.00	5.05	1.69	0.00	0.82	1.05	0.85	0.87
time (sec)	N/A	0.036	0.020	0.022	0.000	0.902	0.166	0.350	0.033

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	236	0	466	605	145	183
normalized size	1	1.00	0.95	1.61	0.00	3.17	4.12	0.99	1.24
time (sec)	N/A	0.137	0.122	0.006	0.000	0.896	1.249	0.291	0.159

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	190	0	383	498	113	151
normalized size	1	1.00	0.95	1.61	0.00	3.25	4.22	0.96	1.28
time (sec)	N/A	0.105	0.085	0.003	0.000	0.946	1.012	0.392	1.396

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	297	381	86	112
normalized size	1	1.00	0.94	1.48	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.086	0.104	0.005	0.000	0.882	0.835	0.257	0.131

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	235	306	67	172
normalized size	1	1.00	1.04	1.44	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.049	0.064	0.003	0.000	0.786	0.597	0.274	1.418

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
normalized size	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.036	0.031	0.003	0.000	0.903	0.318	0.291	0.168

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	120	124	34	46
normalized size	1	1.00	1.06	0.97	0.00	3.33	3.44	0.94	1.28
time (sec)	N/A	0.032	0.007	0.001	0.000	0.806	0.218	0.345	0.046

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	211	564	62	213
normalized size	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.048	0.067	0.006	0.000	0.889	4.080	0.297	1.715

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	269	0	79	339
normalized size	1	1.00	0.95	1.38	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.102	0.079	0.006	0.000	0.947	0.000	0.386	1.814

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	358	0	105	447
normalized size	1	1.00	0.98	1.44	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.153	0.136	0.007	0.000	1.059	0.000	0.314	1.866

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	445	0	136	524
normalized size	1	1.00	0.96	1.56	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.193	0.099	0.010	0.000	1.045	0.000	0.382	1.923

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	163	434	0	1029	1012	188	382
normalized size	1	1.00	0.83	2.21	0.00	5.25	5.16	0.96	1.95
time (sec)	N/A	0.205	0.222	0.013	0.000	0.918	2.369	0.330	1.822

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	352	0	837	842	161	261
normalized size	1	1.00	0.88	2.35	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.148	0.177	0.012	0.000	0.908	1.727	0.369	1.798

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	209	0	635	729	125	279
normalized size	1	1.00	0.96	1.83	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.100	0.140	0.010	0.000	0.906	1.323	0.362	1.859

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	81	97	0	387	280	88	135
normalized size	1	1.00	1.14	1.37	0.00	5.45	3.94	1.24	1.90
time (sec)	N/A	0.042	0.092	0.007	0.000	0.940	0.597	0.379	1.370

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
normalized size	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.034	0.064	0.003	0.000	0.863	0.562	0.371	1.369

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
normalized size	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.033	0.077	0.003	0.000	0.922	0.579	0.395	0.083

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	237	0	781	0	126	620
normalized size	1	1.00	0.99	2.19	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.145	0.185	0.013	0.000	1.084	0.000	0.359	2.096

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	131	328	0	975	0	171	775
normalized size	1	1.00	0.89	2.22	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.182	0.266	0.014	0.000	1.215	0.000	0.423	2.134

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	175	418	0	1226	0	229	914
normalized size	1	1.00	0.87	2.07	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.239	0.343	0.017	0.000	1.450	0.000	0.330	2.298

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	260	1040	0	1926	1714	282	705
normalized size	1	1.00	1.09	4.37	0.00	8.09	7.20	1.18	2.96
time (sec)	N/A	0.291	0.370	0.017	0.000	0.979	4.196	0.379	2.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	221	530	0	1603	1510	245	620
normalized size	1	1.00	1.16	2.79	0.00	8.44	7.95	1.29	3.26
time (sec)	N/A	0.279	0.308	0.016	0.000	0.902	3.208	0.398	2.203

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	174	260	0	953	547	202	343
normalized size	1	1.00	1.57	2.34	0.00	8.59	4.93	1.82	3.09
time (sec)	N/A	0.067	0.174	0.010	0.000	0.969	1.411	0.416	0.195

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	126	223	0	872	513	163	271
normalized size	1	1.00	1.18	2.08	0.00	8.15	4.79	1.52	2.53
time (sec)	N/A	0.050	0.194	0.012	0.000	0.874	1.209	0.308	1.429

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	131	262	0	887	570	154	313
normalized size	1	1.00	1.14	2.28	0.00	7.71	4.96	1.34	2.72
time (sec)	N/A	0.068	0.140	0.010	0.000	0.878	1.282	0.436	1.500

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	130	0	788	481	135	253
normalized size	1	1.00	0.99	1.26	0.00	7.65	4.67	1.31	2.46
time (sec)	N/A	0.041	0.096	0.005	0.000	0.889	1.092	0.409	1.430

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	101	97	129	0	785	474	136	285
normalized size	1	0.98	0.94	1.25	0.00	7.62	4.60	1.32	2.77
time (sec)	N/A	0.041	0.096	0.004	0.000	0.977	1.111	0.337	1.424

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	178	781	0	1985	0	239	1089
normalized size	1	1.00	0.96	4.22	0.00	10.73	0.00	1.29	5.89
time (sec)	N/A	0.220	0.353	0.017	0.000	1.793	0.000	0.315	2.456

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	954	0	2280	0	309	1255
normalized size	1	1.00	0.92	3.99	0.00	9.54	0.00	1.29	5.25
time (sec)	N/A	0.277	0.437	0.023	0.000	2.174	0.000	0.468	2.554

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	30	30	34	32	26
normalized size	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.022	0.006	0.006	0.423	0.765	0.121	0.255	0.046

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	25	27	27	21
normalized size	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.020	0.004	0.004	0.418	0.882	0.119	0.280	1.310

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	20	22	16
normalized size	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.015	0.004	0.006	0.418	0.833	0.116	0.277	0.077

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	19	13
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.011	0.003	0.006	0.420	0.886	0.111	0.275	0.066

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	18	17	17	15	19	8
normalized size	1	1.00	0.91	0.78	0.74	0.74	0.65	0.83	0.35
time (sec)	N/A	0.014	0.003	0.006	0.418	1.008	0.111	0.224	1.373

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	24	24	17
normalized size	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.017	0.004	0.007	0.421	0.870	0.149	0.250	1.385

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
normalized size	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.031	0.004	0.009	0.427	0.953	0.163	0.308	0.044

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	39	36	34	26
normalized size	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.035	0.005	0.007	0.421	0.792	0.173	0.298	1.312

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	36	44	41	39	32
normalized size	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.042	0.005	0.007	0.645	1.034	0.185	0.294	0.047

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	213	701	0	959	0	0	-1
normalized size	1	1.00	1.04	3.44	0.00	4.70	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.522	0.019	0.000	1.710	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	163	334	0	709	0	0	-1
normalized size	1	1.00	1.12	2.30	0.00	4.89	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.250	0.007	0.000	1.391	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	128	121	0	590	0	0	100
normalized size	1	1.00	1.22	1.15	0.00	5.62	0.00	0.00	0.95
time (sec)	N/A	0.083	0.081	0.007	0.000	1.306	0.000	0.000	0.126

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	89	88	0	171	0	0	53
normalized size	1	1.00	1.33	1.31	0.00	2.55	0.00	0.00	0.79
time (sec)	N/A	0.041	0.058	0.005	0.000	1.146	0.000	0.000	1.452

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	138	197	0	465	0	0	-1
normalized size	1	1.00	1.04	1.48	0.00	3.50	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.171	0.008	0.000	1.309	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	256	376	0	1081	0	0	-1
normalized size	1	1.00	1.16	1.71	0.00	4.91	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.378	0.012	0.000	1.451	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	32	40	8	8	0	29	134
normalized size	1	1.00	0.44	0.55	0.11	0.11	0.00	0.40	1.84
time (sec)	N/A	0.036	0.019	0.013	0.785	0.863	0.000	0.341	0.112

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	1059	129	2109	3026
normalized size	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.291	0.117	0.011	0.000	0.914	2.119	1.663	2.079

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	5260	196	0	2280
normalized size	1	1.00	0.11	0.09	0.00	8.34	0.31	0.00	3.61
time (sec)	N/A	1.170	0.036	0.006	0.000	2.596	6.965	0.000	4.540

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	5082	0	0	10382
normalized size	1	1.00	0.19	0.16	0.00	13.52	0.00	0.00	27.61
time (sec)	N/A	0.666	0.043	0.004	0.000	2.058	0.000	0.000	3.776

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	84	0	0	0	0	-1
normalized size	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.056	0.007	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	52	54	53	51	49	44
normalized size	1	1.00	0.72	1.30	1.35	1.32	1.28	1.22	1.10
time (sec)	N/A	0.019	0.025	0.001	0.869	1.198	0.330	0.332	0.038

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	50	23	0	0	45	-1
normalized size	1	1.00	0.67	0.67	0.31	0.00	0.00	0.60	-0.01
time (sec)	N/A	0.040	0.030	0.011	0.956	0.000	0.000	0.397	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	56	109	114	84	0	140	-1
normalized size	1	1.00	0.41	0.80	0.83	0.61	0.00	1.02	-0.01
time (sec)	N/A	0.081	0.049	0.016	0.906	1.280	0.000	0.511	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	56	87	114	61	0	102	-1
normalized size	1	1.00	0.41	0.64	0.83	0.45	0.00	0.74	-0.01
time (sec)	N/A	0.071	0.035	0.002	0.890	0.763	0.000	0.534	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	65	65	114	32	0	64	-1
normalized size	1	1.00	0.47	0.47	0.83	0.23	0.00	0.47	-0.01
time (sec)	N/A	0.056	0.032	0.003	0.914	1.387	0.000	0.368	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	43	43	114	10	0	26	71
normalized size	1	1.00	0.49	0.49	1.30	0.11	0.00	0.30	0.81
time (sec)	N/A	0.040	0.009	0.003	0.884	1.161	0.000	0.389	1.561

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	103	36	33	0	61	-1
normalized size	1	1.00	0.44	0.70	0.24	0.22	0.00	0.41	-0.01
time (sec)	N/A	0.070	0.036	0.018	0.876	1.391	0.000	0.489	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	72	92	55	113	0	64	-1
normalized size	1	1.00	0.55	0.71	0.42	0.87	0.00	0.49	-0.01
time (sec)	N/A	0.072	0.045	0.008	0.617	1.321	0.000	0.532	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	58	54	53	136	0	43	53
normalized size	1	1.00	0.43	0.40	0.39	1.01	0.00	0.32	0.39
time (sec)	N/A	0.075	0.036	0.009	0.716	1.101	0.000	0.545	2.804

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	53	209	0	43	53
normalized size	1	1.00	0.42	0.39	0.39	1.53	0.00	0.31	0.39
time (sec)	N/A	0.078	0.035	0.007	0.538	1.149	0.000	0.589	3.228

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	53	275	0	43	53
normalized size	1	1.00	0.42	0.39	0.39	2.01	0.00	0.31	0.39
time (sec)	N/A	0.079	0.035	0.007	0.721	1.167	0.000	0.649	3.654

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	58	54	53	343	0	43	53
normalized size	1	1.00	0.42	0.39	0.39	2.50	0.00	0.31	0.39
time (sec)	N/A	0.078	0.037	0.007	0.543	1.223	0.000	0.743	4.344

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.030	0.030	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	207	0	362	579	0	1564	777
normalized size	1	1.00	0.44	0.00	0.77	1.24	0.00	3.34	1.66
time (sec)	N/A	0.224	0.216	0.009	0.704	1.479	0.000	0.627	3.515

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	143	0	198	297	0	745	390
normalized size	1	1.00	0.45	0.00	0.63	0.94	0.00	2.37	1.24
time (sec)	N/A	0.140	0.174	0.007	0.640	1.430	0.000	0.481	2.166

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	83	0	77	110	0	229	138
normalized size	1	1.00	0.58	0.00	0.54	0.77	0.00	1.61	0.97
time (sec)	N/A	0.068	0.049	0.006	0.798	1.112	0.000	0.483	1.542

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.013	0.009	0.000	0.742	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.015	0.008	0.000	1.281	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	162	101	0	0	82	0	0	69
normalized size	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.47
time (sec)	N/A	0.099	0.082	0.011	0.000	1.317	0.000	0.000	1.647

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	93	114	114	147	0	0	-1
normalized size	1	1.00	0.53	0.65	0.65	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.069	0.016	0.576	9.056	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	121	174	119	0	0	237	-1
normalized size	1	1.00	0.45	0.65	0.44	0.00	0.00	0.88	-0.00
time (sec)	N/A	0.151	0.096	0.016	0.878	0.000	0.000	0.866	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	66	68	0	0	0	80	-1
normalized size	1	1.00	0.37	0.38	0.00	0.00	0.00	0.45	-0.01
time (sec)	N/A	0.092	0.034	0.030	0.000	0.000	0.000	0.390	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	125	115	79	0	0	173	-1
normalized size	1	1.00	0.32	0.29	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.187	0.068	0.028	0.598	0.000	0.000	0.541	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	99	91	57	0	0	128	-1
normalized size	1	1.00	0.34	0.31	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.137	0.057	0.010	0.635	0.000	0.000	0.505	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	77	69	30	0	0	79	-1
normalized size	1	1.00	0.41	0.37	0.16	0.00	0.00	0.42	-0.01
time (sec)	N/A	0.091	0.029	0.005	0.663	0.000	0.000	0.412	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	49	50	10	0	0	34	39
normalized size	1	1.00	0.56	0.57	0.11	0.00	0.00	0.39	0.44
time (sec)	N/A	0.053	0.014	0.005	0.733	0.000	0.000	0.397	1.426

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	86	78	44	0	0	77	-1
normalized size	1	1.00	0.45	0.41	0.23	0.00	0.00	0.41	-0.01
time (sec)	N/A	0.118	0.040	0.006	0.743	0.000	0.000	0.592	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	126	141	97	0	0	121	-1
normalized size	1	1.00	0.42	0.47	0.32	0.00	0.00	0.40	-0.00
time (sec)	N/A	0.187	0.086	0.011	1.068	0.000	0.000	0.660	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	152	199	139	0	0	141	-1
normalized size	1	1.00	0.37	0.49	0.34	0.00	0.00	0.34	-0.00
time (sec)	N/A	0.268	0.130	0.013	0.854	0.000	0.000	0.717	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	98	94	57	0	0	126	-1
normalized size	1	1.00	0.34	0.33	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.138	0.052	0.036	0.995	0.000	0.000	0.486	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	103	91	52	0	0	125	-1
normalized size	1	1.00	0.35	0.31	0.18	0.00	0.00	0.43	-0.00
time (sec)	N/A	0.137	0.048	0.026	0.940	0.000	0.000	0.481	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	98	152	99	302	0	84	-1
normalized size	1	1.00	0.44	0.68	0.45	1.36	0.00	0.38	-0.00
time (sec)	N/A	0.126	0.087	0.012	1.022	0.827	0.000	0.590	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	124	116	79	0	0	172	-1
normalized size	1	1.00	0.32	0.30	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.180	0.066	0.032	0.947	0.000	0.000	0.461	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	62	45	38	42	0	-1
normalized size	1	1.00	0.83	1.35	0.98	0.83	0.91	0.00	-0.02
time (sec)	N/A	0.036	0.028	0.030	0.934	1.058	19.826	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	33	32	24	26	0	-1
normalized size	1	1.00	0.93	1.18	1.14	0.86	0.93	0.00	-0.04
time (sec)	N/A	0.026	0.014	0.024	0.872	0.864	13.890	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	19	15	37	0	-1
normalized size	1	1.00	1.00	1.20	1.27	1.00	2.47	0.00	-0.07
time (sec)	N/A	0.012	0.003	0.022	0.885	0.867	11.280	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	26	27	22	66	25	20
normalized size	1	1.00	0.96	1.13	1.17	0.96	2.87	1.09	0.87
time (sec)	N/A	0.019	0.007	0.020	0.896	0.926	16.485	0.251	1.369

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	69	58	59	0	0	-1
normalized size	1	1.00	0.86	1.21	1.02	1.04	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.066	0.024	0.911	0.620	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	88	71	72	73	0	-1
normalized size	1	1.00	0.82	1.16	0.93	0.95	0.96	0.00	-0.01
time (sec)	N/A	0.047	0.078	0.031	0.910	0.593	52.911	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	75	105	84	85	88	0	-1
normalized size	1	1.00	0.81	1.13	0.90	0.91	0.95	0.00	-0.01
time (sec)	N/A	0.054	0.095	0.032	0.921	0.806	105.734	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	34	54	0	272	0	203	-1
normalized size	1	1.00	0.14	0.23	0.00	1.15	0.00	0.86	-0.00
time (sec)	N/A	0.206	0.009	0.121	0.000	0.572	0.000	0.348	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	212	0	136	-1
normalized size	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	-0.01
time (sec)	N/A	0.132	0.009	0.069	0.000	0.738	0.000	0.419	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	32	79	0	151	36	38	-1
normalized size	1	1.00	0.64	1.58	0.00	3.02	0.72	0.76	-0.02
time (sec)	N/A	0.035	0.007	0.073	0.000	0.837	12.999	0.375	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	97	0	161	58	0	-1
normalized size	1	1.00	0.50	1.43	0.00	2.37	0.85	0.00	-0.01
time (sec)	N/A	0.041	0.008	0.095	0.000	1.049	24.919	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	34	73	0	171	0	0	-1
normalized size	1	1.00	0.19	0.41	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.008	0.079	0.000	0.827	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	34	73	0	259	0	0	-1
normalized size	1	1.00	0.13	0.29	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.008	0.092	0.000	0.684	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	43	59	0	0	-1
normalized size	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	-0.03
time (sec)	N/A	0.040	0.017	0.098	1.106	0.681	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	59	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.019	0.070	0.000	0.795	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	40	208	74	74	0	0	-1
normalized size	1	1.00	0.36	1.86	0.66	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.056	0.047	0.922	0.731	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	40	135	48	48	0	0	-1
normalized size	1	1.00	0.36	1.21	0.43	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.037	0.026	0.922	0.764	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	44	64	22	22	0	0	-1
normalized size	1	1.00	0.44	0.65	0.22	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.023	0.023	0.890	0.724	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	46	71	32	24	0	0	-1
normalized size	1	1.00	0.51	0.79	0.36	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.032	0.030	0.894	0.745	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	35	37	41	41	0	0	-1
normalized size	1	1.00	0.73	0.77	0.85	0.85	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.016	0.026	0.915	0.677	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	69	69	0	0	-1
normalized size	1	1.00	0.45	0.42	0.78	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.032	0.030	0.947	0.660	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	97	97	0	0	-1
normalized size	1	1.00	0.45	0.42	1.10	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.034	0.032	0.975	0.640	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	132	47	57	0	173	-1
normalized size	1	1.00	0.51	1.22	0.44	0.53	0.00	1.60	-0.01
time (sec)	N/A	0.042	0.033	0.043	0.940	0.861	0.000	0.350	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	25	28	0	53	-1
normalized size	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01
time (sec)	N/A	0.028	0.022	0.016	0.949	0.828	0.000	0.258	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	25	28	0	53	-1
normalized size	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01
time (sec)	N/A	0.025	0.021	0.015	0.883	0.598	0.000	0.295	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	39	56	19	20	0	25	-1
normalized size	1	1.00	0.44	0.64	0.22	0.23	0.00	0.28	-0.01
time (sec)	N/A	0.019	0.014	0.016	1.043	0.723	0.000	0.219	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	37	54	13	15	0	0	-1
normalized size	1	1.00	0.44	0.64	0.15	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.015	0.017	0.797	0.892	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	42	61	22	23	0	0	-1
normalized size	1	1.00	0.45	0.65	0.23	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.025	0.017	0.665	0.567	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	47	61	22	23	0	0	-1
normalized size	1	1.00	0.49	0.64	0.23	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.026	0.020	0.911	0.876	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	90	532	276	390	0	2719	-1
normalized size	1	1.00	0.38	2.24	1.16	1.64	0.00	11.42	-0.00
time (sec)	N/A	0.098	0.107	0.059	0.993	0.815	0.000	0.955	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	123	146	108	144	0	292	-1
normalized size	1	1.00	0.58	0.69	0.51	0.68	0.00	1.38	-0.00
time (sec)	N/A	0.062	0.078	0.017	0.928	0.743	0.000	0.434	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	124	145	109	145	0	292	-1
normalized size	1	1.00	0.59	0.69	0.52	0.69	0.00	1.38	-0.00
time (sec)	N/A	0.058	0.070	0.017	0.937	0.786	0.000	0.373	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	122	138	101	130	0	263	-1
normalized size	1	1.00	0.59	0.67	0.49	0.63	0.00	1.28	-0.00
time (sec)	N/A	0.052	0.075	0.016	0.945	0.756	0.000	0.489	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	68	127	43	44	0	0	-1
normalized size	1	1.00	0.35	0.65	0.22	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.037	0.019	0.902	0.936	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	124	147	101	131	0	0	-1
normalized size	1	1.00	0.58	0.69	0.48	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.070	0.091	0.025	0.945	0.752	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	124	145	101	134	0	0	-1
normalized size	1	1.00	0.57	0.67	0.46	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.070	0.087	0.026	1.172	0.559	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.023	0.070	0.000	0.773	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.014	0.021	0.000	0.644	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.014	0.023	0.000	0.814	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.011	0.020	0.000	0.558	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	42	66	27	22	0	0	-1
normalized size	1	1.00	0.49	0.78	0.32	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.015	0.020	1.066	0.743	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.014	0.023	0.000	0.463	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.014	0.023	0.000	0.689	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.022	0.040	0.000	0.710	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.018	0.083	0.000	0.749	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.017	0.077	0.000	0.584	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.012	0.089	0.000	0.758	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	76	104	70	106	0	0	-1
normalized size	1	1.00	0.48	0.65	0.44	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.063	0.021	1.162	1.043	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.015	0.079	0.000	0.863	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.016	0.078	0.000	0.743	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	0	0	79	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	1.52	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.023	0.279	0.000	0.704	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	51	0	45	0	0	-1
normalized size	1	1.00	0.74	1.19	0.00	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.062	0.043	0.000	0.950	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	80	0	0	103	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.062	0.282	0.000	0.881	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	59	0	0	82	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.045	0.111	0.000	0.947	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	124	75	0	0	165	0	0	-1
normalized size	1	1.06	0.64	0.00	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.035	0.052	0.000	0.819	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	54	148	59	78	0	0	-1
normalized size	1	1.00	0.52	1.44	0.57	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.029	0.073	1.233	0.937	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	93	973	0	353	0	0	-1
normalized size	1	1.00	0.84	8.77	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.194	0.164	0.000	0.980	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	664	0	285	0	0	-1
normalized size	1	1.00	0.92	7.63	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.127	0.127	0.000	0.852	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	402	0	231	0	0	-1
normalized size	1	1.00	0.91	5.91	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.066	0.101	0.000	0.933	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	113	0	159	0	39	39
normalized size	1	1.00	1.00	2.90	0.00	4.08	0.00	1.00	1.00
time (sec)	N/A	0.033	0.059	0.063	0.000	0.621	0.000	0.414	1.467

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	135	658	0	333	0	0	-1
normalized size	1	1.00	1.38	6.71	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.665	0.148	0.000	0.896	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	112	958	0	429	0	0	-1
normalized size	1	1.00	0.89	7.60	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.333	0.174	0.000	0.759	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	143	1300	0	522	0	0	-1
normalized size	1	1.00	0.87	7.93	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.443	0.201	0.000	0.963	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	340	280	0	4426	0	0	-1
normalized size	1	1.00	0.96	0.79	0.00	12.54	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.898	0.747	0.000	1.791	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	526	260	0	4699	0	0	-1
normalized size	1	1.00	0.86	0.43	0.00	7.70	0.00	0.00	-0.00
time (sec)	N/A	1.152	0.763	0.461	0.000	1.432	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	114	0	801	0	1035	-1
normalized size	1	1.00	0.86	0.67	0.00	4.74	0.00	6.12	-0.01
time (sec)	N/A	0.192	0.258	0.194	0.000	0.711	0.000	1.846	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	127	268	0	1229	0	0	-1
normalized size	1	1.00	0.62	1.31	0.00	6.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	0.179	0.348	0.000	0.741	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	699	127	534	0	6279	0	0	-1
normalized size	1	1.00	0.18	0.76	0.00	8.98	0.00	0.00	-0.00
time (sec)	N/A	1.493	0.128	0.750	0.000	5.400	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	127	630	0	5712	0	0	-1
normalized size	1	1.00	0.31	1.52	0.00	13.80	0.00	0.00	-0.00
time (sec)	N/A	0.787	0.125	1.215	0.000	2.699	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	265	0	0	0	0	0	-1
normalized size	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.654	0.042	0.000	0.945	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	263	0	0	0	0	0	-1
normalized size	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.575	0.033	0.000	0.946	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	261	0	0	0	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.532	0.026	0.000	0.899	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	397	0	259	0	0	224
normalized size	1	1.00	1.00	5.36	0.00	3.50	0.00	0.00	3.03
time (sec)	N/A	0.066	0.139	0.096	0.000	0.631	0.000	0.000	1.608

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	240	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.384	0.036	0.000	0.769	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	258	0	0	0	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.424	0.040	0.000	0.767	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	365	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.754	0.083	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.715	0.064	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	364	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.654	0.069	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0	-1
normalized size	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.649	0.068	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	110	125	0	658	0	0	-1
normalized size	1	1.00	0.92	1.05	0.00	5.53	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.148	0.115	0.000	1.123	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	365	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.636	0.069	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	365	0	0	0	0	0	-1
normalized size	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.611	0.069	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	518	0	0	0	0	0	-1
normalized size	1	1.00	3.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	1.490	0.054	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	524	0	0	0	0	0	-1
normalized size	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.464	0.054	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	520	0	0	0	0	0	-1
normalized size	1	1.00	3.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.491	0.053	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	513	0	0	0	0	0	-1
normalized size	1	1.00	3.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.524	0.055	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	158	209	0	827	0	0	-1
normalized size	1	1.00	0.91	1.21	0.00	4.78	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.282	0.047	0.000	0.963	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	526	0	0	0	0	0	-1
normalized size	1	1.00	3.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.388	0.061	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	520	0	0	0	0	0	-1
normalized size	1	1.00	3.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	1.372	0.060	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.159	0.019	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.142	0.019	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.135	0.018	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.108	0.018	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	148	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	3.15	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.063	0.036	0.000	0.887	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.144	0.019	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	175	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.146	0.019	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	-1
normalized size	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.859	0.013	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	-1
normalized size	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.840	0.014	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	-1
normalized size	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.839	0.013	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0	-1
normalized size	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.944	0.013	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	449	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.310	0.013	0.000	1.142	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	395	0	0	0	0	0	-1
normalized size	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.784	0.016	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	399	0	0	0	0	0	-1
normalized size	1	1.00	2.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.798	0.015	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	137	3798	273	2303	0	0	1734
normalized size	1	1.00	0.75	20.87	1.50	12.65	0.00	0.00	9.53
time (sec)	N/A	0.156	0.355	0.113	1.299	0.853	0.000	0.000	2.156

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	1065	152	706	0	5454	543
normalized size	1	1.00	0.74	9.10	1.30	6.03	0.00	46.62	4.64
time (sec)	N/A	0.069	0.151	0.071	1.265	0.938	0.000	0.803	1.617

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	205	65	142	1239	557	83
normalized size	1	1.00	0.71	3.53	1.12	2.45	21.36	9.60	1.43
time (sec)	N/A	0.024	0.065	0.046	1.124	0.921	42.999	0.403	1.406

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	307	0	0	0	0	0	-1
normalized size	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.845	0.034	0.000	0.670	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	3515	0	0	0	0	0	-1
normalized size	1	1.00	10.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.961	6.360	0.042	0.000	0.644	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	12289	0	0	0	0	0	-1
normalized size	1	1.00	19.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	10.724	6.861	0.055	0.000	0.933	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	618	0	0	0	0	0	-1
normalized size	1	1.00	3.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	3.211	0.263	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	388	0	0	0	0	0	-1
normalized size	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.691	0.147	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	183	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.159	0.034	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	428	0	0	0	0	0	-1
normalized size	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	1.457	0.020	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.358	0.052	0.000	0.908	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	150	298	142	175	178	169	141
normalized size	1	1.00	3.26	6.48	3.09	3.80	3.87	3.67	3.07
time (sec)	N/A	0.054	0.039	0.002	1.018	0.780	0.106	0.408	0.076

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	401	1314	403	571	559	493	383
normalized size	1	1.00	4.51	14.76	4.53	6.42	6.28	5.54	4.30
time (sec)	N/A	0.187	0.114	0.003	1.061	0.671	0.194	0.410	1.479

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	797	7550	872	1335	1314	1109	777
normalized size	1	1.00	5.78	54.71	6.32	9.67	9.52	8.04	5.63
time (sec)	N/A	0.373	0.275	0.003	1.129	0.768	0.356	0.617	1.657

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	154	349	166	229	240	213	164
normalized size	1	1.00	2.80	6.35	3.02	4.16	4.36	3.87	2.98
time (sec)	N/A	0.053	0.010	0.001	1.077	0.774	0.112	0.300	0.077

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	405	1413	439	715	722	615	419
normalized size	1	1.00	3.89	13.59	4.22	6.88	6.94	5.91	4.03
time (sec)	N/A	0.164	0.074	0.000	1.034	0.754	0.212	0.433	1.472

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	801	7697	920	1635	1654	1360	825
normalized size	1	1.00	5.04	48.41	5.79	10.28	10.40	8.55	5.19
time (sec)	N/A	0.315	0.042	0.003	1.164	0.759	0.392	0.514	1.650

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	219	158	0	1231	178	1194	3988
normalized size	1	1.00	1.13	0.82	0.00	6.38	0.92	6.19	20.66
time (sec)	N/A	0.438	0.138	0.069	0.000	0.856	3.065	0.431	2.324

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	151	0	434	280	130	278
normalized size	1	1.00	0.95	1.86	0.00	5.36	3.46	1.60	3.43
time (sec)	N/A	0.129	0.044	0.003	0.000	0.709	1.638	0.405	1.764

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	175	140	0	703	104	1285	590
normalized size	1	1.00	1.07	0.85	0.00	4.29	0.63	7.84	3.60
time (sec)	N/A	0.155	0.091	0.006	0.000	0.842	1.350	0.455	1.737

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	129	0	272	168	53	61
normalized size	1	1.00	1.07	3.00	0.00	6.33	3.91	1.23	1.42
time (sec)	N/A	0.061	0.016	0.007	0.000	0.867	1.036	0.452	0.094

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	128	184	0	468	320	274	2173
normalized size	1	1.00	1.36	1.96	0.00	4.98	3.40	2.91	23.12
time (sec)	N/A	0.132	0.078	0.011	0.000	0.883	6.044	1.147	2.504

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	206	168	0	1339	211	0	3844
normalized size	1	1.00	1.06	0.86	0.00	6.87	1.08	0.00	19.71
time (sec)	N/A	0.286	0.358	0.010	0.000	0.880	4.269	0.000	2.389

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	154	213	0	810	464	102	4950
normalized size	1	1.00	1.27	1.76	0.00	6.69	3.83	0.84	40.91
time (sec)	N/A	0.198	0.134	0.013	0.000	1.111	146.464	0.402	5.855

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	235	188	0	2044	347	1243	5214
normalized size	1	1.00	1.05	0.84	0.00	9.12	1.55	5.55	23.28
time (sec)	N/A	0.497	0.209	0.010	0.000	0.933	12.774	0.495	2.833

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	263	323	0	2454	573	1304	7327
normalized size	1	1.00	0.97	1.20	0.00	9.09	2.12	4.83	27.14
time (sec)	N/A	0.579	0.468	0.020	0.000	0.963	8.966	0.524	4.755

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	100	276	0	1021	495	171	427
normalized size	1	1.00	1.03	2.85	0.00	10.53	5.10	1.76	4.40
time (sec)	N/A	0.135	0.131	0.021	0.000	0.921	4.875	0.565	1.770

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	247	319	0	2474	578	1312	7200
normalized size	1	1.00	0.97	1.26	0.00	9.74	2.28	5.17	28.35
time (sec)	N/A	0.388	0.975	0.022	0.000	0.780	18.532	0.529	3.953

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	96	98	270	0	1042	495	172	417
normalized size	1	0.98	1.00	2.76	0.00	10.63	5.05	1.76	4.26
time (sec)	N/A	0.122	0.122	0.023	0.000	0.921	4.675	0.462	1.720

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	271	364	0	3228	0	1357	9056
normalized size	1	1.00	0.91	1.22	0.00	10.80	0.00	4.54	30.29
time (sec)	N/A	0.702	0.889	0.018	0.000	1.166	0.000	0.463	4.846

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	235	693	0	2476	0	454	11072
normalized size	1	1.00	1.45	4.28	0.00	15.28	0.00	2.80	68.35
time (sec)	N/A	0.294	0.473	0.033	0.000	1.209	0.000	1.253	11.354

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	339	1304	0	4330	0	847	10556
normalized size	1	1.00	0.97	3.75	0.00	12.44	0.00	2.43	30.33
time (sec)	N/A	1.653	1.632	0.032	0.000	1.180	0.000	0.787	6.384

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	284	1014	0	4562	0	224	12436
normalized size	1	1.00	1.33	4.76	0.00	21.42	0.00	1.05	58.38
time (sec)	N/A	0.390	0.520	0.040	0.000	1.899	0.000	0.430	12.317

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	384	1518	0	5734	0	1987	12239
normalized size	1	1.00	0.94	3.72	0.00	14.05	0.00	4.87	30.00
time (sec)	N/A	3.676	2.983	0.036	0.000	1.922	0.000	0.548	8.725

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	328	704	0	6633	0	1688	12677
normalized size	1	1.00	0.96	2.06	0.00	19.45	0.00	4.95	37.18
time (sec)	N/A	0.951	4.434	0.050	0.000	1.567	0.000	0.746	7.019

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	146	544	0	3739	1671	365	1182
normalized size	1	1.00	0.97	3.63	0.00	24.93	11.14	2.43	7.88
time (sec)	N/A	0.196	0.206	0.046	0.000	1.070	14.453	0.693	3.855

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	382	885	0	7701	0	2295	14584
normalized size	1	1.00	1.05	2.44	0.00	21.21	0.00	6.32	40.18
time (sec)	N/A	1.039	4.861	0.051	0.000	1.522	0.000	0.866	7.429

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	150	147	541	0	3708	1646	365	1157
normalized size	1	0.99	0.97	3.56	0.00	24.39	10.83	2.40	7.61
time (sec)	N/A	0.185	0.184	0.050	0.000	1.126	13.928	0.633	3.802

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	463	1010	0	8554	0	2487	16086
normalized size	1	1.00	1.06	2.31	0.00	19.57	0.00	5.69	36.81
time (sec)	N/A	5.361	6.169	0.052	0.000	1.941	0.000	0.563	7.800

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	391	4477	0	9908	0	1012	19440
normalized size	1	1.00	1.53	17.56	0.00	38.85	0.00	3.97	76.24
time (sec)	N/A	0.486	3.949	0.080	0.000	3.206	0.000	1.716	17.982

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	560	6821	0	10260	0	1412	18112
normalized size	1	1.00	1.16	14.09	0.00	21.20	0.00	2.92	37.42
time (sec)	N/A	1.227	6.237	0.071	0.000	2.499	0.000	1.220	14.376

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	491	5575	0	15165	0	377	21465
normalized size	1	1.00	1.51	17.15	0.00	46.66	0.00	1.16	66.05
time (sec)	N/A	0.585	6.179	0.086	0.000	5.164	0.000	0.639	22.450

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	222	164	0	1346	219	1245	4605
normalized size	1	1.00	1.10	0.81	0.00	6.66	1.08	6.16	22.80
time (sec)	N/A	0.362	0.049	0.005	0.000	0.890	3.623	0.536	1.341

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	154	0	446	332	162	287
normalized size	1	1.00	0.92	1.77	0.00	5.13	3.82	1.86	3.30
time (sec)	N/A	0.127	0.044	0.005	0.000	0.942	1.950	0.429	0.443

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	178	143	0	799	124	1325	683
normalized size	1	1.00	1.05	0.84	0.00	4.70	0.73	7.79	4.02
time (sec)	N/A	0.153	0.101	0.002	0.000	0.890	1.595	0.466	1.792

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	274	189	62	477
normalized size	1	1.00	1.07	2.95	0.00	6.23	4.30	1.41	10.84
time (sec)	N/A	0.063	0.016	0.005	0.000	0.893	1.187	0.397	1.622

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	131	190	0	474	348	285	2520
normalized size	1	1.00	1.27	1.84	0.00	4.60	3.38	2.77	24.47
time (sec)	N/A	0.138	0.069	0.007	0.000	0.923	7.157	1.193	3.462

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	209	174	0	1477	258	0	4339
normalized size	1	1.00	1.02	0.85	0.00	7.24	1.26	0.00	21.27
time (sec)	N/A	0.273	0.342	0.007	0.000	0.942	4.899	0.000	3.872

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	157	222	0	828	532	348	5947
normalized size	1	1.00	1.18	1.67	0.00	6.23	4.00	2.62	44.71
time (sec)	N/A	0.195	0.136	0.008	0.000	1.220	156.628	1.106	6.981

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	238	197	0	2212	411	1249	5771
normalized size	1	1.00	1.01	0.83	0.00	9.37	1.74	5.29	24.45
time (sec)	N/A	0.488	0.103	0.010	0.000	0.946	14.012	0.570	3.173

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	695	0	2578	641	1370	8025
normalized size	1	1.00	0.95	2.49	0.00	9.24	2.30	4.91	28.76
time (sec)	N/A	0.539	0.461	0.018	0.000	0.725	9.821	0.603	5.092

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	500	0	1077	556	211	460
normalized size	1	1.00	1.00	4.85	0.00	10.46	5.40	2.05	4.47
time (sec)	N/A	0.138	0.126	0.020	0.000	0.887	5.286	0.515	1.903

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	250	693	0	2600	646	1378	7835
normalized size	1	1.00	0.95	2.63	0.00	9.89	2.46	5.24	29.79
time (sec)	N/A	0.368	0.939	0.017	0.000	0.871	19.757	0.675	4.398

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	484	0	1066	525	211	442
normalized size	1	1.00	1.01	4.94	0.00	10.88	5.36	2.15	4.51
time (sec)	N/A	0.127	0.123	0.024	0.000	0.665	5.068	0.445	1.905

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	238	714	0	2486	0	476	13434
normalized size	1	1.00	1.37	4.10	0.00	14.29	0.00	2.74	77.21
time (sec)	N/A	0.295	0.436	0.030	0.000	1.722	0.000	1.432	11.689

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	342	1346	0	4520	0	999	12008
normalized size	1	1.00	0.95	3.74	0.00	12.56	0.00	2.78	33.36
time (sec)	N/A	1.599	1.471	0.028	0.000	1.197	0.000	1.021	7.289

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	287	1047	0	4604	0	687	14830
normalized size	1	1.00	1.26	4.59	0.00	20.19	0.00	3.01	65.04
time (sec)	N/A	0.369	0.515	0.034	0.000	4.143	0.000	1.296	13.520

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	387	1569	0	5954	0	2002	13781
normalized size	1	1.00	0.91	3.71	0.00	14.08	0.00	4.73	32.58
time (sec)	N/A	3.550	3.025	0.030	0.000	1.370	0.000	0.714	10.447

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	331	3432	0	6770	0	1844	13840
normalized size	1	1.00	0.94	9.72	0.00	19.18	0.00	5.22	39.21
time (sec)	N/A	0.870	4.445	0.049	0.000	1.245	0.000	0.799	7.520

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	149	2181	0	3843	1794	447	1267
normalized size	1	1.00	0.94	13.72	0.00	24.17	11.28	2.81	7.97
time (sec)	N/A	0.200	0.209	0.046	0.000	1.000	14.583	0.781	4.019

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	385	4751	0	7838	0	2527	16025
normalized size	1	1.00	1.03	12.67	0.00	20.90	0.00	6.74	42.73
time (sec)	N/A	0.972	4.483	0.049	0.000	1.459	0.000	0.794	7.944

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	148	2132	0	3748	1707	445	1199
normalized size	1	1.00	0.97	13.93	0.00	24.50	11.16	2.91	7.84
time (sec)	N/A	0.192	0.179	0.053	0.000	1.027	13.965	0.690	3.994

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	394	4606	0	9926	0	1044	22621
normalized size	1	1.00	1.46	17.06	0.00	36.76	0.00	3.87	83.78
time (sec)	N/A	0.496	3.925	0.075	0.000	4.916	0.000	1.700	18.492

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	575	7019	0	10518	0	1658	20580
normalized size	1	1.00	1.15	14.07	0.00	21.08	0.00	3.32	41.24
time (sec)	N/A	1.094	6.210	0.070	0.000	2.626	0.000	1.419	15.398

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	509	5737	0	15231	0	1735	25334
normalized size	1	1.00	1.48	16.73	0.00	44.41	0.00	5.06	73.86
time (sec)	N/A	0.589	6.157	0.082	0.000	14.755	0.000	1.628	24.912

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.691	1.167	0.054	0.000	0.921	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.687	0.442	0.058	0.000	11.321	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	105	104	104	107	28	29
normalized size	1	1.00	1.00	3.09	3.06	3.06	3.15	0.82	0.85
time (sec)	N/A	0.036	0.011	0.003	0.543	0.745	0.101	0.349	1.584

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	188	175	174	174	187	46	46
normalized size	1	1.00	3.36	3.12	3.11	3.11	3.34	0.82	0.82
time (sec)	N/A	0.096	0.010	0.003	0.774	0.662	0.150	0.415	1.602

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [411] had the largest ratio of [.8000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	1	1	1.00	15	0.067
3	A	1	1	1.00	15	0.067
4	A	3	3	1.00	15	0.200
5	A	8	8	1.00	11	0.727
6	A	3	2	1.00	26	0.077
7	A	3	2	1.00	26	0.077
8	A	3	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	3	2	1.00	24	0.083
11	A	2	1	1.00	22	0.045
12	A	3	2	1.00	26	0.077
13	A	3	2	1.00	26	0.077
14	A	3	2	1.00	26	0.077
15	A	3	2	1.00	26	0.077
16	A	3	2	1.00	26	0.077
17	A	3	2	1.00	26	0.077
18	A	3	2	1.00	26	0.077
19	A	3	2	1.00	26	0.077
20	A	3	2	1.00	26	0.077
21	A	3	2	1.00	26	0.077
22	A	3	2	1.00	26	0.077
23	A	3	2	1.00	26	0.077
24	A	4	3	1.40	26	0.115
25	A	3	2	1.00	26	0.077
26	A	3	2	1.00	26	0.077
27	A	4	3	1.00	26	0.115
28	A	3	2	1.00	26	0.077
29	A	3	2	1.00	26	0.077
30	A	2	2	1.00	26	0.077
31	A	3	2	1.00	24	0.083
32	A	3	2	1.00	22	0.091
33	A	4	3	1.00	26	0.115
34	A	3	2	1.00	26	0.077
35	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	3	1.00	26	0.115
37	A	3	2	1.00	26	0.077
38	A	3	2	1.00	26	0.077
39	A	4	3	1.00	26	0.115
40	A	3	2	1.00	26	0.077
41	A	3	2	1.00	26	0.077
42	A	4	3	1.00	26	0.115
43	A	3	2	1.00	26	0.077
44	A	3	2	1.00	26	0.077
45	A	2	2	1.00	26	0.077
46	A	3	2	1.00	26	0.077
47	A	3	2	1.00	26	0.077
48	A	4	4	1.00	26	0.154
49	A	3	2	1.00	26	0.077
50	A	3	2	1.00	26	0.077
51	A	3	2	1.00	26	0.077
52	A	4	3	1.00	26	0.115
53	A	3	2	1.00	26	0.077
54	A	3	2	1.00	26	0.077
55	A	4	3	1.00	26	0.115
56	A	3	2	1.00	26	0.077
57	A	3	2	1.00	26	0.077
58	A	4	3	1.00	26	0.115
59	A	3	2	1.00	26	0.077
60	A	3	2	1.00	26	0.077
61	A	2	2	1.00	26	0.077
62	A	3	2	1.00	24	0.083
63	A	3	2	1.00	22	0.091
64	A	4	3	1.00	26	0.115
65	A	3	2	1.00	26	0.077
66	A	3	2	1.00	26	0.077
67	A	4	3	1.00	26	0.115
68	A	3	2	1.00	26	0.077
69	A	3	2	1.00	26	0.077
70	A	4	3	1.00	26	0.115
71	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	3	2	1.00	26	0.077
73	A	4	3	1.00	26	0.115
74	A	3	2	1.00	26	0.077
75	A	3	2	1.00	26	0.077
76	A	4	3	1.00	26	0.115
77	A	3	2	1.00	26	0.077
78	A	3	2	1.00	26	0.077
79	A	4	3	1.00	26	0.115
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	26	0.077
82	A	2	2	1.00	26	0.077
83	A	3	2	1.00	26	0.077
84	A	3	2	1.00	26	0.077
85	A	4	4	1.00	26	0.154
86	A	3	2	1.00	26	0.077
87	A	3	2	1.00	26	0.077
88	A	5	4	1.00	26	0.154
89	A	8	8	1.00	26	0.308
90	A	8	8	1.00	26	0.308
91	A	3	3	1.00	26	0.115
92	A	7	7	1.00	24	0.292
93	A	7	7	1.00	22	0.318
94	A	5	5	1.00	26	0.192
95	A	8	8	1.00	26	0.308
96	A	8	8	0.99	26	0.308
97	A	4	3	0.98	26	0.115
98	A	9	9	1.00	26	0.346
99	A	9	9	1.00	26	0.346
100	A	2	2	1.00	26	0.077
101	A	9	8	1.00	24	0.333
102	A	9	8	1.00	22	0.364
103	A	4	3	1.00	26	0.115
104	A	10	9	1.00	26	0.346
105	A	10	9	1.00	26	0.346
106	A	4	3	1.00	26	0.115
107	A	11	9	1.00	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	4	3	1.00	26	0.115
109	A	11	9	1.00	26	0.346
110	A	11	9	1.00	26	0.346
111	A	2	2	1.00	26	0.077
112	A	11	8	1.00	24	0.333
113	A	11	8	1.00	22	0.364
114	A	4	3	1.00	26	0.115
115	A	12	9	1.00	26	0.346
116	A	12	9	1.00	26	0.346
117	A	4	3	1.00	26	0.115
118	A	3	2	1.00	28	0.071
119	A	3	2	1.00	28	0.071
120	A	3	2	1.00	28	0.071
121	A	2	2	1.00	28	0.071
122	A	2	2	1.00	28	0.071
123	A	2	2	1.00	28	0.071
124	A	2	2	1.00	26	0.077
125	A	4	3	1.00	24	0.125
126	A	4	3	1.00	24	0.125
127	A	4	3	1.00	24	0.125
128	A	2	2	1.00	24	0.083
129	A	2	2	1.00	24	0.083
130	A	2	2	1.00	24	0.083
131	A	2	2	1.03	22	0.091
132	A	3	3	1.04	20	0.150
133	A	3	3	1.00	24	0.125
134	A	2	2	1.00	24	0.083
135	A	2	2	1.00	24	0.083
136	A	3	3	1.00	24	0.125
137	A	2	2	1.00	24	0.083
138	A	6	6	1.00	18	0.333
139	A	5	5	1.00	18	0.278
140	A	3	3	1.00	18	0.167
141	A	7	7	1.00	18	0.389
142	A	8	7	1.00	18	0.389
143	A	14	8	1.00	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	14	8	1.00	18	0.444
145	A	13	7	1.00	18	0.389
146	A	13	7	1.00	18	0.389
147	A	13	7	1.00	16	0.438
148	A	13	7	1.00	14	0.500
149	A	14	8	1.00	18	0.444
150	A	14	8	1.00	18	0.444
151	A	6	4	1.00	16	0.250
152	A	5	4	1.00	16	0.250
153	A	4	3	1.00	16	0.188
154	B	4	3	2.10	16	0.188
155	A	6	5	1.00	16	0.312
156	A	4	3	1.00	16	0.188
157	A	4	3	1.00	16	0.188
158	A	15	10	1.00	16	0.625
159	A	15	10	1.00	16	0.625
160	A	14	9	1.00	16	0.562
161	A	14	9	1.00	16	0.562
162	A	13	8	1.00	16	0.500
163	A	13	8	1.00	16	0.500
164	A	13	8	1.00	14	0.571
165	A	13	8	1.00	12	0.667
166	A	14	9	1.00	16	0.562
167	A	14	9	1.00	16	0.562
168	A	15	10	1.00	16	0.625
169	A	15	10	1.00	16	0.625
170	A	14	8	1.00	16	0.500
171	A	5	5	1.00	16	0.312
172	A	13	7	1.00	16	0.438
173	A	13	7	1.00	16	0.438
174	A	3	3	1.00	16	0.188
175	A	13	7	1.00	14	0.500
176	C	13	7	2.02	12	0.583
177	A	7	7	1.00	16	0.438
178	A	14	8	1.00	16	0.500
179	A	14	8	1.00	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	8	7	1.00	16	0.438
181	A	16	10	1.00	16	0.625
182	A	13	7	1.00	10	0.700
183	A	3	3	1.00	14	0.214
184	A	13	7	1.00	14	0.500
185	A	7	7	1.00	20	0.350
186	A	6	6	1.00	20	0.300
187	A	6	6	1.00	20	0.300
188	A	5	5	1.00	20	0.250
189	A	4	4	1.00	20	0.200
190	A	7	6	1.00	20	0.300
191	A	7	6	1.00	20	0.300
192	A	4	4	1.00	20	0.200
193	A	5	5	1.00	20	0.250
194	A	6	6	1.00	20	0.300
195	A	7	7	1.00	20	0.350
196	A	2	2	1.00	20	0.100
197	A	2	2	1.00	18	0.111
198	A	2	2	1.00	16	0.125
199	A	2	2	1.00	20	0.100
200	A	2	2	1.00	20	0.100
201	A	8	7	1.00	20	0.350
202	A	7	6	1.00	20	0.300
203	A	7	6	1.00	20	0.300
204	A	6	5	1.00	20	0.250
205	A	5	4	1.00	20	0.200
206	A	8	7	1.00	20	0.350
207	A	8	7	1.00	20	0.350
208	A	8	7	1.00	20	0.350
209	A	8	7	1.00	20	0.350
210	A	5	4	1.00	20	0.200
211	A	6	5	1.00	20	0.250
212	A	7	6	1.00	20	0.300
213	A	8	7	1.00	20	0.350
214	A	2	2	1.00	20	0.100
215	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	2	2	1.00	16	0.125
217	A	2	2	1.00	20	0.100
218	A	2	2	1.00	20	0.100
219	A	6	6	1.00	20	0.300
220	A	5	5	1.00	20	0.250
221	A	5	5	1.00	20	0.250
222	A	4	4	1.00	20	0.200
223	A	3	3	1.00	20	0.150
224	A	3	3	1.00	20	0.150
225	A	4	4	1.00	20	0.200
226	A	5	5	1.00	20	0.250
227	A	6	6	1.00	20	0.300
228	A	7	6	1.00	20	0.300
229	A	2	2	1.00	20	0.100
230	A	2	2	1.00	18	0.111
231	A	2	2	1.00	16	0.125
232	A	2	2	1.00	20	0.100
233	A	2	2	1.00	20	0.100
234	A	6	6	1.00	20	0.300
235	A	5	5	1.00	20	0.250
236	A	5	5	1.00	20	0.250
237	A	2	2	1.00	20	0.100
238	A	2	2	1.00	20	0.100
239	A	5	5	1.00	20	0.250
240	A	5	5	1.00	20	0.250
241	A	6	6	1.00	20	0.300
242	A	7	6	1.00	20	0.300
243	A	2	2	1.00	20	0.100
244	A	2	2	1.00	18	0.111
245	A	2	2	1.00	16	0.125
246	A	2	2	1.00	20	0.100
247	A	2	2	1.00	20	0.100
248	A	2	1	1.00	20	0.050
249	A	2	1	1.00	18	0.056
250	A	3	2	1.00	20	0.100
251	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	2	2	1.00	22	0.091
253	A	2	2	1.00	22	0.091
254	A	2	2	1.00	22	0.091
255	A	2	2	1.00	22	0.091
256	A	2	2	1.00	20	0.100
257	A	4	4	1.00	18	0.222
258	A	3	3	1.00	18	0.167
259	A	2	2	1.00	18	0.111
260	A	2	2	1.00	18	0.111
261	A	2	2	1.00	18	0.111
262	A	2	2	1.00	16	0.125
263	A	2	2	1.00	14	0.143
264	A	3	3	1.00	18	0.167
265	A	2	2	1.00	18	0.111
266	A	2	2	1.00	18	0.111
267	A	3	3	1.00	18	0.167
268	A	2	2	1.00	18	0.111
269	A	2	2	1.00	18	0.111
270	A	3	3	1.00	18	0.167
271	A	2	2	1.00	16	0.125
272	A	5	5	1.00	16	0.312
273	A	4	3	1.00	16	0.188
274	A	4	4	1.00	16	0.250
275	A	2	2	1.00	16	0.125
276	A	4	4	1.00	14	0.286
277	A	4	3	1.00	16	0.188
278	A	5	5	1.00	16	0.312
279	A	4	3	1.00	16	0.188
280	A	6	5	1.00	16	0.312
281	A	12	9	1.00	16	0.562
282	A	11	8	1.00	16	0.500
283	A	11	8	1.00	16	0.500
284	A	11	8	1.00	16	0.500
285	A	11	8	1.00	12	0.667
286	A	12	9	1.00	16	0.562
287	A	12	9	1.00	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
288	A	13	9	1.00	16	0.562
289	A	13	9	1.00	16	0.562
290	A	2	2	1.00	16	0.125
291	A	5	5	1.00	16	0.312
292	A	4	3	1.00	16	0.188
293	A	4	4	1.00	16	0.250
294	A	2	2	1.00	16	0.125
295	A	4	4	1.00	14	0.286
296	A	4	3	1.00	16	0.188
297	A	5	5	1.00	16	0.312
298	A	4	3	1.00	16	0.188
299	A	6	5	1.00	16	0.312
300	A	6	6	1.00	16	0.375
301	A	5	5	1.00	16	0.312
302	A	5	5	1.00	16	0.312
303	A	5	5	1.00	16	0.312
304	A	5	5	1.00	12	0.417
305	A	6	6	1.00	16	0.375
306	A	6	6	1.00	16	0.375
307	A	7	6	1.00	16	0.375
308	A	7	6	1.00	16	0.375
309	A	3	2	1.00	18	0.111
310	A	6	6	1.00	18	0.333
311	A	5	4	1.00	18	0.222
312	A	5	5	1.00	18	0.278
313	A	4	3	1.00	18	0.167
314	A	3	3	1.00	18	0.167
315	A	4	3	1.00	16	0.188
316	A	7	7	1.00	18	0.389
317	A	5	4	1.00	18	0.222
318	A	8	7	1.00	18	0.389
319	A	8	5	1.00	18	0.278
320	A	8	5	1.00	18	0.278
321	A	7	4	1.00	18	0.222
322	A	7	4	1.00	18	0.222
323	A	7	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	7	4	1.00	14	0.286
325	A	8	5	1.00	18	0.278
326	A	8	5	1.00	18	0.278
327	A	3	2	1.00	14	0.143
328	A	6	6	1.00	14	0.429
329	A	7	5	1.00	14	0.357
330	A	5	5	1.00	14	0.357
331	A	10	7	1.00	14	0.500
332	A	3	3	1.00	14	0.214
333	A	10	6	1.00	12	0.500
334	A	7	7	1.00	14	0.500
335	A	7	5	1.00	14	0.357
336	A	8	7	1.00	14	0.500
337	A	13	10	1.00	14	0.714
338	A	20	7	1.00	14	0.500
339	A	9	6	1.00	14	0.429
340	A	19	7	1.00	14	0.500
341	A	19	6	1.00	14	0.429
342	A	9	6	1.00	10	0.600
343	A	20	8	1.00	14	0.571
344	A	20	7	1.00	14	0.500
345	A	12	9	1.00	14	0.643
346	A	22	10	1.00	14	0.714
347	A	3	2	1.00	16	0.125
348	A	6	6	1.00	16	0.375
349	A	5	4	1.00	16	0.250
350	A	5	5	1.00	16	0.312
351	A	10	7	1.00	16	0.438
352	A	3	3	1.00	16	0.188
353	A	10	6	1.00	14	0.429
354	A	7	7	1.00	16	0.438
355	A	5	4	1.00	16	0.250
356	A	8	7	1.00	16	0.438
357	A	13	10	1.00	16	0.625
358	A	20	7	1.00	16	0.438
359	A	19	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	19	7	1.00	16	0.438
361	A	19	6	1.00	16	0.375
362	A	19	6	1.00	12	0.500
363	A	22	8	1.00	16	0.500
364	A	20	7	1.00	16	0.438
365	A	22	9	1.00	16	0.562
366	A	22	10	1.00	16	0.625
367	A	3	2	1.00	16	0.125
368	A	5	4	1.00	16	0.250
369	A	5	4	1.00	16	0.250
370	A	4	3	1.00	16	0.188
371	A	4	3	1.00	16	0.188
372	A	3	3	1.00	16	0.188
373	A	4	3	1.00	14	0.214
374	A	6	5	1.00	16	0.312
375	A	5	4	1.00	16	0.250
376	A	7	5	1.00	16	0.312
377	A	6	5	1.00	16	0.312
378	A	20	8	0.96	16	0.500
379	A	19	7	1.00	16	0.438
380	A	19	7	1.00	16	0.438
381	A	19	7	1.01	16	0.438
382	A	19	7	1.00	12	0.583
383	A	20	8	1.00	16	0.500
384	A	20	8	1.00	16	0.500
385	A	3	2	1.00	16	0.125
386	A	5	4	1.00	16	0.250
387	A	5	4	1.00	16	0.250
388	A	4	3	1.00	16	0.188
389	A	4	3	1.00	16	0.188
390	A	3	3	1.00	16	0.188
391	A	4	3	1.00	14	0.214
392	A	6	5	1.00	16	0.312
393	A	5	4	1.00	16	0.250
394	A	7	5	1.00	16	0.312
395	A	6	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	8	5	1.00	16	0.312
397	A	7	4	1.00	16	0.250
398	A	7	4	1.00	16	0.250
399	A	7	4	1.14	16	0.250
400	A	7	4	1.00	12	0.333
401	A	8	5	1.00	16	0.312
402	A	8	5	1.00	16	0.312
403	A	9	6	1.00	16	0.375
404	A	9	6	1.00	16	0.375
405	A	4	3	1.00	16	0.188
406	A	5	4	1.00	16	0.250
407	A	5	5	1.00	14	0.357
408	A	3	3	1.00	14	0.214
409	A	7	7	1.00	14	0.500
410	A	8	7	1.00	14	0.500
411	A	8	8	1.00	10	0.800
412	A	7	6	1.00	18	0.333
413	A	7	6	1.00	18	0.333
414	A	7	6	1.00	16	0.375
415	A	6	6	1.00	14	0.429
416	A	5	5	1.00	18	0.278
417	A	3	3	1.00	18	0.167
418	A	7	7	1.00	18	0.389
419	A	8	7	1.00	18	0.389
420	A	8	7	1.00	18	0.389
421	A	8	7	1.00	18	0.389
422	A	8	7	1.00	16	0.438
423	A	8	7	1.00	14	0.500
424	A	7	7	1.00	18	0.389
425	A	4	4	1.00	18	0.222
426	A	4	4	1.00	18	0.222
427	A	4	4	1.00	18	0.222
428	A	8	7	1.00	18	0.389
429	A	8	7	1.00	18	0.389
430	A	8	7	1.00	18	0.389
431	A	9	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	8	8	1.00	18	0.444
433	A	5	4	1.00	18	0.222
434	A	5	5	1.00	18	0.278
435	A	5	5	1.00	18	0.278
436	A	5	5	1.00	18	0.278
437	A	5	4	0.98	18	0.222
438	A	9	8	1.00	18	0.444
439	A	9	8	1.00	18	0.444
440	A	6	4	1.00	18	0.222
441	A	6	4	1.00	16	0.250
442	A	5	4	1.00	14	0.286
443	A	4	3	1.00	18	0.167
444	A	4	3	1.00	18	0.167
445	A	6	5	1.00	18	0.278
446	A	4	3	1.00	18	0.167
447	A	4	3	1.00	18	0.167
448	A	4	3	1.00	18	0.167
449	A	9	7	1.00	16	0.438
450	A	8	7	1.00	16	0.438
451	A	7	6	1.00	16	0.375
452	A	4	4	1.00	16	0.250
453	A	5	5	1.00	16	0.312
454	A	6	6	1.00	16	0.375
455	A	4	3	1.00	22	0.136
456	A	5	4	1.00	14	0.286
457	A	15	9	1.00	14	0.643
458	A	9	6	1.00	14	0.429
459	A	7	6	1.00	20	0.300
460	A	4	3	1.00	23	0.130
461	A	4	4	1.00	22	0.182
462	A	4	3	1.00	26	0.115
463	A	4	3	1.00	26	0.115
464	A	3	2	1.00	26	0.077
465	A	4	3	1.00	26	0.115
466	A	4	3	1.00	26	0.115
467	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
468	A	4	3	1.00	26	0.115
469	A	4	3	1.00	26	0.115
470	A	4	3	1.00	26	0.115
471	A	4	3	1.00	26	0.115
472	A	4	4	1.00	30	0.133
473	A	4	3	1.00	28	0.107
474	A	4	3	1.00	26	0.115
475	A	4	3	1.00	24	0.125
476	A	3	3	1.00	28	0.107
477	A	3	3	1.00	28	0.107
478	C	7	3	1.11	77	0.039
479	A	4	3	1.00	26	0.115
480	A	4	3	1.00	26	0.115
481	A	5	4	1.00	24	0.167
482	A	5	4	1.00	26	0.154
483	A	5	4	1.00	26	0.154
484	A	5	4	1.00	26	0.154
485	A	4	3	1.00	26	0.115
486	A	5	4	1.00	26	0.154
487	A	5	4	1.00	26	0.154
488	A	5	4	1.00	26	0.154
489	A	5	4	1.00	26	0.154
490	A	5	4	1.00	26	0.154
491	A	4	3	1.00	26	0.115
492	A	5	4	1.00	26	0.154
493	A	4	3	1.00	23	0.130
494	A	4	3	1.00	23	0.130
495	A	2	2	1.00	23	0.087
496	A	5	5	1.00	21	0.238
497	A	4	3	1.00	23	0.130
498	A	4	3	1.00	23	0.130
499	A	4	3	1.00	23	0.130
500	A	12	9	1.00	25	0.360
501	A	9	9	1.00	25	0.360
502	A	5	5	1.00	25	0.200
503	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
504	A	11	11	1.00	25	0.440
505	A	14	11	1.00	25	0.440
506	A	1	1	1.00	26	0.038
507	A	1	1	1.00	28	0.036
508	A	4	3	1.00	32	0.094
509	A	4	3	1.00	32	0.094
510	A	3	2	1.00	32	0.062
511	A	4	3	1.00	32	0.094
512	A	2	2	1.00	32	0.062
513	A	4	3	1.00	32	0.094
514	A	4	3	1.00	32	0.094
515	A	5	4	1.00	30	0.133
516	A	3	2	1.00	28	0.071
517	A	3	2	1.00	26	0.077
518	A	2	1	1.00	24	0.042
519	A	3	2	1.00	28	0.071
520	A	3	2	1.00	28	0.071
521	A	3	2	1.00	28	0.071
522	A	9	4	1.00	30	0.133
523	A	3	2	1.00	28	0.071
524	A	3	2	1.00	26	0.077
525	A	3	2	1.00	24	0.083
526	A	4	3	1.00	28	0.107
527	A	3	2	1.00	28	0.071
528	A	3	2	1.00	28	0.071
529	A	2	2	1.00	30	0.067
530	A	2	2	1.00	28	0.071
531	A	2	2	1.00	26	0.077
532	A	2	2	1.00	24	0.083
533	A	5	5	1.00	28	0.179
534	A	2	2	1.00	28	0.071
535	A	2	2	1.00	28	0.071
536	A	2	2	1.00	30	0.067
537	A	2	2	1.00	28	0.071
538	A	2	2	1.00	26	0.077
539	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
540	A	4	3	1.00	28	0.107
541	A	2	2	1.00	28	0.071
542	A	2	2	1.00	28	0.071
543	A	2	2	1.00	36	0.056
544	A	2	2	1.00	31	0.065
545	A	3	3	1.00	34	0.088
546	A	3	3	1.00	33	0.091
547	A	3	3	1.06	35	0.086
548	A	4	3	1.00	30	0.100
549	A	7	6	1.00	24	0.250
550	A	6	6	1.00	24	0.250
551	A	5	5	1.00	24	0.208
552	A	3	3	1.00	22	0.136
553	A	8	7	1.00	24	0.292
554	A	8	7	1.00	24	0.292
555	A	8	7	1.00	24	0.292
556	A	8	5	1.00	26	0.192
557	A	14	8	1.00	26	0.308
558	A	4	3	1.00	26	0.115
559	A	6	5	1.00	26	0.192
560	A	16	10	1.00	26	0.385
561	A	10	7	1.00	26	0.269
562	A	3	2	1.00	20	0.100
563	A	3	2	1.00	18	0.111
564	A	3	2	1.00	16	0.125
565	A	7	7	1.00	20	0.350
566	A	3	2	1.00	20	0.100
567	A	3	2	1.00	20	0.100
568	A	2	2	1.00	22	0.091
569	A	2	2	1.00	22	0.091
570	A	2	2	1.00	20	0.100
571	A	2	2	1.00	18	0.111
572	A	7	6	1.00	22	0.273
573	A	2	2	1.00	22	0.091
574	A	2	2	1.00	22	0.091
575	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
576	A	2	2	1.00	22	0.091
577	A	2	2	1.00	20	0.100
578	A	2	2	1.00	18	0.111
579	A	8	7	1.00	22	0.318
580	A	2	2	1.00	22	0.091
581	A	2	2	1.00	22	0.091
582	A	2	2	1.00	22	0.091
583	A	2	2	1.00	22	0.091
584	A	2	2	1.00	20	0.100
585	A	2	2	1.00	18	0.111
586	A	3	3	1.00	22	0.136
587	A	2	2	1.00	22	0.091
588	A	2	2	1.00	22	0.091
589	A	2	2	1.00	22	0.091
590	A	2	2	1.00	22	0.091
591	A	2	2	1.00	20	0.100
592	A	2	2	1.00	18	0.111
593	A	5	5	1.00	22	0.227
594	A	2	2	1.00	22	0.091
595	A	2	2	1.00	22	0.091
596	A	14	3	1.00	22	0.136
597	A	10	3	1.00	22	0.136
598	A	6	3	1.00	20	0.150
599	A	3	2	1.00	22	0.091
600	A	5	3	1.00	22	0.136
601	A	6	4	1.00	22	0.182
602	A	2	2	1.00	24	0.083
603	A	2	2	1.00	24	0.083
604	A	2	2	1.00	24	0.083
605	A	2	2	1.00	24	0.083
606	A	2	2	1.00	22	0.091
607	A	3	2	1.00	28	0.071
608	A	4	3	1.00	30	0.100
609	A	4	3	1.00	30	0.100
610	A	3	2	1.00	31	0.065
611	A	4	3	1.00	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
612	A	4	3	1.00	33	0.091
613	A	5	4	1.00	30	0.133
614	A	6	6	1.00	30	0.200
615	A	4	3	1.00	30	0.100
616	A	4	4	1.00	28	0.143
617	A	8	8	1.00	30	0.267
618	A	5	4	1.00	30	0.133
619	A	9	8	1.00	30	0.267
620	A	6	5	1.00	30	0.167
621	A	5	4	1.00	30	0.133
622	A	5	5	1.00	30	0.167
623	A	5	4	1.00	30	0.133
624	A	5	5	0.98	28	0.179
625	A	5	4	1.00	22	0.182
626	A	9	8	1.00	30	0.267
627	A	6	5	1.00	30	0.167
628	A	9	8	1.00	30	0.267
629	A	7	5	1.00	30	0.167
630	A	6	5	1.00	30	0.167
631	A	6	6	1.00	30	0.200
632	A	6	5	1.00	30	0.167
633	A	6	5	0.99	28	0.179
634	A	6	5	1.00	22	0.227
635	A	10	9	1.00	30	0.300
636	A	7	6	1.00	30	0.200
637	A	10	9	1.00	30	0.300
638	A	5	4	1.00	33	0.121
639	A	6	6	1.00	33	0.182
640	A	4	3	1.00	33	0.091
641	A	4	4	1.00	31	0.129
642	A	8	8	1.00	33	0.242
643	A	5	4	1.00	33	0.121
644	A	9	8	1.00	33	0.242
645	A	6	5	1.00	33	0.152
646	A	5	4	1.00	33	0.121
647	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
648	A	5	4	1.00	33	0.121
649	A	5	5	1.00	31	0.161
650	A	9	8	1.00	33	0.242
651	A	6	5	1.00	33	0.152
652	A	9	8	1.00	33	0.242
653	A	7	5	1.00	33	0.152
654	A	6	5	1.00	33	0.152
655	A	6	6	1.00	33	0.182
656	A	6	5	1.00	33	0.152
657	A	6	5	1.00	31	0.161
658	A	10	9	1.00	33	0.273
659	A	7	6	1.00	33	0.182
660	A	10	9	1.00	33	0.273
661	A	7	6	1.00	26	0.231
662	A	10	9	1.00	28	0.321
663	A	3	2	1.00	24	0.083
664	A	4	3	1.00	26	0.115

Chapter 3

Listing of integrals

3.1 $\int \left(ax^3+bx^6\right)^{5/3}\,dx$

Optimal. Leaf size=52

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

```
[Out] -3/88*a*(b*x^6+a*x^3)^(8/3)/b^2/x^8+1/11*(b*x^6+a*x^3)^(8/3)/b/x^5
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5
)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \left(ax^3+bx^6\right)^{5/3} dx = \frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5} - \frac{(3a)\int \frac{\left(ax^3+bx^6\right)^{5/3}}{x^3} dx}{11b}$$
$$= -\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8} + \frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{x\left(a+bx^3\right)^3\left(8bx^3-3a\right)}{88b^2\sqrt[3]{x^3\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(5/3),x]
[Out] (x*(a + b*x^3)^3*(-3*a + 8*b*x^3))/(88*b^2*(x^3*(a + b*x^3))^(1/3))
```

fricas [A] time = 1.16, size = 53, normalized size = 1.02

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^6+ax^3\right)^{\frac{2}{3}}}{88\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3)
/(b^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^6+ax^3\right)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^6 + a*x^3)^(5/3), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.75

$$-\frac{\left(b\,x^3+a\right)\left(-8b\,x^3+3a\right)\left(b\,x^6+a\,x^3\right)^{\frac{5}{3}}}{88b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(5/3),x)
[Out] -1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

maxima [A] time = 0.44, size = 46, normalized size = 0.88

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{88\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^2
```

mupad [B] time = 1.23, size = 40, normalized size = 0.77

$$-\frac{\left(b\,x^3+a\right)^2\,\left(b\,x^6+a\,x^3\right)^{2/3}\,\left(3\,a-8\,b\,x^3\right)}{88\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^6)^(5/3),x)
```

```
[Out] -((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^3+bx^6\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(5/3), x)
```

3.2 $\int \left(ax^3+bx^6\right)^{2/3}\,dx$

Optimal. Leaf size=25

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

```
[Out] 1/5*(b*x^6+a*x^3)^(5/3)/b/x^5
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(x^3\left(a+bx^3\right)\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

fricas [A] time = 1.24, size = 28, normalized size = 1.12

$$\frac{\left(bx^6+ax^3\right)^{\frac{2}{3}}\left(bx^3+a\right)}{5\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^6+ax^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(2/3), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(b\,x^{3}+a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}{5b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(2/3),x)
```

```
[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

maxima [A] time = 0.46, size = 14, normalized size = 0.56

$$\frac{\left(bx^{3}+a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

mupad [B] time = 1.15, size = 29, normalized size = 1.16

$$\frac{\left(\frac{a}{5\,b}+\frac{x^{3}}{5}\right)\,\left(b\,x^{6}+a\,x^{3}\right)^{2/3}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^6)^(2/3),x)
```

```
[Out] ((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^(2/3))/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{3}+bx^{6}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(2/3), x)
```

3.3 $\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

```
[Out] -(b*x^6+a*x^3)^(1/3)/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3\left(a+bx^3\right)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

fricas [A] time = 1.19, size = 21, normalized size = 0.91

$$-\frac{\left(bx^6+ax^3\right)^{\frac{1}{3}}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

giac [A] time = 25.00, size = 14, normalized size = 0.61

$$-\frac{\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
[Out] -(b + a/x^3)^(1/3)/a
```

maple [A] time = 0.00, size = 27, normalized size = 1.17

$$-\frac{\left(b\,x^3+a\right)x}{\left(b\,x^6+a\,x^3\right)^{\frac{2}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(2/3),x)
[Out] -x*(b*x^3+a)/a/(b*x^6+a*x^3)^(2/3)
```

maxima [A] time = 0.45, size = 17, normalized size = 0.74

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

mupad [B] time = 1.15, size = 21, normalized size = 0.91

$$-\frac{\left(b\,x^6+a\,x^3\right)^{1/3}}{a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^6)^(2/3),x)
[Out] -(a*x^3 + b*x^6)^(1/3)/(a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^3+bx^6\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(2/3),x)
[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)
```

3.4 $\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx$

Optimal. Leaf size=77

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

```
[Out] 1/2/a/x^2/(b*x^6+a*x^3)^(2/3)-3/4*(b*x^6+a*x^3)^(1/3)/a^2/x^5+9/4*b*(b*x^6+
a*x^3)^(1/3)/a^3/x^2
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2001, 2016, 2000}

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) +
 (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2001

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1
)/(n - j)], 0] && LtQ[p, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx = \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}} + \frac{3\int \frac{1}{x^3\left(ax^3+bx^6\right)^{2/3}}\,dx}{a}$$
$$= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} - \frac{(9b)\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx}{4a^2}$$
$$= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} + \frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.60

$$\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

fricas [A] time = 0.84, size = 54, normalized size = 0.70

$$\frac{\left(9\,b^2x^6+6\,abx^3-a^2\right)\left(bx^6+ax^3\right)^{\frac{1}{3}}}{4\left(a^3bx^8+a^4x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x^
5)
```

giac [A] time = 23.79, size = 52, normalized size = 0.68

$$\frac{b^2}{2\,a^3\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}} - \frac{a^9\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}} - 8\,a^9\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

```
[Out] 1/2*b^2/(a^3*(b + a/x^3)^(2/3)) - 1/4*(a^9*(b + a/x^3)^(4/3) - 8*a^9*(b + a
/x^3)^(1/3)*b)/a^12
```

maple [A] time = 0.00, size = 46, normalized size = 0.60

$$-\frac{\left(b\,x^3+a\right)\left(-9b^2x^6-6b\,x^3a+a^2\right)x}{4\left(b\,x^6+a\,x^3\right)^{\frac{5}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(5/3),x)
```

```
[Out] -1/4*x*(b*x^3+a)*(-9*b^2*x^6-6*a*b*x^3+a^2)/a^3/(b*x^6+a*x^3)^(5/3)
```

maxima [A] time = 0.47, size = 38, normalized size = 0.49

$$\frac{9 b^{2} x^{6} + 6 a b x^{3} - a^{2}}{4 \left(b x^{3} + a\right)^{\frac{2}{3}} a^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

mupad [B] time = 1.28, size = 51, normalized size = 0.66

$$\frac{\left(b\,x^6+a\,x^3\right)^{1/3}\,\left(-a^2+6\,a\,b\,x^3+9\,b^2\,x^6\right)}{4\,a^3\,x^5\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^6)^(5/3),x)
```

```
[Out] ((a*x^3 + b*x^6)^(1/3)*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^3
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x^{3} + b x^{6}\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-5/3), x)
```

3.5 $\int \frac{1}{-x^3+x^6} \, dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2/x^2+1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1593, 325, 200, 31, 634, 618, 204, 628}

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^3+x^6}\,dx &= \int \frac{1}{x^3\left(-1+x^3\right)}\,dx\\
&= \frac{1}{2x^2}+\int \frac{1}{-1+x^3}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

fricas [A] time = 0.84, size = 46, normalized size = 0.96

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+x^2\log\left(x^2+x+1\right)-2\,x^2\log\left(x-1\right)-3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

giac [A] time = 0.32, size = 38, normalized size = 0.79

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x-1)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3),x)
```

```
[Out] -1/6*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/3*ln(x-1)+1/2/x^
2
```

maxima [A] time = 0.95, size = 37, normalized size = 0.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

mupad [B] time = 0.09, size = 51, normalized size = 1.06

$$\frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3 - x^6),x)
```

```
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

sympy [A] time = 0.15, size = 48, normalized size = 1.00

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

3.6 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

```
[Out] 1/6*a*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b*x^9*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(abx^5+b^2x^8\right)dx}{ab+b^2x^3}\\ &= \frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3ax^6+2bx^9\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

fricas [A] time = 0.85, size = 13, normalized size = 0.16

$$\frac{1}{9}\,bx^9 + \frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/9*b*x^9 + 1/6*a*x^6
```

giac [A] time = 0.40, size = 23, normalized size = 0.29

$$\frac{1}{18}\left(2\,bx^9 + 3\,ax^6\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(2b\,x^3 + 3a\right)\sqrt{\left(b\,x^3 + a\right)^2}\;x^6}{18b\,x^3 + 18a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^3+a)^2)^(1/2),x)
[Out] 1/18*x^6*(2*b*x^3+3*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.49, size = 83, normalized size = 1.05

$$-\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,ax^3}{6\,b} - \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,a^2}{6\,b^2} + \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2
```

mupad [B] time = 1.26, size = 59, normalized size = 0.75

$$\frac{\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}\;\left(8\,b^2\,\left(a^2 + b^2\,x^6\right) - 12\,a^2\,b^2 + 4\,a\,b^3\,x^3\right)}{72\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((a + b*x^3)^2)^(1/2),x)
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4*
a*b^3*x^3))/(72*b^4)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**9/9
```

3.7 $\int x^4 \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] 1/5*a*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b*x^8*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(abx^4+b^2x^7\right)dx}{ab+b^2x^3}\\
&= \frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

fricas [A] time = 0.50, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^{8} + \frac{1}{5}\,ax^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

giac [A] time = 0.37, size = 29, normalized size = 0.37

$$\frac{1}{8}\,bx^{8}\mathrm{sgn}\left(bx^{3} + a\right) + \frac{1}{5}\,ax^{5}\mathrm{sgn}\left(bx^{3} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(5b\,x^{3}+8a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}\;x^{5}}{40b\,x^{3}+40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^3+a)^2)^(1/2),x)
[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.47, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^{8} + \frac{1}{5}\,ax^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\,\sqrt{\left(b\,x^{3}+a\right)^{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a + b*x^3)^2)^(1/2),x)
[Out] int(x^4*((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^{5}}{5} + \frac{bx^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**3+a)**2)**(1/2),x)
[Out] a*x**5/5 + b*x**8/8
```

3.8 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] 1/4*a*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b*x^7*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^3+b^2x^6\right)dx}{ab+b^2x^3}\\ &= \frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(7ax^4+4bx^7\right)}{28\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

fricas [A] time = 0.77, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

giac [A] time = 0.39, size = 29, normalized size = 0.37

$$\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{4}\,ax^4\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(4b\,x^3 + 7a\right)\sqrt{\left(b\,x^3 + a\right)^2}\;x^4}{28b\,x^3 + 28a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^3+a)^2)^(1/2),x)
[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.46, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{\left(b\,x^3 + a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a + b*x^3)^2)^(1/2),x)
[Out] int(x^3*((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**3+a)**2)**(1/2),x)
[Out] a*x**4/4 + b*x**7/7
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

```
[Out] 1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^3\right)\\ &= \frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^3)^2}\,(2ax^3+bx^6)}{6(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))
```

fricas [A] time = 0.57, size = 13, normalized size = 0.36

$$\frac{1}{6}bx^6+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

giac [A] time = 0.36, size = 22, normalized size = 0.61

$$\frac{1}{6}\left(bx^6 + 2\,ax^3\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{\left(b\,x^3 + 2a\right)\sqrt{\left(b\,x^3 + a\right)^2}\;x^3}{6b\,x^3 + 6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [B] time = 0.46, size = 52, normalized size = 1.44

$$\frac{1}{6}\,\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,x^3 + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,a}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^
2)*a/b
```

mupad [B] time = 1.23, size = 33, normalized size = 0.92

$$\left(\frac{a}{6\,b} + \frac{x^3}{6}\right)\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] (a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.33

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**6/6
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] 1/2*a*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b*x^5*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 14}

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(abx+b^2x^4\right)dx}{ab+b^2x^3}\\ &= \frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(5ax^2+2bx^5\right)}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

fricas [A] time = 0.83, size = 13, normalized size = 0.16

$$\frac{1}{5}bx^5 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

giac [A] time = 0.33, size = 29, normalized size = 0.37

$$\frac{1}{5}bx^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{2}ax^2\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$\frac{\left(2b\,x^3 + 5a\right)\sqrt{\left(b\,x^3 + a\right)^2}\;x^2}{10b\,x^3 + 10a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.45, size = 13, normalized size = 0.16

$$\frac{1}{5}bx^5 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\left(b\,x^3 + a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^2}{2} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**5/5
```

3.11 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] a*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1343}

$$\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(2ab+2b^2x^3\right)dx}{2ab+2b^2x^3}\\ &= \frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(4ax+bx^4\right)}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

fricas [A] time = 0.79, size = 10, normalized size = 0.14

$$\frac{1}{4}bx^4+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + a*x
```

giac [A] time = 0.38, size = 20, normalized size = 0.27

$$\frac{1}{4}\left(bx^4+4\,ax\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$\frac{\left(b\,x^3+4a\right)\sqrt{\left(b\,x^3+a\right)^2}\;x}{4b\,x^3+4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.44, size = 10, normalized size = 0.14

$$\frac{1}{4}\,bx^4+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + a*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{\left(b\,x^3+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.11

$$ax+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**4/4
```

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx$

Optimal. Leaf size=75

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] 1/3*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+
a)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{ab+b^2x^3}{x} \, dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{ab}{x}+b^2x^2\right) \, dx}{ab+b^2x^3}\\ &= \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3a\log(x)+bx^3\right)}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3 + 3*a*Log[x]))/(3*(a + b*x^3))
```

fricas [A] time = 0.84, size = 11, normalized size = 0.15

$$\frac{1}{3}\,bx^3 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/3*b*x^3 + a*log(x)
```

giac [A] time = 0.29, size = 28, normalized size = 0.37

$$\frac{1}{3}\,bx^3\mathrm{sgn}\left(bx^3+a\right) + a\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(b\,x^3+3a\ln(x)\right)}{3b\,x^3+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x,x)
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(b*x^3+3*a*ln(x))/(b*x^3+a)
```

maxima [A] time = 0.46, size = 96, normalized size = 1.28

$$\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{1}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)
```

mupad [B] time = 1.38, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3} - \frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)\sqrt{a^2}}{3} + \frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^3+a\right)^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x,x)
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a +
b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))
```

sympy [A] time = 0.13, size = 10, normalized size = 0.13

$$a\log(x) + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**3/3
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx$

Optimal. Leaf size=77

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -a*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^2}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^2}+b^2x\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.49

$$\frac{\left(bx^3-2a\right)\sqrt{\left(a+bx^3\right)^2}}{2x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

fricas [A] time = 0.77, size = 14, normalized size = 0.18

$$\frac{bx^3 - 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/2*(b*x^3 - 2*a)/x
```

giac [A] time = 0.32, size = 29, normalized size = 0.38

$$\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{a\mathrm{sgn}\left(bx^3 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
[Out] 1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

maple [A] time = 0.00, size = 36, normalized size = 0.47

$$-\frac{\left(-b\,x^3 + 2a\right)\sqrt{\left(b\,x^3 + a\right)^2}}{2\left(b\,x^3 + a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^2,x)
[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

maxima [A] time = 0.46, size = 14, normalized size = 0.18

$$\frac{bx^3 - 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] 1/2*(b*x^3 - 2*a)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\left(b\,x^3 + a\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^2,x)
[Out] int(((a + b*x^3)^2)^(1/2)/x^2, x)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.10

$$-\frac{a}{x} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x**2/2
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -1/2*a*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^3}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2+\frac{ab}{x^3}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.50

$$-\frac{\left(a-2bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
[Out] -1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))
```

fricas [A] time = 0.86, size = 15, normalized size = 0.20

$$\frac{2\,bx^3 - a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

giac [A] time = 0.40, size = 26, normalized size = 0.35

$$bx\mathrm{sgn}\left(bx^3 + a\right) - \frac{a\mathrm{sgn}\left(bx^3 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
[Out] b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

maple [A] time = 0.00, size = 34, normalized size = 0.46

$$-\frac{\left(-2b\,x^3 + a\right)\sqrt{\left(b\,x^3 + a\right)^2}}{2\left(b\,x^3 + a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^3,x)
[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

maxima [A] time = 0.46, size = 15, normalized size = 0.20

$$\frac{2\,bx^3 - a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\left(b\,x^3 + a\right)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^3,x)
[Out] int(((a + b*x^3)^2)^(1/2)/x^3, x)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.11

$$-\frac{a}{2x^2} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
[Out] -a/(2*x**2) + b*x
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/3*a*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^4}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^4}+\frac{b^2}{x}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a-3bx^3\log(x)\right)}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
[Out] -1/3*(Sqrt[(a + b*x^3)^2]*(a - 3*b*x^3*Log[x]))/(x^3*(a + b*x^3))
```

fricas [A] time = 0.64, size = 17, normalized size = 0.23

$$\frac{3\,bx^3\log(x) - a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

giac [A] time = 0.35, size = 43, normalized size = 0.57

$$b\log\left(|x|\right)\operatorname{sgn}\left(bx^3 + a\right) - \frac{bx^3\operatorname{sgn}\left(bx^3 + a\right) + a\operatorname{sgn}\left(bx^3 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
[Out] b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a)
)/x^3
```

maple [A] time = 0.01, size = 38, normalized size = 0.51

$$\frac{\sqrt{\left(b\,x^3 + a\right)^2}\,\left(3b\,x^3\ln(x) - a\right)}{3\left(b\,x^3 + a\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^4,x)
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(3*b*ln(x)*x^3-a)/(b*x^3+a)/x^3
```

maxima [A] time = 0.46, size = 99, normalized size = 1.32

$$\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) - \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) - 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)/x^3
```

mupad [B] time = 1.38, size = 112, normalized size = 1.49

$$\frac{\ln\left(a\,b + \sqrt{\left(b\,x^3 + a\right)^2}\,\sqrt{b^2} + b^2\,x^3\right)\sqrt{b^2}}{3} - \frac{\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{3\,x^3} - \frac{a\,b\,\ln\left(a\,b + \frac{a^2}{x^3} + \frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^4,x)
```

```
[Out] (log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3)*(b^2)^(1/2))/3 - (a
^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*x^3) - (a*b*log(a*b + a^2/x^3 + ((a^2)^(
1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3))/(3*(a^2)^(1/2))
```

sympy [A] time = 0.16, size = 10, normalized size = 0.13

$$-\frac{a}{3x^3} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

```
[Out] -a/(3*x**3) + b*log(x)
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5} \, dx$

Optimal. Leaf size=77

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -1/4*a*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{ab+b^2x^3}{x^5} \, dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{ab}{x^5}+\frac{b^2}{x^2}\right) \, dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
[Out] -1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))
```

fricas [A] time = 0.78, size = 13, normalized size = 0.17

$$-\frac{4\,bx^3 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

giac [A] time = 0.35, size = 30, normalized size = 0.39

$$-\frac{4\,bx^3\mathrm{sgn}\left(bx^3 + a\right) + a\mathrm{sgn}\left(bx^3 + a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

maple [A] time = 0.00, size = 34, normalized size = 0.44

$$-\frac{\left(4b\,x^3 + a\right)\sqrt{\left(b\,x^3 + a\right)^2}}{4\left(b\,x^3 + a\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^5,x)
[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

maxima [A] time = 0.45, size = 13, normalized size = 0.17

$$-\frac{4\,bx^3 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

mupad [B] time = 1.21, size = 33, normalized size = 0.43

$$-\frac{\left(4\,b\,x^3 + a\right)\,\sqrt{\left(b\,x^3 + a\right)^2}}{4\,x^4\,\left(b\,x^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^5,x)
[Out] -((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.18

$$\frac{-a - 4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] (-a - 4*b*x**3)/(4*x**4)
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -1/5*a*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b*((b*x^3+a)^2)^(1/2)/x^2/(b*x
^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^6}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^3}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
[Out] -1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))
```

fricas [A] time = 0.87, size = 15, normalized size = 0.19

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

giac [A] time = 0.35, size = 31, normalized size = 0.39

$$-\frac{5\,bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
[Out] -1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(5b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^6,x)
[Out] -1/10*(5*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

maxima [A] time = 0.46, size = 15, normalized size = 0.19

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

mupad [B] time = 1.18, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^3+2\,a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\,x^5\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^6,x)
[Out] -((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))
```

sympy [A] time = 0.18, size = 15, normalized size = 0.19

$$\frac{-2a-5bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

```
[Out] (-2*a - 5*b*x**3)/(10*x**5)
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/6*a*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-1/3*b*((b*x^3+a)^2)^(1/2)/x^3/(b*x
^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^7}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^7}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.47

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+2bx^3\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))
```

fricas [A] time = 0.87, size = 13, normalized size = 0.16

$$-\frac{2\,bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
[Out] -1/6*(2*b*x^3 + a)/x^6
```

giac [A] time = 0.34, size = 30, normalized size = 0.38

$$-\frac{2\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
[Out] -1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

maple [A] time = 0.00, size = 34, normalized size = 0.43

$$-\frac{\left(2b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{6\left(b\,x^3+a\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^7,x)
[Out] -1/6*(2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)
```

maxima [A] time = 0.49, size = 86, normalized size = 1.09

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2}{6\,a^2}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b}{6\,ax^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/a^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
+ a^2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)
```

mupad [B] time = 1.18, size = 33, normalized size = 0.42

$$-\frac{\left(2\,b\,x^3+a\right)\,\sqrt{\left(b\,x^3+a\right)^2}}{6\,x^6\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^7,x)
[Out] -((a + 2*b*x^3)*((a + b*x^3)^2)^(1/2))/(6*x^6*(a + b*x^3))
```

sympy [**A**] time = 0.19, size = 14, normalized size = 0.18

$$\frac{-a-2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
```

```
[Out] (-a - 2*b*x**3)/(6*x**6)
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -1/7*a*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b*((b*x^3+a)^2)^(1/2)/x^4/(b*x
^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^8}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^5}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a+7bx^3\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))
```

fricas [A] time = 0.93, size = 15, normalized size = 0.19

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

giac [A] time = 0.34, size = 31, normalized size = 0.39

$$-\frac{7\,bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
[Out] -1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(7b\,x^3+4a\right)\sqrt{\left(b\,x^3+a\right)^2}}{28\left(b\,x^3+a\right)x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^8,x)
[Out] -1/28*(7*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

maxima [A] time = 0.46, size = 15, normalized size = 0.19

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

mupad [B] time = 1.27, size = 35, normalized size = 0.44

$$-\frac{\left(7\,b\,x^3+4\,a\right)\sqrt{\left(b\,x^3+a\right)^2}}{28\,x^7\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^8,x)
[Out] -((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))
```

sympy [A] time = 0.19, size = 15, normalized size = 0.19

$$\frac{-4a-7bx^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

```
[Out] (-4*a - 7*b*x**3)/(28*x**7)
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9} dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -1/8*a*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b*((b*x^3+a)^2)^(1/2)/x^5/(b*x
^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^9}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^9}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(5a+8bx^3\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(x^8*(a + b*x^3))
```

fricas [A] time = 0.87, size = 15, normalized size = 0.19

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

giac [A] time = 0.38, size = 31, normalized size = 0.39

$$-\frac{8\,bx^3\mathrm{sgn}\left(bx^3+a\right)+5\,a\mathrm{sgn}\left(bx^3+a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")
[Out] -1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(8b\,x^3+5a\right)\sqrt{\left(b\,x^3+a\right)^2}}{40\left(b\,x^3+a\right)x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^9,x)
[Out] -1/40*(8*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

maxima [A] time = 0.45, size = 15, normalized size = 0.19

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

mupad [B] time = 1.16, size = 35, normalized size = 0.44

$$-\frac{\left(8\,b\,x^3+5\,a\right)\,\sqrt{\left(b\,x^3+a\right)^2}}{40\,x^8\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^9,x)
[Out] -((5*a + 8*b*x^3)*((a + b*x^3)^2)^(1/2))/(40*x^8*(a + b*x^3))
```

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-5a-8bx^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

```
[Out] (-5*a - 8*b*x**3)/(40*x**8)
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

```
[Out] -1/9*a*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/6*b*((b*x^3+a)^2)^(1/2)/x^6/(b*x
^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{10}}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^7}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+3bx^3\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))
```

fricas [A] time = 0.87, size = 15, normalized size = 0.19

$$-\frac{3\,bx^3+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
[Out] -1/18*(3*b*x^3 + 2*a)/x^9
```

giac [A] time = 0.36, size = 31, normalized size = 0.39

$$-\frac{3\,bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
[Out] -1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(3b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{18\left(b\,x^3+a\right)x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^10,x)
[Out] -1/18*(3*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)
```

maxima [B] time = 0.47, size = 117, normalized size = 1.48

$$-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^3}{6\,a^3}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2}{6\,a^2x^3}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b}{6\,a^3x^6}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{9\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a^3 - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b^2/(a^2*x^3) + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^6) -
 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^9)
```

mupad [B] time = 1.15, size = 35, normalized size = 0.44

$$-\frac{\left(3\,b\,x^3+2\,a\right)\sqrt{\left(b\,x^3+a\right)^2}}{18\,x^9\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^10,x)
[Out] -((2*a + 3*b*x^3)*((a + b*x^3)^2)^(1/2))/(18*x^9*(a + b*x^3))
```

sympy [**A**] time = 0.21, size = 15, normalized size = 0.19

$$\frac{-2a - 3bx^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

```
[Out] (-2*a - 3*b*x**3)/(18*x**9)
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -1/10*a*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b*((b*x^3+a)^2)^(1/2)/x^7/(b
*x^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{11}}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{11}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a+10bx^3\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(x^10*(a + b*x^3))
```

fricas [A] time = 0.79, size = 15, normalized size = 0.19

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

giac [A] time = 0.31, size = 31, normalized size = 0.39

$$-\frac{10\,bx^3\mathrm{sgn}\left(bx^3+a\right)+7\,a\mathrm{sgn}\left(bx^3+a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")
[Out] -1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(10b\,x^3+7a\right)\sqrt{\left(b\,x^3+a\right)^2}}{70\left(b\,x^3+a\right)x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^11,x)
[Out] -1/70*(10*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)
```

maxima [A] time = 0.45, size = 15, normalized size = 0.19

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

mupad [B] time = 1.16, size = 35, normalized size = 0.44

$$-\frac{\left(10\,b\,x^3+7\,a\right)\,\sqrt{\left(b\,x^3+a\right)^2}}{70\,x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^11,x)
[Out] -((7*a + 10*b*x^3)*((a + b*x^3)^2)^(1/2))/(70*x^10*(a + b*x^3))
```

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-7a-10bx^3}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

```
[Out] (-7*a - 10*b*x**3)/(70*x**10)
```

3.23 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+}}{10\left(a+bx^3\right)}$$

```
[Out] 1/10*a^3*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/13*a^2*b*x^13*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+3/16*a*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/19*b^3*x^19*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+}}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(19*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^3dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^9+3a^2b^4x^{12}+3ab^5x^{15}+b^6x^{18}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+}}{16\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(1976a^3+4560a^2bx^3+3705ab^2x^6+1040b^3x^9\right)}{19760\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 104
0*b^3*x^9))/(19760*(a + b*x^3))
```

fricas [A] time = 0.75, size = 35, normalized size = 0.21

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

giac [A] time = 0.39, size = 67, normalized size = 0.40

$$\frac{1}{19}\,b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{16}\,ab^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}\,a^2bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}\,a^3x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/19*b^3*x^19*sgn(b*x^3 + a) + 3/16*a*b^2*x^16*sgn(b*x^3 + a) + 3/13*a^2*b*
x^13*sgn(b*x^3 + a) + 1/10*a^3*x^10*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(1040b^3x^9+3705a\,b^2x^6+4560a^2b\,x^3+1976a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^{10}}{19760\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3+
a)^2)^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.47, size = 35, normalized size = 0.21

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

3.24 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^4}{15b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}{12b^3}$$

```
[Out] 1/12*a^2*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^3-2/15*a*(b*x^3+a)^4*((b*x^3+a)^
2)^(1/2)/b^3+1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 167, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18\left(a+bx^3\right)}+\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*b*x^12*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*x^15*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(18*(a + b*x^3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8 \left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \text{ Subst}\left(\int x^2 \left(ab+b^2x\right)^3 dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \text{ Subst}\left(\int \left(a^3b^3x^2+3a^2b^4x^3+3ab^5x^4+b^6x^5\right) dx, x, x^3\right.}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)} + \frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{ab^2x^{15}\sqrt{a^2+2ab}}{5\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.51

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9
))/(180*(a + b*x^3))
```

fricas [A] time = 0.93, size = 35, normalized size = 0.29

$$\frac{1}{18}b^3x^{18} + \frac{1}{5}ab^2x^{15} + \frac{1}{4}a^2bx^{12} + \frac{1}{9}a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

giac [A] time = 0.30, size = 67, normalized size = 0.56

$$\frac{1}{18}b^3x^{18}\text{sgn}\left(bx^3+a\right) + \frac{1}{5}ab^2x^{15}\text{sgn}\left(bx^3+a\right) + \frac{1}{4}a^2bx^{12}\text{sgn}\left(bx^3+a\right) + \frac{1}{9}a^3x^9\text{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x^
12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.49

$$\frac{\left(10b^3x^9+36a\,b^2x^6+45a^2b\,x^3+20a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^9}{180\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/180*x^9*(10*b^3*x^9+36*a*b^2*x^6+45*a^2*b*x^3+20*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

maxima [A] time = 0.46, size = 114, normalized size = 0.96

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2x^3}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^3}{12\,b^3}-\frac{7\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{90\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3 -
 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

3.25 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b}}{8\left(a+bx^3\right)}$$

```
[Out] 1/8*a^3*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/11*a^2*b*x^11*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+3/14*a*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/17*b^3*x^17*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(17*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^7+3a^2b^4x^{10}+3ab^5x^{13}+b^6x^{16}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2}}{14\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(1309a^3+2856a^2bx^3+2244ab^2x^6+616b^3x^9\right)}{10472\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616*
b^3*x^9))/(10472*(a + b*x^3))
```

fricas [A] time = 0.83, size = 35, normalized size = 0.21

$$\frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

giac [A] time = 0.29, size = 67, normalized size = 0.40

$$\frac{1}{17}\,b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{14}\,ab^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}\,a^2bx^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/17*b^3*x^17*sgn(b*x^3 + a) + 3/14*a*b^2*x^14*sgn(b*x^3 + a) + 3/11*a^2*b*
x^11*sgn(b*x^3 + a) + 1/8*a^3*x^8*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(616b^3x^9+2244a\,b^2x^6+2856a^2b\,x^3+1309a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^8}{10472\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a)
^2)^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.45, size = 35, normalized size = 0.21

$$\frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

3.26 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[Out] 1/7*a^3*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/10*a^2*b*x^10*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+3/13*a*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/16*b^3*x^16*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(16*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(ab+b^2x^3\right)^3dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^6+3a^2b^4x^9+3ab^5x^{12}+b^6x^{15}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2ab}}{13\left(a+bx^3\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(1040a^3+2184a^2bx^3+1680ab^2x^6+455b^3x^9\right)}{7280\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455*
b^3*x^9))/(7280*(a + b*x^3))
```

fricas [A] time = 0.64, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

giac [A] time = 0.36, size = 67, normalized size = 0.40

$$\frac{1}{16}\,b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}\,ab^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}\,a^3x^7\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^3 + a) + 3/13*a*b^2*x^13*sgn(b*x^3 + a) + 3/10*a^2*b*
x^10*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(455b^3x^9+1680a\,b^2x^6+2184a^2b\,x^3+1040a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^7}{7280\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)^
2)^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.45, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

3.27 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

```
[Out] -1/12*a*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^2+1/15*(b*x^3+a)^4*((b*x^3+a)^2)^
(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a + b*x^3)^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^5 \left(ab+b^2x^3\right)^3 dx}{b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \,\text{Subst}\left(\int x \left(ab+b^2x\right)^3 dx, x, x^3\right)}{3b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \,\text{Subst}\left(\int \left(-\frac{a(ab+b^2x)^3}{b}+\frac{(ab+b^2x)^4}{b^2}\right) dx, x, x^3\right)}{3b^2 \left(ab+b^2x^3\right)}$$
$$= -\frac{a \left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2} + \frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.78

$$\frac{x^6 \sqrt{\left(a+bx^3\right)^2} \left(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9\right)}{60 \left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)
)/(60*(a + b*x^3))
```

fricas [A] time = 0.62, size = 35, normalized size = 0.45

$$\frac{1}{15}\,b^3x^{15} + \frac{1}{4}\,ab^2x^{12} + \frac{1}{3}\,a^2bx^9 + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

giac [A] time = 0.44, size = 45, normalized size = 0.58

$$\frac{1}{60}\left(4\,b^3x^{15} + 15\,ab^2x^{12} + 20\,a^2bx^9 + 10\,a^3x^6\right)\text{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 + a
)
```

maple [A] time = 0.01, size = 58, normalized size = 0.74

$$\frac{\left(4b^3x^9+15a\,b^2x^6+20a^2b\,x^3+10a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}} x^6}{60\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/60*x^6*(4*b^3*x^9+15*a*b^2*x^6+20*a^2*b*x^3+10*a^3)*((b*x^3+a)^2)^(3/2)/(
b*x^3+a)^3
```

maxima [A] time = 0.49, size = 83, normalized size = 1.06

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ax^3}{12\,b}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2
```

mupad [B] time = 1.25, size = 46, normalized size = 0.59

$$\frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,\left(-a^2+3\,a\,b\,x^3+4\,b^2\,x^6\right)}{60\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

3.28 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] 1/5*a^3*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/8*a^2*b*x^8*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/11*a*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^3*x^14*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(14*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4 \left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3x^4+3a^2b^4x^7+3ab^5x^{10}+b^6x^{13}\right) dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx}}{11\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9\right)}{3080\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b^
3*x^9))/(3080*(a + b*x^3))
```

fricas [A] time = 0.78, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{8}\,a^2bx^8 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

giac [A] time = 0.37, size = 67, normalized size = 0.40

$$\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{11}\,ab^2x^{11}\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{8}\,a^2bx^8\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{5}\,a^3x^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*x
^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(220b^3x^9 + 840a\,b^2x^6 + 1155a^2b\,x^3 + 616a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}} x^5}{3080\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.47, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{8}\,a^2bx^8 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

3.29 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2}}{4\left(a+bx^3\right)}$$

```
[Out] 1/4*a^3*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/7*a^2*b*x^7*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/10*a*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13*b^3*x^13*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(13*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^3+3a^2b^4x^6+3ab^5x^9+b^6x^{12}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2a}}{10\left(a+b\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9\right)}{1820\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^3
*x^9))/(1820*(a + b*x^3))
```

fricas [A] time = 0.80, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

giac [A] time = 0.45, size = 67, normalized size = 0.40

$$\frac{1}{13}\,b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}\,ab^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}\,a^2bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^3x^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*x
^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(140b^3x^9+546a\,b^2x^6+780a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^4}{1820\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.48, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\left(a + bx^3\right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

3.30 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

```
[Out] 1/12*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.67

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9\right)}{12\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12
*(a + b*x^3))
```

fricas [A] time = 0.87, size = 35, normalized size = 0.97

$$\frac{1}{12}b^3x^{12}+\frac{1}{3}ab^2x^9+\frac{1}{2}a^2bx^6+\frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

giac [A] time = 0.41, size = 44, normalized size = 1.22

$$\frac{1}{12}\left(2\left(bx^6+2\,ax^3\right)a^2+\left(bx^6+2\,ax^3\right)^2b\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 57, normalized size = 1.58

$$\frac{\left(b^3x^9+4a\,b^2x^6+6a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^3}{12\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/12*x^3*(b^3*x^9+4*a*b^2*x^6+6*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3
+a)^3
```

maxima [A] time = 0.45, size = 52, normalized size = 1.44

$$\frac{1}{12}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}x^3+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(3/2)*a/b
```

mupad [B] time = 1.22, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^3+a\,b\right)\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

3.31 $\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] 1/2*a^3*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/5*a^2*b*x^5*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/8*a*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^3*x^11*((b*x^
3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x+3a^2b^4x^4+3ab^5x^7+b^6x^{10}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3}}{8\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3*
x^9))/(440*(a + b*x^3))
```

fricas [A] time = 0.81, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^{3}x^{11}+\frac{3}{8}\,ab^{2}x^{8}+\frac{3}{5}\,a^{2}bx^{5}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

giac [A] time = 0.37, size = 67, normalized size = 0.40

$$\frac{1}{11}\,b^{3}x^{11}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{3}{8}\,ab^{2}x^{8}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{3}{5}\,a^{2}bx^{5}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{1}{2}\,a^{3}x^{2}\mathrm{sgn}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^5
*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 58, normalized size = 0.35

$$\frac{\left(40b^{3}x^{9}+165a\,b^{2}x^{6}+264a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}x^{2}}{440\left(b\,x^{3}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/(b*x^3+a)^3
```

maxima [A] time = 0.44, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^{3}x^{11}+\frac{3}{8}\,ab^{2}x^{8}+\frac{3}{5}\,a^{2}bx^{5}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)
```

3.32 $\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=162

$$\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3\right.}{\left(a+bx\right.}$$

```
[Out] a^3*x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/4*a^2*b*x^4*(b^2*x^6+2*a*
b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/7*a*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b
*x^3+a)^3+1/10*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3
```

Rubi [A] time = 0.03, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 194}

$$\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3\right.}{\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*
b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^
2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2} \int \left(2ab+2b^2x^3\right)^3 \, dx}{\left(2ab+2b^2x^3\right)^3}\\ &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2} \int \left(8a^3b^3+24a^2b^4x^3+24ab^5x^6+8b^6x^9\right) \, dx}{\left(2ab+2b^2x^3\right)^3}\\ &= \frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2a\right.}{7\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.36

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\left(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9\right)}{140\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^9
))/(140*(a + b*x^3))
```

fricas [A] time = 0.72, size = 32, normalized size = 0.20

$$\frac{1}{10} b^3 x^{10} + \frac{3}{7} a b^2 x^7 + \frac{3}{4} a^2 b x^4 + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

giac [A] time = 0.36, size = 64, normalized size = 0.40

$$\frac{1}{10} b^3 x^{10} \operatorname{sgn}\left(b x^3 + a\right) + \frac{3}{7} a b^2 x^7 \operatorname{sgn}\left(b x^3 + a\right) + \frac{3}{4} a^2 b x^4 \operatorname{sgn}\left(b x^3 + a\right) + a^3 x \operatorname{sgn}\left(b x^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^4
*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 56, normalized size = 0.35

$$\frac{\left(14 b^3 x^9 + 60 a\, b^2 x^6 + 105 a^2 b\, x^3 + 140 a^3\right) \left(\left(b\, x^3 + a\right)^2\right)^{\frac{3}{2}} x}{140 \left(b\, x^3 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

maxima [A] time = 0.71, size = 32, normalized size = 0.20

$$\frac{1}{10} b^3 x^{10} + \frac{3}{7} a b^2 x^7 + \frac{3}{4} a^2 b x^4 + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + 2\, a\, b\, x^3 + b^2\, x^6\right)^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

3.33 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx$

Optimal. Leaf size=160

$$\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] a^2*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^2*x^6*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+1/9*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^3*ln(x)*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.05, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/
(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^9\sqrt{a^2+2abx^3+}}{9\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(18a^3\log(x)+bx^3\left(18a^2+9abx^3+2b^2x^6\right)\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[x
]))/(18*(a + b*x^3))
```

fricas [A] time = 0.73, size = 32, normalized size = 0.20

$$\frac{1}{9}\,b^3x^9+\frac{1}{2}\,ab^2x^6+a^2bx^3+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

giac [A] time = 0.43, size = 65, normalized size = 0.41

$$\frac{1}{9}\,b^3x^9\text{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^2x^6\text{sgn}\left(bx^3+a\right)+a^2bx^3\text{sgn}\left(bx^3+a\right)+a^3\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(b
*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 57, normalized size = 0.36

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^9+9a\,b^2x^6+18a^2b\,x^3+18a^3\ln(x)\right)}{18\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+9*a*b^2*x^6+18*a^2*b*x^3+18*a^3*ln(x))/
(b*x^3+a)^3
```

maxima [A] time = 0.52, size = 152, normalized size = 0.95

$$\frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}\,abx^3+\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{1}{2}\,\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*
a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x/a
bs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 + 1/9
*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

3.34 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=165

$$\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -a^3*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+3/2*a^2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+3/5*a*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^3*x^8*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(8*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{\left(ab+b^2x^3\right)^3}{x^2} \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{a^3b^3}{x^2}+3a^2b^4x+3ab^5x^4+b^6x^7\right) \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3}}{5\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9\right)}{40x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(
40*x*(a + b*x^3))
```

fricas [A] time = 0.83, size = 37, normalized size = 0.22

$$\frac{5\,b^3x^9 + 24\,ab^2x^6 + 60\,a^2bx^3 - 40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

giac [A] time = 0.34, size = 67, normalized size = 0.41

$$\frac{1}{8}\,b^3x^8\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{a^3\mathrm{sgn}\left(bx^3 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*s
gn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(-5b^3x^9 - 24a\,b^2x^6 - 60a^2b\,x^3 + 40a^3\right)\left(\left(b\,x^3 + a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^3 + a\right)^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/(
b*x^3+a)^3
```

maxima [A] time = 0.50, size = 37, normalized size = 0.22

$$\frac{5\,b^3x^9 + 24\,ab^2x^6 + 60\,a^2bx^3 - 40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

3.35 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=163

$$\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -1/2*a^3*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+3*a^2*b*x*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+3/4*a*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^3*x^7*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a^2*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(7*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3a^2b^4+\frac{a^3b^3}{x^3}+3ab^5x^3+b^6x^6\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9\right)}{28x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(
28*x^2*(a + b*x^3))
```

fricas [A] time = 0.88, size = 37, normalized size = 0.23

$$\frac{4\,b^3x^9 + 21\,ab^2x^6 + 84\,a^2bx^3 - 14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

giac [A] time = 0.38, size = 65, normalized size = 0.40

$$\frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^3 + a\right) + 3\,a^2bx\mathrm{sgn}\left(bx^3 + a\right) - \frac{a^3\mathrm{sgn}\left(bx^3 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(b
*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-4b^3x^9 - 21a\,b^2x^6 - 84a^2b\,x^3 + 14a^3\right)\left(\left(b\,x^3 + a\right)^2\right)^{\frac{3}{2}}}{28\left(b\,x^3 + a\right)^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x)
```

```
[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^2
/(b*x^3+a)^3
```

maxima [A] time = 0.50, size = 37, normalized size = 0.23

$$\frac{4\,b^3x^9 + 21\,ab^2x^6 + 84\,a^2bx^3 - 14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

3.36 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/3*a^3*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+a*b^2*x^3*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+1/6*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a^2*b*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b^2*x^3*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.39

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-2a^3+18a^2bx^3\log(x)+6ab^2x^6+b^3x^9\right)}{6x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x])
)/(6*x^3*(a + b*x^3))
```

fricas [A] time = 0.88, size = 38, normalized size = 0.24

$$\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log(x)-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

giac [A] time = 0.37, size = 85, normalized size = 0.53

$$\frac{1}{6}\,b^3x^6\text{sgn}\left(bx^3+a\right)+ab^2x^3\text{sgn}\left(bx^3+a\right)+3\,a^2b\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)-\frac{3\,a^2bx^3\text{sgn}\left(bx^3+a\right)+a^3\text{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^3
```

maple [A] time = 0.01, size = 59, normalized size = 0.37

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}\left(b^3x^9+6a\,b^2x^6+18a^2b\,x^3\ln(x)-2a^3\right)}{6\left(b\,x^3+a\right)^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*a*b^2*x^6+18*a^2*b*ln(x)*x^3-2*a^3)/(b*x
^3+a)^3/x^3
```

maxima [A] time = 0.53, size = 156, normalized size = 0.97

$$\frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2x^3+(-1)^{2\,b^2x^3+2\,ab}\,a^2b\log\left(2\,b^2x^3+2\,ab\right)-(-1)^{2\,abx^3+2\,a^2}\,a^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{3}{2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2*x^3 + (-1)^(2*b^2*x^3 + 2*a*b)*a^2*
b*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a^2*b*log(2*a*b*x/abs(x
) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b - 1/3*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

3.37 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5} dx$

Optimal. Leaf size=165

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[Out] -1/4*a^3*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-3*a^2*b*((b*x^3+a)^2)^(1/2)/x/(b
*x^3+a)+3/2*a*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b^3*x^5*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(5*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^5} dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^5}+\frac{3a^2b^4}{x^2}+3ab^5x+b^6x^4\right) dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9\right)}{20x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(2
0*x^4*(a + b*x^3))
```

fricas [A] time = 0.88, size = 37, normalized size = 0.22

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

giac [A] time = 0.36, size = 69, normalized size = 0.42

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{12\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*x
^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(-4b^3x^9-30a\,b^2x^6+60a^2b\,x^3+5a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{20\left(b\,x^3+a\right)^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x)
```

```
[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^4/
(b*x^3+a)^3
```

maxima [A] time = 0.53, size = 37, normalized size = 0.22

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

3.38 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -1/5*a^3*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-3/2*a^2*b*((b*x^3+a)^2)^(1/2)/x^
2/(b*x^3+a)+3*a*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^3*x^4*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(4*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3ab^5+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^3}+b^6x^3\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9\right)}{20x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(2
0*x^5*(a + b*x^3))
```

fricas [A] time = 0.78, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

giac [A] time = 0.29, size = 68, normalized size = 0.42

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^3+a\right)+3\,ab^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{15\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^3\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3*
sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-5b^3x^9-60a\,b^2x^6+30a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{20\left(b\,x^3+a\right)^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^5/
(b*x^3+a)^3
```

maxima [A] time = 0.50, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

3.39 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=162

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

```
[Out] -1/6*a^3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-a^2*b*((b*x^3+a)^2)^(1/2)/x^3/(b
*x^3+a)+1/3*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a*b^2*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^3}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)} + \frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^3+6a^2bx^3-18ab^2x^6\log(x)-2b^3x^9\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x^6*Log
[x]))/(x^6*(a + b*x^3))
```

fricas [A] time = 0.86, size = 39, normalized size = 0.24

$$\frac{2\,b^3x^9+18\,ab^2x^6\log(x)-6\,a^2bx^3-a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

giac [A] time = 0.34, size = 86, normalized size = 0.53

$$\frac{1}{3}\,b^3x^3\text{sgn}\left(bx^3+a\right)+3\,ab^2\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)-\frac{9\,ab^2x^6\text{sgn}\left(bx^3+a\right)+6\,a^2bx^3\text{sgn}\left(bx^3+a\right)+a^3\text{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a*
b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^6
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^9+18a\,b^2x^6\ln(x)-6a^2b\,x^3-a^3\right)}{6\left(b\,x^3+a\right)^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+18*a*b^2*ln(x)*x^6-6*a^2*b*x^3-a^3)/(b*x
^3+a)^3/x^6
```

maxima [A] time = 0.70, size = 220, normalized size = 1.36

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^3x^3}{2\,a}+(-1)^{2\,b^2x^3+2\,ab}\,ab^2\log\left(2\,b^2x^3+2\,ab\right)-(-1)^{2\,abx^3+2\,a^2}\,ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{3}{2}\,\sqrt{b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3*x^3/a + (-1)^(2*b^2*x^3 + 2*a*b)*a*
b^2*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a*b^2*log(2*a*b*x/abs
(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2 + 1/6*(
b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^
(3/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

3.40 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=165

$$-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -1/7*a^3*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-3/4*a^2*b*((b*x^3+a)^2)^(1/2)/x^
4/(b*x^3+a)-3*a*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b^3*x^2*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^5}+\frac{3ab^5}{x^2}+b^6x\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x^
9))/(x^7*(a + b*x^3))
```

fricas [A] time = 0.68, size = 37, normalized size = 0.22

$$\frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

giac [A] time = 0.47, size = 70, normalized size = 0.42

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^3+a\right) - \frac{84\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + 21\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + 4\,a^3\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(-14b^3x^9 + 84a\,b^2x^6 + 21a^2b\,x^3 + 4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{28\left(b\,x^3+a\right)^3 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^7
/(b*x^3+a)^3
```

maxima [A] time = 0.74, size = 37, normalized size = 0.22

$$\frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

3.41 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=162

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}$$

```
[Out] -1/8*a^3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-3/5*a^2*b*((b*x^3+a)^2)^(1/2)/x^
5/(b*x^3+a)-3/2*a*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b^3*x*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(a + b*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^9}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^6+\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^3}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(5a^3+24a^2bx^3+60ab^2x^6-40b^3x^9\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x^
9))/(x^8*(a + b*x^3))
```

fricas [A] time = 0.71, size = 37, normalized size = 0.23

$$\frac{40\,b^3x^9 - 60\,ab^2x^6 - 24\,a^2bx^3 - 5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

giac [A] time = 0.39, size = 67, normalized size = 0.41

$$b^3x\mathrm{sgn}\left(bx^3+a\right) - \frac{60\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + 24\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + 5\,a^3\mathrm{sgn}\left(bx^3+a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sgn
(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-40b^3x^9 + 60a\,b^2x^6 + 24a^2b\,x^3 + 5a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^3+a\right)^3 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^8
/(b*x^3+a)^3
```

maxima [A] time = 0.68, size = 37, normalized size = 0.23

$$\frac{40\,b^3x^9 - 60\,ab^2x^6 - 24\,a^2bx^3 - 5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

3.42 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=161

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

```
[Out] -1/9*a^3*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/2*a^2*b*((b*x^3+a)^2)^(1/2)/x^
6/(b*x^3+a)-a*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b^3*ln(x)*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a\left(2a^2+9abx^3+18b^2x^6\right)-18b^3x^9\log(x)\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a*(2*a^2 + 9*a*b*x^3 + 18*b^2*x^6) - 18*b^3*x^9
*Log[x]))/(x^9*(a + b*x^3))
```

fricas [A] time = 0.85, size = 39, normalized size = 0.24

$$\frac{18\,b^3x^9\log(x)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

giac [A] time = 0.38, size = 85, normalized size = 0.53

$$b^3\log(|x|)\,\text{sgn}\left(bx^3+a\right)-\frac{11\,b^3x^9\text{sgn}\left(bx^3+a\right)+18\,ab^2x^6\text{sgn}\left(bx^3+a\right)+9\,a^2bx^3\text{sgn}\left(bx^3+a\right)+2\,a^3\text{sgn}\left(bx^3+\right.}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^2
*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^
9
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}\left(18b^3x^9\ln(x)-18a\,b^2x^6-9a^2b\,x^3-2a^3\right)}{18\left(b\,x^3+a\right)^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(18*b^3*ln(x)*x^9-18*a*b^2*x^6-9*a^2*b*x^3-2*a^3)/
(b*x^3+a)^3/x^9
```

maxima [B] time = 0.72, size = 253, normalized size = 1.57

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^4x^3}{6\,a^2}+\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^3\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a
 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a^3 - 1/6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*b^2/(a^2*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^
3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

3.43 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=165

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}$$

```
[Out] -1/10*a^3*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-3/7*a^2*b*((b*x^3+a)^2)^(1/2)/
x^7/(b*x^3+a)-3/4*a*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b^3*((b*x^3+a)^2)
^(1/2)/x/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{11}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{11}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^5}+\frac{b^6}{x^2}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(14a^3+60a^2bx^3+105ab^2x^6+140b^3x^9\right)}{140x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -1/140*(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b^
3*x^9))/(x^10*(a + b*x^3))
```

fricas [A] time = 0.94, size = 37, normalized size = 0.22

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

giac [A] time = 0.36, size = 69, normalized size = 0.42

$$-\frac{140\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+105\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+60\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+14\,a^3\mathrm{sgn}\left(bx^3+a\right)}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2*
b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(140b^3x^9+105a\,b^2x^6+60a^2b\,x^3+14a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{140\left(b\,x^3+a\right)^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/140*(140*b^3*x^9+105*a*b^2*x^6+60*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/
x^10/(b*x^3+a)^3
```

maxima [A] time = 0.66, size = 37, normalized size = 0.22

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

mupad [B] time = 1.21, size = 151, normalized size = 0.92

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2}}{7\,x^7\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(7*x^7*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

3.44 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}$$

```
[Out] -1/11*a^3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-3/8*a^2*b*((b*x^3+a)^2)^(1/2)/
x^8/(b*x^3+a)-3/5*a*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b^3*((b*x^3+a
)^2)^(1/2)/x^2/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*x^2*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{12}}\,dx}{b^2(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^9}+\frac{3ab^5}{x^6}+\frac{b^6}{x^3}\right)dx}{b^2(ab+b^2x^3)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2}}{5x^5(a+bx^3)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(40a^3+165a^2bx^3+264ab^2x^6+220b^3x^9)}{440x^{11}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -1/440*(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*b
^3*x^9))/(x^11*(a + b*x^3))
```

fricas [A] time = 0.86, size = 37, normalized size = 0.22

$$-\frac{220\,b^3x^9 + 264\,ab^2x^6 + 165\,a^2bx^3 + 40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

giac [A] time = 0.42, size = 69, normalized size = 0.41

$$-\frac{220\,b^3x^9\mathrm{sgn}\left(bx^3 + a\right) + 264\,ab^2x^6\mathrm{sgn}\left(bx^3 + a\right) + 165\,a^2bx^3\mathrm{sgn}\left(bx^3 + a\right) + 40\,a^3\mathrm{sgn}\left(bx^3 + a\right)}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^2
*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(220b^3x^9 + 264a\,b^2x^6 + 165a^2b\,x^3 + 40a^3\right)\left(\left(b\,x^3 + a\right)^2\right)^{\frac{3}{2}}}{440\left(b\,x^3 + a\right)^3 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/440*(220*b^3*x^9+264*a*b^2*x^6+165*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)
/x^11/(b*x^3+a)^3
```

maxima [A] time = 0.72, size = 37, normalized size = 0.22

$$-\frac{220\,b^3x^9 + 264\,ab^2x^6 + 165\,a^2bx^3 + 40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

mupad [B] time = 1.22, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^12,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(8*x^8*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

3.45 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

```
[Out] -1/12*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^12
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1355, 264\}$

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{13}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9\right)}{12x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/(
x^12*(a + b*x^3))
```

fricas [A] time = 0.82, size = 35, normalized size = 0.85

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

giac [B] time = 0.36, size = 68, normalized size = 1.66

$$-\frac{4\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+6\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+4\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3*
sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

maple [A] time = 0.01, size = 56, normalized size = 1.37

$$-\frac{\left(4b^3x^9+6a\,b^2x^6+4a^2b\,x^3+a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{12\left(b\,x^3+a\right)^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)*((b*x^3+a)^2)^(3/2)/x^12/(b*x
^3+a)^3
```

maxima [B] time = 0.55, size = 148, normalized size = 3.61

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^4}{12\,a^4}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^3}{12\,a^3x^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^2}{12\,a^4x^6}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b}{12\,a^3x^9}-\frac{\left(b^2x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^
4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^12)
```

mupad [B] time = 1.21, size = 151, normalized size = 3.68

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\left(b\,x^3+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^6\left(b\,x^3+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
```

```
+ 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(3*x^9*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

3.46 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}} dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}$$

```
[Out] -1/13*a^3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-3/10*a^2*b*((b*x^3+a)^2)^(1/2)
/x^10/(b*x^3+a)-3/7*a*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b^3*((b*x^3
+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(4*x^4*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{14}} dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{11}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^5}\right) dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2}}{7x^7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(140a^3+546a^2bx^3+780ab^2x^6+455b^3x^9\right)}{1820x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -1/1820*(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 455
*b^3*x^9))/(x^13*(a + b*x^3))
```

fricas [A] time = 0.82, size = 37, normalized size = 0.22

$$-\frac{455\,b^3x^9 + 780\,ab^2x^6 + 546\,a^2bx^3 + 140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

giac [A] time = 0.36, size = 69, normalized size = 0.41

$$-\frac{455\,b^3x^9\mathrm{sgn}\left(bx^3+a\right) + 780\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + 546\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + 140\,a^3\mathrm{sgn}\left(bx^3+a\right)}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a^
2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(455b^3x^9 + 780a\,b^2x^6 + 546a^2b\,x^3 + 140a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{1820\left(b\,x^3+a\right)^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/1820*(455*b^3*x^9+780*a*b^2*x^6+546*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/
2)/x^13/(b*x^3+a)^3
```

maxima [A] time = 0.69, size = 37, normalized size = 0.22

$$-\frac{455\,b^3x^9 + 780\,ab^2x^6 + 546\,a^2bx^3 + 140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

mupad [B] time = 1.19, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^14,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(10*x^10*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

3.47 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}$$

```
[Out] -1/14*a^3*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-3/11*a^2*b*((b*x^3+a)^2)^(1/2)
/x^11/(b*x^3+a)-3/8*a*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b^3*((b*x^3
+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(5*x^5*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{15}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{15}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^9}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(220a^3+840a^2bx^3+1155ab^2x^6+616b^3x^9\right)}{3080x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -1/3080*(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 61
6*b^3*x^9))/(x^14*(a + b*x^3))
```

fricas [A] time = 0.84, size = 37, normalized size = 0.22

$$-\frac{616\,b^3x^9 + 1155\,ab^2x^6 + 840\,a^2bx^3 + 220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

giac [A] time = 0.36, size = 69, normalized size = 0.41

$$-\frac{616\,b^3x^9\mathrm{sgn}\left(bx^3+a\right) + 1155\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + 840\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + 220\,a^3\mathrm{sgn}\left(bx^3+a\right)}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*a
^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(616b^3x^9 + 1155a\,b^2x^6 + 840a^2b\,x^3 + 220a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{3080\left(b\,x^3+a\right)^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/3080*(616*b^3*x^9+1155*a*b^2*x^6+840*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/x^14/(b*x^3+a)^3
```

maxima [A] time = 0.69, size = 37, normalized size = 0.22

$$-\frac{616\,b^3x^9 + 1155\,ab^2x^6 + 840\,a^2bx^3 + 220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

mupad [B] time = 1.21, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^15,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(11*x^11*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

```
[Out] -1/15*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^15+1/60*b*(b*x^3+a)^3*((b*x^3+a)^
2)^(1/2)/a^2/x^12
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b*(a + b*x^
3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^6}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx, x,\right.}{15ab\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.73

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9\right)}{60x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x^
9))/(x^15*(a + b*x^3))
```

fricas [A] time = 0.97, size = 37, normalized size = 0.44

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

giac [A] time = 0.41, size = 69, normalized size = 0.82

$$-\frac{10\,b^3x^9\text{sgn}\left(bx^3+a\right)+20\,ab^2x^6\text{sgn}\left(bx^3+a\right)+15\,a^2bx^3\text{sgn}\left(bx^3+a\right)+4\,a^3\text{sgn}\left(bx^3+a\right)}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

maple [A] time = 0.01, size = 58, normalized size = 0.69

$$-\frac{\left(10b^3x^9+20a\,b^2x^6+15a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{60\left(b\,x^3+a\right)^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/60*(10*b^3*x^9+20*a*b^2*x^6+15*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^15
/(b*x^3+a)^3
```

maxima [B] time = 0.52, size = 179, normalized size = 2.13

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^5}{12\,a^5}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^4}{12\,a^4x^3}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^3}{12\,a^5x^6}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^2}{12\,a^4x^9}+\frac{\left(b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5/a^5 - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^4/(a^4*x^3) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a
^5*x^6) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^9) + 1/12*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(5/2)/(a^2*x^15)
```

mupad [B] time = 1.21, size = 151, normalized size = 1.80

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{15\,x^{15}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,\left(b\,x^3+a\right)}-\frac{a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{12}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(15*x^15*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(4*x^12*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

3.49 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}$$

```
[Out] -1/16*a^3*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-3/13*a^2*b*((b*x^3+a)^2)^(1/2)
/x^13/(b*x^3+a)-3/10*a*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b^3*((b*x
^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1355, 270\}$

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{17}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{17}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{11}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(455a^3+1680a^2bx^3+2184ab^2x^6+1040b^3x^9\right)}{7280x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -1/7280*(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 + 1
040*b^3*x^9))/(x^16*(a + b*x^3))
```

fricas [A] time = 0.88, size = 37, normalized size = 0.22

$$-\frac{1040\,b^3x^9 + 2184\,ab^2x^6 + 1680\,a^2bx^3 + 455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

giac [A] time = 0.30, size = 69, normalized size = 0.41

$$-\frac{1040\,b^3x^9\mathrm{sgn}\left(bx^3+a\right) + 2184\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + 1680\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + 455\,a^3\mathrm{sgn}\left(bx^3+a\right)}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 1680
*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(1040b^3x^9 + 2184a\,b^2x^6 + 1680a^2b\,x^3 + 455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{7280\left(b\,x^3+a\right)^3 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/7280*(1040*b^3*x^9+2184*a*b^2*x^6+1680*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/x^16/(b*x^3+a)^3
```

maxima [A] time = 0.77, size = 37, normalized size = 0.22

$$-\frac{1040\,b^3x^9 + 2184\,ab^2x^6 + 1680\,a^2bx^3 + 455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

mupad [B] time = 1.22, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^17,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(13*x^13*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

3.50 $\int x^{13} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}$$

```
[Out] 1/14*a^5*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a^4*b*x^17*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+1/2*a^3*b^2*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/23*a^2*b^3
*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/26*a*b^4*x^26*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/29*b^5*x^29*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*
(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{13}\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{13}+5a^4b^6x^{16}+10a^3b^7x^{19}+10a^2b^8x^{22}+5ab\right.}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+}}{2\left(a+\right.}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^3\right)^2}\left(147407a^5+606970a^4bx^3+1031849a^3b^2x^6+897260a^2b^3x^9+396865ab^4x^{12}+71162b^5x^{15}\right)}{2063698\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2*
x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*(a
 + b*x^3))
```

fricas [A] time = 0.77, size = 57, normalized size = 0.22

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

giac [A] time = 0.33, size = 105, normalized size = 0.41

$$\frac{1}{29}\,b^5x^{29}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{26}\,ab^4x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{23}\,a^2b^3x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3b^2x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,a^4bx^{17}\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/29*b^5*x^29*sgn(b*x^3 + a) + 5/26*a*b^4*x^26*sgn(b*x^3 + a) + 10/23*a^2*b
^3*x^23*sgn(b*x^3 + a) + 1/2*a^3*b^2*x^20*sgn(b*x^3 + a) + 5/17*a^4*b*x^17*
sgn(b*x^3 + a) + 1/14*a^5*x^14*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(71162b^5x^{15}+396865a\,b^4x^{12}+897260a^2b^3x^9+1031849a^3b^2x^6+606970a^4b\,x^3+147407a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{14}}{2063698\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x^9+1031849
*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.83, size = 57, normalized size = 0.22

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13} \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13} \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**13*((a + b*x**3)**2)**(5/2), x)
```

3.51 $\int x^{12} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x}}{13\left(a+bx^3\right)}$$

```
[Out] 1/13*a^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a^4*b*x^16*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/19*a^3*b^2*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^2*b^
3*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*a*b^4*x^25*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/28*b^5*x^28*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+}}{19\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*
(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{12}\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{12}+5a^4b^6x^{15}+10a^3b^7x^{18}+10a^2b^8x^{21}+5ab^9x\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+}}{19\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{\left(a+bx^3\right)^2}\left(117040a^5+475475a^4bx^3+800800a^3b^2x^6+691600a^2b^3x^9+304304ab^4x^{12}+54340b^5x^{15}\right)}{1521520\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*x
^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(a
+ b*x^3))
```

fricas [A] time = 0.89, size = 57, normalized size = 0.22

$$\frac{1}{28}b^5x^{28}+\frac{1}{5}ab^4x^{25}+\frac{5}{11}a^2b^3x^{22}+\frac{10}{19}a^3b^2x^{19}+\frac{5}{16}a^4bx^{16}+\frac{1}{13}a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

giac [A] time = 0.32, size = 105, normalized size = 0.41

$$\frac{1}{28}b^5x^{28}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}ab^4x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}a^2b^3x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}a^3b^2x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}a^4bx^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/28*b^5*x^28*sgn(b*x^3 + a) + 1/5*a*b^4*x^25*sgn(b*x^3 + a) + 5/11*a^2*b^3
*x^22*sgn(b*x^3 + a) + 10/19*a^3*b^2*x^19*sgn(b*x^3 + a) + 5/16*a^4*b*x^16*
sgn(b*x^3 + a) + 1/13*a^5*x^13*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(54340b^5x^{15}+304304a\,b^4x^{12}+691600a^2b^3x^9+800800a^3b^2x^6+475475a^4b\,x^3+117040a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{13}}{1521520\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x^9+800800*
a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.60, size = 57, normalized size = 0.22

$$\frac{1}{28}b^5x^{28}+\frac{1}{5}ab^4x^{25}+\frac{5}{11}a^2b^3x^{22}+\frac{10}{19}a^3b^2x^{19}+\frac{5}{16}a^4bx^{16}+\frac{1}{13}a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12}\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

3.52 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2}}{}$$

```
[Out] -1/18*a^3*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^4+1/7*a^2*(b*x^3+a)^6*((b*x^3+a
)^2)^(1/2)/b^4-1/8*a*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^4+1/27*(b*x^3+a)^8*(
(b*x^3+a)^2)^(1/2)/b^4
```

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (a^2*(a + b
*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(27*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^{11} \left(ab+b^2x^3\right)^5 dx}{b^4 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \text{ Subst}\left(\int x^3 \left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \text{ Subst}\left(\int \left(-\frac{a^3(ab+b^2x)^5}{b^3} + \frac{3a^2(ab+b^2x)^6}{b^4} - \frac{3a(ab+b^2x)^7}{b^5} + \frac{(ab+b^2}{b^6}\right.\right.}{3b^4 \left(ab+b^2x^3\right)}$$
$$= -\frac{a^3 \left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^4} + \frac{a^2 \left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{7b^4} - \frac{a}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.52

$$\frac{x^{12}\sqrt{\left(a+bx^3\right)^2} \left(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15}\right)}{1512 \left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*
a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))
```

fricas [A] time = 0.86, size = 57, normalized size = 0.36

$$\frac{1}{27}\,b^5x^{27} + \frac{5}{24}\,ab^4x^{24} + \frac{10}{21}\,a^2b^3x^{21} + \frac{5}{9}\,a^3b^2x^{18} + \frac{1}{3}\,a^4bx^{15} + \frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

giac [A] time = 0.35, size = 105, normalized size = 0.66

$$\frac{1}{27}\,b^5x^{27}\text{sgn}\left(bx^3+a\right)+\frac{5}{24}\,ab^4x^{24}\text{sgn}\left(bx^3+a\right)+\frac{10}{21}\,a^2b^3x^{21}\text{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^3b^2x^{18}\text{sgn}\left(bx^3+a\right)+\frac{1}{3}\,a^4bx^{15}\text{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*b
^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15*s
gn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.50

$$\frac{\left(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}} x^{12}}{1512\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1512*x^12*(56*b^5*x^15+315*a*b^4*x^12+720*a^2*b^3*x^9+840*a^3*b^2*x^6+504
*a^4*b*x^3+126*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.72, size = 145, normalized size = 0.91

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^6}{27\,b^2}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3x^3}{18\,b^3}-\frac{11\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}ax^3}{216\,b^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^4}{18\,b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/27*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^6/b^2 - 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*a^3*x^3/b^3 - 11/216*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a*x^3/b
^3 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^4/b^4 + 83/1512*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*a^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{11}\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

3.53 $\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x}}{11\left(a+bx^3\right)}$$

```
[Out] 1/11*a^5*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a^4*b*x^14*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/17*a^3*b^2*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^2*b^3
*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/23*a*b^4*x^23*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/26*b^5*x^26*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+}}{17\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*
(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{10}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{10}+5a^4b^6x^{13}+10a^3b^7x^{16}+10a^2b^8x^{19}+5ab^9x^{}\right.}{b^4\left(ab+b^2x^3\right)}\\&=\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+}}{17\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^3\right)^2}\left(71162a^5+279565a^4bx^3+460460a^3b^2x^6+391391a^2b^3x^9+170170ab^4x^{12}+30107b^5x^{15}\right)}{782782\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x^
6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a +
b*x^3))
```

fricas [A] time = 0.81, size = 57, normalized size = 0.22

$$\frac{1}{26}b^5x^{26}+\frac{5}{23}ab^4x^{23}+\frac{1}{2}a^2b^3x^{20}+\frac{10}{17}a^3b^2x^{17}+\frac{5}{14}a^4bx^{14}+\frac{1}{11}a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

giac [A] time = 0.44, size = 105, normalized size = 0.41

$$\frac{1}{26}b^5x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{23}ab^4x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}a^2b^3x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}a^3b^2x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}a^4bx^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/26*b^5*x^26*sgn(b*x^3 + a) + 5/23*a*b^4*x^23*sgn(b*x^3 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^3 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^3 + a) + 5/14*a^4*b*x^14*
sgn(b*x^3 + a) + 1/11*a^5*x^11*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(30107b^5x^{15}+170170a\,b^4x^{12}+391391a^2b^3x^9+460460a^3b^2x^6+279565a^4b\,x^3+71162a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{11}}{782782\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^9+460460*a
^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.81, size = 57, normalized size = 0.22

$$\frac{1}{26}b^5x^{26}+\frac{5}{23}ab^4x^{23}+\frac{1}{2}a^2b^3x^{20}+\frac{10}{17}a^3b^2x^{17}+\frac{5}{14}a^4bx^{14}+\frac{1}{11}a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10} \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10} \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

3.54 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+\cdots}}{10\left(a+bx^3\right)}$$

```
[Out] 1/10*a^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/13*a^4*b*x^13*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+5/8*a^3*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/19*a^2*b^3
*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/22*a*b^4*x^22*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/25*b^5*x^25*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2ab\cdots}}{8\left(a+bx^3\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2
5*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^9+5a^4b^6x^{12}+10a^3b^7x^{15}+10a^2b^8x^{18}+5ab^9\cdots\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+\cdots}}{8\left(a+\cdots\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(54340a^5+209000a^4bx^3+339625a^3b^2x^6+286000a^2b^3x^9+123500ab^4x^{12}+21736b^5x^{15}\right)}{543400\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x^
6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a +
b*x^3))
```

fricas [A] time = 0.89, size = 57, normalized size = 0.22

$$\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

giac [A] time = 0.40, size = 105, normalized size = 0.41

$$\frac{1}{25}\,b^5x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^13*
sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(21736b^5x^{15}+123500a\,b^4x^{12}+286000a^2b^3x^9+339625a^3b^2x^6+209000a^4b\,x^3+54340a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{10}}{543400\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*a
^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.67, size = 57, normalized size = 0.22

$$\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^9\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

3.55 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^5}{18b^3}$$

```
[Out] 1/18*a^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3-2/21*a*(b*x^3+a)^6*((b*x^3+a)^
2)^(1/2)/b^3+1/24*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^5}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*
x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8\left(ab+b^2x^3\right)^5dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int x^2\left(ab+b^2x\right)^5dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^5}{b^2}-\frac{2a(ab+b^2x)^6}{b^3}+\frac{(ab+b^2x)^7}{b^4}\right)dx,x,x^3\right.}{3b^4\left(ab+b^2x^3\right)}$$
$$=\frac{a^2\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+\frac{(}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.70

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15}\right)}{504\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^
2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))
```

fricas [A] time = 0.85, size = 57, normalized size = 0.48

$$\frac{1}{24}b^5x^{24}+\frac{5}{21}ab^4x^{21}+\frac{5}{9}a^2b^3x^{18}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{12}a^4bx^{12}+\frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

giac [A] time = 0.38, size = 105, normalized size = 0.88

$$\frac{1}{24}b^5x^{24}\text{sgn}\left(bx^3+a\right)+\frac{5}{21}ab^4x^{21}\text{sgn}\left(bx^3+a\right)+\frac{5}{9}a^2b^3x^{18}\text{sgn}\left(bx^3+a\right)+\frac{2}{3}a^3b^2x^{15}\text{sgn}\left(bx^3+a\right)+\frac{5}{12}a^4bx^{12}\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^3
*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*sg
n(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.67

$$\frac{\left(21b^5x^{15}+120ab^4x^{12}+280a^2b^3x^9+336a^3b^2x^6+210a^4bx^3+56a^5\right)\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}x^9}{504\left(bx^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/504*x^9*(21*b^5*x^15+120*a*b^4*x^12+280*a^2*b^3*x^9+336*a^3*b^2*x^6+210*a
^4*b*x^3+56*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.67, size = 114, normalized size = 0.96

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^3}{24\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3}{18\,b^3}-\frac{3\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3 -
 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{8}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

3.56 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[Out] 1/8*a^5*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^4*b*x^11*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+5/7*a^3*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/17*a^2*b^3*x
^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*a*b^4*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)+1/23*b^5*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a
+ b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^7+5a^4b^6x^{10}+10a^3b^7x^{13}+10a^2b^8x^{16}+5ab^9\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+}}{7\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(30107a^5+109480a^4bx^3+172040a^3b^2x^6+141680a^2b^3x^9+60214ab^4x^{12}+10472b^5x^{15}\right)}{240856\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^6
 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a + b*
x^3))
```

fricas [A] time = 0.78, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

giac [A] time = 0.34, size = 105, normalized size = 0.41

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b^
3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11*s
gn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(10472b^5x^{15}+60214a\,b^4x^{12}+141680a^2b^3x^9+172040a^3b^2x^6+109480a^4b\,x^3+30107a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^8}{240856\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^3
*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.77, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

3.57 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x}}{7\left(a+bx^3\right)}$$

```
[Out] 1/7*a^5*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^4*b*x^10*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+10/13*a^3*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^2*b^3*x^
16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/19*a*b^4*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)+1/22*b^5*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3}}{13\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a
+ b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(ab+b^2x^3\right)^5dx}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^6+5a^4b^6x^9+10a^3b^7x^{12}+10a^2b^8x^{15}+5ab^9x^{18}\right.}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2ab}}{13\left(a+bx^3\right.}$$

Mathematica [A]   time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(21736a^5+76076a^4bx^3+117040a^3b^2x^6+95095a^2b^3x^9+40040ab^4x^{12}+6916b^5x^{15}\right)}{152152\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6
+ 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x^3
))
```

fricas [A]   time = 0.87, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

giac [A]   time = 0.34, size = 105, normalized size = 0.41

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10*s
gn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)
```

maple [A]   time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(6916b^5x^{15}+40040a\,b^4x^{12}+95095a^2b^3x^9+117040a^3b^2x^6+76076a^4b\,x^3+21736a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^7}{152152\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*b
^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A]   time = 0.71, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

3.58 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

```
[Out] -1/18*a*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^2+1/21*(b*x^3+a)^6*((b*x^3+a)^2)^
(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^2) + ((a + b*x^3)^
6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^5 \left(ab+b^2x^3\right)^5 dx}{b^4 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int x \left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(-\frac{a(ab+b^2x)^5}{b} + \frac{(ab+b^2x)^6}{b^2}\right) dx, x, x^3\right)}{3b^4 \left(ab+b^2x^3\right)}$$
$$= -\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2} + \frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.06

$$\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15}\right)}{126\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*
b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))
```

fricas [A] time = 0.82, size = 57, normalized size = 0.73

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{18}\,ab^4x^{18}+\frac{2}{3}\,a^2b^3x^{15}+\frac{5}{6}\,a^3b^2x^{12}+\frac{5}{9}\,a^4bx^9+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9
*a^4*b*x^9 + 1/6*a^5*x^6
```

giac [A] time = 0.29, size = 67, normalized size = 0.86

$$\frac{1}{126}\left(6\,b^5x^{21}+35\,ab^4x^{18}+84\,a^2b^3x^{15}+105\,a^3b^2x^{12}+70\,a^4bx^9+21\,a^5x^6\right)\text{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 70
*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 1.03

$$\frac{\left(6b^5x^{15}+35a\,b^4x^{12}+84a^2b^3x^9+105a^3b^2x^6+70a^4b\,x^3+21a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^6}{126\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/126*x^6*(6*b^5*x^15+35*a*b^4*x^12+84*a^2*b^3*x^9+105*a^3*b^2*x^6+70*a^4*b
*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.63, size = 83, normalized size = 1.06

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}ax^3}{18\,b}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

3.59 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x}}{5\left(a+bx^3\right)}$$

```
[Out] 1/5*a^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^4*b*x^8*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+10/11*a^3*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^2*b^3*x^1
4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a*b^4*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+
a)+1/20*b^5*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3}}{11\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a
 + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^4+5a^4b^6x^7+10a^3b^7x^{10}+10a^2b^8x^{13}+5ab^9x^{16}\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2a}}{11\left(a+bx^3\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+2618b^5x^{15}\right)}{52360\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6 +
 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3))
```

fricas [A] time = 0.77, size = 57, normalized size = 0.22

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

giac [A] time = 0.42, size = 105, normalized size = 0.41

$$\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*sg
n(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(2618b^5x^{15}+15400a\,b^4x^{12}+37400a^2b^3x^9+47600a^3b^2x^6+32725a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^5}{52360\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^2
*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.69, size = 57, normalized size = 0.22

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

3.60 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=252

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^5x^4\sqrt{a^2+2abx^3+}}{4\left(a+bx^3\right)}$$

```
[Out] 1/4*a^5*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^4*b*x^7*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+a^3*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/13*a^2*b^3*x^13*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a*b^4*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1
/19*b^5*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x
^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^3+5a^4b^6x^6+10a^3b^7x^9+10a^2b^8x^{12}+5ab^9x^{1}\right.}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2a}}{a+bx}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b^5x^{15}\right)}{27664\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

fricas [A] time = 0.80, size = 56, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

giac [A] time = 0.32, size = 104, normalized size = 0.41

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^3b^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^4bx^7\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*b
^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(b*
x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{\left(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^4}{27664\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2*
x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.51, size = 56, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

3.61 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

```
[Out] 1/18*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 82, normalized size = 2.28

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15}\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^
3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

fricas [A] time = 0.89, size = 57, normalized size = 1.58

$$\frac{1}{18}b^5x^{18}+\frac{1}{3}ab^4x^{15}+\frac{5}{6}a^2b^3x^{12}+\frac{10}{9}a^3b^2x^9+\frac{5}{6}a^4bx^6+\frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6*
a^4*b*x^6 + 1/3*a^5*x^3
```

giac [B] time = 0.37, size = 66, normalized size = 1.83

$$\frac{1}{18}\left(3\left(bx^6+2\,ax^3\right)a^4+3\left(bx^6+2\,ax^3\right)^2a^2b+\left(bx^6+2\,ax^3\right)^3b^2\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a*
x^3)^3*b^2)*sgn(b*x^3 + a)
```

maple [B] time = 0.01, size = 79, normalized size = 2.19

$$\frac{\left(b^5x^{15}+6a\,b^4x^{12}+15a^2b^3x^9+20a^3b^2x^6+15a^4b\,x^3+6a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^3}{18\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/18*x^3*(b^5*x^15+6*a*b^4*x^12+15*a^2*b^3*x^9+20*a^3*b^2*x^6+15*a^4*b*x^3+
6*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.66, size = 52, normalized size = 1.44

$$\frac{1}{18}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(5/2)*a/b
```

mupad [B] time = 1.24, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^3+a\,b\right)\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{18\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2))/(18*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

3.62 $\int x \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=252

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2}}{2\left(a+bx^3\right)}$$

```
[Out] 1/2*a^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^4*b*x^5*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+5/4*a^3*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/11*a^2*b^3*x^11*((b
*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a*b^4*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/
17*b^5*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1355, 270}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^
3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x \left(ab+b^2x^3\right)^5 \, dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^5b^5x+5a^4b^6x^4+10a^3b^7x^7+10a^2b^8x^{10}+5ab^9x^{13}+b\right.}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3}}{4\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15}\right)}{5236\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 47
60*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

fricas [A] time = 0.91, size = 56, normalized size = 0.22

$$\frac{1}{17}b^5x^{17}+\frac{5}{14}ab^4x^{14}+\frac{10}{11}a^2b^3x^{11}+\frac{5}{4}a^3b^2x^8+a^4bx^5+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

giac [A] time = 0.37, size = 104, normalized size = 0.41

$$\frac{1}{17}b^5x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}ab^4x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}a^2b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}a^3b^2x^8\mathrm{sgn}\left(bx^3+a\right)+a^4bx^5\mathrm{sgn}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b*x
^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 80, normalized size = 0.32

$$\frac{\left(308b^5x^{15}+1870a\,b^4x^{12}+4760a^2b^3x^9+6545a^3b^2x^6+5236a^4b\,x^3+2618a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^2}{5236\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6+
5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 1.10, size = 56, normalized size = 0.22

$$\frac{1}{17}b^5x^{17}+\frac{5}{14}ab^4x^{14}+\frac{10}{11}a^2b^3x^{11}+\frac{5}{4}a^3b^2x^8+a^4bx^5+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(5/2), x)
```

3.63 $\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=247

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{a^5x\left(a^2+2ab\right.}{\left(a+b\right.}$$

```
[Out] a^5*x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/4*a^4*b*x^4*(b^2*x^6+2*a*
b*x^3+a^2)^(5/2)/(b*x^3+a)^5+10/7*a^3*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)
/(b*x^3+a)^5+a^2*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/13*a*
b^4*x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+1/16*b^5*x^16*(b^2*x^6+2
*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5
```

Rubi [A] time = 0.05, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1343, 194}

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2\right.}{7\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2
*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x
^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x
^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)
)/(16*(a + b*x^3)^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(2ab+2b^2x^3\right)^5 dx}{\left(2ab+2b^2x^3\right)^5}\\ &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(32a^5b^5+160a^4b^6x^3+320a^3b^7x^6+320a^2b^8x^9+16\right.}{\left(2ab+2b^2x^3\right)^5}\\ &= \frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+\right.}{7\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.33

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\left(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15}\right)}{1456\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1456
*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

fricas [A] time = 0.86, size = 53, normalized size = 0.21

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

giac [A] time = 0.38, size = 101, normalized size = 0.41

$$\frac{1}{16}\,b^5x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^2b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^4bx^4\mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^1
0*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*x^
3 + a) + a^5*x*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 78, normalized size = 0.32

$$\frac{\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x}{1456\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+1820
*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 1.02, size = 53, normalized size = 0.21

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + 2abx^{3} + b^{2}x^{6}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

3.64 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\left(a+bx^3\right)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{a^5\log(x)\sqrt{a^2+2abx^3}}{a+bx^3}$$

```
[Out] 5/3*a^4*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/3*a^3*b^2*x^6*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/9*a^2*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/12*a*b^4*x^
12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/15*b^5*x^15*((b*x^3+a)^2)^(1/2)/(b*x^3+a
)+a^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\left(a+bx^3\right)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+}}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(15*(a + b*x^3)) + (a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x}dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x}dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+}}{9\left(a+\right.}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(180a^5\log(x)+bx^3\left(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12}\right)\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 75
*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

fricas [A] time = 0.86, size = 55, normalized size = 0.22

$$\frac{1}{15}b^5x^{15}+\frac{5}{12}ab^4x^{12}+\frac{10}{9}a^2b^3x^9+\frac{5}{3}a^3b^2x^6+\frac{5}{3}a^4bx^3+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + a^5*log(x)
```

giac [A] time = 0.35, size = 104, normalized size = 0.41

$$\frac{1}{15}b^5x^{15}\text{sgn}\left(bx^3+a\right)+\frac{5}{12}ab^4x^{12}\text{sgn}\left(bx^3+a\right)+\frac{10}{9}a^2b^3x^9\text{sgn}\left(bx^3+a\right)+\frac{5}{3}a^3b^2x^6\text{sgn}\left(bx^3+a\right)+\frac{5}{3}a^4bx^3\text{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn(b
*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 79, normalized size = 0.31

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{15}+75a\,b^4x^{12}+200a^2b^3x^9+300a^3b^2x^6+300a^4b\,x^3+180a^5\ln(x)\right)}{180\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+75*a*b^4*x^12+200*a^2*b^3*x^9+300*a^
3*b^2*x^6+300*a^4*b*x^3+180*a^5*ln(x))/(b*x^3+a)^5
```

maxima [A] time = 1.16, size = 206, normalized size = 0.82

$$\frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}\,a^3bx^3+\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^5\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b*x
/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*b*
x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

3.65 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x}}{x\left(a+bx^3\right)}$$

```
[Out] -a^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a^4*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+2*a^3*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/4*a^2*b^3*x^8*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)+5/11*a*b^4*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^5
*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(1
1*(a + b*x^3)) + (b^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^2}+5a^4b^6x+10a^3b^7x^4+10a^2b^8x^7+5ab^9x^{10}+b^{10}\right.}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+22b^5x^{15}\right)}{308x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*
b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

fricas [A] time = 0.86, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

giac [A] time = 0.33, size = 105, normalized size = 0.42

$$\frac{1}{14}\,b^5x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^2b^3x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^3
*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*x^
3 + a) - a^5*sgn(b*x^3 + a)/x
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-22b^5x^{15}-140a\,b^4x^{12}-385a^2b^3x^9-616a^3b^2x^6-770a^4b\,x^3+308a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{308\left(b\,x^3+a\right)^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^4
*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

maxima [A] time = 0.82, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

3.66 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+$$

```
[Out] -1/2*a^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a^4*b*x*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+5/2*a^3*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/7*a^2*b^3*x^7*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^4*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13
*b^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a^4*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*
(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5a^4b^6+\frac{a^5b^5}{x^3}+10a^3b^7x^3+10a^2b^8x^6+5ab^9x^9+b^{10}x^{12}\right)\,d}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b^5x^{15}\right)}{182x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b
^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

fricas [A] time = 0.62, size = 59, normalized size = 0.24

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

giac [A] time = 0.36, size = 103, normalized size = 0.41

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^4x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^3+a\right)+5\,a^4bx\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^3
*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^3
+ a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-14b^5x^{15}-91a\,b^4x^{12}-260a^2b^3x^9-455a^3b^2x^6-910a^4b\,x^3+91a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{182\left(b\,x^3+a\right)^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x)
```

```
[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4*
b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

maxima [A] time = 1.23, size = 59, normalized size = 0.24

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

3.67 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/3*a^5*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+10/3*a^3*b^2*x^3*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+5/3*a^2*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/9*a*b^4*x^9*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/12*b^5*x^12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5
*a^4*b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3}}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^3*b^2*x^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(12*(a + b*x^3)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^2}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3}}{3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-12a^5+180a^4bx^3\log(x)+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}\right)}{36x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^4
*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))
```

fricas [A] time = 0.65, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log(x)-12\,a^5}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a
^4*b*x^3*log(x) - 12*a^5)/x^3
```

giac [A] time = 0.31, size = 124, normalized size = 0.49

$$\frac{1}{12}\,b^5x^{12}\text{sgn}\left(bx^3+a\right)+\frac{5}{9}\,ab^4x^9\text{sgn}\left(bx^3+a\right)+\frac{5}{3}\,a^2b^3x^6\text{sgn}\left(bx^3+a\right)+\frac{10}{3}\,a^3b^2x^3\text{sgn}\left(bx^3+a\right)+5\,a^4b\log\left(|x|\right)\text{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))*s
gn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^3
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(3b^5x^{15}+20a\,b^4x^{12}+60a^2b^3x^9+120a^3b^2x^6+180a^4b\,x^3\ln(x)-12a^5\right)}{36\left(b\,x^3+a\right)^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+20*a*b^4*x^12+60*a^2*b^3*x^9+120*a^3*b
^2*x^6+180*a^4*b*ln(x)*x^3-12*a^5)/(b*x^3+a)^5/x^3
```

maxima [A] time = 1.20, size = 214, normalized size = 0.85

$$\frac{5}{6}\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2b^2x^3+\frac{5}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^4b\log\left(2\,b^2x^3+2\,ab\right)-\frac{5}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*a
*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log(2
*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2
)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

3.68 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=249

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-5$$

```
[Out] -1/4*a^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a^4*b*((b*x^3+a)^2)^(1/2)/x/(b
*x^3+a)+5*a^3*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+2*a^2*b^3*x^5*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)+5/8*a*b^4*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^5*x
^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [**A**] time = 0.06, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b
*x^3)) + (b^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^2}+10a^3b^7x+10a^2b^8x^4+5ab^9x^7+b^{10}x^{10}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2}}{a+bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5x^{15}\right)}{88x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b
^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

fricas [A] time = 0.86, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

giac [A] time = 0.35, size = 107, normalized size = 0.43

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sgn}\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^5
*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-8b^5x^{15}-55a\,b^4x^{12}-176a^2b^3x^9-440a^3b^2x^6+440a^4b\,x^3+22a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{88\left(b\,x^3+a\right)^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x)
```

```
[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b*
x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

maxima [A] time = 0.97, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

3.69 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6} \, dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -1/5*a^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a^4*b*((b*x^3+a)^2)^(1/2)/x^
2/(b*x^3+a)+10*a^3*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/2*a^2*b^3*x^4*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a*b^4*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/10*
b^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7
*(a + b*x^3)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^3b^7+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^3}+10a^2b^8x^3+5ab^9x^6+b^{10}x^9\right)d}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5x^{15}\right)}{70x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b
^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

fricas [A] time = 0.61, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

giac [A] time = 0.34, size = 106, normalized size = 0.42

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{25\,a^4bx^3\mathrm{sgn}\left(bx^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*x
^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(b*
x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{70\left(b\,x^3+a\right)^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b*
x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

maxima [A] time = 0.67, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

3.70 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-$$

```
[Out] -1/6*a^5*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a^4*b*((b*x^3+a)^2)^(1/2)/x^
3/(b*x^3+a)+10/3*a^2*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/6*a*b^4*x^6*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b^5*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^
3*b^2*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9
*(a + b*x^3)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^7}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^3}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3}}{3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-3a^5-30a^4bx^3+180a^3b^2x^6\log(x)+60a^2b^3x^9+15ab^4x^{12}+2b^5x^{15}\right)}{18x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15*a*b^4*x^1
2 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a + b*x^3))
```

fricas [A] time = 0.66, size = 61, normalized size = 0.24

$$\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2x^6\log(x)-30\,a^4bx^3-3\,a^5}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
- 30*a^4*b*x^3 - 3*a^5)/x^6
```

giac [A] time = 0.37, size = 126, normalized size = 0.50

$$\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{30\,a^3b^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x^
3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^2*
x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^6
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(2b^5x^{15}+15a\,b^4x^{12}+60a^2b^3x^9+180a^3b^2x^6\ln(x)-30a^4b\,x^3-3a^5\right)}{18\left(b\,x^3+a\right)^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*b
^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

maxima [A] time = 1.24, size = 282, normalized size = 1.12

$$\frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}\,ab^3x^3+\frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3b^2\log\left(2\,b^2x^3+2\,ab\right)-\frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2*l
og(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(
3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/a
^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(7/2)/(a^2*x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

3.71 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8} \, dx$

Optimal. Leaf size=248

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-$$

```
[Out] -1/7*a^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a^4*b*((b*x^3+a)^2)^(1/2)/x^
4/(b*x^3+a)-10*a^3*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5*a^2*b^3*x^2*((b*x^
3+a)^2)^(1/2)/(b*x^3+a)+a*b^4*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^5*x^8
*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b
*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{\left(ab+b^2x^3\right)^5}{x^8} \, dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^2}+10a^2b^8x+5ab^9x^4+b^{10}x^7\right) \, dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15}\right)}{56x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3
*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

fricas [A] time = 0.90, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

giac [A] time = 0.33, size = 107, normalized size = 0.43

$$\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^3+a\right)+ab^4x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+35\,a^4bx^3\mathrm{sgn}\left(bx^3+\right.}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(b
*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3 +
 a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{15}-56a\,b^4x^{12}-280a^2b^3x^9+560a^3b^2x^6+70a^4b\,x^3+8a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{56\left(b\,x^3+a\right)^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*x
^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5
```

maxima [A] time = 1.10, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

3.72 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-$$

```
[Out] -1/8*a^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a^4*b*((b*x^3+a)^2)^(1/2)/x^5/(b
*x^3+a)-5*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+10*a^2*b^3*x*((b*x^3+a)
^2)^(1/2)/(b*x^3+a)+5/4*a*b^4*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^5*x^7
*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a +
b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^9}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^2b^8+\frac{a^5b^5}{x^9}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^3}+5ab^9x^3+b^{10}x^6\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15}\right)}{56x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3
*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

fricas [A] time = 0.87, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

giac [A] time = 0.34, size = 105, normalized size = 0.43

$$\frac{1}{7}\,b^5x^7\text{sgn}\left(bx^3+a\right)+\frac{5}{4}\,ab^4x^4\text{sgn}\left(bx^3+a\right)+10\,a^2b^3x\text{sgn}\left(bx^3+a\right)-\frac{40\,a^3b^2x^6\text{sgn}\left(bx^3+a\right)+8\,a^4bx^3\text{sgn}\left(bx\right.}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*sg
n(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3 +
 a) + a^5*sgn(b*x^3 + a))/x^8
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-8b^5x^{15}-70a\,b^4x^{12}-560a^2b^3x^9+280a^3b^2x^6+56a^4b\,x^3+7a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{56\left(b\,x^3+a\right)^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x)
```

```
[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*x
^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^8/(b*x^3+a)^5
```

maxima [A] time = 1.33, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

3.73 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+}}{9x^9\left(a+bx^3\right.}$$

```
[Out] -1/9*a^5*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/6*a^4*b*((b*x^3+a)^2)^(1/2)/x^
6/(b*x^3+a)-10/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+5/3*a*b^4*x^3*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/6*b^5*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^
2*b^3*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6
*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{10}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx,x\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x}}{3x^3\left(a+bx^3\right)}$$

<u>**Mathematica**</u> [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-2a^5-15a^4bx^3-60a^3b^2x^6+180a^2b^3x^9\log(x)+30ab^4x^{12}+3b^5x^{15}\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30*a*b^4*x^1
2 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a + b*x^3))
```

<u>**fricas**</u> [A] time = 0.78, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log(x)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
- 15*a^4*b*x^3 - 2*a^5)/x^9
```

<u>**giac**</u> [A] time = 0.40, size = 127, normalized size = 0.50

$$\frac{1}{6}\,b^5x^6\text{sgn}\left(bx^3+a\right)+\frac{5}{3}\,ab^4x^3\text{sgn}\left(bx^3+a\right)+10\,a^2b^3\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)-\frac{110\,a^2b^3x^9\text{sgn}\left(bx^3+a\right)+60\,a^3b^2x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log(
abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^
9
```

<u>**maple**</u> [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(3b^5x^{15}+30a\,b^4x^{12}+180a^2b^3x^9\ln(x)-60a^3b^2x^6-15a^4b\,x^3-2a^5\right)}{18\left(b\,x^3+a\right)^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x)*x^9-60
*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

maxima [A] time = 0.80, size = 313, normalized size = 1.24

$$\frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}\,b^4x^3+\frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^2b^3\log\left(2\,b^2x^3+2\,ab\right)-\frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a}{x^2|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b)
*a^2*b^3*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^2*b^3*log
(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/
2)*b^4*x^3/a^2 + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3 + 35/18*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(3/2)*b^3/a + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/
a^3 - 11/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^2*x^3) - 1/18*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/
2)/(a^2*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

3.74 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=253

$$\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-5a$$

```
[Out] -1/10*a^5*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a^4*b*((b*x^3+a)^2)^(1/2)/
x^7/(b*x^3+a)-5/2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-10*a^2*b^3*((b*
x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a*b^4*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5
*b^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+5a$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3
))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{11}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^5}+\frac{10a^2b^8}{x^2}+5ab^9x+b^{10}x^4\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a^5+50a^4bx^3+175a^3b^2x^6+700a^2b^3x^9-175ab^4x^{12}-14b^5x^{15}\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a^
2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(x^10*(a + b*x^3))
```

fricas [A] time = 0.64, size = 59, normalized size = 0.23

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

giac [A] time = 0.40, size = 108, normalized size = 0.43

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{700\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+175\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+50\,a^4bx^3\mathrm{sg}}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^3 + a) + 5/2*a*b^4*x^2*sgn(b*x^3 + a) - 1/70*(700*a^2*b
^3*x^9*sgn(b*x^3 + a) + 175*a^3*b^2*x^6*sgn(b*x^3 + a) + 50*a^4*b*x^3*sgn(b
*x^3 + a) + 7*a^5*sgn(b*x^3 + a))/x^10
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-14b^5x^{15}-175a\,b^4x^{12}+700a^2b^3x^9+175a^3b^2x^6+50a^4b\,x^3+7a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{70\left(b\,x^3+a\right)^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x)
```

```
[Out] -1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*b
*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^10/(b*x^3+a)^5
```

maxima [A] time = 1.01, size = 59, normalized size = 0.23

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

3.75 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-5a$$

```
[Out] -1/11*a^5*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a^4*b*((b*x^3+a)^2)^(1/2)/
x^8/(b*x^3+a)-2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5*a^2*b^3*((b*x^3
+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a*b^4*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^5*
x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5ab^9+\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^3}+b^{10}x^3\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b}}{x^5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(8a^5+55a^4bx^3+176a^3b^2x^6+440a^2b^3x^9-440ab^4x^{12}-22b^5x^{15}\right)}{88x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -1/88*(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a^
2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(x^11*(a + b*x^3))
```

fricas [A] time = 0.75, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

giac [A] time = 0.35, size = 106, normalized size = 0.43

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^3+a\right)-\frac{440\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+176\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+55\,a^4bx^3\mathrm{sgn}\left(b\right.}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^3 + a) + 5*a*b^4*x*sgn(b*x^3 + a) - 1/88*(440*a^2*b^3*x
^9*sgn(b*x^3 + a) + 176*a^3*b^2*x^6*sgn(b*x^3 + a) + 55*a^4*b*x^3*sgn(b*x^3
 + a) + 8*a^5*sgn(b*x^3 + a))/x^11
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-22b^5x^{15}-440a\,b^4x^{12}+440a^2b^3x^9+176a^3b^2x^6+55a^4b\,x^3+8a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{88\left(b\,x^3+a\right)^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*b
*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^11/(b*x^3+a)^5
```

maxima [A] time = 0.97, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

3.76 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}} \, dx$

Optimal. Leaf size=252

$$\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}$$

```
[Out] -1/12*a^5*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-5/9*a^4*b*((b*x^3+a)^2)^(1/2)/
x^9/(b*x^3+a)-5/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-10/3*a^2*b^3*((
b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+1/3*b^5*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5
*a*b^4*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(3*(a + b*x^3)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{13}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^5}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx,\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b}}{3x^6\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(3a^5+20a^4bx^3+60a^3b^2x^6+120a^2b^3x^9-180ab^4x^{12}\log(x)-12b^5x^{15}\right)}{36x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -1/36*(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 120*a^2
*b^3*x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(x^12*(a + b*x^3))
```

fricas [A] time = 0.86, size = 61, normalized size = 0.24

$$\frac{12\,b^5x^{15}+180\,ab^4x^{12}\log(x)-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^
6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

giac [A] time = 0.38, size = 125, normalized size = 0.50

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{125\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+120\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+60\,a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(125
*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^
6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x^12
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{15}+180ab^4x^{12}\ln(x)-120a^2b^3x^9-60a^3b^2x^6-20a^4bx^3-3a^5\right)}{36\left(bx^3+a\right)^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+180*a*b^4*ln(x)*x^12-120*a^2*b^3*x^9-
60*a^3*b^2*x^6-20*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^12
```

maxima [A] time = 1.10, size = 342, normalized size = 1.36

$$\frac{5\sqrt{b^2x^6+2\,abx^3+a^2}\,b^5x^3}{6\,a}+\frac{5}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,ab^4\log\left(2\,b^2x^3+2\,ab\right)-\frac{5}{3}\,(-1)^{2\,abx^3+2\,a^2}\,ab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*
b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/a^
4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)
*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{13}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

3.77 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\cdots}{\cdots}$$

```
[Out] -1/13*a^5*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a^4*b*((b*x^3+a)^2)^(1/2)/
x^10/(b*x^3+a)-10/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/2*a^2*b^3*(
(b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a*b^4*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/
2*b^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(x*(a + b*x^3)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{14}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^5}+\frac{5ab^9}{x^2}+b^{10}x\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^{\cdots}}}{7x^7\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(14a^5+91a^4bx^3+260a^3b^2x^6+455a^2b^3x^9+910ab^4x^{12}-91b^5x^{15}\right)}{182x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -1/182*(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455*
a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(x^13*(a + b*x^3))
```

fricas [A] time = 0.60, size = 59, normalized size = 0.23

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

giac [A] time = 0.33, size = 108, normalized size = 0.43

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{910\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+455\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+260\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+91\,a^4bx^3\mathrm{s}}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^2
*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*sgn
(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-91b^5x^{15}+910a\,b^4x^{12}+455a^2b^3x^9+260a^3b^2x^6+91a^4b\,x^3+14a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{182\left(b\,x^3+a\right)^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4*
b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^13/(b*x^3+a)^5
```

maxima [A] time = 0.99, size = 59, normalized size = 0.23

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{14}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

3.78 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a}}{11\ldots}$$

```
[Out] -1/14*a^5*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a^4*b*((b*x^3+a)^2)^(1/2)
/x^11/(b*x^3+a)-5/4*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-2*a^2*b^3*((b
*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+
b^5*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-5ab\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(2*x^2*(a + b*x^3)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{15}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}+\frac{a^5b^5}{x^{15}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^3}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}\ldots
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(22a^5+140a^4bx^3+385a^3b^2x^6+616a^2b^3x^9+770ab^4x^{12}-308b^5x^{15}\right)}{308x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -1/308*(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 616
*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(x^14*(a + b*x^3))
```

fricas [A] time = 0.91, size = 59, normalized size = 0.24

$$\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

giac [A] time = 0.35, size = 105, normalized size = 0.42

$$b^5x\mathrm{sgn}\left(bx^3+a\right)-\frac{770\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+616\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+385\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+140\,a^4bx^3}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*x
^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*x^
3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-308b^5x^{15}+770a\,b^4x^{12}+616a^2b^3x^9+385a^3b^2x^6+140a^4b\,x^3+22a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{308\left(b\,x^3+a\right)^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a^
4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

maxima [A] time = 0.99, size = 59, normalized size = 0.24

$$\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

3.79 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$\frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-$$

```
[Out] -1/15*a^5*((b*x^3+a)^2)^(1/2)/x^15/(b*x^3+a)-5/12*a^4*b*((b*x^3+a)^2)^(1/2)
/x^12/(b*x^3+a)-10/9*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/3*a^2*b^3*
((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a*b^4*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+
a)+b^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(3*x^3*(a + b*x^3)) + (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx, x, x$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x}}{9x^9\left(a+bx^3\right)}$$

Mathematica [**A**] time = 0.03, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a\left(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12}\right)-180b^5x^{15}\log(x)\right)}{180x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -1/180*(Sqrt[(a + b*x^3)^2]*(a*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*b^2*x^6 + 3
00*a*b^3*x^9 + 300*b^4*x^12) - 180*b^5*x^15*Log[x]))/(x^15*(a + b*x^3))
```

fricas [**A**] time = 0.74, size = 61, normalized size = 0.24

$$\frac{180\,b^5x^{15}\log(x)-300\,ab^4x^{12}-300\,a^2b^3x^9-200\,a^3b^2x^6-75\,a^4bx^3-12\,a^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2
*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

giac [**A**] time = 0.37, size = 123, normalized size = 0.49

$$b^5\log(|x|)\,\text{sgn}\left(bx^3+a\right)-\frac{137\,b^5x^{15}\text{sgn}\left(bx^3+a\right)+300\,ab^4x^{12}\text{sgn}\left(bx^3+a\right)+300\,a^2b^3x^9\text{sgn}\left(bx^3+a\right)+200\,a^3}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*a
*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x^6
*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x^15
```

maple [**A**] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(180b^5x^{15}\ln(x)-300a\,b^4x^{12}-300a^2b^3x^9-200a^3b^2x^6-75a^4b\,x^3-12a^5\right)}{180\left(b\,x^3+a\right)^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(180*b^5*ln(x)*x^15-300*a*b^4*x^12-300*a^2*b^3*x^
9-200*a^3*b^2*x^6-75*a^4*b*x^3-12*a^5)/(b*x^3+a)^5/x^15
```

maxima [B] time = 1.10, size = 374, normalized size = 1.49

$$\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^6x^3}{6\,a^2} + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^5\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^5\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{(b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^
6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 + 2*a
*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/a^
5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x^6 + 2
*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7
/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^12) - 1
/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

3.80 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=251

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{}}{1}$$

```
[Out] -1/16*a^5*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-5/13*a^4*b*((b*x^3+a)^2)^(1/2)
/x^13/(b*x^3+a)-a^3*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-10/7*a^2*b^3*((b
*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a*b^4*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-
b^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(4*x^4*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{17}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{17}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^5}+\frac{b^{10}}{x^2}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(91a^5+560a^4bx^3+1456a^3b^2x^6+2080a^2b^3x^9+1820ab^4x^{12}+1456b^5x^{15}\right)}{1456x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -1/1456*(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 + 2
080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(x^16*(a + b*x^3))
```

fricas [**A**] time = 0.58, size = 59, normalized size = 0.24

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

giac [**A**] time = 0.34, size = 107, normalized size = 0.43

$$-\frac{1456\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+1820\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+2080\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+1456\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right.}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 20
80*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4*b
*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16
```

maple [**A**] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(1456b^5x^{15}+1820a\,b^4x^{12}+2080a^2b^3x^9+1456a^3b^2x^6+560a^4b\,x^3+91a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/1456*(1456*b^5*x^15+1820*a*b^4*x^12+2080*a^2*b^3*x^9+1456*a^3*b^2*x^6+56
0*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^16/(b*x^3+a)^5
```

maxima [**A**] time = 0.99, size = 59, normalized size = 0.24

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

mupad [B] time = 1.26, size = 231, normalized size = 0.92

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(13*x^13*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1
/2))/(7*x^7*(a + b*x^3)) - (a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x^1
0*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{17}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**17, x)
```

3.81 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{\ldots}}{1\ldots}$$

```
[Out] -1/17*a^5*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/14*a^4*b*((b*x^3+a)^2)^(1/2)
/x^14/(b*x^3+a)-10/11*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/4*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a*b^4*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
-1/2*b^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b
*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{18}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{15}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^3}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+\ldots}}{11x^{11}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(308a^5+1870a^4bx^3+4760a^3b^2x^6+6545a^2b^3x^9+5236ab^4x^{12}+2618b^5x^{15}\right)}{5236x^{17}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -1/5236*(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6 +
 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(x^17*(a + b*x^3))
```

fricas [A] time = 0.84, size = 59, normalized size = 0.23

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

giac [A] time = 0.45, size = 107, normalized size = 0.42

$$-\frac{2618\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+5236\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+6545\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+4760\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 65
45*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^4*
b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(2618b^5x^{15}+5236a\,b^4x^{12}+6545a^2b^3x^9+4760a^3b^2x^6+1870a^4b\,x^3+308a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{5236\left(b\,x^3+a\right)^5x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/5236*(2618*b^5*x^15+5236*a*b^4*x^12+6545*a^2*b^3*x^9+4760*a^3*b^2*x^6+18
70*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x^17/(b*x^3+a)^5
```

maxima [A] time = 1.16, size = 59, normalized size = 0.23

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

mupad [B] time = 1.32, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^5\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(x^5*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(14*x^14*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(4*x^8*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(1
1*x^11*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

3.82 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

```
[Out] -1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^18
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 264}

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{19}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.98

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15}\right)}{18x^{18}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^3
*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))
```

fricas [B] time = 0.92, size = 57, normalized size = 1.39

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

giac [B] time = 0.29, size = 106, normalized size = 2.59

$$-\frac{6\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+15\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+20\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+15\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^4bx^3}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b^
3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

maple [B] time = 0.01, size = 78, normalized size = 1.90

$$-\frac{\left(6b^5x^{15}+15a\,b^4x^{12}+20a^2b^3x^9+15a^3b^2x^6+6a^4b\,x^3+a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{18\left(b\,x^3+a\right)^5x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a^4*b*x^3+a
^5)*((b*x^3+a)^2)^(5/2)/x^18/(b*x^3+a)^5
```

maxima [B] time = 1.10, size = 210, normalized size = 5.12

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^6}{18\,a^6}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^5}{18\,a^5x^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^4}{18\,a^6x^6}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^3}{18\,a^5x^9}-\frac{\left(b^2x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^
6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^18)
```

mupad [B] time = 1.22, size = 231, normalized size = 5.63

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\left(b\,x^3+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x}}{3\,x^{15}\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b*x
^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
6*x^12*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

3.83 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\cdots}{1\cdots}$$

```
[Out] -1/19*a^5*((b*x^3+a)^2)^(1/2)/x^19/(b*x^3+a)-5/16*a^4*b*((b*x^3+a)^2)^(1/2)
/x^16/(b*x^3+a)-10/13*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-a^2*b^3*((
b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a*b^4*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a
)-1/4*b^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a +
b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{20}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{17}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{11}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^5}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+\cdots}}{13x^{13}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(1456a^5+8645a^4bx^3+21280a^3b^2x^6+27664a^2b^3x^9+19760ab^4x^{12}+6916b^5x^{15}\right)}{27664x^{19}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -1/27664*(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x^
6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(x^19*(a + b*x^3
))
```

fricas [A] time = 0.87, size = 59, normalized size = 0.23

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

giac [A] time = 0.36, size = 107, normalized size = 0.42

$$-\frac{6916\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+19760\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+27664\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+21280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+\right.}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) +
27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 8645*
a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(6916b^5x^{15}+19760a\,b^4x^{12}+27664a^2b^3x^9+21280a^3b^2x^6+8645a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{27664\left(b\,x^3+a\right)^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/27664*(6916*b^5*x^15+19760*a*b^4*x^12+27664*a^2*b^3*x^9+21280*a^3*b^2*x^
6+8645*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/x^19/(b*x^3+a)^5
```

maxima [A] time = 1.04, size = 59, normalized size = 0.23

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

mupad [B] time = 1.31, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2}}{16\,x^{16}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(x^10*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
13*x^13*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

3.84 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{}}{1}$$

```
[Out] -1/20*a^5*((b*x^3+a)^2)^(1/2)/x^20/(b*x^3+a)-5/17*a^4*b*((b*x^3+a)^2)^(1/2)
/x^17/(b*x^3+a)-5/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-10/11*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a*b^4*((b*x^3+a)^2)^(1/2)/x^8/(b*x
^3+a)-1/5*b^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-5a$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(8*x^8*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(
a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{21}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{21}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{15}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^6}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(2618a^5+15400a^4bx^3+37400a^3b^2x^6+47600a^2b^3x^9+32725ab^4x^{12}+10472b^5x^{15}\right)}{52360x^{20}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -1/52360*(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*x
^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(x^20*(a + b*x
^3))
```

fricas [A] time = 0.94, size = 59, normalized size = 0.23

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

giac [A] time = 0.32, size = 107, normalized size = 0.42

$$-\frac{10472\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+32725\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+47600\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+37400\,a^3b^2x^6\mathrm{sgn}\left(bx\right.}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a) +
 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 1540
0*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(10472b^5x^{15}+32725a\,b^4x^{12}+47600a^2b^3x^9+37400a^3b^2x^6+15400a^4b\,x^3+2618a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{52360\left(b\,x^3+a\right)^5x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/52360*(10472*b^5*x^15+32725*a*b^4*x^12+47600*a^2*b^3*x^9+37400*a^3*b^2*x
^6+15400*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/x^20/(b*x^3+a)^5
```

maxima [A] time = 0.98, size = 59, normalized size = 0.23

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

mupad [B] time = 1.24, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{20\,x^{20}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^21,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(20*x^20*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(7*x^14*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{21}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**21, x)
```

3.85 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

```
[Out] -1/21*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^21+1/126*b*(b*x^3+a)^5*((b*x^3+a)
^2)^(1/2)/a^2/x^18
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx, x,\right.}{21ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.99

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15}\right)}{126x^{21}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a^
2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))
```

fricas [A] time = 0.80, size = 59, normalized size = 0.70

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

giac [A] time = 0.32, size = 107, normalized size = 1.27

$$-\frac{21\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+70\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+105\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+84\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+35\,a^4bx}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^2
*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(
b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

maple [A] time = 0.01, size = 80, normalized size = 0.95

$$-\frac{\left(21b^5x^{15}+70a\,b^4x^{12}+105a^2b^3x^9+84a^3b^2x^6+35a^4b\,x^3+6a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{126\left(b\,x^3+a\right)^5x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/126*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*x^6+35*a^4*b*x
^3+6*a^5)*((b*x^3+a)^2)^(5/2)/x^21/(b*x^3+a)^5
```

maxima [B] time = 1.16, size = 241, normalized size = 2.87

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^7}{18\,a^7}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^6}{18\,a^6x^3}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^5}{18\,a^7x^6}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^4}{18\,a^6x^9}+\frac{\left(b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a
^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^12) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(7/2)*b^2/(a^4*x^15) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^
18) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^21)
```

mupad [B] time = 1.22, size = 231, normalized size = 2.75

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2}}{18\,x^{18}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(6*x^12*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(3*x^15*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

3.86 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a}}{19}$$

```
[Out] -1/22*a^5*((b*x^3+a)^2)^(1/2)/x^22/(b*x^3+a)-5/19*a^4*b*((b*x^3+a)^2)^(1/2)
/x^19/(b*x^3+a)-5/8*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-10/13*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^10/(b*
x^3+a)-1/7*b^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])/(2*x^10*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a
 + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{23}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{23}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{17}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{11}}+\frac{b^{10}}{x^8}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(6916a^5+40040a^4bx^3+95095a^3b^2x^6+117040a^2b^3x^9+76076ab^4x^{12}+21736b^5x^{15}\right)}{152152x^{22}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -1/152152*(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2*
x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(x^22*(a + b
*x^3))
```

fricas [A] time = 0.90, size = 59, normalized size = 0.23

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

giac [A] time = 0.33, size = 107, normalized size = 0.42

$$-\frac{21736\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+76076\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+117040\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+95095\,a^3b^2x^6\mathrm{sgn}\left(b\right.}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a)
+ 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) + 40
040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(21736b^5x^{15}+76076a\,b^4x^{12}+117040a^2b^3x^9+95095a^3b^2x^6+40040a^4b\,x^3+6916a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{152152\left(b\,x^3+a\right)^5x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/152152*(21736*b^5*x^15+76076*a*b^4*x^12+117040*a^2*b^3*x^9+95095*a^3*b^2
*x^6+40040*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/x^22/(b*x^3+a)^5
```

maxima [A] time = 0.98, size = 59, normalized size = 0.23

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

mupad [B] time = 1.22, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{22\,x^{22}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^{10}\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(22*x^22*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^10*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(13*x^13*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2
))/(8*x^16*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**23, x)
```

3.87 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b}{}$$

```
[Out] -1/23*a^5*((b*x^3+a)^2)^(1/2)/x^23/(b*x^3+a)-1/4*a^4*b*((b*x^3+a)^2)^(1/2)/
x^20/(b*x^3+a)-10/17*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/7*a^2*b^3
*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a*b^4*((b*x^3+a)^2)^(1/2)/x^11/(b*
x^3+a)-1/8*b^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(11*x^11*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a
 + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{24}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{21}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{15}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^9}\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2}}{17x^{17}\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(10472a^5+60214a^4bx^3+141680a^3b^2x^6+172040a^2b^3x^9+109480ab^4x^{12}+30107b^5x^{15}\right)}{240856x^{23}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -1/240856*(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b^
2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(x^23*(a
+ b*x^3))
```

fricas [A] time = 0.92, size = 59, normalized size = 0.23

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

giac [A] time = 0.34, size = 107, normalized size = 0.42

$$-\frac{30107\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+109480\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+172040\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+141680\,a^3b^2x^6\mathrm{sgn}\left(b\right.}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a)
 + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a) +
60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(30107b^5x^{15}+109480a\,b^4x^{12}+172040a^2b^3x^9+141680a^3b^2x^6+60214a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{240856\left(b\,x^3+a\right)^5x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/240856*(30107*b^5*x^15+109480*a*b^4*x^12+172040*a^2*b^3*x^9+141680*a^3*b
^2*x^6+60214*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/x^23/(b*x^3+a)^5
```

maxima [A] time = 0.83, size = 59, normalized size = 0.23

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

mupad [B] time = 1.23, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{23\,x^{23}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{20}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(23*x^23*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(4*x^20*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(7*x^14*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(17*x^17*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**24, x)
```

3.88 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}} dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}}-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}$$

```
[Out] -1/24*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^24+1/84*b*(b*x^3+a)^5*((b*x^3+a)^
2)^(1/2)/a^2/x^21-1/504*b^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a^3/x^18
```

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{25}}dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}dx,\right.}{12ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} + \frac{\sqrt{a^2+}}{}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2\left(a\right.}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.65

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15}\right)}{504x^{24}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 336
*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))
```

fricas [A] time = 0.87, size = 59, normalized size = 0.46

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

giac [A] time = 0.39, size = 107, normalized size = 0.84

$$-\frac{56\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+210\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+336\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+12}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a^
2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3*s
gn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

maple [A] time = 0.01, size = 80, normalized size = 0.62

$$-\frac{\left(56b^{5}x^{15}+210a\,b^{4}x^{12}+336a^{2}b^{3}x^{9}+280a^{3}b^{2}x^{6}+120a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/504*(56*b^5*x^15+210*a*b^4*x^12+336*a^2*b^3*x^9+280*a^3*b^2*x^6+120*a^4*
b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/x^24/(b*x^3+a)^5
```

maxima [B] time = 1.02, size = 272, normalized size = 2.12

$$\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{8}}{18\,a^{8}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{7}}{18\,a^{7}x^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{6}}{18\,a^{8}x^{6}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{5}}{18\,a^{7}x^{9}}-\frac{\left(b^{2}x^{6}+\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(a^
8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x
^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)
```

mupad [B] time = 1.22, size = 231, normalized size = 1.80

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{24\,x^{24}\left(b\,x^{3}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{9\,x^{9}\left(b\,x^{3}+a\right)}-\frac{5\,a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{12\,x^{12}\left(b\,x^{3}+a\right)}-\frac{5\,a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{21\,x^{21}\left(b\,x^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(9*x^18*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=240

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(\right.}{6b^{5/3}\sqrt{a^2}}$$

```
[Out] 1/2*x^2*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^
(1/3)*x)/b^(5/3)/((b*x^3+a)^2)^(1/2)-1/6*a^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)/((b*x^3+
a)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 321, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a\right.}{\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x
^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{x^4}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int}{3b^2\sqrt{a^2+2}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{}{a^{2/3}}}{6b^{8/3}\sqrt{a^2+2ab}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}\right.}{6b^{5/3}\sqrt{a^2+2a}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}\right.}{3b^{5/3}\sqrt{a^2+2abx^3+}}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.55

$$\frac{\left(a+bx^3\right)\left(-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3b^{2/3}x^2\right)}{6b^{5/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.65, size = 123, normalized size = 0.51

$$\frac{3\,x^2-2\,\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)\right.}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

giac [A] time = 0.37, size = 146, normalized size = 0.61

$$\frac{x^2\mathrm{sgn}\left(bx^3+a\right)}{2\,b}+\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}\left(bx^3+a\right)}{3\,b}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}\left(bx^3+a\right)}{3\,b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(
b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

maple [A] time = 0.01, size = 113, normalized size = 0.47

$$\frac{\left(b\,x^3+a\right)\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}b\,x^2+2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(3*x^2*b*(a/b)^(1/3)+2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/
(a/b)^(1/3))*3^(1/2)*a+2*ln(x+(a/b)^(1/3))*a-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(1/3)
```

maxima [A] time = 2.18, size = 109, normalized size = 0.45

$$\frac{x^2}{2\,b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.21, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**
2/(2*b)
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=235

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a\right.}{6b^{4/3}\sqrt{a^2+}}$$

```
[Out] x*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)-1/3*a^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*
x)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/6*a^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(1/3)*(b*x^3+a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^
(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 321, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+\right.}{\sqrt{3}\,b^{4/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(ab+b^2x^3\right)\int \frac{x^3}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \cdots}{3b^{5/3}\sqrt{a^2+2\cdots}}\\
&= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \cdots}{6b^{7/3}\sqrt{a^2+2abx\cdots}}\\
&= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\cdots\right.}{6b^{4/3}\sqrt{a^2+2ab\cdots}}\\
&= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\cdots\right.}{3b^{4/3}\sqrt{a^2+2abx^3+\cdots}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 128, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}x\right)}{6b^{4/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.85, size = 106, normalized size = 0.45

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

giac [A] time = 0.41, size = 143, normalized size = 0.61

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}\left(bx^3+a\right)}{3b}+\frac{x\operatorname{sgn}\left(bx^3+a\right)}{b}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}\left(bx^3+a\right)}{3b^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
+ a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

maple [A] time = 0.01, size = 110, normalized size = 0.47

$$\frac{\left(bx^3+a\right)\left(2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+6\left(\frac{a}{b}\right)^{\frac{2}{3}}bx\right)}{6\sqrt{\left(bx^3+a\right)^2}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(6*x*b*(a/b)^(2/3)+2*3^(1/2)*a*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))-2*a*ln(x+(a/b)^(1/3))+a*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3)))/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(2/3)
```

maxima [A] time = 1.36, size = 106, normalized size = 0.45

$$\frac{x}{b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^
2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2
/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^3/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.20, size = 22, normalized size = 0.09

$$\mathrm{RootSum}\left(27t^3b^4 + a, \left(t \mapsto t\log\left(-3tb + x\right)\right)\right) + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1352, 608, 31}

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^3\right)\\ &= \frac{(ab+b^2x^3)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.80

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.54, size = 13, normalized size = 0.30

$$\frac{\log\left(bx^3 + a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(b*x^3 + a)/b
```

giac [A] time = 0.34, size = 22, normalized size = 0.50

$$\frac{\log\left(\left|bx^3 + a\right|\right)\operatorname{sgn}\left(bx^3 + a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

maple [A] time = 0.01, size = 32, normalized size = 0.73

$$\frac{\left(b\,x^3 + a\right)\ln\left(b\,x^3 + a\right)}{3\sqrt{\left(b\,x^3 + a\right)^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^3+a)^2)^(1/2),x)
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

maxima [A] time = 0.93, size = 15, normalized size = 0.34

$$\frac{\log\left(x^3 + \frac{a}{b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*log(x^3 + a/b)/b
```

mupad [B] time = 1.39, size = 33, normalized size = 0.75

$$\frac{\ln\left(b^2\,x^3 + a\,b\right)\,\operatorname{sign}\left(2\,b^2\,x^3 + 2\,a\,b\right)}{3\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2)^(1/2),x)
[Out] (log(a*b + b^2*x^3)*sign(2*a*b + 2*b^2*x^3))/(3*(b^2)^(1/2))
```

sympy [A] time = 0.17, size = 10, normalized size = 0.23

$$\frac{\log\left(a + bx^3\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x**3)/(3*b)
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/
6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)/((b*x
^3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(1/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1355, 292, 31, 634, 617, 204, 628}

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(ab+b^2x^3\right)\int \frac{x}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\,\sqrt[3]{b}+b^{2/3}x}\,dx}{3\sqrt[3]{a}\,b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}\,bx+b^{4/3}x^2}\,dx}{3\sqrt[3]{a}\,b\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{-\sqrt[3]{a}\,b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}\,bx+b^{4/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2\cdots\right.}{2b^{2\cdots}} \\
&= -\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab\cdots}{\cdots} \\
&= -\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\cdots}{6\sqrt[3]{a}\,b^{2\cdots}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(
6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.65, size = 304, normalized size = 1.50

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}b\right.}{6\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 0.42, size = 124, normalized size = 0.61

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}\left(bx^3+a\right)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}\left(bx^3+a\right)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{s}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3
 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x
^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*
x^3 + a)/a
```

maple [A] time = 0.00, size = 97, normalized size = 0.48

$$-\frac{\left(b\,x^3+a\right)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(1/3)
```

maximA [A] time = 1.28, size = 98, normalized size = 0.49

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(1
/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{\left(bx^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.18, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)-1/6
*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)/((b*x^
3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(2/3)/b^(1/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1343, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(2ab+2b^2x^3\right)\int \frac{1}{2ab+2b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(2ab+2b^2x^3\right)\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}+\sqrt[3]{2}b^{2/3}x}\,dx}{3\,2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2ab+2b^2x^3\right)\int \frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{2}b^{2/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/}}}{3\,2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(2ab+2b^2x^3\right)\int \frac{-2^{2/3}\sqrt[3]{a}b+2\,2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{12a^{2/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2ab}{}$$

$$= -\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log}{6a^{2/3}\sqrt[3]{}}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 0.54

$$-\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
))/(a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.79, size = 299, normalized size = 1.48

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(\right.\right.}{6\,a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 0.36, size = 122, normalized size = 0.60

$$-\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right)\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1
/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 97, normalized size = 0.48

$$\frac{\left(b\,x^3+a\right)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(2/3)
```

maxima [A] time = 2.73, size = 98, normalized size = 0.49

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\left(bx^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.19, size = 20, normalized size = 0.10

$$\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (b*x^3+a)*ln(x)/a/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a/((b*x^3+a
)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log
[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(ab+b^2x^3\right)\operatorname{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(ab+b^2x^3\right)\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b\left(ab+b^2x^3\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(a+bx^3\right)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\left(3\log(x)-\log\left(a+bx^3\right)\right)}{3a\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
[Out] ((a + b*x^3)*(3*Log[x] - Log[a + b*x^3]))/(3*a*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.69, size = 18, normalized size = 0.22

$$-\frac{\log\left(bx^3+a\right)-3\log(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

giac [A] time = 0.34, size = 32, normalized size = 0.40

$$-\frac{1}{3}\left(\frac{\log\left(\left|bx^3+a\right|\right)}{a}-\frac{3\log\left(|x|\right)}{a}\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] -1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\left(b\,x^3+a\right)\left(3\ln(x)-\ln\left(b\,x^3+a\right)\right)}{3\sqrt{\left(b\,x^3+a\right)^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^3+a)^2)^(1/2),x)
[Out] 1/3*(b*x^3+a)*(3*ln(x)-ln(b*x^3+a))/((b*x^3+a)^2)^(1/2)/a
```

maxima [A] time = 1.15, size = 43, normalized size = 0.54

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
```

mupad [B] time = 1.39, size = 48, normalized size = 0.60

$$-\frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] -log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/(
3*(a^2)^(1/2))
```

sympy [A] time = 0.28, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=238

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log}{6a^{4/3}\sqrt{a^2}}$$

```
[Out] (-b*x^3-a)/a/x/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)
*x)/a^(4/3)/((b*x^3+a)^2)^(1/2)-1/6*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)/((b*x^3+a)^2)
^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\sqrt[3]{\ldots}}{a^{2/3}b^{2/3}-\ldots}}{3a^{4/3}\sqrt{a^2+2abx^3\ldots}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(ab+b^2x^3\right)\int \frac{-\ldots}{a^{2/3}b^{2/\ldots}}}{6a^{4/3}b^{2/3}\sqrt{a^2+2\ldots}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\ldots\right.}{6a^{4/3}\sqrt{a^2+\ldots}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\ldots\right.}{3a^{4/3}\sqrt{a^2+2abx\ldots}}$$

Mathematica [A] time = 0.03, size = 133, normalized size = 0.56

$$-\frac{\left(a+bx^3\right)\left(\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\right)}{6a^{4/3}x\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*Sqrt[(a + b*x^3)^2]
)
```

fricas [A] time = 0.63, size = 103, normalized size = 0.43

$$-\frac{2\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right.}{6\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

giac [A] time = 0.37, size = 131, normalized size = 0.55

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}+\frac{2\,\sqrt{3}\,\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{a^{2}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn(b*
x^3 + a)
```

maple [A] time = 0.01, size = 111, normalized size = 0.47

$$-\frac{\left(b\,x^{3}+a\right)\left(-2\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2x\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+x\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+6\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3
))*x+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x-2*ln(x+(a/b)^(1/3))*x+6*(a/b)^(1/3
))/((b*x^3+a)^2)^(1/2)/a/x/(a/b)^(1/3)
```

maxima [A] time = 1.74, size = 106, normalized size = 0.45

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)
```

sympy [A] time = 0.23, size = 29, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(\frac{9t^2a^3}{b}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=243

$$\frac{-a-bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log}{6a^{5/3}\sqrt{}}$$

```
[Out] 1/2*(-b*x^3-a)/a/x^2/((b*x^3+a)^2)^(1/2)-1/3*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b
^(1/3)*x)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/6*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(2/3)*(b*x^3+a
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)/((b*x^3
+a)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^
3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\left(ab+b^2\right.\right.}{3a^{5/3}\sqrt{}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{}{a^{2/3}}}{6a^{5/3}\sqrt[3]{b}\sqrt{a^2+}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}\left(a+bx^3\right)\log}{6a^{5/3}\sqrt{a}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log}{3a^{5/3}\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.03, size = 140, normalized size = 0.58

$$-\frac{\left(a+bx^3\right)\left(-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.\right.}{6a^{5/3}x^2\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*x^
2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a + b
*x^3)^2])
```

fricas [A] time = 0.68, size = 143, normalized size = 0.59

$$\frac{2\,\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

giac [A] time = 0.33, size = 125, normalized size = 0.51

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{2\,\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)^(
1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3 +
a)
```

maple [A] time = 0.01, size = 117, normalized size = 0.48

$$\frac{\left(b\,x^3+a\right)\left(2\sqrt{3}\;x^2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2x^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+x^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\sqrt{\left(b\,x^3+a\right)^2}\;\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))
*x^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2-2*ln(x+(a/b)^(1/3))*x^2-3*(a/b)^
(2/3))/((b*x^3+a)^2)^(1/2)/x^2/a/(a/b)^(2/3)
```

maxima [A] time = 2.21, size = 106, normalized size = 0.44

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)
```

sympy [A] time = 0.26, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(-\frac{3ta^2}{b}+x\right)\right)\right)-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*
a*x**2)
```

3.97 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=125

$$\frac{-a-bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(-b*x^3-a)/a/x^3/((b*x^3+a)^2)^(1/2)-b*(b*x^3+a)*ln(x)/a^2/((b*x^3+a)^2
)^(1/2)+1/3*b*(b*x^3+a)*ln(b*x^3+a)/a^2/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])
/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\left(a+bx^3\right)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.43

$$-\frac{\left(a+bx^3\right)\left(-bx^3\log\left(a+bx^3\right)+a+3bx^3\log(x)\right)}{3a^2x^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/3*((a + b*x^3)*(a + 3*b*x^3*Log[x] - b*x^3*Log[a + b*x^3]))/(a^2*x^3*Sqr
t[(a + b*x^3)^2])
```

fricas [A] time = 0.72, size = 33, normalized size = 0.26

$$\frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log(x)-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

giac [A] time = 0.37, size = 50, normalized size = 0.40

$$\frac{1}{3}\left(\frac{b\log\left(\left|bx^3+a\right|\right)}{a^2}-\frac{3\,b\log\left(|x|\right)}{a^2}+\frac{bx^3-a}{a^2x^3}\right)\text{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^3
))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 51, normalized size = 0.41

$$-\frac{\left(b\,x^3+a\right)\left(3b\,x^3\ln(x)-b\,x^3\ln\left(b\,x^3+a\right)+a\right)}{3\sqrt{\left(b\,x^3+a\right)^2}\;a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/3*(b*x^3+a)*(3*b*x^3*ln(x)-b*ln(b*x^3+a)*x^3+a)/((b*x^3+a)^2)^(1/2)/a^2/
x^3
```

maxima [A] time = 1.20, size = 73, normalized size = 0.58

$$\frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^2}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^2
 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)
```

mupad [B] time = 1.40, size = 75, normalized size = 0.60

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^3}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\right)}{3\,{\left(a^2\right)}^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)))
)/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)
```

sympy [A] time = 0.36, size = 31, normalized size = 0.25

$$-\frac{1}{3ax^3}-\frac{b\log{(x)}}{a^2}+\frac{b\log{\left(\frac{a}{b}+x^3\right)}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

3.98 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+2}}$$

```
[Out] 1/9*x^2/a/b/((b*x^3+a)^2)^(1/2)-1/6*x^2/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/2
7*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/54*
(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)/((b*x^3
+a)^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(4/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}\right.}{54a^{4/3}b^{5/3}\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{}{ab+}}{9ab\sqrt{a^2+2abx^3}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(ab+b^2x^3\right)\int \frac{}{\sqrt[3]{a}}}{27a^{4/3}b^2\sqrt{a^2+2a}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{}\right.}{27a^{4/3}b^{5/3}\sqrt{a^2+2}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{}\right.}{27a^{4/3}b^{5/3}\sqrt{a^2+2}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\tan^{-1}}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+}}$$

Mathematica [A] time = 0.08, size = 235, normalized size = 0.84

$$\frac{-3a^{4/3}b^{2/3}x^2 + 2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) + b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) + a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\right.}{54a^{4/3}b^{5/3}\left(a+bx^3\right)\sqrt{\left(a+b\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*x^3
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a + b*
x^3)^2])
```

fricas [A] time = 0.70, size = 512, normalized size = 1.83

$$\left[\frac{6\,ab^3x^5 - 3\,a^2b^2x^2 + 3\,\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{bx^3+a}\right.}{54\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3
+ a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*
a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2
*a^3*b^4*x^3 + a^4*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 299, normalized size = 1.07

$$\frac{\left(-2\sqrt{3}\, b^{2}x^{6}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2b^{2}x^{6}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+b^{2}x^{6}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}x^{5}-4\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*b^2
-2*ln(x+(a/b)^(1/3))*x^6*b^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*b^2+6*(a
/b)^(1/3)*x^5*b^2-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a*b-4*ln(x+(a/b)^(1/3))*x^3*a*b+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*x^3*a*b-3*(a/b)^(1/3)*x^2*a*b-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*a^2-2*ln(x+(a/b)^(1/3))*a^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.81, size = 149, normalized size = 0.53

$$\frac{2\,bx^{5}-ax^{2}}{18\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/
```

```
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log(x
+ (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

3.99 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} dx$

Optimal. Leaf size=276

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{a}}$$

```
[Out] 1/18*x/a/b/((b*x^3+a)^2)^(1/2)-1/6*x/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/27*(
b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-1/54*(b*
x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)/((b*x^3+a)
^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(5/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a\right.}{54a^{5/3}b^{4/3}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) +
((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{\left(ab+b^2x^3\right)^2}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int}{9ab\sqrt{a^2+2ab}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)}{27a^{5/3}b^{5/3}\sqrt{a^2}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log}{27a^{5/3}b^{4/3}\sqrt{a^2}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log}{27a^{5/3}b^{4/3}\sqrt{a^2}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\tan}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{}}$$

Mathematica [A] time = 0.07, size = 235, normalized size = 0.85

$$\frac{3a^{2/3}b^{4/3}x^4 - 2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) - b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) - 6a^{5/3}\sqrt[3]{b}\,x - a^2\log\left(a^2\right.}{54a^{5/3}b^{4/3}\left(a+bx^3\right)\sqrt{\left(a+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) + b^
(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^3*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^
3)^2])
```

fricas [A] time = 0.75, size = 503, normalized size = 1.82

$$\left[\frac{3a^2b^2x^4 - 6a^3bx + 3\sqrt{\frac{1}{3}}\left(ab^3x^6+2a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right.}{bx^3+a}\right.}{54\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 +
 a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*
x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*
b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^
2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2
*a^4*b^3*x^3 + a^5*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 299, normalized size = 1.08

$$\frac{\left(-2\sqrt{3}\, b^2 x^6 \arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2b^2x^6 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4\sqrt{3}\, ab\, x^3 \arctan\left(\frac{\sqrt{3}}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*3^(1/2)*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))
+2*b^2*x^6*ln(x+(a/b)^(1/3))-b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+3*(a
/b)^(2/3)*x^4*b^2-4*3^(1/2)*a*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))+4*a*b*x^3*ln(x+(a/b)^(1/3))-2*a*b*x^3*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))-6*(a/b)^(2/3)*x*a*b-2*3^(1/2)*a^2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(
1/3))/(a/b)^(1/3))+2*a^2*ln(x+(a/b)^(1/3))-a^2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3)))*(b*x^3+a)/(a/b)^(2/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.70, size = 146, normalized size = 0.53

$$\frac{bx^4-2\,ax}{18\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/54
```

```
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x +
(a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

3.100 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/6/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 607}

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^3\right)\\ &= -\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{6b\left(\left(a+bx^3\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/6*(a + b*x^3)/(b*((a + b*x^3)^2)^(3/2))
```

fricas [A] time = 0.61, size = 26, normalized size = 0.68

$$-\frac{1}{6\left(b^3x^6+2\,ab^2x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{b\,x^3+a}{6\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
[Out] -1/6*(b*x^3+a)/b/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 0.89, size = 16, normalized size = 0.42

$$-\frac{1}{6\left(x^3+\frac{a}{b}\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/6/((x^3 + a/b)^2*b^3)
```

mupad [B] time = 1.19, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,b\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

3.101 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\tan^{-1}}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a^2+2}}$$

```
[Out] 2/9*x^2/a^2/((b*x^3+a)^2)^(1/2)+1/6*x^2/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-2/2
7*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/27*
(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)/((b*x^3
+a)^2)^(1/2)-2/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(7/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}\right.}{27a^{7/3}b^{2/3}\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2b\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2\left(ab+b^2x^3\right)\right.}{9a^2\sqrt{a^2+2a}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(2\left(ab+b^2x^3\right)\right.}{27a^{7/3}b\sqrt{a^2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\log}{27a^{7/3}b^{2/3}\sqrt{a^2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\log}{27a^{7/3}b^{2/3}\sqrt{a^2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\tan}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a}}$$

Mathematica [A] time = 0.07, size = 237, normalized size = 0.86

$$\frac{21a^{4/3}b^{2/3}x^2 + 4abx^3\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 2b^2x^6\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 2a^2\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\right.}{54a^{7/3}b^{2/3}\left(a + bx^3\right)\sqrt{(a +}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

fricas [A] time = 0.70, size = 514, normalized size = 1.86

$$\left[\frac{12\,ab^3x^5 + 21\,a^2b^2x^2 + 6\sqrt{\frac{1}{3}}\left(ab^3x^6 + 2\,a^2b^2x^3 + a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\sqrt{\frac{1}{3}}\left(abx + 2\left(-ab^2\right)^{\frac{2}{3}}x^2 + \left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)}{a}}}{bx^3 + a}\right.}{54}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a
*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*
x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x
 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 301, normalized size = 1.09

$$\frac{\left(-4\sqrt{3}\,b^2x^6\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-4b^2x^6\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5-8\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-4*3^(1/2)*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))
-4*b^2*x^6*ln(x+(a/b)^(1/3))+2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+12
*(a/b)^(1/3)*b^2*x^5-8*3^(1/2)*a*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3)
)/(a/b)^(1/3))-8*a*b*x^3*ln(x+(a/b)^(1/3))+4*a*b*x^3*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+21*(a/b)^(1/3)*a*b*x^2-4*3^(1/2)*a^2*arctan(1/3*3^(1/2)*(-2*x+(
a/b)^(1/3))/(a/b)^(1/3))-4*a^2*ln(x+(a/b)^(1/3))+2*a^2*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(1/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.36, size = 147, normalized size = 0.53

$$\frac{4\,bx^5+7\,ax^2}{18\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) + 1/
27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log(x
+ (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)
```

3.102 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\tan^{-1}}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\left(a^2+2a\right.}$$

```
[Out] 1/6*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)+5/18*x*(b*x^3+a)^2/a^2/(b^2
*x^6+2*a*b*x^3+a^2)^(3/2)+5/27*(b*x^3+a)^3*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^
(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/54*(b*x^3+a)^3*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/27*(b*x
^3+a)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)/(
b^2*x^6+2*a*b*x^3+a^2)^(3/2)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\log\left(a^{2/3}\right.}{54a^{8/3}\sqrt[3]{b}\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^
2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a
*b*x^3 + b^2*x^6)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(2ab+2b^2x^3\right)^3 \int \frac{1}{\left(2ab+2b^2x^3\right)^3}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^2}\,dx}{12ab\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int}{36a^2b^2\left(a^2+2abx^3\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int}{108\,2^{2/3}a^{8/3}b^{8/3}\left(a^2\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}\right.}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}\right.}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3\right.}$$

$$= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} - \frac{5\left(a+bx^3\right)^3\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\left(a^2+2ab\right.}$$

Mathematica [A] time = 0.07, size = 235, normalized size = 0.82

$$\frac{15a^{2/3}b^{4/3}x^4 - 10abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) - 5b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) + 24a^{5/3}\sqrt[3]{b}\,x - 5a^2\log}{54a^{8/3}\sqrt[3]{b}\left(a+bx^3\right)\sqrt{\left(a+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*a*b
*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*Sqrt[(
a + b*x^3)^2])
```

fricas [A] time = 0.59, size = 499, normalized size = 1.74

$$\left[\frac{15\,a^2b^2x^4 + 24\,a^3bx + 15\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}\right.}{bx^3+a}\right.}{54\left(a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3
+ a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)
*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b
^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 299, normalized size = 1.05

$$\left(-10\sqrt{3}\, b^2x^6 \arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 10b^2x^6\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 5b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 20\sqrt{3}\, ab\, x^3 \,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-10*3^(1/2)*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+10*b^2*x^6*ln(x+(a/b)^(1/3))-5*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
15*(a/b)^(2/3)*b^2*x^4-20*3^(1/2)*a*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))+20*a*b*x^3*ln(x+(a/b)^(1/3))-10*a*b*x^3*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+24*(a/b)^(2/3)*a*b*x-10*3^(1/2)*a^2*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))+10*a^2*ln(x+(a/b)^(1/3))-5*a^2*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(2/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.92, size = 145, normalized size = 0.51

$$\frac{5\,bx^4+8\,ax}{18\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/54
```

```
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x +
(a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

3.103 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3/a^2/((b*x^3+a)^2)^(1/2)+1/6/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^3+a)*l
n(x)/a^3/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^3/((b*x^3+a)^2)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log}{a^3\sqrt{a^2+2abx^3}}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.50

$$\frac{a\left(3a+2bx^3\right)+6\log(x)\left(a+bx^3\right)^2-2\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log[a + b*x^3
])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.70, size = 90, normalized size = 0.61

$$\frac{2\,abx^3+3\,a^2-2\left(b^2x^6+2\,abx^3+a^2\right)\log\left(bx^3+a\right)+6\left(b^2x^6+2\,abx^3+a^2\right)\log(x)}{6\left(a^3b^2x^6+2\,a^4bx^3+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(
b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 107, normalized size = 0.73

$$\frac{\left(6b^2x^6\ln(x)-2b^2x^6\ln\left(b\,x^3+a\right)+12ab\,x^3\ln(x)-4ab\,x^3\ln\left(b\,x^3+a\right)+2ab\,x^3+6a^2\ln(x)-2a^2\ln\left(b\,x^3+a\right)+\right.}{6\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/6*(6*ln(x)*x^6*b^2-2*ln(b*x^3+a)*x^6*b^2+12*ln(x)*x^3*a*b-4*ln(b*x^3+a)*x
^3*a*b+2*a*b*x^3+6*ln(x)*a^2-2*ln(b*x^3+a)*a^2+3*a^2)*(b*x^3+a)/a^3/((b*x^3
+a)^2)^(3/2)
```

maxima [A] time = 1.15, size = 88, normalized size = 0.60

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3}+\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2}+\frac{1}{6\left(x^3+\frac{a}{b}\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3
+ 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

3.104 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{14\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\left(a+bx\right.}{9\sqrt{3}\,a^{10/3}\sqrt{}}$$

```
[Out] 7/18/a^2/x/((b*x^3+a)^2)^(1/2)+1/6/a/x/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-14/9*(
b*x^3+a)/a^3/x/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/
3)*x)/a^(10/3)/((b*x^3+a)^2)^(1/2)-7/27*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+
a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)/((b*x
^3+a)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{14\sqrt[3]{b}\left(a+bx^3\right)\log}{27a^{10/3}\sqrt{a^2+2a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^
(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^2}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(14\left(ab+b\right.\right.}{9a^2\sqrt{a^2}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

Mathematica [A] time = 0.08, size = 260, normalized size = 0.82

$$\frac{-14b^{7/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-28ab^{4/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-147a^{4/3}bx^3-54a^{7/3}-14a^2\sqrt[3]{b}}{54a^{10/3}x\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)*
x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*x*
(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.64, size = 201, normalized size = 0.64

$$-\frac{84\,b^2x^6+147\,abx^3+28\,\sqrt{3}\left(b^2x^7+2\,abx^4+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\left(b^2x^7+2\,abx^4+a^2x\right.}{54\left(a^3b^2x^7+2\,a^4bx^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*
(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7 +
 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)
) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) +
 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 316, normalized size = 1.00

$$-\frac{\left(-28\sqrt{3}\,b^2x^7\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-28b^2x^7\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+14b^2x^7\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-28*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^7*b
^2-28*ln(x+(a/b)^(1/3))*x^7*b^2+14*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^7*b^
2+84*(a/b)^(1/3)*x^6*b^2-56*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*x^4*a*b-56*ln(x+(a/b)^(1/3))*x^4*a*b+28*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^4*a*b+147*(a/b)^(1/3)*x^3*a*b-28*3^(1/2)*arctan(1/3*3^(1/2)*(-
2*x+(a/b)^(1/3))/(a/b)^(1/3))*x*a^2-28*ln(x+(a/b)^(1/3))*x*a^2+14*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))*x*a^2+54*(a/b)^(1/3)*a^2)*(b*x^3+a)/(a/b)^(1/3)/x/
a^3/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.95, size = 148, normalized size = 0.47

$$-\frac{28\,b^2x^6+49\,abx^3+18\,a^2}{18\left(a^3b^2x^7+2\,a^4bx^4+a^5x\right)}-\frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x
) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*
(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)
) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

3.105 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{20b^{2/3}\left(a\right.}{9\sqrt{3}\,a^{1}}$$

```
[Out] 4/9/a^2/x^2/((b*x^3+a)^2)^(1/2)+1/6/a/x^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-10/
9*(b*x^3+a)/a^3/x^2/((b*x^3+a)^2)^(1/2)-20/27*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+
b^(1/3)*x)/a^(11/3)/((b*x^3+a)^2)^(1/2)+10/27*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/((b*x^3+a)^2)^(1/2)+20/27*b^(2/3)*(
b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}-\frac{20b^{2/3}\left(a+bx^3\right.}{27a^{11/3}\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(4b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(20\left(ab\right.\right.}{9a^2}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

Mathematica [A] time = 0.09, size = 266, normalized size = 0.84

$$\frac{20b^{8/3}x^8\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+40ab^{5/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-60a^{2/3}b^2x^6-96a^{5/3}bx^3-27}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*x
^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)*x
^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.62, size = 242, normalized size = 0.77

$$-\frac{60\,b^2x^6+96\,abx^3-40\,\sqrt{3}\left(b^2x^8+2\,abx^5+a^2x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+20\left(b^2x^8+2\,abx^5+a\right.}{54\left(a^3b^2x^8+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)
*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b
) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x
*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a^2*x
^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*x^8 +
 2*a^4*b*x^5 + a^5*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 322, normalized size = 1.02

$$-\frac{\left(-40\sqrt{3}\,b^2x^8\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+40b^2x^8\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-20b^2x^8\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-80\sqrt{3}\,ab\,x^5\,\mathrm{arc}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^8*b
^2+40*ln(x+(a/b)^(1/3))*x^8*b^2-20*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^8*b^
2+60*(a/b)^(2/3)*x^6*b^2-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*x^5*a*b+80*ln(x+(a/b)^(1/3))*x^5*a*b-40*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^5*a*b+96*(a/b)^(2/3)*x^3*a*b-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2*a^2+40*ln(x+(a/b)^(1/3))*x^2*a^2-20*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2*a^2+27*(a/b)^(2/3)*a^2)*(b*x^3+a)/(a/b)^(2/
3)/x^2/a^3/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.80, size = 150, normalized size = 0.47

$$-\frac{20\,b^2x^6+32\,abx^3+9\,a^2}{18\left(a^3b^2x^8+2\,a^4bx^5+a^5x^2\right)}-\frac{20\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^
2) - 20/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3
*(a/b)^(2/3)) + 10/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/
3)) - 20/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

3.106 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -2/3*b/a^3/((b*x^3+a)^2)^(1/2)-1/6*b/a^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/3*
(-b*x^3-a)/a^3/x^3/((b*x^3+a)^2)^(1/2)-3*b*(b*x^3+a)*ln(x)/a^4/((b*x^3+a)^2
)^(1/2)+b*(b*x^3+a)*ln(b*x^3+a)/a^4/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{3a^3x^3\sqrt{a^2}}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 0.52

$$\frac{-a\left(2a^2+9abx^3+6b^2x^6\right)-18bx^3\log(x)\left(a+bx^3\right)^2+6bx^3\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^4x^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*Log[x] + 6*b
*x^3*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^4*x^3*(a + b*x^3)*Sqrt[(a + b*x^3)^
2])
```

fricas [A] time = 0.60, size = 119, normalized size = 0.63

$$-\frac{6\,ab^2x^6+9\,a^2bx^3+2\,a^3-6\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(bx^3+a\right)+18\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log(x)}{6\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*
x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 133, normalized size = 0.71

$$-\frac{\left(18b^3x^9\ln(x)-6b^3x^9\ln\left(b\,x^3+a\right)+36a\,b^2x^6\ln(x)-12a\,b^2x^6\ln\left(b\,x^3+a\right)+6a\,b^2x^6+18a^2b\,x^3\ln(x)-6a^2\right.}{6\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(18*b^3*x^9*ln(x)-6*ln(b*x^3+a)*x^9*b^3+36*a*b^2*x^6*ln(x)-12*ln(b*x^3
+a)*x^6*a*b^2+6*a*b^2*x^6+18*a^2*b*x^3*ln(x)-6*ln(b*x^3+a)*x^3*a^2*b+9*a^2*
b*x^3+2*a^3)*(b*x^3+a)/x^3/a^4/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 0.99, size = 117, normalized size = 0.62

$$\frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{a^4}-\frac{b}{\sqrt{b^2x^6+2\,abx^3+a^2}\,a^3}-\frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^2b}-\frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^4 - b
/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/(s
qrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

3.107 $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\ldots$$

```
[Out] 5/486*x/a^2/b^2/((b*x^3+a)^2)^(1/2)-1/12*x^4/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)-1/27*x/b^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/162*x/a/b^2/(b*x^3+a)/((b*
x^3+a)^2)^(1/2)+5/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(7/3)/((b*x
^3+a)^2)^(1/2)-5/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3
)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(14
58*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^6}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^4}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\left(a\right.}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\left(a\right.}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\left(a\right.}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\left(a\right.}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\left(a\right.}$$

Mathematica [A] time = 0.13, size = 218, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-\frac{10\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{20\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}+\frac{20\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{8/3}}+\frac{30\sqrt[3]{b}x\left(a+bx^3\right)}{a^2}\right.}{2916b^{7/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*(
a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a +
b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^3)^2)
^(5/2))
```

fricas [A] time = 0.66, size = 723, normalized size = 2.01

$$\left[\frac{30\,a^{2}b^{4}x^{10}+108\,a^{3}b^{3}x^{7}-225\,a^{4}b^{2}x^{4}-60\,a^{5}bx+30\,\sqrt{\frac{1}{3}}\left(ab^{5}x^{12}+4\,a^{2}b^{4}x^{9}+6\,a^{3}b^{3}x^{6}+4\,a^{4}b^{2}x^{3}+a^{5}b\right)\sqrt{-\frac{(a^{2}\cdots}{b}}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x +
 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 +
 a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)
^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^
5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7
 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9
+ 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 10*
(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(
2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 519, normalized size = 1.45

$$\frac{\left(-20\sqrt{3}\,b^{4}x^{12}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+20b^{4}x^{12}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-10b^{4}x^{12}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-80\sqrt{3}\,a\,b^{3}x^{9}\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^12
*b^4+20*ln(x+(a/b)^(1/3))*x^12*b^4-10*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^1
2*b^4+30*(a/b)^(2/3)*x^10*b^4-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*x^9*a*b^3+80*ln(x+(a/b)^(1/3))*x^9*a*b^3-40*ln(x^2-(a/b)^(
```

```
1/3)*x+(a/b)^(2/3))*x^9*a*b^3+108*(a/b)^(2/3)*x^7*a*b^3-120*3^(1/2)*arctan(
1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2+120*ln(x+(a/b)^(1/3
))*x^6*a^2*b^2-60*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b^2-225*(a/b)^(
2/3)*x^4*a^2*b^2-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a^3*b+80*ln(x+(a/b)^(1/3))*x^3*a^3*b-40*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*x^3*a^3*b-60*(a/b)^(2/3)*x*a^3*b-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*a^4+20*ln(x+(a/b)^(1/3))*a^4-10*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^3/a^2/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 1.35, size = 195, normalized size = 0.54

$$\frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\left(a^2b^6x^{12}+4\,a^3b^5x^9+6\,a^4b^4x^6+4\,a^5b^3x^3+a^6b^2\right)}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)\right.}{1458\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12
+ 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) -
 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) + 5/72
9*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

3.108 $\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/12*a/b^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-1/9/b^2/(b*x^3+a)^2/((b*x^3+a)^2
)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^5}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.50

$$\frac{-a-4bx^3}{36b^2\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (-a - 4*b*x^3)/(36*b^2*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.72, size = 58, normalized size = 0.74

$$-\frac{4\,bx^3+a}{36\left(b^6x^{12}+4\,ab^5x^9+6\,a^2b^4x^6+4\,a^3b^3x^3+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^3
 + a^4*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 32, normalized size = 0.41

$$-\frac{\left(b\,x^3+a\right)\left(4b\,x^3+a\right)}{36\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(b*x^3+a)*(4*b*x^3+a)/b^2/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 0.86, size = 43, normalized size = 0.55

$$-\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2}+\frac{a}{12\left(x^3+\frac{a}{b}\right)^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)
```

mupad [B] time = 1.28, size = 42, normalized size = 0.54

$$-\frac{\left(4\,b\,x^3+a\right)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{36\,b^2\,\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] -((a + 4*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(36*b^2*(a + b*x^3)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

3.109 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=368

$$\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b}}$$

```
[Out] 7/243*x^2/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x^2/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+1/54*x^2/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+7/324*x^2/a^2/b/(b*x^3+a)/
((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(5/3)/
((b*x^3+a)^2)^(1/2)+7/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(10/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2
)
```

Rubi [A] time = 0.19, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(
a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} -$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} -$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54ab\left(a+\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54ab\left(a+\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54ab\left(a+\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54ab\left(a+\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54ab\left(a+\right.}$$

Mathematica [A] time = 0.14, size = 229, normalized size = 0.62

$$\frac{\left(a+bx^3\right)\left(-243a^{10/3}b^{2/3}x^2+63a^{4/3}b^{2/3}x^2\left(a+bx^3\right)^2+54a^{7/3}b^{2/3}x^2\left(a+bx^3\right)+14\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}\right.\right.}{2916a^{10/3}b^{5/3}\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^3
) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*x^
3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((a + b
*x^3)^2)^(5/2))
```

fricas [A] time = 0.83, size = 734, normalized size = 1.99

$$\left[\frac{84\,ab^{5}x^{11} + 315\,a^{2}b^{4}x^{8} + 432\,a^{3}b^{3}x^{5} - 42\,a^{4}b^{2}x^{2} + 42\,\sqrt{\frac{1}{3}}\left(ab^{5}x^{12} + 4\,a^{2}b^{4}x^{9} + 6\,a^{3}b^{3}x^{6} + 4\,a^{4}b^{2}x^{3} + a^{5}b\right)\sqrt{\frac{(-a}{}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2
 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)
^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*
a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 315*a^2*b^4
*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*
b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*ar
ctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^
4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(
1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 521, normalized size = 1.42

$$\frac{\left(-28\sqrt{3}\,b^{4}x^{12}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 28b^{4}x^{12}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 14b^{4}x^{12}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}x^{11}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-28*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1
/3))-28*b^4*x^12*ln(x+(a/b)^(1/3))+14*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+84*(a/b)^(1/3)*x^11*b^4-112*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)*(-2*
x+(a/b)^(1/3))/(a/b)^(1/3))-112*a*b^3*x^9*ln(x+(a/b)^(1/3))+56*a*b^3*x^9*ln
```

```
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+315*(a/b)^(1/3)*x^8*a*b^3-168*3^(1/2)*a^2*b
^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-168*a^2*b^2*x^6*l
n(x+(a/b)^(1/3))+84*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+432*(a/b)
^(1/3)*x^5*a^2*b^2-112*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))-112*a^3*b*x^3*ln(x+(a/b)^(1/3))+56*a^3*b*x^3*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))-42*(a/b)^(1/3)*x^2*a^3*b-28*3^(1/2)*a^4*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-28*a^4*ln(x+(a/b)^(1/3))+14*a^4*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(1/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

maxima [A] time = 1.77, size = 195, normalized size = 0.53

$$\frac{28\,b^3x^{11} + 105\,ab^2x^8 + 144\,a^2bx^5 - 14\,a^3x^2}{972\left(a^3b^5x^{12} + 4\,a^4b^4x^9 + 6\,a^5b^3x^6 + 4\,a^6b^2x^3 + a^7b\right)} + \frac{7\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right.}{1458\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*x
^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
 + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 7/
729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

3.110 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=360

$$\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 5/243*x/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
1/108*x/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/81*x/a^2/b/(b*x^3+a)/((b*x^3+
a)^2)^(1/2)+10/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(4/3)/((b*x^3
+a)^2)^(1/2)-5/729*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
1/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-10/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12b\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*
Sqrt[3]*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(72
9*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{\left(ab+b^2x^3\right)^4}\,dx}{12\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108ab\left(a+b\cdots\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108ab\left(a+b\cdots\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108ab\left(a+b\cdots\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108ab\left(a+b\cdots\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{108ab\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.13, size = 221, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-20\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+60a^{2/3}\sqrt[3]{b}\,x\left(a+bx^3\right)^3+36a^{5/3}\sqrt[3]{b}\,x\left(a+bx^3\right)^2+27a^{8/3}\cdots\right.}{2916a^{11/3}b^{4/3}\left(\left(a+\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 + 4
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
+ 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)
^(5/2))
```

fricas [A] time = 0.58, size = 723, normalized size = 2.01

$$\left[\frac{60\,a^2b^4x^{10} + 216\,a^3b^3x^7 + 279\,a^4b^2x^4 - 120\,a^5bx + 60\,\sqrt{\frac{1}{3}}\left(ab^5x^{12} + 4\,a^2b^4x^9 + 6\,a^3b^3x^6 + 4\,a^4b^2x^3 + a^5b\right)\sqrt{\cdots}}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x
+ 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
+ a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
+ 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)
^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^
3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b
)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b
^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^
7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x
^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sq
rt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) -
20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^
3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a
^9*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 519, normalized size = 1.44

$$\frac{\left(-40\sqrt{3}\,b^4x^{12}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 40b^4x^{12}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 20b^4x^{12}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 160\sqrt{3}\,a\,b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-40*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1
/3))+40*b^4*x^12*ln(x+(a/b)^(1/3))-20*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+60*(a/b)^(2/3)*b^4*x^10-160*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)*(-2*
x+(a/b)^(1/3))/(a/b)^(1/3))+160*a*b^3*x^9*ln(x+(a/b)^(1/3))-80*a*b^3*x^9*ln
```

```
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+216*(a/b)^(2/3)*a*b^3*x^7-240*3^(1/2)*a^2*b
^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+240*a^2*b^2*x^6*l
n(x+(a/b)^(1/3))-120*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+279*(a/b
)^(2/3)*a^2*b^2*x^4-160*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))+160*a^3*b*x^3*ln(x+(a/b)^(1/3))-80*a^3*b*x^3*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))-120*(a/b)^(2/3)*a^3*b*x-40*3^(1/2)*a^4*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+40*a^4*ln(x+(a/b)^(1/3))-20*a^4*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(2/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

maxima [A] time = 2.28, size = 193, normalized size = 0.54

$$\frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\,\left(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b\right)}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12
+ 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) -
5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10/729
*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

3.111 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

```
[Out] -1/12/b/(b*x^3+a)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 607}

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^3\right)\\ &= -\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{12b\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/12*(a + b*x^3)/(b*((a + b*x^3)^2)^(5/2))
```

fricas [A] time = 0.56, size = 48, normalized size = 1.26

$$-\frac{1}{12\left(b^{5}x^{12}+4\,ab^{4}x^{9}+6\,a^{2}b^{3}x^{6}+4\,a^{3}b^{2}x^{3}+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{b\,x^{3}+a}{12\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
[Out] -1/12*(b*x^3+a)/b/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 0.51, size = 16, normalized size = 0.42

$$-\frac{1}{12\left(x^{3}+\frac{a}{b}\right)^{4}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/12/((x^3 + a/b)^4*b^5)
```

mupad [B] time = 1.26, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{12\,b\,\left(b\,x^{3}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

3.112 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-$$

```
[Out] 35/243*x^2/a^4/((b*x^3+a)^2)^(1/2)+1/12*x^2/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/
2)+5/54*x^2/a^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+35/324*x^2/a^3/(b*x^3+a)/((
b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)/(
(b*x^3+a)^2)^(1/2)+35/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(13/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/
2)
```

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(5b^3\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

Mathematica [A] time = 0.12, size = 219, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(\frac{70\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + 315a^{4/3}x^2\left(a+bx^3\right)^2 + 270a^{7/3}x^2\left(a+bx^3\right) + 243a^{10/3}x^2 - \frac{140\left(a+bx\right.}{}\right.}{2916a^{13/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)*
x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(a +
 b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*x^3)^
2)^(5/2))
```

fricas [A] time = 0.80, size = 734, normalized size = 2.04

$$\left[\frac{420\,ab^5x^{11}+1575\,a^2b^4x^8+2160\,a^3b^3x^5+1248\,a^4b^2x^2+210\,\sqrt{\frac{1}{3}}\left(ab^5x^{12}+4\,a^2b^4x^9+6\,a^3b^3x^6+4\,a^4b^2x^3+a^5b\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^
2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b
^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(
a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3
*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^
6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x
^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x^11 + 157
5*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/3)*(a*b^5*
x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2
)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a
)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*
b^3*x^3 + a^9*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 521, normalized size = 1.45

$$\frac{\left(-140\sqrt{3}\,b^4x^{12}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-140b^4x^{12}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+70b^4x^{12}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+420\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-140*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(
1/3))-140*b^4*x^12*ln(x+(a/b)^(1/3))+70*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+420*(a/b)^(1/3)*b^4*x^11-560*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)*(
-2*x+(a/b)^(1/3))/(a/b)^(1/3))-560*a*b^3*x^9*ln(x+(a/b)^(1/3))+280*a*b^3*x^
```

```
9*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1575*(a/b)^(1/3)*a*b^3*x^8-840*3^(1/2)*
a^2*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-840*a^2*b^2*
x^6*ln(x+(a/b)^(1/3))+420*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+216
0*(a/b)^(1/3)*a^2*b^2*x^5-560*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a
/b)^(1/3))/(a/b)^(1/3))-560*a^3*b*x^3*ln(x+(a/b)^(1/3))+280*a^3*b*x^3*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1248*(a/b)^(1/3)*a^3*b*x^2-140*3^(1/2)*a^4*arc
tan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-140*a^4*ln(x+(a/b)^(1/3))+7
0*a^4*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(1/3)/b/a^4/((b*x^
3+a)^2)^(5/2)
```

maxima [A] time = 2.18, size = 191, normalized size = 0.53

$$\frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}+\frac{35\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{35\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^4
*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) +
35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)
```

3.113 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110\left(a+bx^3\right)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{110\left(a+bx^3\right)}{243\sqrt{3}\,a^{14/3}\sqrt[3]{b}\,(}$$

```
[Out] 1/12*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/108*x*(b*x^3+a)^2/a^2/(
b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/81*x*(b*x^3+a)^3/a^3/(b^2*x^6+2*a*b*x^3+a^2
)^(5/2)+55/243*x*(b*x^3+a)^4/a^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+110/729*(b*x
^3+a)^5*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2
)-55/729*(b*x^3+a)^5*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(
1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)-110/729*(b*x^3+a)^5*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2
)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{55x\left(a+bx^3\right)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3
)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81
*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3
 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(1
4/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^3\right)^5 \int \frac{1}{\left(2ab+2b^2x^3\right)^5}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^4}\,dx}{24ab\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5\right.}{54a^2b^2\left(a^2+2a\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

Mathematica [A] time = 0.11, size = 211, normalized size = 0.58

$$\frac{\left(a+bx^3\right)\left(-\frac{220\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + 660a^{2/3}x\left(a+bx^3\right)^3 + 396a^{5/3}x\left(a+bx^3\right)^2 + 297a^{8/3}x\left(a+bx^3\right) + 2\right.}{2916a^{14/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(a
 + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*ArcT
an[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^3)^
4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2
))
```

fricas [A] time = 0.73, size = 719, normalized size = 1.98

$$\left[\frac{660\,a^2b^4x^{10} + 2376\,a^3b^3x^7 + 3069\,a^4b^2x^4 + 1596\,a^5bx + 660\,\sqrt{\frac{1}{3}}\left(ab^5x^{12} + 4\,a^2b^4x^9 + 6\,a^3b^3x^6 + 4\,a^4b^2x^3 + \cdots\right.}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*
b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2
*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
+ 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 +
6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10 + 2376*a^
3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b^5*x^12 +
4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)
/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 440*(b^4*
x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a
*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^
9*b^2*x^3 + a^10*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 519, normalized size = 1.43

$$\frac{\left(-440\sqrt{3}\,b^4x^{12}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 440b^4x^{12}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 220b^4x^{12}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 1760\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-440*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(
1/3))+440*b^4*x^12*ln(x+(a/b)^(1/3))-220*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+660*(a/b)^(2/3)*b^4*x^10-1760*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+1760*a*b^3*x^9*ln(x+(a/b)^(1/3))-880*a*b^3
```

```
*x^9*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2376*(a/b)^(2/3)*a*b^3*x^7-2640*3^(1
/2)*a^2*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+2640*a^2
*b^2*x^6*ln(x+(a/b)^(1/3))-1320*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+3069*(a/b)^(2/3)*a^2*b^2*x^4-1760*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(
-2*x+(a/b)^(1/3))/(a/b)^(1/3))+1760*a^3*b*x^3*ln(x+(a/b)^(1/3))-880*a^3*b*x
^3*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1596*(a/b)^(2/3)*a^3*b*x-440*3^(1/2)*a
^4*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+440*a^4*ln(x+(a/b)^(1
/3))-220*a^4*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(2/3)/b/a^4
/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 2.15, size = 189, normalized size = 0.52

$$\frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}+\frac{110\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{55\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4*
x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3)) -
55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

3.114 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\mathrm{l}}{3a^5\sqrt{a^2+\cdots}}$$

```
[Out] 1/3/a^4/((b*x^3+a)^2)^(1/2)+1/12/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+1/9/a^2/
(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/6/a^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^
3+a)*ln(x)/a^5/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^5/((b*x^3+a)
^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^
3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x
^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x} - \frac{1}{ab^4(a+bx)^5} - \frac{1}{a^2b^4(a+bx)^4} - \frac{1}{a^3b^4(a+bx)^3} - \frac{1}{a^4b^4(a+bx}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{9a^2\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.43

$$\frac{a\left(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9\right)+36\log(x)\left(a+bx^3\right)^4-12\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*L
og[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*
x^3)^2])
```

fricas [A] time = 0.62, size = 178, normalized size = 0.80

$$\frac{12\,ab^3x^9+42\,a^2b^2x^6+52\,a^3bx^3+25\,a^4-12\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log\left(bx^3+a\right)+36\left(b^4x\right.}{36\left(a^5b^4x^{12}+4\,a^6b^3x^9+6\,a^7b^2x^6+4\,a^8bx^3+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
+ 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b^4
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^4*x
^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 193, normalized size = 0.87

$$\frac{\left(36b^4x^{12}\ln(x)-12b^4x^{12}\ln\left(b\,x^3+a\right)+144a\,b^3x^9\ln(x)-48a\,b^3x^9\ln\left(b\,x^3+a\right)+12a\,b^3x^9+216a^2b^2x^6\ln(x)-7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/36*(36*ln(x)*x^12*b^4-12*ln(b*x^3+a)*x^12*b^4+144*ln(x)*x^9*a*b^3-48*ln(b
*x^3+a)*x^9*a*b^3+12*x^9*a*b^3+216*ln(x)*x^6*a^2*b^2-72*ln(b*x^3+a)*x^6*a^2
*b^2+42*x^6*a^2*b^2+144*ln(x)*x^3*a^3*b-48*ln(b*x^3+a)*x^3*a^3*b+52*x^3*a^3
*b+36*ln(x)*a^4-12*ln(b*x^3+a)*a^4+25*a^4)*(b*x^3+a)/a^5/((b*x^3+a)^2)^(5/2
)
```

maxima [A] time = 0.88, size = 132, normalized size = 0.59

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^5}+\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}+\frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}\,a^4}+\frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^3b^2}+\frac{}{12\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^5
+ 1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2) + 1/3/(sqrt(b^2*x^6 + 2*a*b*x
^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3*b^2) + 1/12/((x^3 + a/b)^4*a*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

3.115 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^2}+\frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^3}+\frac{455\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 455/972/a^4/x/((b*x^3+a)^2)^(1/2)+1/12/a/x/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
13/108/a^2/x/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+65/324/a^3/x/(b*x^3+a)/((b*x^3
+a)^2)^(1/2)-455/243*(b*x^3+a)/a^5/x/((b*x^3+a)^2)^(1/2)+455/729*b^(1/3)*(b
*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/((b*x^3+a)^2)^(1/2)-455/1458*b^(1/3)
*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/((b*x^3+a)^2)
^(1/2)+455/729*b^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(16/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{455\left(a+bx^3\right)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)}+\frac{}{108a^2x\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6]) - (455*(a + b*x^3))/(243*a^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (45
5*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(13b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^4}\,dx}{12a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

Mathematica [A] time = 0.13, size = 242, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-910\sqrt[3]{b}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-243a^{10/3}bx^2-1179a^{4/3}bx^2\left(a+bx^3\right)^2-594a^{7/3}bx^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a^
(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1/3
)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) +
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) +
b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

fricas [A] time = 1.04, size = 311, normalized size = 0.78

$$-\frac{5460\,b^4x^{12}+20475\,ab^3x^9+28080\,a^2b^2x^6+16224\,a^3bx^3+2916\,a^4+1820\,\sqrt{3}\left(b^4x^{13}+4\,ab^3x^{10}+6\,a^2b^2x^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b*
x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*
a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(
b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^13 +
4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a
*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4
 + a^9*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 536, normalized size = 1.35

$$-\frac{\left(-1820\sqrt{3}\,b^4x^{13}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-1820b^4x^{13}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+910b^4x^{13}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+546\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x
^13*b^4-1820*ln(x+(a/b)^(1/3))*x^13*b^4+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))*x^13*b^4+5460*(a/b)^(1/3)*x^12*b^4-7280*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*x^10*a*b^3-7280*ln(x+(a/b)^(1/3))*x^10*a*b^3+364
0*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^10*a*b^3+20475*(a/b)^(1/3)*x^9*a*b^3-
10920*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^7*a^2*b^
2-10920*ln(x+(a/b)^(1/3))*x^7*a^2*b^2+5460*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*x^7*a^2*b^2+28080*(a/b)^(1/3)*x^6*a^2*b^2-7280*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^4*a^3*b-7280*ln(x+(a/b)^(1/3))*x^4*a^3*b
+3640*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^4*a^3*b+16224*(a/b)^(1/3)*x^3*a^3
*b-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x*a^4-18
20*ln(x+(a/b)^(1/3))*x*a^4+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x*a^4+2916
*(a/b)^(1/3)*a^4)*(b*x^3+a)/(a/b)^(1/3)/x/a^5/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 2.33, size = 192, normalized size = 0.48

$$-\frac{1820\,b^4x^{12}+6825\,ab^3x^9+9360\,a^2b^2x^6+5408\,a^3bx^3+972\,a^4}{972\left(a^5b^4x^{13}+4\,a^6b^3x^{10}+6\,a^7b^2x^7+4\,a^8bx^4+a^9x\right)}-\frac{455\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{455\log\left(x^2\right.}{1458}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1820*b^4*x^12 + 6825*a*b^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3
+ 972*a^4)/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 + a
^9*x) - 455/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a^5*(a/b)^(1/3)) - 455/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(
a/b)^(1/3)) + 455/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

3.116 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^2}+\frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^3}-\frac{770b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^{\cdots}}}$$

```
[Out] 154/243/a^4/x^2/((b*x^3+a)^2)^(1/2)+1/12/a/x^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+7/54/a^2/x^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+77/324/a^3/x^2/(b*x^3+a)/(
(b*x^3+a)^2)^(1/2)-385/243*(b*x^3+a)/a^5/x^2/((b*x^3+a)^2)^(1/2)-770/729*b^
(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/((b*x^3+a)^2)^(1/2)+385/729*
b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)/((b*x^
3+a)^2)^(1/2)+770/729*b^(2/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(17/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)}+\frac{\cdots}{54a^2x^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)
*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b}}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

Mathematica [A] time = 0.14, size = 234, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(1540b^{2/3}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-3162a^{2/3}bx\left(a+bx^3\right)^3-1314a^{5/3}bx\left(a+bx^3\right)^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/3
)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a + b
*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3
)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

fricas [A] time = 0.91, size = 352, normalized size = 0.88

$$-\frac{4620\,b^{4}x^{12}+16632\,ab^{3}x^{9}+21483\,a^{2}b^{2}x^{6}+11172\,a^{3}bx^{3}+1458\,a^{4}-3080\,\sqrt{3}\left(b^{4}x^{14}+4\,ab^{3}x^{11}+6\,a^{2}b^{2}x^{8}+4\,\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b*
x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^
(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a
^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) +
 a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b
^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 542, normalized size = 1.36

$$-\frac{\left(-3080\sqrt{3}\,b^{4}x^{14}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+3080b^{4}x^{14}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-1540b^{4}x^{14}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-1232\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x
^14*b^4+3080*ln(x+(a/b)^(1/3))*x^14*b^4-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*x^14*b^4+4620*(a/b)^(2/3)*x^12*b^4-12320*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*x^11*a*b^3+12320*ln(x+(a/b)^(1/3))*x^11*a*b^3-
6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^11*a*b^3+16632*(a/b)^(2/3)*x^9*a*b
^3-18480*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^8*a^2
*b^2+18480*ln(x+(a/b)^(1/3))*x^8*a^2*b^2-9240*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*x^8*a^2*b^2+21483*(a/b)^(2/3)*x^6*a^2*b^2-12320*3^(1/2)*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^5*a^3*b+12320*ln(x+(a/b)^(1/3))*x^5*
a^3*b-6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^5*a^3*b+11172*(a/b)^(2/3)*x^
3*a^3*b-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2
*a^4+3080*ln(x+(a/b)^(1/3))*x^2*a^4-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
x^2*a^4+1458*(a/b)^(2/3)*a^4)*(b*x^3+a)/(a/b)^(2/3)/x^2/a^5/((b*x^3+a)^2)^(
5/2)
```

maxima [A] time = 2.06, size = 194, normalized size = 0.49

$$-\frac{1540\,b^{4}x^{12}+5544\,ab^{3}x^{9}+7161\,a^{2}b^{2}x^{6}+3724\,a^{3}bx^{3}+486\,a^{4}}{972\,\left(a^{5}b^{4}x^{14}+4\,a^{6}b^{3}x^{11}+6\,a^{7}b^{2}x^{8}+4\,a^{8}bx^{5}+a^{9}x^{2}\right)}-\frac{770\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{385\log\left(x^{2}\right.}{729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3
+ 486*a^4)/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a
^9*x^2) - 770/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^5*(a/b)^(2/3)) + 385/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*
(a/b)^(2/3)) - 770/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

3.117 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -4/3*b/a^5/((b*x^3+a)^2)^(1/2)-1/12*b/a^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-2
/9*b/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-1/2*b/a^4/(b*x^3+a)/((b*x^3+a)^2)^
(1/2)+1/3*(-b*x^3-a)/a^5/x^3/((b*x^3+a)^2)^(1/2)-5*b*(b*x^3+a)*ln(x)/a^6/((
b*x^3+a)^2)^(1/2)+5/3*b*(b*x^3+a)*ln(b*x^3+a)/a^6/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{1355, 266, 44\}$

$$-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) -
 (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)
*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b
*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+b}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{9a^3\left(a\right.}$$

Mathematica [A] time = 0.05, size = 119, normalized size = 0.44

$$\frac{-a\left(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12}\right)-180bx^3\log(x)\left(a+bx^3\right)^4+60bx^3\left(a+bx^3\right)^4\log\left(\right.}{36a^6x^3\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^1
2)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x^3
])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

fricas [A] time = 0.92, size = 207, normalized size = 0.77

$$-\frac{60\,ab^4x^{12}+210\,a^2b^3x^9+260\,a^3b^2x^6+125\,a^4bx^3+12\,a^5-60\left(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4b\right.}{36\left(a^6b^4x^{15}+4\,a^7b^3x^{12}+6\,a^8b^2x^9+4\,a^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*
b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^
3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b^2*x
^9 + 4*a^9*b*x^6 + a^10*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 219, normalized size = 0.81

$$-\frac{\left(180b^5x^{15}\ln(x)-60b^5x^{15}\ln\left(b\,x^3+a\right)+720a\,b^4x^{12}\ln(x)-240a\,b^4x^{12}\ln\left(b\,x^3+a\right)+60a\,b^4x^{12}+1080a^2b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(180*b^5*x^15*ln(x)-60*ln(b*x^3+a)*x^15*b^5+720*a*b^4*x^12*ln(x)-240*
ln(b*x^3+a)*x^12*a*b^4+60*a*b^4*x^12+1080*a^2*b^3*x^9*ln(x)-360*ln(b*x^3+a)
*x^9*a^2*b^3+210*a^2*b^3*x^9+720*a^3*b^2*x^6*ln(x)-240*ln(b*x^3+a)*x^6*a^3*
b^2+260*a^3*b^2*x^6+180*a^4*b*x^3*ln(x)-60*ln(b*x^3+a)*x^3*a^4*b+125*a^4*b*
x^3+12*a^5)*(b*x^3+a)/x^3/a^6/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 0.85, size = 163, normalized size = 0.61

$$\frac{5\,(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^6}-\frac{5\,b}{9\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^3}-\frac{5\,b}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^5}-\frac{1}{3\,\left(b^2x^6+2\,abx^3+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^6
 - 5/9*b/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3) - 5/3*b/(sqrt(b^2*x^6 + 2*
a*b*x^3 + a^2)*a^5) - 1/3/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3) - 5/6
/((x^3 + a/b)^2*a^4*b) - 1/12/((x^3 + a/b)^4*a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=313

$$\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+16}}{d^{16}(m+16)\left(a+bx^3\right)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+13}}{d^{13}(m+13)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^5}{}$$

```
[Out] a^5*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+5*a^4*b*(d*x)^(4+m)*(
(b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+10*a^3*b^2*(d*x)^(7+m)*((b*x^3+a)^2)
^(1/2)/d^7/(7+m)/(b*x^3+a)+10*a^2*b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10
/(10+m)/(b*x^3+a)+5*a*b^4*(d*x)^(13+m)*((b*x^3+a)^2)^(1/2)/d^13/(13+m)/(b*x
^3+a)+b^5*(d*x)^(16+m)*((b*x^3+a)^2)^(1/2)/d^16/(16+m)/(b*x^3+a)
```

Rubi [A] time = 0.14, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7
*(7 + m)*(a + b*x^3)) + (10*a^2*b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(d^10*(10 + m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int (dx)^m \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int \left(a^5b^5(dx)^m+\frac{5a^4b^6(dx)^{3+m}}{d^3}+\frac{10a^3b^7(dx)^{6+m}}{d^6}+\frac{10a^2b^8(d}{d^9}\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{10}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.35

$$\frac{x\left(\left(a+bx^3\right)^2\right)^{5/2}(dx)^m\left(\frac{a^5}{m+1}+\frac{5a^4bx^3}{m+4}+\frac{10a^3b^2x^6}{m+7}+\frac{10a^2b^3x^9}{m+10}+\frac{5ab^4x^{12}}{m+13}+\frac{b^5x^{15}}{m+16}\right)}{\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10
*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m)
 + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

fricas [A] time = 0.83, size = 369, normalized size = 1.18

$$\frac{\left(\left(b^5m^5+35\,b^5m^4+445\,b^5m^3+2485\,b^5m^2+5714\,b^5m+3640\,b^5\right)x^{16}+5\left(ab^4m^5+38\,ab^4m^4+511\,ab^4m^3+296\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6 + 51*m^
5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

giac [B] time = 0.67, size = 900, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*sgn
(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b^4*
m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555*(d*
x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 + a) +
 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2*x^13*
sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840*(d*x)^
m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b*x^3 + a
) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m^5*x^7*sg
n(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 440*(d*x)^m*
a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sgn(b*x^3 + a
) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m*a^2*b^3*x^1
0*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47260*(d*x)^m*a
^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn(b*x^3 + a) +
123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a^4*b*m^3*x^4*sg
n(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (d*x)^m*a^5*m^5*x
*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 + a) + 50*(d*x)^m*a
^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(b*x^3 + a) + 955*(
d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^4*sgn(b*x^3 + a) +
8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^m*a^5*m*x*sgn(b*x^3 + a
```

```
) + 58240*(d*x)^m*a^5*x*sgn(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3
 + 45474*m^2 + 95064*m + 58240)
```

maple [A] time = 0.01, size = 453, normalized size = 1.45

$$\frac{\left(b^5m^5x^{15} + 35b^5m^4x^{15} + 445b^5m^3x^{15} + 5a\,b^4m^5x^{12} + 2485b^5m^2x^{15} + 190a\,b^4m^4x^{12} + 5714b^5m\,x^{15} + 2555a\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5*
m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*x^
15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b^4*
m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a^2*b
^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+582
40*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+1239
20*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^4*b*m
^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650*a^5*m
^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(m+1)/(m+4)/(m+7)/(m+
10)/(m+13)/(16+m)/(b*x^3+a)^5
```

maxima [A] time = 0.95, size = 243, normalized size = 0.78

$$\frac{\left(\left(m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640\right)b^5d^mx^{16} + 5\left(m^5 + 38\,m^4 + 511\,m^3 + 2962\,m^2 + 6968\,m\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
+ 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 + 4
1*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 + 44
*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m
^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 +
955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m
^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

<u>**Optimal**</u>. Leaf size=205

$$\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^3\sqrt{a^2+}}{d(m}$$

```
[Out] a^3*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+3*a^2*b*(d*x)^(4+m)*(
(b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+3*a*b^2*(d*x)^(7+m)*((b*x^3+a)^2)^(1
/2)/d^7/(7+m)/(b*x^3+a)+b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10/(10+m)/(b
*x^3+a)
```

<u>**Rubi**</u> **[A]** time = 0.09, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^3\sqrt{a^2+}}{d(m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7
 + m)*(a + b*x^3)) + (b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
d^10*(10 + m)*(a + b*x^3))
```

<u>Rule 270</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

<u>Rule 1355</u>

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

<u>Rubi steps</u>

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3(dx)^m+\frac{3a^2b^4(dx)^{3+m}}{d^3}+\frac{3ab^5(dx)^{6+m}}{d^6}+\frac{b^6(dx)^{9+m}}{d^9}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{3ab^2(}{}\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.64

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\,(dx)^m\left(a^3\left(m^3+21m^2+138m+280\right)+3a^2b\left(m^3+18m^2+87m+70\right)x^3+3ab^2\left(m^3+15m^2+\right.\right.}{(m+1)(m+4)(m+7)(m+10)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b*
(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 + b
^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a +
b*x^3))
```

fricas [A] time = 0.90, size = 159, normalized size = 0.78

$$\frac{\left(\left(b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3\right)x^{10}+3\left(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2\right)x^7+3\left(a^2bm^3+18\,a^2bm^2\right.\right.}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*
m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m
 + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)^m/
(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

giac [B] time = 0.52, size = 384, normalized size = 1.87

$$\frac{(dx)^m\,b^3m^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+12\,(dx)^m\,b^3m^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+39\,(dx)^m\,b^3mx^{10}\mathrm{sgn}\left(bx^3+a\right)+3\,(dx)^m\,ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b*x
^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*sgn
(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*m^3*
x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m*a^2*
b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m
*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 21*(d*x)
^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) + 280*(d*x
)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

maple [A] time = 0.02, size = 199, normalized size = 0.97

$$\frac{\left(b^3m^3x^9+12b^3m^2x^9+39b^3m\,x^9+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162a\,b^2m\,x^6+3a^2b\,m^3x^3+120a\,b^2x^6\right.}{(m+10)\,(m+7)\,(m+4)\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a*
b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+
261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x)^m
*((b*x^3+a)^2)^(3/2)/(m+10)/(m+7)/(m+4)/(m+1)/(b*x^3+a)^3
```

maxima [A] time = 1.04, size = 119, normalized size = 0.58

$$\frac{\left(\left(m^3+12\,m^2+39\,m+28\right)b^3d^mx^{10}+3\left(m^3+15\,m^2+54\,m+40\right)ab^2d^mx^7+3\left(m^3+18\,m^2+87\,m+70\right)a^2bd^mx}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*b
^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 + 1
38*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\, dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

```
[Out] a*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+b*(d*x)^(4+m)*((b*x^3+a
)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) +
 (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(ab(dx)^m+\frac{b^2(dx)^{3+m}}{d^3}\right)dx}{ab+b^2x^3}\\ &= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.55

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\,(dx)^m\left(a(m+4)+b(m+1)x^3\right)}{(m+1)(m+4)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 + m
)*(a + b*x^3))
```

fricas [A] time = 0.86, size = 35, normalized size = 0.36

$$\frac{\left((bm+b)x^4+(am+4\,a)x\right)(dx)^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

giac [A] time = 0.45, size = 83, normalized size = 0.86

$$\frac{(dx)^m\,bmx^4\mathrm{sgn}\left(bx^3+a\right)+(dx)^m\,bx^4\mathrm{sgn}\left(bx^3+a\right)+(dx)^m\,amx\mathrm{sgn}\left(bx^3+a\right)+4\,(dx)^m\,ax\mathrm{sgn}\left(bx^3+a\right)}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a*
m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

maple [A] time = 0.00, size = 56, normalized size = 0.58

$$\frac{\left(bm\,x^3+b\,x^3+am+4a\right)\sqrt{\left(b\,x^3+a\right)^2}\;x\,(dx)^m}{(m+4)\,(m+1)\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

```
[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(m+4)/(m+1)/(b*x^3+a)
```

maxima [A] time = 1.03, size = 35, normalized size = 0.36

$$\frac{\left(bd^m(m+1)x^4+ad^m(m+4)x\right)x^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\,\sqrt{\left(a+bx^3\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

3.121 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^3\right)(dx)^{m+1} {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/d/(1
+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1355, 364\}$

$$\frac{\left(a+bx^3\right)(dx)^{m+1} {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx &= \frac{\left(ab+b^2x^3\right)\int \frac{(dx)^m}{ab+b^2x^3} \, dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(dx)^{1+m}\left(a+bx^3\right) {}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.85

$$\frac{x\left(a+bx^3\right)(dx)^m {}_2F_1\left(1,\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b*
x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2ab\,x^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{\left(a + bx^3\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

3.122 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^3\right)(dx)^{m+1} {}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/d/
(1+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{\left(a+bx^3\right)(dx)^{m+1} {}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \, dx &= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^3\right)^3} \, dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(dx)^{1+m}\left(a+bx^3\right) {}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)(dx)^m {}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3(m+1)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,(dx)^m}{b^4x^{12} + 4\,ab^3x^9 + 6\,a^2b^2x^6 + 4\,a^3bx^3 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^4*x^12 + 4*a*b^3*x^9 +
6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2ab\,x^3 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

3.123 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([5, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^5/d/
(1+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 364}

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(dx)^{1+m}\left(a+bx^3\right)\,{}_2F_1\left(5,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)(dx)^m\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5(m+1)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,(dx)^m}{b^6x^{18} + 6\,ab^5x^{15} + 15\,a^2b^4x^{12} + 20\,a^3b^3x^9 + 15\,a^4b^2x^6 + 6\,a^5bx^3 + a^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^6*x^18 + 6*a*b^5*x^15 +
 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2ab\,x^3 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

3.124 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=77

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

```
[Out] (d*x)^(1+m)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2*p, 1/3+1/3*m],[4/3+1/3*
m],-b*x^3/a)/d/(1+m)/((1+b*x^3/a)^(2*p))
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1356, 364}

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3, -
2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int (dx)^m\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1+m}{3},-2p;\frac{4+m}{3};-\frac{bx^3}{a}\right)}{d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.86

$$\frac{x(dx)^m\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx)^m \left(b^2x^6 + 2ab\,x^3 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Timed out
```

3.125 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=172

$$\frac{\left(a+bx^3\right)^4 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3 \left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-a^3\left(\right.$$

```
[Out] -1/3*a^3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+2*p)+1/2*a^2*(b*x^3+a)^
2*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+p)-a*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2)
^p/b^4/(3+2*p)+1/6*(b*x^3+a)^4*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(2+p)
```

Rubi [A] time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^4 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3 \left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-a^3\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)) + (a^2*(
a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x^3
)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2 +
2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^{11}\left(1+\frac{bx^3}{a}\right)^{2p}\,dx$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\frac{bx}{a}\right.}{b^3}\right.\right.$$
$$=-\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\frac{a\left(\right.}{}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 0.64

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(-3a^3+3a^2b(2p+1)x^3-3ab^2\left(2p^2+3p+1\right)x^6+b^3\left(4p^3+12p^2+11p+3\right)x^9\right)}{6b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

fricas [A] time = 0.94, size = 163, normalized size = 0.95

$$\frac{\left(\left(4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4\right)x^{12}+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^9+6\,a^3bpx^3-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^6-3\,a\right.}{6\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3*
a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^
6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p
^2 + 25*b^4*p + 6*b^4)
```

giac [B] time = 0.53, size = 375, normalized size = 2.18

$$\frac{4\left(b^2x^6+2\,abx^3+a^2\right)^pb^4p^3x^{12}+12\left(b^2x^6+2\,abx^3+a^2\right)^pb^4p^2x^{12}+11\left(b^2x^6+2\,abx^3+a^2\right)^pb^4px^{12}+4\left(b^2x^6+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p^2*
x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*a*b*x^
```

```
3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4*p^4 +
20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

maple [A] time = 0.01, size = 150, normalized size = 0.87

$$-\frac{\left(-4b^3p^3x^9-12b^3p^2x^9-11b^3p\,x^9-3b^3x^9+6a\,b^2p^2x^6+9a\,b^2p\,x^6+3a\,b^2x^6-6a^2bp\,x^3-3a^2b\,x^3+3a^3\right)\left(b\,x\right.}{6\left(4p^4+20p^3+35p^2+25p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9-
3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b*x
^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

maxima [A] time = 0.81, size = 115, normalized size = 0.67

$$\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^{12}+2\left(2\,p^3+3\,p^2+p\right)ab^3x^9-3\left(2\,p^2+p\right)a^2b^2x^6+6\,a^3bpx^3-3\,a^4\right)\left(bx^3+a\right)^{2\,p}}{6\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^9
 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/((4
*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

mupad [B] time = 1.31, size = 207, normalized size = 1.20

$$\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^{12}\,\left(4\,p^3+12\,p^2+11\,p+3\right)}{6\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{a^4}{2\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{}{b^3\,\left(4\,p\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^12*(11*p + 12*p^2 + 4*p^3 + 3))/(6*(25*p
+ 35*p^2 + 20*p^3 + 4*p^4 + 6)) - a^4/(2*b^4*(25*p + 35*p^2 + 20*p^3 + 4*p^
4 + 6)) + (a^3*p*x^3)/(b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a*p*x^9
*(3*p + 2*p^2 + 1))/(3*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (a^2*p*x^6
*(2*p + 1))/(2*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**12*(a**2)**p/12, Eq(b, 0)), (6*a**3*log(-(-1)**(1/3)*a**(1/3)
*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*
b**7*x**9) + 6*a**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)
*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a
*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x*
*3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b**4 + 54*a**2*b**5*
x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*log(-(-1)**(1/3)*a**
(1/3)*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6
+ 18*b**7*x**9) + 18*a**2*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) +
```

```
4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**
5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a**2*b*x**3*log(2)/(18*a**3*b*
*4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 27*a**2*b*x**3/(1
8*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**
2*x**6*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*
b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(4*(-1)**(2/
3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/
(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b
**2*x**6*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**
7*x**9) + 18*a*b**2*x**6/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6
 + 18*b**7*x**9) + 6*b**3*x**9*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/
(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3
*x**9*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1
/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 1
8*b**7*x**9) - 12*b**3*x**9*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a
*b**6*x**6 + 18*b**7*x**9), Eq(p, -2)), (Integral(x**11/((a + b*x**3)**2)**
(3/2), x), Eq(p, -3/2)), (6*a**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x
)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3)
 + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a*b**4 + 6*b**5*x**3)
 - 12*a**3*log(2)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3/(6*a*b**4 + 6*b**5*x**3
) + 6*a**2*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(6*a*b**4 + 6
*b**5*x**3) + 6*a**2*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1
)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a*b**4 + 6*b**5*x**3) - 12*a*
*2*b*x**3*log(2)/(6*a*b**4 + 6*b**5*x**3) - 3*a*b**2*x**6/(6*a*b**4 + 6*b**
5*x**3) + b**3*x**9/(6*a*b**4 + 6*b**5*x**3), Eq(p, -1)), (Integral(x**11/s
qrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**3 + b**2*
x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b*
*4) + 6*a**3*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 12
0*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 6*a**2*b**2*p**2*x**6
*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**
4*p**2 + 150*b**4*p + 36*b**4) - 3*a**2*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b*
*2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36
*b**4) + 4*a*b**3*p**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**
4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 6*a*b**3*p**2*x
**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*
b**4*p**2 + 150*b**4*p + 36*b**4) + 2*a*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b*
*2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36
*b**4) + 4*b**4*p**3*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4
 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 12*b**4*p**2*x**
12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b
**4*p**2 + 150*b**4*p + 36*b**4) + 11*b**4*p*x**12*(a**2 + 2*a*b*x**3 + b**
2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*
b**4) + 3*b**4*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120
*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4), True))
```

3.126 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

```
[Out] 1/3*a^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+2*p)-1/3*a*(b*x^3+a)^2*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+p)+1/3*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2)^
p/b^3/(3+2*p)
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a +
 b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(
a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^8\left(1+\frac{bx^3}{a}\right)^{2p}\,dx$$

$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2\left(1+\frac{bx}{a}\right)^{1}}{b^2}\right.\right.$$

$$=\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+2p)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+p)}+\frac{\left(a+b\right.}{}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(a^2-ab(2p+1)x^3+b^2\left(2p^2+3p+1\right)x^6\right)}{3b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2*
p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

fricas [A] time = 0.88, size = 108, normalized size = 0.83

$$\frac{\left(\left(2\,b^3p^2+3\,b^3p+b^3\right)x^9-2\,a^2bpx^3+\left(2\,ab^2p^2+ab^2p\right)x^6+a^3\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{3\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^2
*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

giac [A] time = 0.44, size = 235, normalized size = 1.81

$$\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^pb^3p^2x^9+3\left(b^2x^6+2\,abx^3+a^2\right)^pb^3px^9+\left(b^2x^6+2\,abx^3+a^2\right)^pb^3x^9+2\left(b^2x^6+2\,abx^3+a\right.}{3\left(4\,b^3p^3+12\,b^3p^2+11\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x^6
- 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

maple [A] time = 0.01, size = 96, normalized size = 0.74

$$\frac{\left(b\,x^3+a\right)\left(2b^2p^2x^6+3b^2p\,x^6+b^2x^6-2abp\,x^3-ab\,x^3+a^2\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{3\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

maxima [A] time = 1.22, size = 79, normalized size = 0.61

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^9+\left(2\,p^2+p\right)ab^2x^6-2\,a^2bpx^3+a^3\right)\left(bx^3+a\right)^{2\,p}}{3\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a^
3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

mupad [B] time = 1.22, size = 137, normalized size = 1.05

$$\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^9\,\left(\frac{2\,p^2}{3}+p+\frac{1}{3}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{3\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{2\,a^2\,p\,x^3}{3\,b^2\,\left(4\,p^3+12\,p^2+11\,p+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^9*(p + (2*p^2)/3 + 1/3))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(3*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x^3)/(3*b^
2*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^6*(2*p + 1))/(3*b*(11*p + 12*p^2 +
4*p^3 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^9\left(a^2\right)^p}{9} \\ \int\frac{x^8}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx \\ -\frac{2a^2\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^3+3b^4x^3}-\frac{2a^2\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3ab^3+3b^4x^3}-\frac{2a^2}{3ab^3+3b^4x^3}+\frac{4a^2\log(2)}{3ab^3+3b^4x^3}-\frac{2abx^3\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}\right.}{3ab^3+3b^4x^3} \\ \int\frac{x^8}{\sqrt{\left(a+bx^3\right)^2}}\,dx \\ \frac{a^3\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}-\frac{2a^2bpx^3\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{2ab^2p^2x^6\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{ab^2px^6\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{2b^3p^2x^9\left(a^2+2\right.}{12b^3p^3+36b^3p} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**9*(a**2)**p/9, Eq(b, 0)), (Integral(x**8/((a + b*x**3)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x
)/(3*a*b**3 + 3*b**4*x**3) - 2*a**2*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3)
 + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**3 + 3*b**4*x**3)
 - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b**3 + 3*b**4*x**3)
```

```
 - 2*a*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**3 + 3*b**
4*x**3) - 2*a*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3
)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**3 + 3*b**4*x**3) + 4*a*b*x**3*l
og(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x**6/(3*a*b**3 + 3*b**4*x**3), Eq(p,
-1)), (Integral(x**8/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a**3*(a**2 +
 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b*
*3) - 2*a**2*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36
*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*a*b**2*p**2*x**6*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + a*b**2*p
*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*
b**3*p + 9*b**3) + 2*b**3*p**2*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*
b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 3*b**3*p*x**9*(a**2 + 2*a*
b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) +
 b**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2
+ 33*b**3*p + 9*b**3), True))
```

3.127 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

```
[Out] -1/3*a*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^2/(1+2*p)+1/6*(b*x^3+a)^2*(b^2
*x^6+2*a*b*x^3+a^2)^p/b^2/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p)) + ((a + b*
x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^5\left(1+\frac{bx^3}{a}\right)^{2p} dx$$

$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p} dx, x, x^3\right)$$

$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2}}{b}\right.\right.$$

$$= -\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(b(2p+1)x^3-a\right)}{6b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 +
2*p))
```

fricas [A] time = 0.88, size = 70, normalized size = 0.83

$$\frac{\left(\left(2\,b^2p+b^2\right)x^6+2\,abpx^3-a^2\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{6\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

giac [A] time = 0.48, size = 132, normalized size = 1.57

$$\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^p b^2px^6+\left(b^2x^6+2\,abx^3+a^2\right)^p b^2x^6+2\left(b^2x^6+2\,abx^3+a^2\right)^p abpx^3-\left(b^2x^6+2\,abx^3+a^2\right)^p}{6\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*b*
x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.71

$$-\frac{\left(-2x^3pb-b\,x^3+a\right)\left(b\,x^3+a\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{6\left(2p^2+3p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-2*b*p*x^3-b*x^3+a)*(b*x^3+a)/b^2/(2*p^2+3*
p+1)
```

maxima [A] time = 0.63, size = 54, normalized size = 0.64

$$\frac{\left(b^2(2p+1)x^6+2\,abpx^3-a^2\right)\left(bx^3+a\right)^{2p}}{6\left(2p^2+3p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

mupad [B] time = 1.19, size = 85, normalized size = 1.01

$$\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^6\,\left(2\,p+1\right)}{6\,\left(2\,p^2+3\,p+1\right)}-\frac{a^2}{6\,b^2\,\left(2\,p^2+3\,p+1\right)}+\frac{a\,p\,x^3}{3\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2/
(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6\left(a^2\right)^p}{6} \\ \frac{a\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^2+3b^3x^3}+\frac{a\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3ab^2+3b^3x^3}-\frac{2a\log(2)}{3ab^2+3b^3x^3}+\frac{a}{3ab^2+3b^3x^3}+\frac{bx^3\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^2+3b^3x^3}+ \\ \int\frac{x^5}{\sqrt{\left(a+bx^3\right)^2}}\,dx \\ -\frac{a^2\left(a^2+2abx^3+b^2x^6\right)^p}{12b^2p^2+18b^2p+6b^2}+\frac{2abpx^3\left(a^2+2abx^3+b^2x^6\right)^p}{12b^2p^2+18b^2p+6b^2}+\frac{2b^2px^6\left(a^2+2abx^3+b^2x^6\right)^p}{12b^2p^2+18b^2p+6b^2}+\frac{b^2x^6\left(a^2+2abx^3+b^2x^6\right)^p}{12b^2p^2+18b^2p+6b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(-(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + x)/(3*a*b**2 + 3*b**3*x**3) + a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**
(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*
x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3*b**3*x**3) +
b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**2 + 3*b**3*x**3)
 + b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*
x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*x**3) - 2*b*x**3*log(2)/(3*a*b*
*2 + 3*b**3*x**3), Eq(p, -1)), (Integral(x**5/sqrt((a + b*x**3)**2), x), Eq
(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b*
*2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p
**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**
p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + b**2*x**6*(a**2 + 2*a*b*x**3 + b**2
*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2), True))
```

3.128 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

```
[Out] 1/5*x^5*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([5/3, -2*p],[8/3],-b*x^3/a)/((1
+b*x^3/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x
^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^4\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3)/a)])/(5*
(1 + (b*x^3)/a)^(2*p))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^4 \left(b^2x^6 + 2ab\,x^3 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral(x**4*((a + b*x**3)**2)**p, x)
```

3.129 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

```
[Out] 1/4*x^4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([4/3, -2*p],[7/3],-b*x^3/a)/((1
+b*x^3/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x
^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^3\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.85

$$\frac{1}{4}x^4\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3)/a)])/(4*
(1 + (b*x^3)/a)^(2*p))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^3\left(b^2x^6 + 2ab\,x^3 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\left(a + bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral(x**3*((a + b*x**3)**2)**p, x)
```

3.130 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^p\,dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

fricas [A] time = 0.75, size = 37, normalized size = 0.90

$$\frac{\left(bx^3+a\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{3\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

giac [A] time = 0.48, size = 58, normalized size = 1.41

$$\frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p bx^3 + \left(b^2x^6 + 2\,abx^3 + a^2\right)^p a}{3\left(2\,bp + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a)
/(2*b*p + b)
```

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$\frac{\left(b\,x^3 + a\right)\left(b^2x^6 + 2ab\,x^3 + a^2\right)^p}{3\left(2p + 1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

maxima [A] time = 0.95, size = 30, normalized size = 0.73

$$\frac{\left(bx^3 + a\right)\left(bx^3 + a\right)^{2\,p}}{3\,b\left(2\,p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

mupad [B] time = 1.16, size = 46, normalized size = 1.12

$$\left(\frac{x^3}{3\,\left(2\,p + 1\right)} + \frac{a}{3\,b\,\left(2\,p + 1\right)}\right)\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3}{3\sqrt{a^2}} & \text{for}\: b = 0 \wedge p = -\frac{1}{2} \\ \frac{x^3\left(a^2\right)^p}{3} & \text{for}\: b = 0 \\ \int \frac{x^2}{\sqrt{\left(a+bx^3\right)^2}}\, dx & \text{for}\: p = -\frac{1}{2} \\ \frac{a\left(a^2+2abx^3+b^2x^6\right)^p}{6bp+3b} + \frac{bx^3\left(a^2+2abx^3+b^2x^6\right)^p}{6bp+3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3,
 Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a**
2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(6*b*p + 3*b), True))
```

3.131 $\int x \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=58

$$\frac{x^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 5/3+2*p],[5/3],-b
*x^3/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1356, 364}

$$\frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x
^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{2}x^2\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.88

$$\frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(2*p))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(b^2x^6 + 2ab\,x^3 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a + bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**p, x)
```

3.132 $\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(1,2p+\frac{4}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a}$$

```
[Out] x*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 4/3+2*p],[4/3],-b*x^3/a
)/a
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1343, 246, 245}

$$x\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3, -((b*x^3
)/a)])/(1 + (b*x^3)/a)^(2*p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(2ab+2b^2x^3\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \left(2ab+2b^2x^3\right)^{2p}\,dx\\ &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= x\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [C] time = 0.17, size = 204, normalized size = 3.85

$$\frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^p F_1\left(2p+1;-2p,-2p;2(p+1);-\frac{i\left(\sqrt[3]{b}x+(-1)\cdots\right.}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt[3]{b}(2p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 + 2*p, -2*p,
 -2*p, 2*(1 + p), ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3))
, (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(4^p*b^(1/3)*
(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^(2*
p)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(2*p))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6+2\,abx^3+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

3.133 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x}\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

```
[Out] -1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 1+2*p],[2+2*p],1+b*x
^3/a)/a/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 65}

$$-\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1 + 2*p, 2
*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x}\,dx$$
$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx,x,x^3\right)$$
$$= -\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.86

$$-\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\,{}_2F_1\left(1,2p+1;2p+2;\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -1/3*((a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
1 + (b*x^3)/a])/(a*(1 + 2*p))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
[Out] Integral(((a + b*x**3)**2)**p/x, x)
```

3.134 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

```
[Out] -(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-1/3, -2*p],[2/3],-b*x^3/a)/x/((1+b*x
^3/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^
3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^2}\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^2}\,dx\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.84

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(2*p)))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**2, x)
```

3.135 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

```
[Out] -1/2*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2/3, -2*p],[1/3],-b*x^3/a)/x^2/(
(1+b*x^3/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3
)/a)])/(2*x^2*(1 + (b*x^3)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^3} dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^3} dx\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p} {}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -1/2*(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/a)])/(
x^2*(1 + (b*x^3)/a)^(2*p))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2ab\,x^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**3, x)
```

3.136 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$

Optimal. Leaf size=64

$$\frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

```
[Out] 1/3*b*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([2, 1+2*p],[2+2*p],1+b*
x^3/a)/a^2/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 65}

$$\frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^4}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^4}\,dx$$

$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx,x,x^3\right)$$

$$= \frac{b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.86

$$\frac{b\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\,{}_2F_1\left(2,2p+1;2p+2;\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6+2\,abx^3+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
[Out] Integral(((a + b*x**3)**2)**p/x**4, x)
```

3.137 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^5}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

```
[Out] -1/4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-4/3, -2*p],[-1/3],-b*x^3/a)/x^4/
((1+b*x^3/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1356, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^
3)/a)])/(4*x^4*(1 + (b*x^3)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^5}\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^5}\,dx\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^3\right)^2\right)^p\left(\frac{bx^3}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -1/4*(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)/a)])/
(x^4*(1 + (b*x^3)/a)^(2*p))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2ab\,x^3 + a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**5, x)
```

3.138 $\int \frac{x^8}{a+bx^3+cx^6} dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-1/3*(-2*a*c+b^2)*arctanh((2*c*x^3+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^3\right)\\ &= \frac{x^3}{3c}+\frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}\\ &= \frac{x^3}{3c}-\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}+\frac{(b^2-2ac)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}\\ &= \frac{x^3}{3c}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}-\frac{(b^2-2ac)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c^2}\\ &= \frac{x^3}{3c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^3+cx^6\right)+2cx^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

fricas [A] time = 0.93, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^3-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^6+bx^3+\right.}{6\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b
*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1
/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.14, size = 75, normalized size = 0.93

$$\frac{x^3}{3\,c}-\frac{b\log\left(cx^6+bx^3+a\right)}{6\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2*
c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 111, normalized size = 1.37

$$\frac{x^{3}}{3c}-\frac{2a\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}\;c}+\frac{b^{2}\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}\;c^{2}}-\frac{b\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-2/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))*a+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*
a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.98, size = 1758, normalized size = 21.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b^
2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*a*
b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c
^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^
3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(36
*a*c^3 - 9*b^2*c^2)) - ((((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (2
7*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)
^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)
*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3*b
^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2
*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((
45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 -
9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a
*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 -
9*b^2*c^2)) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/2)) + (((6*a^2*
c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 3
6*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(
2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2))))/(4*
a^2*c*(4*a*c - b^2)^(1/2))))/(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)
 - (c^2*(2*a*c - b^2)*(4*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((36*a^2*c^5 - 7
2*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*
```

```
(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2
*a*c - b^2))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 -
9*b^2*c^2)) - (((15*a*b^3*c^2 - 12*a^2*b*c^3)/c^3 - ((3*b^3 - 12*a*b*c)*((3
6*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 -
 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^
2)^(1/2)) + (a*b*(2*a*c - b^2)^3)/(2*c^3*(4*a*c - b^2)^(3/2))))/(a^2*(b^6 -
 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)) + (b*c^2*(4*a*c - b^2)^(3/2)*((a*
b^4 - a^2*b^2*c)/c^3 + ((3*b^3 - 12*a*b*c)*((15*a*b^3*c^2 - 12*a^2*b*c^3)/c
^3 - ((3*b^3 - 12*a*b*c)*((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*
b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(
36*a*c^3 - 9*b^2*c^2)) + (((((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*
(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c - b^2))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/
2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3
*a*b*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(2*c*(4*a*c - b^2)*(36*a*c^3 - 9*b
^2*c^2))))/(a^2*(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)))*(2*a*c - b
^2))/(3*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.70, size = 316, normalized size = 3.90

$$\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right.}{2ac-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*
log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 + (-a*b
 - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a
*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

3.139 $\int \frac{x^5}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(6*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c}$$
$$= \frac{\log\left(a+bx^3+cx^6\right)}{6c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^3+cx^6\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^3 + c*x^6])/(6*c)
```

fricas [A] time = 1.12, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right) + \left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c-4\,ac^2\right)}, \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 1.19, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\sqrt{-b^2+4\,ac}\,c} + \frac{\log\left(cx^6+bx^3+a\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
6*log(c*x^6 + b*x^3 + a)/c
```

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c} + \frac{\ln\left(c\,x^6+b\,x^3+a\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.80, size = 1199, normalized size = 19.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6),x)
```

```
[Out] (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*ata
n((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2
))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12*a*
c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3*(12
*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^
2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*
a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*c)*(4*a
*c - b^2))))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)) + ((12
*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9
*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2))/(2*(36*a
*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b*(12*b^
2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(1
2*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c))))/(6*c*(4*a*c - b^2)^(1/2))))/(4*a
^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c - b^2)^(3/2)*
(a*b + ((((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12*a*c - 3*b^2))/(
2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2)
)/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c -
 3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)))
)/(a^2*b^2*c) + ((2*a*c - b^2)*(4*a*c - b^2)*((a*b^4)/(2*(4*a*c - b^2)^(3/2
)) + (((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
)/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9
*b^2*c)*(4*a*c - b^2)^(1/2)))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) +
(b*(((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(12*
a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c))/(6*c*(4*a*c - b^2)^(1/2
))))/(a^2*b^3*c)))/(3*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.37, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c
*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(-b
*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2
)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

3.140 $\int \frac{x^2}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

```
[Out] -2/3*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{2\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

fricas [A] time = 1.03, size = 129, normalized size = 3.39

$$\left[\frac{\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,\left(b^2-4\,ac\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.98, size = 36, normalized size = 0.95

$$\frac{2\,\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 37, normalized size = 0.97

$$\frac{2\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] 2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.23, size = 174, normalized size = 4.58

$$-\frac{2\,\mathrm{atan}\left(\frac{\frac{x^3\,{\left(4\,a\,c-b^2\right)}^4}{2}+a\,b\,{\left(4\,a\,c-b^2\right)}^3+a\,b^3\,{\left(4\,a\,c-b^2\right)}^2+b^2\,x^3\,{\left(4\,a\,c-b^2\right)}^3+\frac{b^4\,x^3\,{\left(4\,a\,c-b^2\right)}^2}{2}}{b^2\,\left(32\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}-4\,a^2\,b^2\,c\,\sqrt{4\,a\,c-b^2}\right)-64\,a^4\,c^3\,\sqrt{4\,a\,c-b^2}}\right)}{3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6),x)
```

```
[Out] -(2*atan(((x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b^
2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2)/(b^2*(32*a^3*
c^2*(4*a*c - b^2)^(1/2) - 4*a^2*b^2*c*(4*a*c - b^2)^(1/2)) - 64*a^4*c^3*(4*
a*c - b^2)^(1/2))))/(3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.64, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3
```

3.141 $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^3\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a}+\frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^3\right)}{3a}\\
&= \frac{\log(x)}{a}-\frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\operatorname{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

fricas [A] time = 0.91, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4\,ac\right)\log(x)}{6\left(ab^2-4\,a^2c\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 1.00, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a} - \frac{\log\left(cx^6+bx^3+a\right)}{6\,a} + \frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a} + \frac{\ln(x)}{a} - \frac{\ln\left(c\,x^6+b\,x^3+a\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a),x)
```

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a-1/3/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b
)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.92, size = 1362, normalized size = 19.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(x)/a + (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c
)) - (b*atan((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(27*
b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(216*a
^3*(4*a*c - b^2)^(3/2)) + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 - 36*
a^2*c)^3*(4*a*c - b^2)^(1/2)) - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(9*a*b
^2 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(12*a*c - 3*b^2)^2*(27*b^3*c^3 - (
27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*a*(9*a*b^2 - 3
6*a^2*c)^2*(4*a*c - b^2)^(1/2))))/(b^3*c^6*(49*a*c - 12*b^2)) - (3*(4*a*c -
 b^2)^(3/2)*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)^3*(27*b^
3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*(9*a*
b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(12*a*c - 3*
b^2)*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))
))/(24*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)) + (9*b^5*c^3*(12*a*c - 3*b^2
)^2)/(16*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2))))/(b^3*c^6*(49*a*c - 12*b^
2)) + (48*a^4*x^3*(((4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(63*b^2*c^4 - ((
108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(
```

```
216*a^3*(4*a*c - b^2)^(3/2)) + (b*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3
*b^2)^3)/(48*a*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^(1/2)) - (b^3*(108*b^4*
c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(144*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c
 - b^2)^(3/2)) - (b*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c -
3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^2)/(8*a*(9*a*b^2 - 36*a^
2*c)^2*(4*a*c - b^2)^(1/2))))/(16*a^4*c^3*(49*a*c - 12*b^2)) - ((4*b^5 + 29
*a^2*b*c^2 - 23*a*b^3*c)*(((63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12
*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^3)/(8*(9*a*b^2 -
36*a^2*c)^3) - (b^4*(108*b^4*c^3 - 378*a*b^2*c^4))/(1296*a^4*(4*a*c - b^2)^
2) + (b^2*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2)^2)/(48*a^2*(9*a*b^
2 - 36*a^2*c)^2*(4*a*c - b^2)) - (b^2*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b
^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2))/(24*a
^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(4
9*a*c - 12*b^2)))*(4*a*c - b^2)^2)/(b^3*c^3)))/(3*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 6.73, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(b*s
qrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

3.142 $\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-1/3*(-2*a*c+b^2)*arctanh
((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^3\right)\\
&= -\frac{1}{3ax^3}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^3\right)}{3a}\\
&= -\frac{1}{3ax^3}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)}{3a}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^3\right)}{3a^2}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\, d\cdots\right.}{6a^2}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\cdots\right.}{3a^2}\\
&= -\frac{1}{3ax^3}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 92, normalized size = 1.03

$$\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2
)
```

fricas [A] time = 1.25, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^3-4\,abc\right)x^3\log\left(cx^6+bx^3+a\right)+6\left(b^3-\right.}{6\left(a^2b^2-4\,a^3c\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2
 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

giac [A] time = 1.14, size = 93, normalized size = 1.04

$$\frac{b\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^3-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*ar
ctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*x^
3 - a)/(a^2*x^3)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{2c\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a}+\frac{b^2\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c\,x^6+b\,x^3+a\right)}{6a^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-2/3/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*c+1/3/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.03, size = 4281, normalized size = 48.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (atan((48*a^8*x^3*(((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*c)
*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4
 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*
c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c
- b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36
*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 33*a^2*
b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 -
((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c -
 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b
^2)^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((((((2*a*c - b^2)*((252*a^4*b*c^
5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^
2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*
b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*
(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c -
 b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a
^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/
(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((42*a^3*
c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^
3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^
4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (12*b*c^6)/a^3))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^3*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(432*a^10*(4*a*c -
b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(16*a^4*c^3*(a^2*c^2 - 12*b^4 + 48*a*b
^2*c)) + ((4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^4*c)*(((3*b^3 - 12*a
*b*c)*(((((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 1
2*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)
)))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^
4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2
*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2
*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(3
6*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - c^7/a^4 - ((3*b^3 - 12
*a*b*c)*(((3*b^3 - 12*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 -
 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108
*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^
3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (12*b*c^6)/a^3))/(2*(36*a^
3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((2
52*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 -
 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1
/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4
))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*
a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))
)*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
+ ((2*a*c - b^2)^4*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(1296*a^12*(4*a*c -
 b^2)^2)))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 12*b^4 + 48*a*b^2*c))
)*(4*a*c - b^2)^2)/(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5) + (
3*a^4*(4*a*c - b^2)^2*(4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*
c)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^
3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2
*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (
(2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((2
7*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*
(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)
^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*
```

```
b^3 - 12*a*b*c)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(
(27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(
2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((((((27*a^3*b^4*c^
3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c -
9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3
 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)
))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c -
b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) + (b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^
2)^3)/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(c^3*(a^2*c^2 -
 12*b^4 + 48*a*b^2*c)*(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5))
 + (3*a^4*(4*a*c - b^2)^(3/2)*(4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^
4*c)*(((3*b^3 - 12*a*b*c)*((((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*
a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6
*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4
*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*
c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*
a*c - b^2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (b*c^6)/a
^4 + ((3*b^3 - 12*a*b*c)*((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)
*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((27*a^3*b^4*c^3
 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9
*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(
36*a^3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4
*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c
 - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*
b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b
^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b
^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 -
(27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c
 - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
+ (b^3*c^3*(2*a*c - b^2)^4)/(48*a^7*(4*a*c - b^2)^2)))/(c^3*(a^2*c^2 - 12*b
^4 + 48*a*b^2*c)*(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5)))*(2*
a*c - b^2))/(3*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/a^2 - (log(a + b*x^3 +
 c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)) - 1/(3*a*x^3)
```

sympy [B] time = 113.79, size = 345, normalized size = 3.88

$$\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right)\log\left(x^3 + \frac{-12a^3c\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right) + 3a^2b^2\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right) + 3abc}{2ac^2-b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] (b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*l
og(x**3 + (-12*a**3*c*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) +
(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*l
og(x**3 + (-12*a**3*c*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) -
1/(3*a*x**3) - b*log(x)/a**2
```

3.143 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=636

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\right.}{}$$

```
[Out] 1/2*x^2/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-
b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-
4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)
/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/
c^(5/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-
4*a*c+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*
x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2
)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arct
an(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2
*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/3)
```

Rubi [A] time = 1.25, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1367, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sq
rt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{x^2}{2c} - \frac{\int \frac{x(2a+2bx^3)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\;2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\,\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c\cdots}}{3\;2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\;2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2}\cdots}\right.}{3\;2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2}\cdots}}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\;2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2}\cdots}\right.}{3\;2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2}\cdots}}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(b-\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.11

$$\frac{3x^2-2\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

fricas [B] time = 3.29, size = 5601, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*
c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5
 - 4*a*c^6))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^
6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^
12 - 64*a^3*c^13)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2
```

```
*c^3))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*sqrt
((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2/3)*((b^8*c^5 - 13*
a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 1
0*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 1
2*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c + 52*a^
2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2
 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^
4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c +
 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^
7)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*
a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3
*c + 5*a^5*b*c^2)) - (1/2)^(1/3)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*
b^2*c^7 - 32*a^3*c^8)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b
^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x)*
((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c +
 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) + sqrt(3)*(
a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)
) - 4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^
6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 -
 4*a*c^6))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6
+ 32*a^2*b^2*c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)) + sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c
^3))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^
8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*sqrt((
2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a*
b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2*
b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 -
 (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*
c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 2
5*a^4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)
*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4
)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*
b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b
^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3*c
 + 5*a^5*b*c^2)) - (1/2)^(1/3)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^
2*c^7 - 32*a^3*c^8)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4
*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)) + sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x)*((
b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) - sqrt(3)*(a^
2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))
+ (1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^1
0 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^1
```

```
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(
1/3)*log(2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2/3)*((b^8*c^
5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c
+ 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c +
a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*
b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a
^4*b*c^7)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^
4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b
^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35
*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) + (1/2)^(1/3)
*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*
c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(2*(a
^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a*b^6*
c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*a*b^
8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2*b^6*
c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^
2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3
+ 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 25*a^
4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*sqr
t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*
c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) - 2*(1/2)^(1/3)*c*((b^4 -
3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*
b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^
10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 - (b^8*c
^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^
10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a
^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*
a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a
^5*b*c^2)*x) - 2*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a
*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^
5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 9
5*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 -
 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))
^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) - 3*x^2)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.14, size = 61, normalized size = 0.10

$$\frac{x^2}{2c}-\frac{\left(\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^4 b+\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)+x\right)}{3c\left(2\,\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^5 c+\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6
*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2\,c}-\frac{\int\frac{bx^4+ax}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 12.15, size = 4069, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1
/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1
/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))
^(2/3))/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*
c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*
b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4
 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^
6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(
1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)*(-(b^8 +
16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))
)^(1/3) + log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c
- b^2)^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))
/6 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 +
16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)
*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a
```

```
^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2
*b^2*c^7)))^(1/3) + x^2/(2*c) - log((a^4*x*(a*c - b^2))/c^2 - (2^(1/3)*(3^(
1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*
b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4
*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2
*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8
*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2
*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36)*
((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a*c - b^2))/c^2 - (2^
(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(b^4 + 8*a^2*
c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b
^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4
+ b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6
*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b
^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2
) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(
2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 -
 b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) - log((a^4*x*(a*c - b^2))
/c^2 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^
4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c -
b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2
- 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 1
6*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
- 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3
+ 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^
2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c -
b^2)^3))^(2/3))/36)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a
*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a
*c - b^2))/c^2 - (2^(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a
^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)
*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^
2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1
/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*
(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^
3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 1
2*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(
4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^
2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5
```

```
*(4*a*c - b^2)^3))^(2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 -
b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c
 - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2
))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3)
```

sympy [A] time = 14.58, size = 279, normalized size = 0.44

$$\mathrm{RootSum}\Big(t^6\left(46656a^3c^8 - 34992a^2b^2c^7 + 8748ab^4c^6 - 729b^6c^5\right) + t^3\left(432a^4c^4 - 1512a^3b^2c^3 + 1107a^2b^4c^2 - 297$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b**
4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_t**
5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 + 315
9*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*
a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*
_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*b**5)
))) + x**2/(2*c)
```

3.144 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\right.}{}$$

```
[Out] x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(
-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3
)
```

Rubi [A] time = 1.02, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/\cdots}}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-\cdots}}\right.}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-\cdots}\right.}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-\cdots}}\right.}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-\cdots}\right.}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\#1^3 b\log(x-\#1)+a\log(x-\#1)}{2\#1^5 c+\#1^2 b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

fricas [B] time = 2.54, size = 5260, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6
- 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 +
```

```
24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*
(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^
4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a
*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)
*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16
*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4
*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c
+ 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c
+ 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*sqrt(3)*(1/2)^(1/3)*
c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(s
qrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*
c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2
*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^
6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^
2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(
2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 1
0*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*
b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3
)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^
3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c +
 2*a^4*c^2)) - 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*
a^4*b^2*c + 2*a^5*c^2)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 -
 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*
```

```
a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1
/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*
c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*
c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3
+ 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^
4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^
(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b
^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log
(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*
a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*
b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*
a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*
c^5))^(1/3)) + 6*x)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 59, normalized size = 0.09

$$\frac{x}{c} + \frac{\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^3 b - a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{3c\left(2\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + \mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^6+b*x^3+a),x)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3
*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c} - \frac{\int \frac{bx^3+a}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 3.40, size = 2280, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
```

```
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

sympy [A] time = 6.90, size = 196, normalized size = 0.31

$$\mathrm{RootSum}\left(t^6\left(46656a^3c^7 - 34992a^2b^2c^6 + 8748ab^4c^5 - 729b^6c^4\right) + t^3\left(864a^3bc^3 - 864a^2b^3c^2 + 270ab^5c - 27b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
 x/c
```

3.145 $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log}{}$$

```
[Out] 1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/2
))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(
1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(
b-(-4*a*c+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/6*arctan(1
/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-4*a*c
+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)-1/6*ln(2^(1/3
)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(2/3)*2^(1
/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*
x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-4*a*c+b^2
)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*(1-2*2^(1/
3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b^2)^(1/2))^
(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c])
 - ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]
) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2
/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^
2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}}}{3\,2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b}}\right.}{3\,2^{2/3}c^{2/3}\sqrt{b^2-}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b}}\right.}{3\,2^{2/3}c^{2/3}\sqrt{b^2-}}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}} + \frac{(b}{}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^2\log(x-\#1)}{2\#1^3c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) & ]/3
```

fricas [B] time = 1.61, size = 3799, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^
2 - 4*a*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^
3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^
5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2))*
sqrt((2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2
)*x)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 1
2*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3)
 - 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(
-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^
4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3))/(a*b
```

```
^2 - 2*a^2*c))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b
^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c
^3))^(1/3) - (1/2)^(1/3)*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6
 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)*(-((b^2*c^2 - 4*
a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3) - sqrt(3)*(a*b^2 - 2
*a^2*c))/(a*b^2 - 2*a^2*c)) - 2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^
6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(
sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*(b
^4 - 6*a*b^2*c + 8*a^2*c^2))*sqrt((2*(a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (
b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b
)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*
c^2 - 4*a*c^3))^(1/3))/(a*b^2 - 2*a^2*c))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c
^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3) - (1/2)^(1/3)*(sqrt(3)*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^
2*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(
1/3) + sqrt(3)*(a*b^2 - 2*a^2*c))/(a*b^2 - 2*a^2*c)) - 1/6*(1/2)^(1/3)*(-((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-2*(
a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^
5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2
)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(-((b^2*c^
2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)) - 1/6*(1/2)^(
1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*
log(-2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)
*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) -
 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)) + 1/3*(
1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c
^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))
^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - (b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^
6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x)
 + 1/3*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/
```

```
(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a
*c^3))^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + (b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^
2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^2 - 4*
a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c
)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.08

$$\frac{\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^4 \ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + 3\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 8.11, size = 2695, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2
)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/2)*((b^5 + b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c))/18 +
 a^2*b*c*x)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
 - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3
 - 48*a^2*b^2*c^4)))^(1/3) + log((2^(1/3)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*
```

```
a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^
3))^(2/3))/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a
^2*b^2*c^2 + 9*a*b^4*c))/18 + a^2*b*c*x)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(
1/2)*1i - 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - 45*a^2*b^2*c^2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4
*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2
*(4*a*c - b^2)^3))^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4
)))^(1/3) + log((2^(1/3)*(3^(1/2)*1i + 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(
1/2)*1i - 1)*(54*a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i
+ 1)*(4*a*c - b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)
*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 +
12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*
c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^
2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c
^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3)
)^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5 - b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))
/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) + log((
2^(1/3)*(3^(1/2)*1i + 1)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3
)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(1/2)*1i - 1)*(54*
a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)*((3^(1/2)*1i)/2
- 1/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*
a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48
*a^2*b^2*c^4)))^(1/3)
```

sympy [A] time = 2.18, size = 175, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{3}c^{5} - 34992a^{2}b^{2}c^{4} + 8748ab^{4}c^{3} - 729b^{6}c^{2}\right) + t^{3}\left(-432a^{2}bc^{2} + 216ab^{3}c - 27b^{5}\right) + a^{2}, \left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2,
```

```
Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**4 +
 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*
_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

3.146 $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,\sqrt[3]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\,\log\left(-\right.}{}$$

```
[Out] -1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/
2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^
(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*
(b-(-4*a*c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-4*a*
c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)+1/6*ln(2^(1/
3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(1/3)*2^(
2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)
*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-4*a*c+b^
2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*(1-2*2^(1
/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b^2)^(1/2))
^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,\sqrt[3]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\,\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]
) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*
a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1
/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^
2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}}{3\sqrt[3]{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{}}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{}}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-}}$$

$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b}}{}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1\log(x-\#1)}{2\#1^3c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

fricas [B] time = 1.24, size = 2551, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*arctan(-1/
3*((1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4
)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - sqrt(3
)*(b^4 - 4*a*b^2*c))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3)*sqrt(-((1/2)^(
1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) - b
*x^2 - (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3))
/b) - sqrt(3)*a*b - (1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))*x - sqrt(3)*(b^4 - 4*a*b^2*c)*x)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
```

```
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))
^(2/3))/(a*b)) + 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))
^(1/3)*arctan(-1/3*((1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)) + sqrt(3)*(b^4 - 4*a*b^2*c))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2
/3)*sqrt(((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4
*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4*a*c^2)*s
qrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c
 - 4*a*c^2))^(2/3))/b) + sqrt(3)*a*b - (1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*x + sqrt(3)*(b^4 - 4*a*b^2*c)*x)*(-((b^2*c - 4*
a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1
)/(b^2*c - 4*a*c^2))^(2/3))/(a*b)) - 1/6*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sq
rt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c
- 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*
a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1
)/(b^2*c - 4*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c -
 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
+ 1)/(b^2*c - 4*a*c^2))^(2/3)) - 1/6*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4
*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c
^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(
b^2*c - 4*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4
*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) -
1)/(b^2*c - 4*a*c^2))^(2/3)) + 1/3*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2
/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*
c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sq
rt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c
- 4*a*c^2))^(1/3) + b*x) + 1/3*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2
))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqr
t(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c -
 4*a*c^2))^(1/3) + b*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.08

$$\frac{\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^3 \ln\left(-\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)+x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^5 c+3\,\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 7.71, size = 2129, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(2/3)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/
(c*(4*a*c - b^2)^3))^(1/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(
4*a*c - b^2)^2*(x - (2^(2/3)*b*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b*(-(4*a*c - b^2)^3)^(1
/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2))/6 + 3*a
*c^2*x*(2*a*c - b^2))*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) +
log((2^(2/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(x - (2^(2/3)*b
*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b
^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a
^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2)*((b*(-(4*a*c - b^2)^3)^(
1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/6 + 3*a*c^
2*x*(2*a*c - b^2))*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b
^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + lo
g((2^(2/3)*(3^(1/2)*1i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(3^(1/2)
*1i + 1)*(81*a*c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)
*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16
*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*(-(b*(-(4*a*c - b^2)^
3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3
*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2)
 + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3)))^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^
2*b*c^3 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
 b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c
 - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3)
)/36)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a
*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(
(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64
*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + log((2^(2/3)*(3^(1/2)*1
i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(81*a*c^3*x*(4*a*c - b^2)^2 -
 (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3
)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(3^(
1/2)*1i + 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(
c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*
c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*
((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8
*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3)
```

```
- log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (2^(1/3)*(81*
a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c
- b^2)^3))^(1/3))/4)*(3^(1/2)*1i - 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 -
3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/
2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)))^(1/3)
```

sympy [A] time = 1.80, size = 122, normalized size = 0.22

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^4 - 34992a^2b^2c^3 + 8748ab^4c^2 - 729b^6c\right) + t^3\left(432a^2c^2 - 216ab^2c + 27b^4\right) + a, \left(t \mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(_t
, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*b**
4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

3.147 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\right.}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{}}$$

```
[Out] -1/3*2^(1/3)*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-
(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)
*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^
(2/3))*2^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(1/3)*
c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(
1/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/3*2^(1/3)*c
^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c
^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(1/3)
/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/3*2^(1/3)*c^(1/3)*arctan
(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] time = 0.47, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\right.}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)
)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1
/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{\left(\sqrt[3]{2}\,c^{2/3}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}\,c^{2/3}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\,\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \ldots$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\ldots\right.}{3\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}}\ldots$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\ldots\right.}{3\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}}\ldots$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b\ldots}\right.}{3\sqrt{b^2-4a\ldots}}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

fricas [B] time = 1.34, size = 2875, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-1
/3*(2*sqrt(3)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(-((a*b^2 - 4*a^2*c)*s
qrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2
 - 4*a^2*c))^(1/3) - 2*sqrt(3)*(1/2)^(5/6)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*(-((a*b
^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*sqrt((2*b*c*x^2 + (1/2)^(2/3)*((a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-((a*b^2 - 4*a^2
```

```
*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(
a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c - (a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3))/(b*c)) + sqrt(3)*b)/b) +
2/3*sqrt(3)*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-1/3
*(2*sqrt(3)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(((a*b^2 - 4*a^2*c)*sqrt
(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 -
4*a^2*c))^(1/3) - 2*sqrt(3)*(1/2)^(5/6)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*
sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*(((a*b^2 -
 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))
- 1)/(a*b^2 - 4*a^2*c))^(1/3)*sqrt((2*b*c*x^2 - (1/2)^(2/3)*((a*b^6 - 12*a^
2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b^4 - 4*a*b^2*c)*x)*(((a*b^2 - 4*a^2*c)*s
qrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2
 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3))/(b*c)) - sqrt(3)*b)/b) - 1/6*(1
/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 + (1/2)^(2
/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-
((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c - (a*b^
5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)) - 1/6*(1/2
)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 - (1/2)^(2/3)
*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b^4 - 4*a*b^2*c)*x)*(((a*
b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)) + 1/3*(1/2)^(1/
3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4
- 4*a*b^2*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2
/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2
*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(
a*b^2 - 4*a^2*c))^(2/3)) + 1/3*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c)
)^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c + (a*b^6 - 12*a^2*b^4*c
+ 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 41, normalized size = 0.07

$$\frac{\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)+x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^5 c+3\,\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a),x)
[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 5.39, size = 1543, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6),x)
[Out] log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*
(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c
)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2)))^(1/3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((b*(-(4*
a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3
 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) - log(c^4*x - ((27*c^3*x*(b^4 + 8
*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^
2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c -
b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)
^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*
a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b*(-(4*a*
c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3
))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(54*a*(4
*a*c - b^2)^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4
+ 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2)))^(1/3) - log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (2
7*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^
(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(
4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^
3))*((3^(1/2)*1i)/2 + 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2
+ 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/
3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b
*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^
```

```
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)
*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/
(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3)
```

sympy [**A**] time = 1.53, size = 158, normalized size = 0.28

$$\text{RootSum}\left(t^6\left(46656a^4c^3 - 34992a^3b^2c^2 + 8748a^2b^4c - 729ab^6\right) + t^3\left(-432a^2c^2 + 216ab^2c - 27b^4\right) + c, \left(t \mapsto t\,\text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda(_
t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 2916*_
t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*
c + 9*_t**2*b**4)/(b*c))))
```

3.148 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}\right.}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\right.}$$

```
[Out] 1/3*2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-(
-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*
x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(
2/3))*2^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/3)*c
^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1
/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/3)*c^
(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^
(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/3*2^(2/3)*c^(2/3)*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/(
-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 0.60, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}\right.}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2
/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(2^{2/3}c\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(2^{2/\cdots\right.}{3\sqrt{b^2\cdots}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}\cdots$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}\cdots$$

$$= -\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\log\cdots}{3\sqrt{b^2\cdots}}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

fricas [B] time = 1.38, size = 3978, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b
^3*c + 8*a^2*b*c^2)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*
c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
- 4*a^3*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b
^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c
^2))*sqrt((2*(b^2*c^2 - 2*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^
```

```
2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*
b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c^2 - 2*a*c^3))
*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5
*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + 2*
sqrt(3)*(b^2*c - 2*a*c^2))/(b^2*c - 2*a*c^2)) - 2/3*sqrt(3)*(1/2)^(1/3)*(-(
(a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*arctan(
-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*
a^6*b^2*c^2 - 64*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-(
(a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)
^(1/6)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 6
4*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*sqrt((2*(b^2*c^2 - 2
*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 +
(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(
-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*
b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/
2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b
^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) -
 b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c^2 - 2*a*c^3))*(-((a^2*b^2 - 4*a^3*c)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - 2*sqrt(3)*(b^2*c - 2*a*c
^2))/(b^2*c - 2*a*c^2)) - 1/6*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c
^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)
^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b
^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/2)^(1/3)*((a^2*b^5*c
 - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*
b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 +
 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*
c))^(1/3)) - 1/6*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c +
 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a
^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)^(2/3)*(b^6
- 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^
4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4
 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^
3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^
5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)
*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12
*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
 + 1/3*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
```

```
/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*
a^3*c))^(1/3)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8
*a^2*c^2 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4
*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b
^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/3*(1/2)^
(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 -
12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3
)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 + (
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*
c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*
c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 40, normalized size = 0.07

$$\frac{\ln\left(-\operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{6\operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + 3\operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 8.49, size = 2597, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(6*c^5*x + (2^(2/3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)
*(36*a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1
/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
```

```
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b
^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x + (2
^(2/3)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2
*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b
^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a
*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c
 - b^2)^3))^(2/3))/2))/6)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3
- 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x - (2^(2/3)*(3^(1/2)*
1i - 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c -
2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4 - 3
6*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4
*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(2
/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) - log(6*c^5*x - (2^(2/3
)*(3^(1/2)*1i + 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36
*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*c^3*x*(4*a*c - b^2)^2
- (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c -
b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*
x - (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3)
)^(1/3)*(9*b^2*c^4 - 36*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*
c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^
2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) -
 log(6*c^5*x - (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*
c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*
c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^
2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)))^(1/3)
```

sympy [A] time = 4.35, size = 155, normalized size = 0.28

$$\operatorname{RootSum}\left(t^6\left(46656a^5c^3-34992a^4b^2c^2+8748a^3b^4c-729a^2b^6\right)+t^3\left(432a^2bc^2-216ab^3c+27b^5\right)+c^2,\left(t\mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2, L
ambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c - 81*
_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 -
 b**2*c))))
```

3.149 $\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=610

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\cdots\right.}{}$$

```
[Out] -1/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(1+b/
(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(
2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a
*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1
/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1
/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(
1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/
3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*
a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] time = 0.82, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^
2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2
- 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{ax} + \frac{\int \frac{x\left(-b-cx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{1}{ax} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{1}{ax} + \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt[3]{b-}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}}}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2}}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\right.}{3\,2^{2/3}a\sqrt[3]{}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\right.}{3\,2^{2/3}a\sqrt[3]{}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a} - \frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

fricas [B] time = 2.37, size = 5266, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*a
rctan(1/3*((1/2)^(5/6)*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 -
 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c
```

```
+ 18*a^2*b^2*c^2 - 8*a^3*c^3))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*s
qrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8 - 13
*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2
 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(
2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a
^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
+ 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sq
rt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/
3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)) - (1/2)^(1/3)*(sqrt(3)*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*x)*((b^3 -
2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) + sqrt(3)*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c
^3))/(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)) - 4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 -
 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^1
1*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*arctan(1/3*((1/2)^(5/6)*(sqrt(3)*(a^4*b
^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3))*((b^3 -
 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^1
1*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*sqrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*
c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*
b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2
*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3))
 + (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(
(b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^
2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2
*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b
^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64
*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5
)) - (1/2)^(1/3)*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18
*a^2*b^2*c^2 - 8*a^3*c^3)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) - s
qrt(3)*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3))/(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)
) - (1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c
^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c +
60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
```

```
0*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3
*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^
(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a^4*b^6*c - 10
*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*
b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2
*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) - (1/2)^(
1/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c^3 - 4*a*b^
2*c^4 + 2*a^2*c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
 64*a^11*c^3)) + (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a
^4*b*c^4)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*
c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4*c^
2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) + 2*(1/2)^(1/3)*a*x*
((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*
c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 - (a^4*b^8 - 13*a^5*b^6*
c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2
*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) + 2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c
 - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))
/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*
c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 6
4*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a
*b^2*c^4 + 2*a^2*c^5)*x) - 6)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

maple [C] time = 0.01, size = 61, normalized size = 0.10

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^4 c+\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right) b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)+x\right)}{3a\left(2\,\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^5 c+\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^2 b\right)}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*
b+a))-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx^4+bx}{cx^6+bx^3+a}\,dx}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

mupad [B] time = 6.89, size = 2978, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b
^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*a*
c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^
3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*a*c -
 b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/
3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/
3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3
*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c -
 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*((b^4
*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(a^4*(4*a*c - b^2)^3))^(1/3))/6)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - 1/(a*x) + log(36*a^9*c^6 + 9*a^7*b^4*c
^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(b^6 - 8*a^3*
c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)
*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-(4
*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a
*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^7 + b^4*(-(4*a*c - b^2)
```

```
^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^
7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b^4
*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(b^6 - 8*a^
3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i -
1)*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 3
2*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-
(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*
a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b
^4*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*
b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2
)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i + 1)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*
b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2
*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^
2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i - 1)*((b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(
4*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)^3
)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 6
4*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
```

sympy [A] time = 3.19, size = 252, normalized size = 0.41

$$\operatorname{RootSum}\left(t^6\left(46656a^7c^3-34992a^6b^2c^2+8748a^5b^4c-729a^4b^6\right)+t^3\left(-864a^3bc^3+864a^2b^3c^2-270ab^5c+27b^7\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**5*
c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 27216*
_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c
- 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c**3 +
378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 -
 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

3.150 $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=612

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\Big(}{}$$

```
[Out] -1/2/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(
1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)
*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(
-4*a*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2
)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2
))^(1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1
/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))
^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+
(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+
(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 0.82, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\Big(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(2*a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^
(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b - S
qrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sq
rt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[
b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])
^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*
x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^
2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4
*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{2ax^2}+\frac{\int \frac{-2b-2cx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+c\ldots}}{2a}$$

$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b\ldots}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}\ldots}}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-\ldots}\right.}$$

$$= -\frac{1}{2ax^2}-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\ldots}{3\sqrt[3]{2}\ldots}$$

$$= -\frac{1}{2ax^2}-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\ldots}{3\sqrt[3]{2}\ldots}$$

$$= -\frac{1}{2ax^2}+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\ldots}{\sqrt[3]{b+\ldots}}}{\ldots}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b+\sqrt{b^2-4ac}\right)\ldots}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x -
#1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

fricas [B] time = 2.63, size = 5771, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 -
4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^2 - 4*a^6*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^8 - 13*a^
6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*x*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*
```

a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3)*sqrt((2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)) + 2*sqrt(3)*(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5))/(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)) - 4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3)*sqrt((2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2

```
- 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2
*b*c^6)) - 2*sqrt(3)*(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5))/(b^5*c^3 - 5*a*
b^3*c^4 + 5*a^2*b*c^5)) - (1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 +
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c
^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^
6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3
+ 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2
 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^
2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*s
qrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a
^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4
- 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3
*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b
^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) - (1/2)^(1/3)*a*x^2
*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^
5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 6
3*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9
 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c +
 a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^6*c^2 -
10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4
 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sq
rt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^
6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2
- 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a
^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a
^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + (1
/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (a^5*b^6 - 1
0*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2
*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a
^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^
2 - 4*a^6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (
a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^
3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4
)*x + (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 + (a^5
*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b
^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 2
5*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
/(a^5*b^2 - 4*a^6*c))^(1/3)) - 3)/(a*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

maple [C] time = 0.00, size = 62, normalized size = 0.10

$$\frac{\left(-\operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^3 c - b\right)\ln\left(-\operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{3a\left(2\operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + \operatorname{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b\right)} - \frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a),x)
[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a
))-1/2/a/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}\,dx}{a} - \frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] -integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

mupad [B] time = 10.65, size = 4063, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)),x)
[Out] log((2^(2/3)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4
*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b
*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^1
0*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3
))/2)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 -
56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^
3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*
c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 +
 b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*
c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/
2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) + lo
g((2^(2/3)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c
^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b*c
^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
```

```
b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))
/2)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56
*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*
c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*c^
4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 - b
^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c
- 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)
)/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) - 1/(2
*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c -
 b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2
*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
 - b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)
*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a
^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*
c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*
a*c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2
)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/
3))/36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16
*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 -
 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b
^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -
 b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(
3^(1/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^1
0*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*
c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3)
)/36))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2
)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b
^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)*(3^
(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41
*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/
36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4
*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*
a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^
3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3
) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2
)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(3^(1
```

```
/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^10*b*
c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^
2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -
b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a
^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36
))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c
^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*
b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)
^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3)
```

sympy [A] time = 52.50, size = 241, normalized size = 0.39

$$\mathrm{RootSum}\left(t^6\left(46656a^8c^3-34992a^7b^2c^2+8748a^6b^4c-729a^5b^6\right)+t^3\left(-432a^4c^4+1512a^3b^2c^3-1107a^2b^4c^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**8*c**3 - 34992*a**7*b**2*c**2 + 8748*a**6*b**4*c -
729*a**5*b**6) + _t**3*(-432*a**4*c**4 + 1512*a**3*b**2*c**3 - 1107*a**2*b*
*4*c**2 + 297*a*b**6*c - 27*b**8) + c**5, Lambda(_t, _t*log(x + (-2592*_t**
4*a**8*c**3 + 2592*_t**4*a**7*b**2*c**2 - 810*_t**4*a**6*b**4*c + 81*_t**4*
a**5*b**6 + 12*_t*a**4*c**4 - 75*_t*a**3*b**2*c**3 + 78*_t*a**2*b**4*c**2 -
 27*_t*a*b**6*c + 3*_t*b**8)/(5*a**2*b*c**4 - 5*a*b**3*c**3 + b**5*c**2))))
 - 1/(2*a*x**2)
```

3.151 $\int \frac{x^{11}}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=35

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

```
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 701, 632, 31}

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{3+4x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{3+4x+x^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-4+x+\frac{12+13x}{3+4x+x^2}\right)dx,x,x^3\right)\\ &= -\frac{4x^3}{3}+\frac{x^6}{6}+\frac{1}{3}\,\text{Subst}\left(\int \frac{12+13x}{3+4x+x^2}\,dx,x,x^3\right)\\ &= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)+\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\ &= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(3 + 4*x^3 + x^6),x]
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

fricas [A] time = 1.09, size = 27, normalized size = 0.77

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

giac [A] time = 0.39, size = 29, normalized size = 0.83

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{\ln\left(x^3+1\right)}{6}+\frac{9\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6+4*x^3+3),x)
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

maxima [A] time = 0.49, size = 27, normalized size = 0.77

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

mupad [B] time = 1.25, size = 27, normalized size = 0.77

$$\frac{9\,\ln\left(x^3+3\right)}{2}-\frac{\ln\left(x^3+1\right)}{6}-\frac{4\,x^3}{3}+\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(4*x^3 + x^6 + 3),x)
[Out] (9*log(x^3 + 3))/2 - log(x^3 + 1)/6 - (4*x^3)/3 + x^6/6
```

sympy [A] time = 0.13, size = 29, normalized size = 0.83

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{9\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**6+4*x**3+3),x)
[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

3.152 $\int \frac{x^8}{3+4x^3+x^6} dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

```
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{3+4x+x^2}\,dx,x,x^3\right)$$
$$= \frac{x^3}{3} + \frac{1}{3}\text{Subst}\left(\int \frac{-3-4x}{3+4x+x^2}\,dx,x,x^3\right)$$
$$= \frac{x^3}{3} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)$$
$$= \frac{x^3}{3} + \frac{1}{6}\log\left(1+x^3\right) - \frac{3}{2}\log\left(3+x^3\right)$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(3 + 4*x^3 + x^6),x]
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

fricas [A] time = 1.06, size = 22, normalized size = 0.79

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

giac [A] time = 0.34, size = 24, normalized size = 0.86

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{x^3}{3} + \frac{\ln\left(x^3+1\right)}{6} - \frac{3\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+4*x^3+3),x)
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

maxima [A] time = 0.49, size = 22, normalized size = 0.79

$$\frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

mupad [B] time = 0.05, size = 22, normalized size = 0.79

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{3\,\ln\left(x^3+3\right)}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3
```

sympy [A] time = 0.12, size = 22, normalized size = 0.79

$$\frac{x^3}{3}+\frac{\log\left(x^3+1\right)}{6}-\frac{3\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+4*x**3+3),x)
[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

3.153 $\int \frac{x^5}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

```
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{3+4x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{3+4x+x^2}\,dx,x,x^3\right)\\ &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\ &= -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

fricas [A] time = 0.92, size = 17, normalized size = 0.81

$$\frac{1}{2}\,\log\left(x^{3}+3\right)-\frac{1}{6}\,\log\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

giac [A] time = 0.36, size = 19, normalized size = 0.90

$$\frac{1}{2}\,\log\left(\left|x^{3}+3\right|\right)-\frac{1}{6}\,\log\left(\left|x^{3}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln\left(x^{3}+1\right)}{6}+\frac{\ln\left(x^{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+4*x^3+3),x)
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

maxima [A] time = 0.52, size = 17, normalized size = 0.81

$$\frac{1}{2}\,\log\left(x^{3}+3\right)-\frac{1}{6}\,\log\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.81

$$\frac{\ln\left(x^{3}+3\right)}{2}-\frac{\ln\left(x^{3}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 3)/2 - log(x^3 + 1)/6
```

sympy [A] time = 0.12, size = 15, normalized size = 0.71

$$-\frac{\log\left(x^{3}+1\right)}{6}+\frac{\log\left(x^{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+4*x**3+3),x)
[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

3.154 $\int \frac{x^2}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\tanh^{-1}\left(x^3+2\right)$$

```
[Out] -1/3*arctanh(x^3+2)
```

Rubi [B] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 2.10, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{3+4x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{3+4x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1+x^3\right)-\frac{1}{6}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 21, normalized size = 2.10

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

fricas [B] time = 1.13, size = 17, normalized size = 1.70

$$-\frac{1}{6}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

giac [B] time = 0.33, size = 19, normalized size = 1.90

$$-\frac{1}{6}\,\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
[Out] -1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

maple [B] time = 0.00, size = 18, normalized size = 1.80

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+4*x^3+3),x)
[Out] 1/6*ln(x^3+1)-1/6*ln(x^3+3)
```

maxima [B] time = 0.47, size = 17, normalized size = 1.70

$$-\frac{1}{6}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

mupad [B] time = 0.38, size = 16, normalized size = 1.60

$$\frac{\operatorname{atanh}\left(\frac{9}{2\,\left(8\,x^3+6\right)}+\frac{5}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^3 + x^6 + 3),x)
[Out] atanh(9/(2*(8*x^3 + 6)) + 5/4)/3
```

sympy [A] time = 0.11, size = 15, normalized size = 1.50

$$\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+4*x**3+3),x)
[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

3.155 $\int \frac{1}{x(3+4x^3+x^6)} \, dx$

Optimal. Leaf size=27

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

```
[Out] 1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\ &= \frac{1}{9}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{3+4x+x^2}\,dx,x,x^3\right)\\ &= \frac{\log(x)}{3}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)\\ &= \frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{18}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

fricas [A] time = 1.05, size = 21, normalized size = 0.78

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

giac [A] time = 0.35, size = 24, normalized size = 0.89

$$\frac{1}{18}\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\log\left(\left|x^3+1\right|\right)+\frac{1}{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

maple [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^3+3\right)}{18}-\frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+4*x^3+3),x)
```

```
[Out] 1/3*ln(x)-1/6*ln(x+1)+1/18*ln(x^3+3)-1/6*ln(x^2-x+1)
```

maxima [A] time = 0.45, size = 23, normalized size = 0.85

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{9}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

mupad [B] time = 1.26, size = 21, normalized size = 0.78

$$\frac{\ln\left(x^3+3\right)}{18}-\frac{\ln\left(x^3+1\right)}{6}+\frac{\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3
```

sympy [A] time = 0.14, size = 20, normalized size = 0.74

$$\frac{\log\left(x\right)}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+4*x**3+3),x)
```

```
[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

3.156 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

```
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 709, 800}

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{9x^3}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{9x^3}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x}+\frac{3}{2(1+x)}-\frac{1}{6(3+x)}\right)dx,x,x^3\right)\\
&= -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

fricas [A] time = 0.90, size = 35, normalized size = 1.03

$$-\frac{x^3\log\left(x^3+3\right)-9\,x^3\log\left(x^3+1\right)+24\,x^3\log(x)+6}{54\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

giac [A] time = 0.30, size = 36, normalized size = 1.06

$$\frac{4\,x^3-3}{27\,x^3}-\frac{1}{54}\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\log\left(\left|x^3+1\right|\right)-\frac{4}{9}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/9
*log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 1.06

$$-\frac{4\ln(x)}{9}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^3+3\right)}{54}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+4*x^3+3),x)
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x+1)-1/54*ln(x^3+3)+1/6*ln(x^2-x+1)
```

maxima [A] time = 0.68, size = 28, normalized size = 0.82

$$-\frac{1}{9\,x^3}-\frac{1}{54}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)-\frac{4}{27}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

mupad [B] time = 1.23, size = 26, normalized size = 0.76

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{54}-\frac{4\,\ln(x)}{9}-\frac{1}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)
```

sympy [A] time = 0.17, size = 29, normalized size = 0.85

$$-\frac{4\log(x)}{9}+\frac{\log(x^3+1)}{6}-\frac{\log(x^3+3)}{54}-\frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+4*x**3+3),x)
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

3.157 $\int \frac{1}{x^7(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

```
[Out] -1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 709, 800}

$$\frac{4}{27x^3}-\frac{1}{18x^6}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x^2\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x^2}+\frac{13}{9x}-\frac{3}{2(1+x)}+\frac{1}{18(3+x)}\right)dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

fricas [A] time = 0.95, size = 40, normalized size = 0.98

$$\frac{x^6\log\left(x^3+3\right)-27\,x^6\log\left(x^3+1\right)+78\,x^6\log(x)+24\,x^3-9}{162\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9)
/x^6
```

giac [A] time = 0.37, size = 41, normalized size = 1.00

$$-\frac{13\,x^6-8\,x^3+3}{54\,x^6}+\frac{1}{162}\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\log\left(\left|x^3+1\right|\right)+\frac{13}{27}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
+ 1)) + 13/27*log(abs(x))
```

maple [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^3+3\right)}{162}-\frac{\ln\left(x^2-x+1\right)}{6}+\frac{4}{27x^3}-\frac{1}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+4*x^3+3),x)
```

```
[Out] -1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x+1)+1/162*ln(x^3+3)-1/6*ln(x^2-x+1)
```

maxima [A] time = 0.59, size = 35, normalized size = 0.85

$$\frac{8\,x^3-3}{54\,x^6}+\frac{1}{162}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{13}{81}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x^
3)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.78

$$\frac{\ln\left(x^3+3\right)}{162}-\frac{\ln\left(x^3+1\right)}{6}+\frac{13\,\ln(x)}{27}+\frac{\frac{4\,x^3}{27}-\frac{1}{18}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 3)/162 - log(x^3 + 1)/6 + (13*log(x))/27 + ((4*x^3)/27 - 1/18)/x^
6
```

sympy [A] time = 0.19, size = 34, normalized size = 0.83

$$\frac{13\log(x)}{27} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{162} + \frac{8x^3-3}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+4*x**3+3),x)
```

```
[Out] 13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6)
```

3.158 $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=124

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}$$

```
[Out] -2*x^2+1/5*x^5-9/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-3/
2*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x
^2)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1367, 1502, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[
(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2
 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{x^5}{5}-\frac{1}{5}\int \frac{x^4\left(15+20x^3\right)}{3+4x^3+x^6}\,dx$$
$$= -2x^2+\frac{x^5}{5}+\frac{1}{10}\int \frac{x\left(120+130x^3\right)}{3+4x^3+x^6}\,dx$$
$$= -2x^2+\frac{x^5}{5}-\frac{1}{2}\int \frac{x}{1+x^3}\,dx+\frac{27}{2}\int \frac{x}{3+x^3}\,dx$$
$$= -2x^2+\frac{x^5}{5}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{1}{2}\left(3\,3^{2/3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx+\frac{1}{2}\left(3\,3^{2/3}\right)\int$$
$$= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx$$
$$= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\right.$$
$$= -2x^2+\frac{x^5}{5}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.95

$$\frac{1}{60}\left(12x^5-120x^2-5\log\left(x^2-x+1\right)+45\,3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+10\log(x+1)-90\,3^{2/3}\log\left(3^{2/3}x+3\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt[
3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3)*
x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/60
```

fricas [A] time = 0.82, size = 102, normalized size = 0.82

$$\frac{1}{5}x^5-2x^2+\frac{3}{2}\sqrt{3}\,(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\,(-9)^{\frac{1}{3}}x+3\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{4}(-9)^{\frac{1}{3}}\log\left(3x^2-(-9)^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/2*sqrt(3)*(-9)^(1/3)*arctan(1/9*sqrt(3)*(2*(-9)^(1/3)*x
 + 3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*(-9)^(1/3)*log(3*x
^2 - (-9)^(2/3)*x - 3*(-9)^(1/3)) + 3/2*(-9)^(1/3)*log(3*x + (-9)^(2/3)) -
1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.37, size = 96, normalized size = 0.77

$$\frac{1}{5}x^5-2x^2+\frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(abs(x + 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(
1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(
abs(x + 1))
```

maple [A] time = 0.01, size = 94, normalized size = 0.76

$$\frac{x^5}{5}-2x^2+\frac{9\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}-\frac{3\,3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}+\frac{3\,3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6+4*x^3+3),x)
```

```
[Out] 1/5*x^5-2*x^2+1/6*ln(x+1)-3/2*3^(2/3)*ln(3^(1/3)+x)+3/4*3^(2/3)*ln(3^(2/3)-
3^(1/3)*x+x^2)+9/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^
2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.23, size = 94, normalized size = 0.76

$$\frac{1}{5}x^5-2x^2+\frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6)*
arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
)
```

mupad [B] time = 0.24, size = 124, normalized size = 1.00

$$\frac{\ln(x+1)}{6}-\frac{3\,3^{2/3}\ln\left(x+3^{1/3}\right)}{2}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-2x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 -
((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)^(1/
3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2
```

sympy [C] time = 0.62, size = 144, normalized size = 1.16

$$\frac{x^5}{5}-2x^2+\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{3872\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5}{3281}+\frac{3188648\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2}{88587}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/12
 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1/1
2 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872*(-
1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*log(38
72*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

3.159 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=122

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}$$

```
[Out] -4*x+1/4*x^4-3/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+3/2*
3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1367, 1502, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3
^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 +
 Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{3+4x^3+x^6}\,dx &= \frac{x^4}{4}-\frac{1}{4}\int \frac{x^3\left(12+16x^3\right)}{3+4x^3+x^6}\,dx\\
&= -4x+\frac{x^4}{4}+\frac{1}{4}\int \frac{48+52x^3}{3+4x^3+x^6}\,dx\\
&= -4x+\frac{x^4}{4}-\frac{1}{2}\int \frac{1}{1+x^3}\,dx+\frac{27}{2}\int \frac{1}{3+x^3}\,dx\\
&= -4x+\frac{x^4}{4}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx+\frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{}{3^{2/}}\\
&= -4x+\frac{x^4}{4}-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx\\
&= -4x+\frac{x^4}{4}-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{}\right.\\
&= -4x+\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\end{aligned}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 0.93

$$\frac{1}{12}\left(3x^4+\log\left(x^2-x+1\right)-9\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-48x-2\log(x+1)+18\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-18\,3^{5/6}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] + Lo
g[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

fricas [A] time = 0.99, size = 90, normalized size = 0.74

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x-\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

giac [A] time = 0.43, size = 94, normalized size = 0.77

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(ab
s(x + 1))
```

maple [A] time = 0.01, size = 92, normalized size = 0.75

$$\frac{x^4}{4}-4x+\frac{3\,3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3\,3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}-\frac{3\,3^{\frac{1}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6+4*x^3+3),x)
```

```
[Out] 1/4*x^4-4*x-1/6*ln(x+1)+3/2*3^(1/3)*ln(x+3^(1/3))-3/4*3^(1/3)*ln(x^2-3^(1/3
)*x+3^(2/3))+3/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-
x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.33, size = 92, normalized size = 0.75

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.42, size = 119, normalized size = 0.98

$$\frac{3\,3^{1/3}\ln\left(x+3^{1/3}\right)}{2}-\frac{\ln(x+1)}{6}-4\,x+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3*3^(1/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - 4*x + log(x - (3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/12 - 1/12) + x^4/4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*((3*3^(1/3))/
4 + (3^(5/6)*3i)/4) + 3^(1/3)*log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*((3^(1/2)
*3i)/4 - 3/4)
```

sympy [C] time = 0.61, size = 129, normalized size = 1.06

$$\frac{x^4}{4}-4x-\frac{\log(x+1)}{6}+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{9841}{19692}-\frac{9841\sqrt{3}\,i}{19692}+\frac{360\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4}{547}\right)+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{984}{1969}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 98
41*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*I/
12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)*I/
19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_t/16
41 + x)))
```

3.160 $\int \frac{x^7}{3+4x^3+x^6} dx$

Optimal. Leaf size=119

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan$$

```
[Out] 1/2*x^2+3/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/2*3^(2/
3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6
*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1367, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 -
x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{3+4x^3+x^6}\,dx &= \frac{x^2}{2}-\frac{1}{2}\int \frac{x\left(6+8x^3\right)}{3+4x^3+x^6}\,dx\\
&= \frac{x^2}{2}+\frac{1}{2}\int \frac{x}{1+x^3}\,dx-\frac{9}{2}\int \frac{x}{3+x^3}\,dx\\
&= \frac{x^2}{2}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{1}{2}3^{2/3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}3^{2/3}\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+\cdots}\\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{9}{4}\cdots\\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+\cdots\right.\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\cdots
\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.93

$$\frac{1}{12}\left(6x^2+\log\left(x^2-x+1\right)-3\,3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+6\,3^{2/3}\log\left(3^{2/3}x+3\right)+18\sqrt[6]{3}\tan\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x +
 x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

fricas [A] time = 1.12, size = 99, normalized size = 0.83

$$\frac{1}{2}x^2-\frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}x-\frac{1}{3}\sqrt{3}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{2}\cdot 9^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3)) +
 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2/3
)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1) -
 1/6*log(x + 1)
```

giac [A] time = 0.34, size = 91, normalized size = 0.76

$$\frac{1}{2}x^2-\frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(abs(
x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arc
tan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x +
1))
```

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{x^2}{2}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{4}+\frac{\ln(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+4*x^3+3),x)
```

```
[Out] 1/2*x^2-1/6*ln(x+1)+1/2*3^(2/3)*ln(x+3^(1/3))-1/4*3^(2/3)*ln(x^2-3^(1/3)*x+
3^(2/3))-3/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)
+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.31, size = 89, normalized size = 0.75

$$\frac{1}{2}x^2-\frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 0.19, size = 118, normalized size = 0.99

$$\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{2}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{x^2}{2}-\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/4 - (3^(1/6)
*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/4 + (3^(1/6)*3i)/4)
```

sympy [C] time = 0.61, size = 134, normalized size = 1.13

$$\frac{x^2}{2}-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{6562\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2}{183}-\frac{1872\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1872\left(\frac{1}{12}\right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)*
I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*lo
g(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/183)
 + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/183 +
 x)))
```

3.161 $\int \frac{x^6}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan$$

```
[Out] x+1/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/2*3^(1/3)*ln(
3^(1/3)+x)-1/12*ln(x^2-x+1)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1367, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)
/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^
2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{3+4x^3+x^6}\,dx &= x-\int \frac{3+4x^3}{3+4x^3+x^6}\,dx\\
&= x+\frac{1}{2}\int \frac{1}{1+x^3}\,dx-\frac{9}{2}\int \frac{1}{3+x^3}\,dx\\
&= x+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x\ldots}\\
&= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\ldots\\
&= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+\ldots\right.\\
&= x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\text{lo}\ldots
\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.98

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+12x+2\log(x+1)-6\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+6\,3^{5/6}\tan\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x
^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

fricas [A] time = 1.13, size = 88, normalized size = 0.78

$$\frac{1}{2}\sqrt{3}\left(-3\right)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(-3\right)^{\frac{2}{3}}x-3\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{4}\left(-3\right)^{\frac{1}{3}}\log\left(x^{2}+\left(-3\right)^{\frac{1}{3}}x+\left(-3\right)^{\frac{2}{3}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(-3)^(1/3)*arctan(1/9*sqrt(3)*(2*(-3)^(2/3)*x - 3)) + 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x +
(-3)^(2/3)) + 1/2*(-3)^(1/3)*log(x - (-3)^(1/3)) + x - 1/12*log(x^2 - x + 1
) + 1/6*log(x + 1)
```

giac [A] time = 0.45, size = 87, normalized size = 0.77

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(abs(x + 3^(1/3))) + x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 85, normalized size = 0.75

$$x-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x+1\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}+\frac{3^{\frac{1}{3}}\ln\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{4}-\frac{\ln\left(x^{2}-\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+4*x^3+3),x)
```

```
[Out] x+1/6*ln(x+1)-1/2*3^(1/3)*ln(x+3^(1/3))+1/4*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3
))-1/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+1)+1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.23, size = 85, normalized size = 0.75

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 0.16, size = 104, normalized size = 0.92

$$x+\frac{\ln\left(x+1\right)}{6}-\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{2}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(4*x^3 + x^6 + 3),x)
```

```
[Out] x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*1i)
/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2
```

sympy [C] time = 0.61, size = 126, normalized size = 1.12

$$x+\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{121}{246}-\frac{121\sqrt{3}\,i}{246}+\frac{864\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{121}{246}+\frac{86}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/2
46 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 121
/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8*_t
**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

3.162 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

```
[Out] -1/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/6*3^(2/3)*ln(3
^(1/3)+x)-1/12*ln(x^2-x+1)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1374, 292, 31, 634, 617, 204, 628, 618}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(
5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/1
2 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{3+4x^3+x^6}\,dx &= -\left(\frac{1}{2}\int \frac{x}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{x}{3+x^3}\,dx\\ &= \frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\sqrt[3]{3}}\\ &= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{4}\int \frac{}{3^{2/3}-}\\ &= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}+\frac{1}{2}\text{Subst}\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+2\log(x+1)-2\,3^{2/3}\log\left(3^{2/3}x+3\right)-6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}}{3}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] +
3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

fricas [A] time = 0.94, size = 106, normalized size = 0.95

$$-\frac{1}{12}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/12*3^(2/3)*(-1)^(1/3)*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)^(1/
3)) + 1/6*3^(2/3)*(-1)^(1/3)*log(3^(1/3)*(-1)^(2/3) + x) - 1/6*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*(-1)^(1/3)*arctan(1/3*3^(1/6)*(2*(-
1)^(1/3)*x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.30, size = 86, normalized size = 0.77

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(abs(x + 3^(1/
3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{6} + \frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{12} - \frac{\ln\left(x^2-x\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x+1)-1/6*3^(2/3)*ln(x+3^(1/3))+1/12*3^(2/3)*ln(x^2-3^(1/3)*x+3^(2/3)
)+1/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+1)-1/6*3^
(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.61, size = 84, normalized size = 0.75

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1/6
)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 1.37, size = 114, normalized size = 1.02

$$\frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1i}{12}\right) - \frac{(-1)^{1/3}\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/6))
/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/12 + ((-1)^(1/3)*3^(2/3)*log(x + (-
1)^(2/3)*3^(1/3)))/6
```

sympy [C] time = 0.60, size = 134, normalized size = 1.20

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5}{5}+\frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2}{5}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{168\left(-\frac{1}{1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)**
5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 168
*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootSum(
24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

3.163 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

```
[Out] -1/6*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/6*3^(1/3)*ln(3
^(1/3)+x)+1/12*ln(x^2-x+1)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1374, 200, 31, 634, 617, 204, 628, 618}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*
3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/1
2 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{3+4x^3+x^6}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{1}{3+x^3}\,dx\\ &= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right)-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\,3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\,3^{2/3}}\\ &= -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\,3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+}}{4\,3^{2/3}}\\ &= -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\,3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\,3^{2/3}}+\frac{1}{2}\text{Subs}\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\,3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\end{aligned}$$

Mathematica [A] time = 0.02, size = 106, normalized size = 0.95

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-}{3^{5/6}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] -
3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

fricas [A] time = 1.12, size = 102, normalized size = 0.91

$$\frac{1}{6}\cdot 9^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{36}\cdot 9^{\frac{2}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{18}\cdot 9^{\frac{2}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)-\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/6*9^(1/6)*sqrt(3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*sq
rt(3))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*lo
g(3*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(x^2
 - x + 1) - 1/6*log(x + 1)
```

giac [A] time = 0.34, size = 86, normalized size = 0.77

$$\frac{1}{6}\cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3} \arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x - 1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(abs(x + 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{\frac{1}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{6} - \frac{3^{\frac{1}{3}} \ln\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right)}{12} + \frac{\ln\left(x^2 - x\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x+1)+1/6*3^(1/3)*ln(x+3^(1/3))-1/12*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3
))+1/6*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)-1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.71, size = 84, normalized size = 0.75

$$\frac{1}{6}\cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3} \arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x - 1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{1/3}\ \ln\left(x + 3^{1/3}\right)}{6} - \frac{\ln(x+1)}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\ 1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\ 1\mathrm{i}}{12}\right) - \ln\left(x - \frac{3^{1}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/6 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
- 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/12 + (3^(5/6)*1i)/12
) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/12 - (3^(5/6)*1i)/12)
```

sympy [C] time = 0.59, size = 110, normalized size = 0.98

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4+\frac{\sqrt{3}\,i}{4}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/1
2)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sqrt
(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(648*_
t**4 - 3*_t + x)))
```

3.164 $\int \frac{x}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

```
[Out] 1/6*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/18*3^(2/3)*ln(3
^(1/3)+x)+1/12*ln(x^2-x+1)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1375, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2
*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/
12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{3+4x^3+x^6}\,dx &= \frac{1}{2}\int \frac{x}{1+x^3}\,dx - \frac{1}{2}\int \frac{x}{3+x^3}\,dx\\
&= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) + \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\sqrt[3]{3}}\\
&= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{}{3^{2/3}-}\\
&= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}} - \frac{1}{2}\text{Subs}\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\,3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) -
\end{aligned}$$

Mathematica [A] time = 0.03, size = 108, normalized size = 0.96

$$\frac{1}{36}\left(3\log\left(x^2-x+1\right) - 3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right) - 6\log(x+1) + 2\,3^{2/3}\log\left(3^{2/3}x+3\right) + 6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}}{3^{5}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2] -
 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

fricas [A] time = 1.02, size = 84, normalized size = 0.75

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3^(
1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

giac [A] time = 0.36, size = 86, normalized size = 0.77

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(abs(x + 3^(
1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

maple [A] time = 0.00, size = 84, normalized size = 0.75

$$-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{36}+\frac{\ln\left(x^2-x\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x+1)+1/18*3^(2/3)*ln(x+3^(1/3))-1/36*3^(2/3)*ln(x^2-3^(1/3)*x+3^(2/
3))-1/6*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)+1/6*
3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.60, size = 84, normalized size = 0.75

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{18}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/18 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/36 - (3^(1/6)*1i)/1
2) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/36 + (3^(1/6)*1i)/12)
```

sympy [C] time = 1.86, size = 119, normalized size = 1.06

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+11664\left(\frac{1}{12}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2 +
 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1/1
2 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3 - 1
, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

3.165 $\int \frac{1}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

```
[Out] 1/18*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/18*3^(1/3)*ln(
3^(1/3)+x)-1/12*ln(x^2-x+1)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arct
an(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1347, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6
*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/
12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3+4x^3+x^6}\,dx &= \frac{1}{2}\int \frac{1}{1+x^3}\,dx - \frac{1}{2}\int \frac{1}{3+x^3}\,dx\\
&= \frac{1}{6}\int \frac{1}{1+x}\,dx + \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx - \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\,3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\,3^{2/3}}\\
&= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\,3^{2/3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x}}{12\,3^{2/3}}\\
&= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\,3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\,3^{2/3}} - \frac{1}{2}\mathrm{Subst}\Big(\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\,3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) +
\end{aligned}$$

Mathematica [A] time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{36}\left(-3\log\left(x^2-x+1\right)+\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+6\log(x+1)-2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+2\,3^{5/6}\tan^{-1}\left(\sqrt[3]{3}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2] +
 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

fricas [A] time = 1.00, size = 124, normalized size = 1.11

$$\frac{1}{18}\cdot 9^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{108}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x+3x^2+3\cdot 9^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*9^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*(-1)
^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 1/108*9^(2/3)*(-1)^(1/3)*log(9^(2/3)*(-1)^
(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 1/54*9^(2/3)*(-1)^(1/3)*log(-9^(2
/3)*(-1)^(1/3) + 3*x) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*lo
g(x^2 - x + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.34, size = 86, normalized size = 0.77

$$-\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{18}+\frac{3^{\frac{1}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{36}-\frac{\ln\left(x^2-x\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x+1)-1/18*3^(1/3)*ln(x+3^(1/3))+1/36*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3
))-1/18*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+1)+1/6*
3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.28, size = 84, normalized size = 0.75

$$-\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 0.23, size = 110, normalized size = 0.98

$$\frac{\ln(x+1)}{6}-\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{18}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\frac{(-1)^{1/3}\,3^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
```

```
 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(1/3
)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^
(5/6)*1i))/36
```

sympy [C] time = 1.82, size = 124, normalized size = 1.11

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{13}{10}-\frac{13\sqrt{3}\,i}{10}+\frac{23328\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{4}}{5}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{13}{10}+\frac{23328}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 233
28*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 + 23
328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3 + 1
, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

3.166 $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}$$

```
[Out] -1/3/x-1/18*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/54*3^(2
/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+
1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1
 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{3x}+\frac{1}{3}\int \frac{x\left(-4-x^3\right)}{3+4x^3+x^6}\,dx \\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{x}{3+x^3}\,dx-\frac{1}{2}\int \frac{x}{1+x^3}\,dx \\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{3}} \\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}} \\
&= -\frac{1}{3x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-
\end{aligned}$$

Mathematica [A] time = 0.04, size = 118, normalized size = 0.99

$$-\frac{9x\log\left(x^2-x+1\right)-3^{2/3}x\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18x\log(x+1)+2\ 3^{2/3}x\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}\,x\tan^{-1}\left(}{108x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x]
+ 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x
```

fricas [A] time = 0.85, size = 117, normalized size = 0.98

$$-\frac{3^{\frac{2}{3}} (-1)^{\frac{1}{3}} x \log \left(-3^{\frac{1}{3}} (-1)^{\frac{2}{3}} x + x^2 - 3^{\frac{2}{3}} (-1)^{\frac{1}{3}}\right) - 2 \cdot 3^{\frac{2}{3}} (-1)^{\frac{1}{3}} x \log \left(3^{\frac{1}{3}} (-1)^{\frac{2}{3}} + x\right) + 18 \sqrt{3} x \arctan \left(\frac{1}{3} \sqrt{3} (2 x - \right.}{108 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/108*(3^(2/3)*(-1)^(1/3)*x*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)
^(1/3)) - 2*3^(2/3)*(-1)^(1/3)*x*log(3^(1/3)*(-1)^(2/3) + x) + 18*sqrt(3)*x
*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*(-1)^(1/3)*x*arctan(1/3*3^(1/6)*
(2*(-1)^(1/3)*x + 3^(1/3))) + 9*x*log(x^2 - x + 1) - 18*x*log(x + 1) + 36)/
x
```

giac [A] time = 0.41, size = 91, normalized size = 0.76

$$\frac{1}{108} \cdot 3^{\frac{2}{3}} \log \left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{54} \cdot 3^{\frac{2}{3}} \log \left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) + \frac{1}{18} \cdot 3^{\frac{1}{6}} \arctan \left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(abs(x + 3^(
1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/
3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x
+ 1))
```

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{3^{\frac{1}{6}} \arctan \left(\frac{\sqrt{3}\left(\frac{2 3^{\frac{2}{3}} x}{3}-1\right)}{3}\right)}{18} - \frac{\sqrt{3}\ \arctan \left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln (x+1)}{6} - \frac{3^{\frac{2}{3}} \ln \left(x + 3^{\frac{1}{3}}\right)}{54} + \frac{3^{\frac{2}{3}} \ln \left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right)}{108} - \frac{\ln \left(x^2 - x \right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x+1)-1/3/x-1/54*3^(2/3)*ln(x+3^(1/3))+1/108*3^(2/3)*ln(x^2-3^(1/3)*x
+3^(2/3))+1/18*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+
1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.09, size = 89, normalized size = 0.75

$$\frac{1}{108} \cdot 3^{\frac{2}{3}} \log \left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{54} \cdot 3^{\frac{2}{3}} \log \left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) + \frac{1}{18} \cdot 3^{\frac{1}{6}} \arctan \left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3)
) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3^(
1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 1.38, size = 119, normalized size = 1.00

$$\frac{\ln(x+1)}{6}-\frac{3^{2/3}\,\ln\left(x+3^{1/3}\right)}{54}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{3\,x}-\frac{(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/54 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - 1/(3*x) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/
6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/108 + ((-1)^(1/3)*3^(2/3
)*log(x + (-1)^(2/3)*3^(1/3)))/54
```

sympy [C] time = 1.83, size = 139, normalized size = 1.17

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{5}}{41}+\frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{2}}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{39}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/12
)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*log(
x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)**5/
41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39384*
_t**2/41 + x))) - 1/(3*x)
```

3.167 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

```
[Out] -1/6/x^2-1/54*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/54*3^
(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)
/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log
[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{-8-2x^3}{3+4x^3+x^6}\,dx \\
&= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{1}{3+x^3}\,dx-\frac{1}{2}\int \frac{1}{1+x^3}\,dx \\
&= -\frac{1}{6x^2}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\ 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\ 3^{2/3}} \\
&= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx- \\
&= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right.}{36\ 3^{2/3}} \\
&= -\frac{1}{6x^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1\right.
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 113, normalized size = 0.95

$$
\frac{1}{108}\left(-\frac{18}{x^2}+9\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\ 3^{5/6}\tan\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[1
- x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

fricas [A] time = 1.08, size = 126, normalized size = 1.06

$$\frac{6\cdot 9^{\frac{1}{6}}\sqrt{3}\,x^2\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-9^{\frac{2}{3}}x^2\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+2\cdot 9^{\frac{2}{3}}x^2\log\left(3\,x+9^{\frac{2}{3}}\right)-}{324\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9
^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/
3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) +
27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

giac [A] time = 0.41, size = 91, normalized size = 0.76

$$\frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(abs(
x + 1))
```

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(x+1\right)}{6}+\frac{3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{54}-\frac{3^{\frac{1}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{108}+\frac{\ln\left(x^2-x\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x+1)+1/54*3^(1/3)*ln(x+3^(1/3))-1/108*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2
/3))+1/54*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6/x^2+1/12*ln(x^2
-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.27, size = 89, normalized size = 0.75

$$\frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.36, size = 118, normalized size = 0.99

$$\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{54}-\frac{\ln\left(x+1\right)}{6}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{6\,x^2}-\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/54 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 - 1/12) - 1/(6*x^2) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/108 + (
3^(5/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/108 - (3^(5
/6)*1i)/108)
```

sympy [C] time = 1.74, size = 128, normalized size = 1.08

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}\,i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1093}{244}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/244
 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 1093
/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSum(5
2488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) - 1/(
6*x**2)
```

3.168 $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\,3^{5/6}}$$

```
[Out] -1/12/x^4+4/9/x+1/54*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+
1/162*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/
3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(
1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(
1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{12x^4}+\frac{1}{12}\int \frac{-16-4x^3}{x^2\left(3+4x^3+x^6\right)}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{36}\int \frac{x\left(-52-16x^3\right)}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{18}\int \frac{x}{3+x^3}\,dx+\frac{1}{2}\int \frac{x}{1+x^3}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\sqrt[3]{3}}-\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{3}}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}-\frac{1}{36}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx+\frac{1}{12}\int \frac{-1}{1-}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x\right.}{108\sqrt[3]{3}}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.94

$$\frac{1}{324}\left(-\frac{27}{x^4}+27\log\left(x^2-x+1\right)-3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+\frac{144}{x}-54\log(x+1)+2\ 3^{2/3}\log\left(3^{2/3}x+3\right)+6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*A
rcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] +
27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

fricas [A] time = 1.06, size = 112, normalized size = 0.89

$$-\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\right.}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*arc
tan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log(x
+ 1) - 144*x^3 + 27)/x^4
```

giac [A] time = 0.36, size = 98, normalized size = 0.78

$$-\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x + 3
^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(
1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1
) - 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 94, normalized size = 0.75

$$-\frac{3^{\frac{1}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}} x}{3}-1\right)}{3}\right)}{54}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{324}+\frac{\ln\left(x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+4*x^3+3),x)
```

```
[Out] -1/12/x^4+4/9/x-1/6*ln(x+1)+1/162*3^(2/3)*ln(x+3^(1/3))-1/324*3^(2/3)*ln(x^
2-3^(1/3)*x+3^(2/3))-1/54*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/1
2*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.20, size = 96, normalized size = 0.76

$$-\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1/
3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1) - 1
/6*log(x + 1)
```

mupad [B] time = 0.19, size = 124, normalized size = 0.98

$$\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{162}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{4x^3}{9}-}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/162 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) + ((4*x^3)/9 - 1/12)/x^4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3
^(2/3)/324 - (3^(1/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/
3)/324 + (3^(1/6)*1i)/108)
```

sympy [C] time = 1.84, size = 141, normalized size = 1.12

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{547}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12)
**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12)*
log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3)*I
/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_t**5
/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

3.169 $\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right.}{54\cdots}$$

```
[Out] -1/15/x^5+2/9/x^2+1/162*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+
x)-1/162*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/324*3^(1/3)*ln(3^(2/3)-3^
(1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{2}{9x^2}-\frac{1}{15x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right.}{54\sqrt[6]{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3
^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3
^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{15x^5}+\frac{1}{15}\int \frac{-20-5x^3}{x^3\left(3+4x^3+x^6\right)}\,dx$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{90}\int \frac{-130-40x^3}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{18}\int \frac{1}{3+x^3}\,dx+\frac{1}{2}\int \frac{1}{1+x^3}\,dx$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\,3^{2/3}}-\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\,3^{2/3}}$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\,3^{2/3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+}$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\,3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{}\right.}{108\,3^{2}}$$
$$= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\,3^{2/3}}-\frac{1}{12}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.94

$$\frac{-\frac{108}{x^5}+\frac{360}{x^2}-135\log\left(x^2-x+1\right)+5\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+270\log(x+1)-10\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+1}{1620}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2/3
)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/1
620
```

fricas [A] time = 1.07, size = 153, normalized size = 1.21

$$\frac{30\cdot 9^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-5\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x+3x^2+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4860*(30*9^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/27*9^(1/6)*(2*9^(2/3)*sq
rt(3)*(-1)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 5*9^(2/3)*(-1)^(1/3)*x^5*log(9^(
2/3)*(-1)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 10*9^(2/3)*(-1)^(1/3)*x
^5*log(-9^(2/3)*(-1)^(1/3) + 3*x) + 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 405*x^5*log(x^2 - x + 1) + 810*x^5*log(x + 1) + 1080*x^3 - 324)/x^
5
```

giac [A] time = 0.36, size = 98, normalized size = 0.78

$$-\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(abs(x + 3^(1/3))) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x +
1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 94, normalized size = 0.75

$$-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{162}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{162}+\frac{3^{\frac{1}{3}}\ln\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{324}-\frac{\ln\left(x^{2}-x\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+4*x^3+3),x)
```

```
[Out] -1/15/x^5+2/9/x^2+1/6*ln(x+1)-1/162*3^(1/3)*ln(x+3^(1/3))+1/324*3^(1/3)*ln(
x^2-3^(1/3)*x+3^(2/3))-1/162*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-
1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.66, size = 96, normalized size = 0.76

$$-\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1) +
1/6*log(x + 1)
```

mupad [B] time = 1.40, size = 121, normalized size = 0.96

$$\frac{\ln(x+1)}{6}-\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{162}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\frac{\frac{2\,x^{3}}{9}-\frac{1}{15}}{x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/162 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)
*3^(1/3)))/162 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3
^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/324
```

sympy [C] time = 1.78, size = 136, normalized size = 1.08

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{88573}{6562}-\frac{88573\sqrt{3}\,i}{6562}+\frac{119042784\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{4}}{3281}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I/
6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)*l
og(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqrt(3
)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**4/32
81 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

3.170 $\int \frac{x^6}{1-x^3+x^6}\,dx$

Optimal. Leaf size=412

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2
^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(
1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2
))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/1
8*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*
(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+
I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A] time = 0.43, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(
3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*S
qrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^
(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3
)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
 - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x - \int \frac{1-x^3}{1-x^3+x^6}\,dx$$
$$= x - \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx + \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$
$$= x + \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\cdots}{9\sqrt[3]{2}\cdots}$$
$$= x + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int\cdots}{\cdots}$$
$$= x + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\mathrm{lo}\cdots}{\cdots}$$
$$= x + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\cdots}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\cdots\right.}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.14

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
2*#1^5) & ]/3
```

fricas [B] time = 0.93, size = 1028, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 2/27*18^(2/3)*12
^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
- 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3)
- 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
```

```
an(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(
3) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)
*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) -
2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) +
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
- 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + x
```

giac [B] time = 0.52, size = 638, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6
*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi
)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*s
in(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*si
n(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*
pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)
*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i
+ 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*co
s(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)
^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi)
- 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) -
1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*p
i)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sq
rt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi
))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos
(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + si
n(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9
*pi) + cos(1/9*pi))*x + x^2 + 1) + x
```

maple [C] time = 0.01, size = 44, normalized size = 0.11

$$x+\frac{\left(\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{3}-1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-3\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6-x^3+1),x)
```

```
[Out] x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+\int\frac{x^{3}-1}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.82, size = 320, normalized size = 0.78

$$x+\frac{\ln\left(x+\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\ln\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6 - x^3 + 1),x)
```

```
[Out] x + (log(x + (((3^(1/2)*9i)/2 - 27/2)*(3^(1/2)*12i + 36)^(1/3))/54)*(3^(1/2
)*12i + 36)^(1/3))/18 + (log(x - (((3^(1/2)*9i)/2 + 27/2)*(36 - 3^(1/2)*12i
)^(1/3))/54)*(36 - 3^(1/2)*12i)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(3 -
3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^
(5/6)*1i)^3)/16 - 27))/108)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i
)*((3*(3^(1/2)*1i + 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(3^(1/2)*1i
 + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5/6)*
(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(
1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$x+\mathrm{RootSum}\left(19683t^{6}-243t^{3}+1,\left(t\mapsto t\log\left(729t^{4}-9t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t
 + x)))
```

3.171 $\int \frac{x^5}{1-x^3+x^6} \, dx$

Optimal. Leaf size=39

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^3 + x^6),x]

[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$
\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

fricas [A] time = 0.76, size = 32, normalized size = 0.82

$$
\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

giac [A] time = 0.43, size = 32, normalized size = 0.82

$$
\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$
\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^6-x^3+1\right)}{6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6-x^3+1),x)
[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 1.04, size = 32, normalized size = 0.82

$$
\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

mupad [B] time = 1.21, size = 34, normalized size = 0.87

$$\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

sympy [A] time = 0.13, size = 37, normalized size = 0.95

$$\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.172 $\int \frac{x^4}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(1/3)/(1+I*3^(1/2))^(1/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(1
/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2)
)^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/18
*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/
3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(
3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I
*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)
```

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^3 + x^6),x]
```

```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - (
(3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\left(-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{(3-\cdots}{\cdots}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(-3-i\sqrt{3}\right)\cdots}{\cdots}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\cdots}{\cdots}$$

$$= \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\cdots}\right.}{9\ 2^{2/3}\sqrt[3]{1-i\cdots}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/3
```

fricas [B] time = 1.35, size = 1583, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) +
2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3
*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2
))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3
) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(sq
rt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) +
2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4))*sin(2/3*arctan(sqrt(3) + 2)) - 1
/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(
1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2
))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt
(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x -
 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2
+ 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(
sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))
^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*ar
ctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 +
 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*
x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(
2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(
3) + 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arcta
n(sqrt(3) + 2))))/(3*cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqr
t(3) + 2))^2*sin(2/3*arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2)
)^4)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(
2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/432*(6*18^(2/3)*12^(2
/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arctan(sqrt(3) + 2
```

```
))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2)
)^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36
*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)))) - 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6
)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3
) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)
) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan
(sqrt(3) + 2))^2 + 36*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arcta
n(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arc
tan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)
```

giac [B] time = 0.69, size = 824, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(
4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)
*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/
9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^5
- 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*p
i)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*si
n(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)
*sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*
sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin
(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/
9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*
pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)
*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(10*sqrt(3
)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sq
rt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi
)^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1
) - 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*s
in(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3
*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2
/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt
(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3
)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*c
os(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.10

$$\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 - x^3 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.72, size = 304, normalized size = 0.74

$$\frac{\ln\left(x+\left(162\,x+\frac{27\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\left(162\,x+\frac{27\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(\frac{1}{162}+\right.\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i - 3
6)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18 - (
2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3
^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/
12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3^(1/2)*1
i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*1i - 3)^(
1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**
2 + x)))
```

3.173 $\int \frac{x^3}{1-x^3+x^6} \, dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{\right.\right.}}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] 1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(
2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(1/
2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))
^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*
ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3
)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3
+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*
3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{\right.\right.}}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^3 + x^6),x]
```

```
[Out] -((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$=\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\int \cdots}{9\sqrt[3]{2}\left(1\cdots\right.}$$

$$=\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \cdots}{18\sqrt[3]{\cdots}}$$

$$=\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\cdots\right.}{\cdots}$$

$$=-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}\cdots}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2\cdots}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

fricas [B] time = 1.27, size = 1031, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(2*18^(2/3)*12^(1/6)
*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18
*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^
(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3
*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin
(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqr
t(3) + 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) -
 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/108*(6*18^(1/3)*
12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*sqrt(3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 18*(18^(1/
3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2
)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^
(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3
*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^
2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2))
 - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arct
an(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/27*(18^(2/3)*12
^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arc
tan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arct
an(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(3)
*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arct
an(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sq
rt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan
(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^
(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*
sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*
sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*
arctan(sqrt(3) + 2))^2)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(
sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/
3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*
cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) +
 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) + 1/1
08*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1
/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) +
 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*s
in(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

giac [B] time = 0.57, size = 637, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9
*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/
9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^4
- 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(
2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) +
 sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*
sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin
(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(
1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)
*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(8*sqrt(3)
*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/
9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/
9*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1
) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)
^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2
/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3
)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi
)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((sqrt(3)*i*c
os(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.10

$$\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3 \ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.84, size = 327, normalized size = 0.80

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2}{\ }\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i
```

```
 - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1/2)*
1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3
)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3))/12)*
(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/
3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3
)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2
/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(
3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36
```

sympy [A] time = 0.18, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(-1458t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t +
 x)))
```

3.174 $\int \frac{x^2}{1-x^3+x^6} dx$

Optimal. Leaf size=23

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^3+x^6} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3\right)\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^3 + x^6),x]
[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.89, size = 18, normalized size = 0.78

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

giac [A] time = 0.49, size = 18, normalized size = 0.78

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-x^3+1),x)
[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 1.14, size = 18, normalized size = 0.78

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

mupad [B] time = 1.22, size = 20, normalized size = 0.87

$$-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - x^3 + 1),x)
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

<u>**sympy**</u> **[A]** time = 0.12, size = 27, normalized size = 1.17

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.175 $\int \frac{x}{1-x^3+x^6} \, dx$

Optimal. Leaf size=375

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}\right.}{3\sqrt{3}\sqrt[3]{}}$$

```
[Out] 1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1-I*
3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))
*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/
3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2
))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/9*I*2^(
1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2)+1/18*I*
ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^(1/3)/(
1+I*3^(1/2))^(1/3)*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}\right.}{3\sqrt{3}\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^3 + x^6),x]
```

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[
3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]
)/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))
^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{1-x^3+x^6}\,dx = -\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}\,x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{\cdots}{\left(\frac{1}{2}(1+i\cdots\right.}}{3\sqrt{\cdots}}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{1}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}\,x+x^2}\,dx}{2\sqrt{3}} - \cdots$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)\cdots}\right.}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{\cdots}}\right.}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^3 + x^6),x]
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

fricas [B] time = 1.32, size = 1583, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(
1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt
(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x -
 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2
- 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(
sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))
^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*ar
ctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 +
 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*
x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(
2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(
3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arcta
n(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqr
t(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2)
)^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(
2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(2/3)*12^(2
/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(sqrt(3) - 2
```

```
))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2)
)^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)))) + 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6
)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3
) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)
) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arc
tan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)
```

giac [B] time = 0.57, size = 812, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*
pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*sin(4
/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
- 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*sqr
t(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 - 5*
cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)^5
- sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*sin(2/9*pi)
)*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi))
) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1
/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/9*pi)^2 - sqrt(3)*si
n(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi
) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin
(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 +
 cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)
^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log
(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*si
n(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi
)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)^2 + sin(2
/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*(5
*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3
 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 -
 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) - cos(1/9*
pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 +
1)
```

maple [C] time = 0.01, size = 38, normalized size = 0.10

$$\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 - x^3 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^3 + 1), x)
```

mupad [B] time = 0.45, size = 304, normalized size = 0.81

$$\frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1}{486}\right.\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3
)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3
 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 - (2^(1/3
)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3
^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/
6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^
(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^{6} - 243t^{3} + 1, \left(t \mapsto t\log\left(6561t^{5} - 27t^{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t**
2 + x)))
```

3.176 $\int \frac{1}{1-x^3+x^6}\,dx$

Optimal. Leaf size=186

$$-\frac{(-1)^{5/18}\left(3\log\left(\sqrt[9]{-1}-x\right)+\log(2)\right)}{9\sqrt{3}}+\frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}\left(x+(-1)^{8/9}\right)\right)}{3\sqrt{3}}-\frac{(-1)^{13/18}\log\left(-2^{2/3}\left(\left((-1)^{8/9}-x\right)x+(\right.\right.}{6\sqrt{3}}$$

```
[Out] -1/3*(-1)^(13/18)*arctan(1/3*(1+2*(-1)^(1/9)*x)*3^(1/2))+1/3*(-1)^(5/18)*ar
ctan(1/3*(1-2*(-1)^(8/9)*x)*3^(1/2))-1/27*(-1)^(5/18)*(ln(2)+3*ln((-1)^(1/9
)-x))*3^(1/2)+1/9*(-1)^(13/18)*ln(-2^(1/3)*((-1)^(8/9)+x))*3^(1/2)-1/18*(-1
)^(13/18)*ln(-2^(2/3)*((-1)^(7/9)+((-1)^(8/9)-x)*x))*3^(1/2)+1/18*(-1)^(5/1
8)*ln(2^(2/3)*((-1)^(2/9)+x*((-1)^(1/9)+x)))*3^(1/2)
```

Rubi [C] time = 0.24, antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(-\sqrt[3]{2}x\right.}{3\sqrt{3}\left(\frac{1}{2}(1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] ((-I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt
[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]
])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x
])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3
])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1
- I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int}{\cdots}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,d\cdots}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{\cdots}\right.}\right.}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{\cdots}}$$

$$= -\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1\cdots}\right.}{3\sqrt{3}\left(\frac{1}{2}\cdots\right.}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.23

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

fricas [B] time = 1.29, size = 1027, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) -
 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2
/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)
) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3
)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(
2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arcta
n(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3
) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*
cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2
)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
+ 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)
```

giac [B] time = 0.50, size = 629, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
```

```
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
 - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqr
t(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*
sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*p
i))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi
))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 +
sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9
*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9
*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*
sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4
/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*
cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*p
i)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi)
+ cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/
18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3
)*sin(1/9*pi) + cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x +
x^2 + 1)
```

maple [C] time = 0.01, size = 37, normalized size = 0.20

$$\frac{\ln\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(1/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.79, size = 327, normalized size = 1.76

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/}}{12}\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x + (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3
)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*
(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3
 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3
))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x +
 (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(
```

```
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)
^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [**A**] time = 0.18, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

3.177 $\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
+ 2*#1^3) & ]/3
```

fricas [A] time = 1.05, size = 34, normalized size = 0.83

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

giac [A] time = 0.35, size = 35, normalized size = 0.85

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)
```

maxima [A] time = 1.45, size = 38, normalized size = 0.93

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{6}\,\log\left(x^6-x^3+1\right)+\frac{1}{3}\,\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

mupad [B] time = 1.23, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

sympy [A] time = 0.15, size = 41, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

3.178 $\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2)
)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*
3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-
1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3
))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/
(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Rubi [A] time = 0.30, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x}+\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$
$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{(3}{}$$
$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{(3-i}{}$$
$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{(3-i}{}$$
$$= -\frac{1}{x}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\right.}{9\,2^{2/3}\sqrt[3]{}}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.15

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#
1 + 2*#1^4) & ]/3
```

fricas [B] time = 1.31, size = 1598, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin
(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*x*arctan(-1/432
*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*
arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 -
12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sq
```

```
rt(3) + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(
3) + 2))))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) + 2
)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arct
an(sqrt(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3
) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt
(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2
/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan
(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18
^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(
3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)
*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))
^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18
^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*ar
ctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqr
t(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 4*(18^(2/3)*12^(1/6)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(s
qrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sq
rt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*si
n(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arc
tan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*a
rctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 + 12*(18^(2/3)*12^(2/
3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(
2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) +
2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(
1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) +
6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)
*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqr
t(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) +
2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/
3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*
cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*
arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + (18^(2/3)*12^
(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4
+ 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*s
qrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(
1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) +
 2))^2 + 36*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)
) + 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)
*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
+ 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2
/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(
1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) - 108)/x
```

giac [B] time = 0.55, size = 826, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
```

```
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*p
i) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*
sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 -
 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)
^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/9*pi)*sin(
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi
)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 1
0*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2
*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)
*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/
9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(
4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*
sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin
(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1/18*
(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)
^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2
 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + 2*cos(
2/9*pi)^2 - 2*sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x +
 x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi
)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 - 4*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) +
cos(1/9*pi))*x + x^2 + 1) - 1/x
```

maple [C] time = 0.01, size = 50, normalized size = 0.12

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^4-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^4-x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.66, size = 286, normalized size = 0.69

$$\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{1}{x}+\frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,(-3\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 - 1/x + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)
^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) -
```

```
3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2
^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/
24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
(2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i -
 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1
i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
+ 3^(5/6)*1i))/36
```

sympy [A] time = 0.20, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-27t^2 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) - 1
/x
```

3.179 $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\,\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\sqrt[3]{2}\,\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I
*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2
/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1
/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A] time = 0.34, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\,\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\sqrt[3]{2}\,\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2}+\frac{1}{2}\int \frac{2-2x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+$$

$$= -\frac{1}{2x^2}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log}{9\sqrt[3]{2}}$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.16

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/
(-#1^2 + 2*#1^5) & ]/3
```

fricas [B] time = 1.43, size = 1066, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*1
2^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(
2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))
^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 8*18^(2
/3)*12^(1/6)*x^2*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arcta
n(sqrt(3) + 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*
sin(2/3*arctan(sqrt(3) + 2))^2 - 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arcta
n(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqr
t(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(
sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(
1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*s
```

```
qrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*s
in(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*a
rctan(sqrt(3) + 2))^2))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)
*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x^2*sin(2/3*a
rctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*ar
ctan(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(
3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*ar
ctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*
sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*1
8^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6
)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2
)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/
3*arctan(sqrt(3) + 2))^2)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arcta
n(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) + 2)))*arcta
n(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)
*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3
) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) -
6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*a
rctan(sqrt(3) + 2))) + (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3
) + 2)) - 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(2*18^(2/3
)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos
(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) +
2))^2 + 18*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) + 2
)) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(
1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3
*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2
+ 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 54)/x^2
```

giac [B] time = 0.55, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*
pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*
sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)
^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*si
n(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin
(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*
sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i +
 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos
(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^
4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1
/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqr
t(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi)
)*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(
1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin
(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*
pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

maple [C] time = 0.01, size = 50, normalized size = 0.12

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^3+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^2}-\int\frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.72, size = 324, normalized size = 0.78

$$\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{1}{2\,x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2
^(2/3)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)
*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
 + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/
108)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (
2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i -
3)^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.21, size = 31, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(729t^4+9t+x\right)\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t + x
))) - 1/(2*x**2)
```

3.180 $\int \frac{1}{x^4\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3
 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 +
2*#1^3) & ]/3
```

fricas [A] time = 0.89, size = 51, normalized size = 1.06

$$-\frac{2\sqrt{3}\,x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3x^3\log\left(x^6-x^3+1\right)-18x^3\log(x)+6}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
+ 1) - 18*x^3*log(x) + 6)/x^3
```

giac [A] time = 0.42, size = 45, normalized size = 0.94

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{x^3+1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log(
x^6 - x^3 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3+ln(x)
```

maxima [A] time = 1.13, size = 43, normalized size = 0.90

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
+ 1) + 1/3*log(x^3)
```

mupad [B] time = 0.06, size = 41, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}x^3}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9 - 1/(3*x^3)
```

sympy [A] time = 0.17, size = 48, normalized size = 1.00

$$\log(x)-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9 - 1/(3*x**3)
```

3.181 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal. Leaf size=423

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/4/x^4-1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I
-3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3
))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*
(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2)
)^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*
3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/
2))^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^
(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/
3)
```

Rubi [A] time = 0.37, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 12, 1374, 292, 31, 634, 617, 204, 628}

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(4*x^4) - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^
(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[
(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^
(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 -
 I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1-x^3+x^6\right)}\,dx &= -\frac{1}{4x^4}+\frac{1}{4}\int \frac{4-4x^3}{x^2\left(1-x^3+x^6\right)}\,dx\\
&= -\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{4}\int \frac{4x^4}{1-x^3+x^6}\,dx\\
&= -\frac{1}{4x^4}-\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx\\
&= -\frac{1}{4x^4}-\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx\\
&= -\frac{1}{4x^4}-\frac{1}{x}+-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\\
&= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\\
&= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\\
&= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right.}{}\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{4x^4}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 
2*#1^3) & ]/3
```

fricas [B] time = 1.33, size = 1623, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12
^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sq
rt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))
*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3
) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3
)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6
```

```
)*x^4*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt
(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sq
rt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos
(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arct
an(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 +
18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqr
t(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/
3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3
*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 -
18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2
/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*
arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(s
qrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3)
- 2)) - 108*x^3 - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*arctan(sqrt(3) -
 2)) - 18^(2/3)*12^(1/6)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*
18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*co
s(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 -
 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2
/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3
*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) -
2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqr
t(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(
2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(
sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*
arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^
2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/
3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqr
t(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4
 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin
(2/3*arctan(sqrt(3) - 2))^4)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*ar
ctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*ar
ctan(-1/432*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 2
16*sin(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
 - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/
3*arctan(sqrt(3) - 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*a
rctan(sqrt(3) - 2)))) - (18^(2/3)*12^(1/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(
3) - 2)) + 18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqr
t(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(
1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + (18^(2/3)*12^(1
/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^4*cos(2/
3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^
4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)
*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqr
t(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x
^2) - 27)/x^4
```

giac [B] time = 0.55, size = 836, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^5 -
 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*sin
(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)*
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin(
1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/9
*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(10*sqrt(3)
*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqr
t(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)
^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*
sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
+ 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/4*(4*x^3 + 1)/x^4
```

maple [C] time = 0.01, size = 51, normalized size = 0.12

$$-\frac{\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^4 \ln\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{3\left(2\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^2\right)}-\frac{1}{x}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^4*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/4/x^4-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,x^3+1}{4\,x^4}-\int \frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.59, size = 318, normalized size = 0.75

$$\frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\cdots\right.\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - (x^
3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12
 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i +
3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i +
 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3
 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
- (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i
+ 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A] time = 0.22, size = 39, normalized size = 0.09

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(-6561t^5 + 54t^2 + x\right)\right)\right) + \frac{-4x^3 - 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t*
*2 + x))) + (-4*x**3 - 1)/(4*x**4)
```

3.182 $\int \frac{1}{2+x^3+x^6}\,dx$

Optimal. Leaf size=381

$$\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{2}\,x+\sqrt[3]{}\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\right.\right.}$$

```
[Out] -1/21*I*2^(2/3)*ln(2^(1/3)*x+(1-I*7^(1/2))^(1/3))/(1-I*7^(1/2))^(2/3)*7^(1/
2)+1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))
*2^(2/3)/(1-I*7^(1/2))^(2/3)*7^(1/2)+1/21*I*2^(2/3)*ln(2^(1/3)*x+(1+I*7^(1/
2))^(1/3))/(1+I*7^(1/2))^(2/3)*7^(1/2)-1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I
*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3))*2^(2/3)/(1+I*7^(1/2))^(2/3)*7^(1/2)+1/
21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))/(1-I*7
^(1/2))^(2/3)*21^(1/2)-1/21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1+I*7^(1/2
))^(1/3))*3^(1/2))/(1+I*7^(1/2))^(2/3)*21^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{2}\,x+\sqrt[3]{}\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 - I
*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[
3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3
) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqr
t[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[
(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3
)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1
 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{i\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}} + \frac{i\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\,x+x^2}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + $$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\,x+x^2}}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-\right.}\right.}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right.}$$

$$= \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+\right.}{3\sqrt{7}\left(\frac{1}{2}\left(\right.\right.}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\log(x-\#1)}{2\#1^5+\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

fricas [B] time = 2.57, size = 1996, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/294*112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*log(112^(1/6)*4
9^(2/3)*sqrt(7)*x*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)
*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 14*49^(1/3)*14^(1/3)*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 + 14*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) +
4/3))^2 + 98*x^2) - 2/147*112^(1/6)*49^(2/3)*arctan(1/2744*(14*112^(5/6)*49
^(1/3)*sqrt(7)*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 2744*sqrt(7)*cos(2/3*
arctan(1/3*sqrt(7) + 4/3))^2 + 2744*sqrt(7)*sin(2/3*arctan(1/3*sqrt(7) + 4/
3))^2 + 98*(112^(5/6)*49^(1/3)*x + 224*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*
sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - sqrt(112^(1/6)*49^(2/3)*sqrt(7)*x*sin(
2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)*x*cos(2/3*arctan(1/3*
sqrt(7) + 4/3)) + 14*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2
 + 14*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 98*x^2)*(112
^(5/6)*49^(1/3)*sqrt(7)*sqrt(2)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^
(5/6)*49^(1/3)*sqrt(2)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))))/(cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 - 7*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2))*sin(2/3*a
rctan(1/3*sqrt(7) + 4/3)) + 1/147*(112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arcta
n(1/3*sqrt(7) + 4/3)) + 112^(1/6)*49^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3
)))*arctan(1/5488*(70*112^(5/6)*49^(1/3)*(sqrt(7)*x + 7*sqrt(3)*x)*cos(2/3*
arctan(1/3*sqrt(7) + 4/3))^3 - 27440*(sqrt(7) + 2*sqrt(3))*cos(2/3*arctan(1
/3*sqrt(7) + 4/3))^4 - 5488*(sqrt(7) - 2*sqrt(3))*sin(2/3*arctan(1/3*sqrt(7
) + 4/3))^4 - 14*(112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3)*x - 7*x) - 1568*(sqrt
(7)*sqrt(3) - 5)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqr
t(7) + 4/3))^3 + 14*(112^(5/6)*49^(1/3)*(13*sqrt(7)*x - 21*sqrt(3)*x)*cos(2
/3*arctan(1/3*sqrt(7) + 4/3)) - 784*(3*sqrt(7) + 4*sqrt(3))*cos(2/3*arctan(
1/3*sqrt(7) + 4/3))^2)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 14*(112^(5/6)
*49^(1/3)*(9*sqrt(7)*sqrt(3)*x + 49*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2
 - 1568*(sqrt(7)*sqrt(3) + 11)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sin(2/
3*arctan(1/3*sqrt(7) + 4/3)) - (5*112^(5/6)*49^(1/3)*(sqrt(7) + 7*sqrt(3))*
cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3
) + 49)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4
/3)) + 112^(5/6)*49^(1/3)*(13*sqrt(7) - 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt
(7) + 4/3))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 112^(5/6)*49^(1/3)*(sqrt
(7)*sqrt(3) - 7)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sqrt(-112^(1/6)*49^(
2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/
6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) +
28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*14^
(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 196*x^2))/(25*cos(2/3*arctan(1
/3*sqrt(7) + 4/3))^4 - 38*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arct
an(1/3*sqrt(7) + 4/3))^2 + sin(2/3*arctan(1/3*sqrt(7) + 4/3))^4)) + 1/147*(
112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*4
9^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)))*arctan(-1/5488*(70*112^(5/6)*49
^(1/3)*(sqrt(7)*x - 7*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 274
```

```
40*(sqrt(7) - 2*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 - 5488*(sqrt(
7) + 2*sqrt(3))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^4 + 14*(112^(5/6)*49^(1/
3)*(sqrt(7)*sqrt(3)*x + 7*x) - 1568*(sqrt(7)*sqrt(3) + 5)*cos(2/3*arctan(1/
3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^3 + 14*(112^(5/6)*49^
(1/3)*(13*sqrt(7)*x + 21*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 78
4*(3*sqrt(7) - 4*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2)*sin(2/3*arc
tan(1/3*sqrt(7) + 4/3))^2 + 14*(112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3)*x - 4
9*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 1568*(sqrt(7)*sqrt(3) - 11)*cos
(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - (5*
112^(5/6)*49^(1/3)*(sqrt(7) - 7*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))
^3 + 112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3) - 49)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 112^(5/6)*49^(1/3)*(13*sqrt
(7) + 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*sin(2/3*arctan(1/3*sqr
t(7) + 4/3))^2 + 112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) + 7)*sin(2/3*arctan(1/
3*sqrt(7) + 4/3))^3)*sqrt(112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(
2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*
x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) +
4/3))^2 + 196*x^2))/(25*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 - 38*cos(2/3*a
rctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + sin(2/3*
arctan(1/3*sqrt(7) + 4/3))^4)) + 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*
arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7)
+ 4/3)))*log(-112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1
/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*a
rctan(1/3*sqrt(7) + 4/3)) + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3))^2 + 28*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 19
6*x^2) - 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3
)) + 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*log(112^(1/6)*4
9^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^
(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))
 + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*
14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 196*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + x^3 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

maple [C] time = 0.01, size = 33, normalized size = 0.09

$$\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right) + x\right)}{6\,\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^5 + 3\,\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(-_R+x),_R=RootOf(_Z^6+_Z^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + x^3 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

mupad [B] time = 2.61, size = 513, normalized size = 1.35

$$\frac{\ln\left(x+\frac{7^{1/3}\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{4}+\frac{7^{5/6}\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}1\mathrm{i}}{28}\right)\left(-49-\sqrt{7}\,21\mathrm{i}\right)^{1/3}}{42}+\frac{\ln\left(x+\frac{7^{1/3}\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{4}-\frac{7^{5/6}\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}1\mathrm{i}}{28}\right)\left(-49\right.}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x + (7^(1/3)*(- 7^(1/2)*3i - 7)^(1/3))/4 + (7^(5/6)*(- 7^(1/2)*3i - 7)
^(1/3)*1i)/28)*(- 7^(1/2)*21i - 49)^(1/3))/42 + (log(x + (7^(1/3)*(7^(1/2)*
3i - 7)^(1/3))/4 - (7^(5/6)*(7^(1/2)*3i - 7)^(1/3)*1i)/28)*(7^(1/2)*21i - 4
9)^(1/3))/42 + (7^(1/3)*log(6*x + (7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i -
 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x + (
567*7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*
(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 + (7^(1/3)*log(6*x + (7^(1/3)
*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(7^(1
/2)*3i - 7)^(2/3)*(3969*x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^
(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3))/84 - (7
^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3)*((7^(2/
3)*(3^(1/2)*1i + 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x - (567*7^(1/3)*(3^(1
/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i + 1)*
(- 7^(1/2)*3i - 7)^(1/3))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)
*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(7^(1/2)*3i - 7)^(2/3)
*(3969*x - (567*7^(1/3)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/2))/7056 +
 63))/84)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/84
```

sympy [A] time = 0.15, size = 24, normalized size = 0.06

$$\mathrm{RootSum}\left(1000188t^6+1323t^3+1,\left(t\mapsto t\log\left(-5292t^4+7t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+x**3+2),x)
```

```
[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*_
t + x)))
```

3.183 $\int \frac{x^2}{2+x^3+x^6} \, dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{2+x^3+x^6} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x+x^2} \, dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-7-x^2} \, dx, x, 1+2x^3\right)\right)\\
&= \frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + x^3 + x^6),x]
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

fricas [A] time = 1.01, size = 18, normalized size = 0.78

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

giac [A] time = 0.50, size = 18, normalized size = 0.78

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{7}\arctan\left(\frac{\left(2x^3+1\right)\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+x^3+2),x)
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 1.36, size = 18, normalized size = 0.78

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

mupad [B] time = 0.05, size = 20, normalized size = 0.87

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + x^6 + 2),x)
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21
```

sympy [A] time = 0.11, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\,\operatorname{atan}\left(\frac{2\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+x**3+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

3.184 $\int \frac{x^3}{2+x^3+x^6}\,dx$

Optimal. Leaf size=399

$$-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2}\right.}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

```
[Out] 1/42*ln(2^(1/3)*x+(1+I*7^(1/2))^(1/3))*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^
(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3)
)*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^(2/3)+1/42*ln(2^(1/3)*x+(1-I*7^(1/2))
^(1/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^(1/2))^(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/
3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^
(1/2))^(2/3)-1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))
*(1-I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)+1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1+I
*7^(1/2))^(1/3))*3^(1/2))*(1+I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2}\right.}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + x^3 + x^6),x]
```

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3)
)/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 +
 I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt
[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/
3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3
)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*L
og[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*
2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx+\frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx$$

$$=\frac{\left(7-i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\,x+x^2}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7+i\sqrt{7}\right)\cdots}{21\sqrt[3]{2}\cdots}$$

$$=\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\int\cdots}{4\cdots}$$

$$=\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{\left(7+i\sqrt{7}\right)\log\cdots}{\cdots}$$

$$=-\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{\left(7+i\sqrt{7}\cdots\right.}{2\cdots}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\#1\log(x-\#1)}{2\#1^3+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + x^3 + x^6),x]
[Out] RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

fricas [B] time = 0.98, size = 1435, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")
[Out] 1/294*98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))*log
(-2*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^
2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 +
 98*x^2) - 2/147*98^(2/3)*56^(1/6)*arctan(1/5488*(98^(1/3)*56^(5/6)*sqrt(7)
*sqrt(2)*sqrt(-2*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7)
 + sqrt(7)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) +
sqrt(7)))^2 + 98*x^2) - 14*98^(1/3)*56^(5/6)*sqrt(7)*x + 5488*sin(2/3*arcta
n(2/7*sqrt(14)*sqrt(7) + sqrt(7))))/cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 1/147*(98^(2/3)
*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3
)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(1/2744*(
14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7
))) + 2744*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 2744
*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 14*(98^(1/3)*5
6^(5/6)*sqrt(7)*sqrt(3)*x + 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - sqrt(98^(2/3)*56^(1
/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^
(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) +
14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 14*
98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 98*x^2
)*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sqrt(2)*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(2)*cos(2/3*arctan(2/7*sq
rt(14)*sqrt(7) + sqrt(7)))))/(cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
))^2 - 3*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2)) + 1/147*(98^(2
/3)*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 98^(
2/3)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(-1/27
44*(14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sq
rt(7))) - 2744*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 -
2744*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 - 14*(98^(1/
3)*56^(5/6)*sqrt(7)*sqrt(3)*x - 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + sqrt(-98^(2/3)*
56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))
+ 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
)) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2
+ 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 9
8*x^2)*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sqrt(2)*sin(2/3*arctan(2/7*sqrt(1
4)*sqrt(7) + sqrt(7))) - 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(2)*cos(2/3*arctan(2
/7*sqrt(14)*sqrt(7) + sqrt(7)))))/(cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sq
rt(7)))^2 - 3*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2)) + 1/588*(
98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) -
```

```
98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*log(98^
(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt
(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7
)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))
^2 + 98*x^2) - 1/588*(98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)
*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7
) + sqrt(7))))*log(-98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*
sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2
/7*sqrt(14)*sqrt(7) + sqrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sq
rt(14)*sqrt(7) + sqrt(7)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(
14)*sqrt(7) + sqrt(7)))^2 + 98*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6+x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

maple [C] time = 0.01, size = 36, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^6+\_Z^3+2\right)^3 \ln\left(-\mathrm{RootOf}\left(\_Z^6+\_Z^3+2\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+\_Z^3+2\right)^5+3\,\mathrm{RootOf}\left(\_Z^6+\_Z^3+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(-_R+x),_R=RootOf(_Z^6+_Z^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6+x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

mupad [B] time = 2.61, size = 351, normalized size = 0.88

$$\frac{\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\left(-196-\sqrt{7}\,28\mathrm{i}\right)^{1/3}}{42}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{42}-\ 2^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x - (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/14)*(- 7^(1/2)*28i -
 196)^(1/3))/42 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7
)^(1/3)*1i)/14)*(7^(1/2)*1i - 7)^(1/3))/42 - (2^(2/3)*7^(1/3)*log(x + (2^(2
/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(- 7
^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(- 7^(1/2)*1i - 7)^(1/3))/84 + (
```

```
2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 +
(2^(2/3)*3^(1/2)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i - 1)*(-
7^(1/2)*1i - 7)^(1/3))/84 + (2^(2/3)*7^(1/3)*log(x - (2^(2/3)*7^(5/6)*(7^(1
/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3))
/28)*(3^(1/2)*1i - 1)*(7^(1/2)*1i - 7)^(1/3))/84 - (2^(2/3)*7^(1/3)*log(x -
 (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 + (2^(2/3)*3^(1/2)*7^(5/6)*
(7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(7^(1/2)*1i - 7)^(1/3))/84
```

sympy [A] time = 0.15, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(250047t^6 + 1323t^3 + 2, \left(t \mapsto t \log\left(7938t^4 + 21t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_t
 + x)))
```

3.185 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=231

$$-\frac{\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}+\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3}}{1536c^5}$$

```
[Out] -1/20*b*x^6*(c*x^6+b*x^3+a)^(3/2)/c^2+1/18*x^9*(c*x^6+b*x^3+a)^(3/2)/c-1/28
80*(7*b*(-28*a*c+15*b^2)-6*c*(-20*a*c+21*b^2)*x^3)*(c*x^6+b*x^3+a)^(3/2)/c^
4-1/3072*(-4*a*c+b^2)*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*(2*c*x^3+b
)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1536*(16*a^2*c^2-56*a*b^2*c+21*
b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

Rubi [A] time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3}}\right)}{3072c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(2880*c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2
*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]
)/(3072*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
```

```
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int x^4\sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} + \frac{\text{Subst}\left(\int x^2\left(-3a-\frac{9bx}{2}\right)\sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{18c}$$

$$= -\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} + \frac{\text{Subst}\left(\int x\left(9ab+\frac{3}{4}\left(21b^2-20ac\right)\right.\right.}{90c^2}$$

$$= -\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} - \frac{\left(7b\left(15b^2-28ac\right)-6c\left(21b^2-20a\right.\right.}{2880c^4}$$

$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a\right.}{}$$

$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a\right.}{}$$

$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a\right.}{}$$

Mathematica [A] time = 0.16, size = 208, normalized size = 0.90

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(16bc^2\left(113a^2-34acx^6+8c^2x^{12}\right)+160c^3x^3\left(-3a^2+2acx^6+8c^2x^{12}\right)+168b^3c\left(cx^6-10a\right)-\right.}{460}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x^
3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a*c
*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) - 15*(2
1*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(46080*c^(11/2))
```

fricas [A] time = 1.06, size = 451, normalized size = 1.95

$$\left[-\frac{15\left(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)
*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*
a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 +
 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*
(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c -
1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*
c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6+bx^3+a}\,x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6+b\,x^3+a}\;x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 2.94, size = 543, normalized size = 2.35

$$\frac{x^9\left(c\,x^6+b\,x^3+a\right)^{3/2}}{18\,c}-\frac{b\left(\frac{x^6\left(c\,x^6+b\,x^3+a\right)^{3/2}}{5\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(8\,c\cdots\right.}{\cdots}\right)}{10\,c}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - (b*((x^6*(a + b*x^3 + c*x^6)^(3/2)
)/(5*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a +
 b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))
/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) -
3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3
+ c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*
c)))/(10*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x
^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)))/(4*c) + (a*((a*((b/(4*c) + x^3
/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x
^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^
(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*
x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(6*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

3.186 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=171

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{\left(-32ac+35b^2\right.}{}$$

```
[Out] 1/15*x^6*(c*x^6+b*x^3+a)^(3/2)/c+1/720*(-42*b*c*x^3-32*a*c+35*b^2)*(c*x^6+b
*x^3+a)^(3/2)/c^3+1/768*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3
+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)-1/384*b*(-12*a*c+7*b^2)*(2*c*x^3
+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*c^4) + (x^
6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a +
b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$

$$= \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{15c}$$

$$= \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3} - \frac{\left(b\left(7b^2-12ac\right)\right)}{}$$

$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-\right.}{}$$

$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-\right.}{}$$

$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-\right.}{}$$

Mathematica [A] time = 0.13, size = 164, normalized size = 0.96

$$\frac{-\frac{\left(32ac-35b^2+42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{5\left(12abc-7b^3\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{256c^{7/2}}+x^6\left(a+bx^3+cx^6\right.}{15c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2) - ((-35*b^2 + 32*a*c + 42*b*c*x^3)*(a + b*x^
3 + c*x^6)^(3/2))/(48*c^2) + (5*(-7*b^3 + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3
)*Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*
Sqrt[a + b*x^3 + c*x^6])]))/(256*c^(7/2)))/(15*c)
```

fricas [A] time = 0.82, size = 367, normalized size = 2.15

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(384c\right.}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4*c
+ 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(
-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*
c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x
^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a}\, x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6 + b\,x^3 + a}\; x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.87, size = 315, normalized size = 1.84

$$\frac{x^6\left(c\,x^6+b\,x^3+a\right)^{3/2}}{15\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b \cdots\right.}{\cdots}\right.}{\cdots}\cdots\right)}{30\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b
*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2)
)/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(30*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^
2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*
x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(15*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

3.187 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=153

$$-\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}+\frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^3}{72c^2}$$

```
[Out] -5/72*b*(c*x^6+b*x^3+a)^(3/2)/c^2+1/12*x^3*(c*x^6+b*x^3+a)^(3/2)/c-1/384*(-
4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(
1/2))/c^(7/2)+1/192*(-4*a*c+5*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A] time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a
 + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c)
- ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
```

```
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c} + \frac{\operatorname{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{12c}$$
$$= -\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c} + \frac{\left(5b^2-4ac\right)\operatorname{Subst}\left(\int\sqrt{a+bx+cx^2}\right.}{48c^2}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3} - \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3} - \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3} - \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(b\left(8c^2x^6-52ac\right)+24c^2x^3\left(a+2cx^6\right)+15b^3-10b^2cx^3\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}}{1152c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 10*b^2*c*x^3 + 24*c^2*x^3*(a +
 2*c*x^6) + b*(-52*a*c + 8*c^2*x^6)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^(7/2))
```

fricas [A] time = 1.04, size = 303, normalized size = 1.98

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+4\left(48\,c^4x^9\right.}{2304\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c
^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a}\, x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6 + b\,x^3 + a}\; x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.59, size = 193, normalized size = 1.26

$$\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{12\,c}-\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}-\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\right.}{24\,c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3 +
 c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*
c - b^2/4))/(2*c^(3/2))))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*
x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2)
 + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{8} \sqrt{a + bx^{3} + cx^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

3.188 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

```
[Out] 1/9*(c*x^6+b*x^3+a)^(3/2)/c+1/48*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(
1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/24*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2
)/c^2
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3 + c*x^6)^(
3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/(48*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$
$$=\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}-\frac{b\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{6c}$$
$$=-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}+\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+c}}\right.}{48c^2}$$
$$=-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}+\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}dx\right.}{24c^2}$$
$$=-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}+\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+}}\right.}{48c^{5/2}}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.92

$$\frac{\left(b^3-4abc\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(8c\left(a+cx^6\right)-3b^2+2bcx^3\right)}{72c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2) +
 ((b^3 - 4*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(48*c^(5/2))
```

fricas [A] time = 1.18, size = 237, normalized size = 2.19

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\left(8\,c^3x^6+2\,bc^2x^3-3\right.}{288\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqr
t(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*c^
2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3 - 4
*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/
(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2
)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

giac [A] time = 0.49, size = 98, normalized size = 0.91

$$\frac{1}{72}\sqrt{cx^6+bx^3+a}\left(2\left(4\,x^3+\frac{b}{c}\right)x^3-\frac{3\,b^2-8\,ac}{c^2}\right)-\frac{\left(b^3-4\,abc\right)\log\left(\left|-2\left(\sqrt{c}\,x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{48\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
1/48*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqr
t(c) - b))/c^(5/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^{6}+b\,x^{3}+a}\;x^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.39, size = 87, normalized size = 0.81

$$\frac{\left(8\,c\,\left(c\,x^{6}+a\right)-3\,b^{2}+2\,b\,c\,x^{3}\right)\,\sqrt{c\,x^{6}+b\,x^{3}+a}}{72\,c^{2}}+\frac{\ln\left(2\,\sqrt{c\,x^{6}+b\,x^{3}+a}+\frac{2\,c\,x^{3}+b}{\sqrt{c}}\right)\,\left(b^{3}-4\,a\,b\,c\right)}{48\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2)
+ (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c)
)/(48*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\sqrt{a+bx^{3}+cx^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

3.189 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=83

$$\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

```
[Out] -1/24*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c
^(3/2)+1/12*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1352, 612, 621, 206}

$$\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24c}$$
$$= \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c}$$
$$= \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Mathematica [A] time = 0.01, size = 87, normalized size = 1.05

$$\frac{1}{3}\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c} - \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2)))/3
```

fricas [A] time = 1.03, size = 197, normalized size = 2.37

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}\left(2\,c^2\right.}{48\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*
(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x
^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]
```

giac [A] time = 0.56, size = 76, normalized size = 0.92

$$\frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{24\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(-2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6+b\,x^3+a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.39, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}}{3}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{6\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)^
(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a + bx^{3} + cx^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

3.190 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))*a^(1/2)+1/6*b*a
rctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/3*(c*x^6+b*
x^3+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(6*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,x^3\right)\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{6}\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6}+\frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)+\frac{1}{3}b\,\text{Subst}\left(\int \frac{}{4c}\right.\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 106, normalized size = 0.97

$$\frac{1}{6}\left(2\sqrt{a+bx^3+cx^6}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3 + c*x^6] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqr
t[a + b*x^3 + c*x^6])] + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
 + c*x^6])])/Sqrt[c])/6
```

fricas [A] time = 1.24, size = 566, normalized size = 5.19

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+2\sqrt{a}\,c\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+b}}{x^6}\right.}{12c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8
```

```
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c*log(
-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(-a)*c*a
rctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3
 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^
3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/6
*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a
*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)
*c)/c]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.36, size = 88, normalized size = 0.81

$$\frac{\sqrt{c\,x^6 + b\,x^3 + a}}{3} - \frac{\sqrt{a}\,\ln\left(\frac{b}{2} + \frac{a}{x^3} + \frac{\sqrt{a}\,\sqrt{c\,x^6 + b\,x^3 + a}}{x^3}\right)}{3} + \frac{b\,\ln\left(\sqrt{c\,x^6 + b\,x^3 + a} + \frac{c\,x^3 + \frac{b}{2}}{\sqrt{c}}\right)}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x,x)
```

```
[Out] (a + b*x^3 + c*x^6)^(1/2)/3 - (a^(1/2)*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^
3 + c*x^6)^(1/2))/x^3))/3 + (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3
)/c^(1/2)))/(6*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

3.191 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/3*a
rctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/3*(c*x^6+b*
x^3+a)^(1/2)/x^3
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3}+\frac{1}{6}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3}+\frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)+\frac{1}{3}c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.\\ &= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)+\frac{1}{3}(2c)\,\text{Subst}\left(\int \frac{}{4c}\right.\\ &= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

fricas [A] time = 0.99, size = 601, normalized size = 5.37

$$\left[\frac{2\,a\sqrt{c}\,x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+\sqrt{a}\,bx^3\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{}}{}\right.}{12\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)
```

```
*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a
*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) -
 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*a*
sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2
*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)]
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

maple [**F**] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [**B**] time = 1.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,x^3}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{6\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)
```

```
[Out] (c^(1/2)*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/3 - (a + b
*x^3 + c*x^6)^(1/2)/(3*x^3) - (b*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*
x^6)^(1/2))/x^3))/(6*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

3.192 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

```
[Out] 1/24*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^
(3/2)-1/12*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a/x^6
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 720, 724, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a*c)*ArcTan
h[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^3\right)\\
&= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24a}\\
&= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a}\\
&= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.01

$$
\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] ((-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

fricas [A] time = 1.14, size = 215, normalized size = 2.44

$$
\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)}{48\,a^2x^6},-\frac{\left(b^2-4\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 +
 b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*x^6
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x
^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{cx^6+bx^3+a}}{x^7}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

3.193 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

```
[Out] -1/9*(c*x^6+b*x^3+a)^(3/2)/a/x^9-1/48*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a
)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/24*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)
^(1/2)/a^2/x^6
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x^3 + c*x^6
)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+c}}\right.}{48a^2}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx\right.}{24a^2}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+}}\right.}{48a^{5/2}}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.93

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}} - \frac{\sqrt{a+bx^3+cx^6}\left(8a^2+2ax^3\left(b+4cx^3\right)-3b^2x^6\right)}{72a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] -1/72*(Sqrt[a + b*x^3 + c*x^6]*(8*a^2 - 3*b^2*x^6 + 2*a*x^3*(b + 4*c*x^3)))
/(a^2*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x
^3 + c*x^6]])])/(48*a^(5/2))
```

fricas [A] time = 1.15, size = 259, normalized size = 2.23

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}\,x^9\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\left(\left(3\,ab^2-8\,a^2c\right)x^6-2\,a^2bx^3-8\,a^3\right.}{288\,a^3x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
+ 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b
^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)
, 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^2*c)*
x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

3.194 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

Optimal. Leaf size=161

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}-\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}$$

```
[Out] -1/12*(c*x^6+b*x^3+a)^(3/2)/a/x^12+5/72*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^9+1/3
84*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3
+a)^(1/2))/a^(7/2)-1/192*(-4*a*c+5*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a
^3/x^6
```

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*a^3*x^6) - (a
 + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*a
^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}-\frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^3\right)}{12a}\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}+\frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^3\right)}{48a^2}\\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}-\\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}+\\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}}+\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}+
\end{aligned}$$

Mathematica [A] time = 0.09, size = 139, normalized size = 0.86

$$\frac{\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}-\frac{\sqrt{a+bx^3+cx^6}\left(48a^3+8a^2x^3\left(b+3cx^3\right)-2abx^6\left(5b+26cx^3\right)\right.}{576a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -1/576*(Sqrt[a + b*x^3 + c*x^6]*(48*a^3 + 15*b^3*x^9 + 8*a^2*x^3*(b + 3*c*x
^3) - 2*a*b*x^6*(5*b + 26*c*x^3)))/(a^3*x^12) + ((5*b^4 - 24*a*b^2*c + 16*a
^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^
(7/2))
```

fricas [A] time = 1.19, size = 325, normalized size = 2.02

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{a}\,x^{12}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\left(\left(15\,ab^3-52\,a^2bc\right)x^{9}\right.}{2304\,a^4x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*
a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a
)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 - 52*a^2
*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6
 + b*x^3 + a))/(a^4*x^12)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

3.195 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

Optimal. Leaf size=199

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{\left(35b^2-32ac\right)}{7}$$

```
[Out] -1/15*(c*x^6+b*x^3+a)^(3/2)/a/x^15+7/120*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1
/720*(-32*a*c+35*b^2)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^9-1/768*b*(-12*a*c+7*b^2)
*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(9/2
)+1/384*b*(-12*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^6
```

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh}{768a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(12
0*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) -
 (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
+ b*x^3 + c*x^6])])/(768*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
```

```
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)}{15a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,d\right.}{60a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} - \frac{(b}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3}}{120a^2x^{12}}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3}}{120a^2x^{12}}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3}}{120a^2x^{12}}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.87

$$-\frac{b\left(48a^2c^2-40ab^2c+7b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}} - \frac{\sqrt{a+bx^3+cx^6}\left(384a^4+16a^3\left(3bx^3+8cx^6\right)-8a^2x^6\left(7\right.\right.}{576}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] -1/5760*(Sqrt[a + b*x^3 + c*x^6]*(384*a^4 - 105*b^4*x^12 + 10*a*b^2*x^9*(7*
b + 46*c*x^3) + 16*a^3*(3*b*x^3 + 8*c*x^6) - 8*a^2*x^6*(7*b^2 + 29*b*c*x^3
+ 32*c^2*x^6)))/(a^4*x^15) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(
2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(768*a^(9/2))
```

fricas [A] time = 1.44, size = 389, normalized size = 1.95

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{a}\,x^{15}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\left(\left(105\,ab^4-460\,a^2\right.\right.}{23040\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2
*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384
*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b^3*c
+ 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 2
```

```
56*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a
^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

3.196 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,-1/2,-1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx^3+cx^6}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.56, size = 358, normalized size = 2.56

$$\frac{x\left(3x^3\left(16ac-5b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-24ab\sqrt{\frac{-\sqrt{b^2-4ac}+b+}{b-\sqrt{b^2-4a}}}\right.}{448c\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(3*b + 8*c*x^3)*(a + b*x^3 + c*x^6) - 24*a*b*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*(-5*b^2 + 16*a*c
)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/
2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])]))/(448*c*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^6+bx^3+a}\,x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^6+bx^3+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\sqrt{c\,x^6+b\,x^3+a}\;x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^6+bx^3+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^3*(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

3.197 $\int x\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,-1/2,-1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^3+cx^6}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.41, size = 337, normalized size = 2.41

$$\frac{x^2\left(3bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+15a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\ldots}\right.}{50\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
[Out] (x^2*(10*(a + b*x^3 + c*x^6) + 15*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(50*Sqrt[a + b*x^3
+ c*x^6])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^6+bx^3+a}\,x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^6+bx^3+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\sqrt{c\,x^6+b\,x^3+a}\;x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^6+bx^3+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c\,x^6+b\,x^3+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x*(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

3.198 $\int \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] x*AppellF1(1/3,-1/2,-1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{x\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+bx^3+cx^6}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int \sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{x\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [B] time = 0.34, size = 335, normalized size = 2.48

$$\frac{x\left(3bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+24a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4a}}{\sqrt{b^2-}}}\right.}{32\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(a + b*x^3 + c*x^6) + 24*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(32*Sqrt[a + b*x^3 + c
*x^6])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^6+bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6), x)
```

3.199 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -AppellF1(-1/3,-1/2,-1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= -\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.35, size = 340, normalized size = 2.46

$$\frac{15bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+12cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}\ldots}{20x\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 15*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 12*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*x*Sqrt[a + b*x^
3 + c*x^6])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^2,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

3.200 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -1/2*AppellF1(-2/3,-1/2,-1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.32, size = 340, normalized size = 2.43

$$\frac{6bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+3cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+\cdots}{\sqrt{b^2-4a\cdots}}}}{8x^2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) + 6*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^3,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

3.201 $\int x^{14} \left(a+bx^3+cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=293

$$\frac{\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{}}{16384c^6}$$

```
[Out] 1/6144*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^5
-11/336*b*x^6*(c*x^6+b*x^3+a)^(5/2)/c^2+1/24*x^9*(c*x^6+b*x^3+a)^(5/2)/c-1/
13440*(3*b*(-124*a*c+77*b^2)-10*c*(-28*a*c+33*b^2)*x^3)*(c*x^6+b*x^3+a)^(5/
2)/c^4+1/32768*(-4*a*c+b^2)^2*(16*a^2*c^2-72*a*b^2*c+33*b^4)*arctanh(1/2*(2
*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(13/2)-1/16384*(-4*a*c+b^2)*(16*
a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^6
```

Rubi [A] time = 0.40, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3}}{16384c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b
*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x
^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(
5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 -
124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^
4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
```

```
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{1}{3}\text{Subst}\left(\int x^4\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)$$

$$=\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{11bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)}{24c}$$

$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x\left(11ab+\frac{3}{4}\left(33b\right.\right.\right.}{}$$

$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}-\frac{\left(3b\left(77b^2-124ac\right)-10c\left(\right.\right.}{}$$

$$=\frac{\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{11bx^6\left(a+bx^3+cx^6\right)}{336c^2}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72\right.}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72\right.}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72\right.}{}$$

Mathematica [A] time = 0.35, size = 241, normalized size = 0.82

$$\frac{\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{4096c^{11/2}}+\frac{\left(372abc-280ac^2x^3-231b^3+\dots\right.}{560\dots}}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((-11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(14*c) + x^9*(a + b*x^3 + c*x^6)^(5/
2) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x^3 - 280*a*c^2*x^3)*(a + b*x^3 + c
*x^6)^(5/2))/(560*c^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b
+ 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6
)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6])]))/(4096*c^(11/2)))/(24*c)
```

fricas [A] time = 1.09, size = 641, normalized size = 2.19

$$\left[\frac{105\left(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\dots\right.}{\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c^7*
x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*
b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3
*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b
^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/3440640*(105*(
33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
+ b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 +
84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*
a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3
*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*
x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{14} \left(c x^6 + b x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.202 $\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$-\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-\right.}{}$$

```
[Out] -1/384*b*(-4*a*c+3*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^4+1/21*x^6*(c*x
^6+b*x^3+a)^(5/2)/c+1/840*(-30*b*c*x^3-16*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)
/c^3-1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)
/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*
x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(10
24*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*
x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
```

```
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)\\ &= \frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\operatorname{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{21c}\\ &= \frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-16ac-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{\left(b\left(3b^2-4\right.\right.}{}\\ &= -\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-\right.}{}\\ &= \frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(\right.}{384c^4}\\ &= \frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(\right.}{384c^4}\\ &= \frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(\right.}{384c^4}\end{aligned}$$

Mathematica [A] time = 0.19, size = 192, normalized size = 0.86

$$\frac{-\frac{\left(16ac-21b^2+30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{40c^2}+\frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+c}}\right.\right.}{2048c^{9/2}}}{21c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^3)*(a + b*x^
3 + c*x^6)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3)
*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^
2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(
2048*c^(9/2)))/(21*c)
```

fricas [A] time = 0.99, size = 535, normalized size = 2.40

$$\left[-\frac{105\left(3b^7 - 28ab^5c + 80a^2b^3c^2 - 64a^3bc^3\right)\sqrt{c}\log\left(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a}\left(2cx^3 + b\right)\sqrt{c} - 4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 + 64
*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4*c^2
 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sqrt(c*x
^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 -
64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128
*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c -
2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}} x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11}\left(c\,x^6 + b\,x^3 + a\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.203 $\int x^8 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(b^2-4ac\right)^2 \left(7b^2-4ac\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)}{}$$

```
[Out] 1/576*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^3-7/180*b*(c*x^6+b
*x^3+a)^(5/2)/c^2+1/18*x^3*(c*x^6+b*x^3+a)^(5/2)/c+1/3072*(-4*a*c+b^2)^2*(-
4*a*c+7*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)
-1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(153
6*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3
) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5
/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)$$

$$=\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}+\frac{\text{Subst}\left(\int\left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)}{18c}$$

$$=-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}+\frac{\left(7b^2-4ac\right)\text{Subst}\left(\int\left(a+bx+\right.\right.}{72c^2}$$

$$=\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3\right.}{18c}$$

$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a\right.}{576c^3}$$

$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a\right.}{576c^3}$$

$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a\right.}{576c^3}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 0.86

$$\frac{\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{512c^{7/2}}+x^3\left(a+bx^3+cx^6\right)^{5/2}-\frac{7b(a-}{}}{18c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((-7*b*(a + b*x^3 + c*x^6)^(5/2))/(10*c) + x^3*(a + b*x^3 + c*x^6)^(5/2) +
((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 +
 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(512*c^(7/2)))/(18*c)
```

fricas [A] time = 1.06, size = 451, normalized size = 2.21

$$\left[-\frac{15\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 140*
a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1
296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760
*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}} x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \left(c\,x^6 + b\,x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.204 $\int x^5 \left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$-\frac{b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}+\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}$$

```
[Out] -1/48*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^2+1/15*(c*x^6+b*x^3+a)^(5/2)/c-
1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/c^(7/2)+1/128*b*(-4*a*c+b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b +
 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(
15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{1}{3}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2} dx, x, x^3\right)$$
$$= \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} - \frac{b\,\text{Subst}\left(\int \left(a+bx+cx^2\right)^{3/2} dx, x, x^3\right)}{6c}$$
$$= -\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\right.}{32}$$
$$= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{(a+}{}$$
$$= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{(a+}{}$$
$$= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{(a+}{}$$

Mathematica [A] time = 0.15, size = 149, normalized size = 0.99

$$-\frac{b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}} - \frac{b\left(b+2cx^3\right)\left(a+bx^3+\right.}{48c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/48*(b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/c^2 + (a + b*x^3 + c*x^6)
^(5/2)/(15*c) - (b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 +
 c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c
*x^6])]))/(256*c^(7/2))
```

fricas [A] time = 1.11, size = 361, normalized size = 2.41

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(128c\right.}{76}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*
a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/
2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))
 + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6
+ b*x^3 + a))/c^4]
```

giac [A] time = 0.56, size = 172, normalized size = 1.15

$$\frac{1}{1920}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\left(8\,cx^3+11\,b\right)x^3+\frac{b^2c^3+32\,ac^4}{c^4}\right)x^3-\frac{5\,b^3c^2-28\,abc^3}{c^4}\right)x^3+\frac{15\,b^4c-100\,ab^2c^2+1\cdots}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 + 3
2*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a*b
^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs
(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(7/2)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}} x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.58, size = 223, normalized size = 1.49

$$\frac{\left(c\,x^6+b\,x^3+a\right)^{5/2}}{15\,c}-\frac{b\left(\frac{3\,a\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^3+b\right)\sqrt{c\,x^6+b\,x^3+a}}{4\,c}\right)}{4}+\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4}-\frac{3\,b^2\left(\ln\left(\sqrt{c\,x^6+b\,x^3\cdots}\right.\right.}{\cdots}\right.}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*((3*a*(log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)
*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/4 + (x^3*(a + b*x^3 + c*x^6)^(3/2))/4 -
 (3*b^2*(log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^3 + c*x^6)^(3/2))/(8*c)))/(6*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.205 $\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=124

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

```
[Out] 1/24*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c+1/128*(-4*a*c+b^2)^2*arctanh(1/2*(
2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/64*(-4*a*c+b^2)*(2*c*x^
3+b)*(c*x^6+b*x^3+a)^(1/2)/c^2
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1352, 612, 621, 206}

$$-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) + ((b + 2*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{1}{3}\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)$$
$$=\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}-\frac{\left(b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{16c}$$
$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-\right.}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-\right.}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-\right.}{}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 1.02

$$\frac{\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)}{8c^{3/2}}+2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(
b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(8*c^(3/2)))/(48*c)
```

fricas [A] time = 1.14, size = 297, normalized size = 2.40

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}\,-4\,ac\right)+4\left(16\,c^4x^9\right.}{768\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3
 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^3)*
sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b
*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

giac [A] time = 0.61, size = 135, normalized size = 1.09

$$\frac{1}{192}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,cx^3+3\,b\right)x^3+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^3-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*c
^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(
5/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.44, size = 115, normalized size = 0.93

$$\frac{\left(c\,x^3+\frac{b}{2}\right)\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{12\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] ((b/2 + c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(12*c) + ((3*a*c - (3*b^2)/4)*((b
/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(12*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.206 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x} \, dx$

Optimal. Leaf size=155

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+}}{24c}$$

```
[Out] 1/9*(c*x^6+b*x^3+a)^(3/2)-1/3*a^(3/2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^
6+b*x^3+a)^(1/2))-1/48*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/c^(3/2)+1/24*(2*b*c*x^3+8*a*c+b^2)*(c*x^6+b*x^3+a)^(1/2)
/c
```

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c
*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6])])/(48*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx,x,x^3\right)$$

$$= \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{6}\,\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^3\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)}{x\sqrt{a+bx+cx^2}}\right.}{24c}$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}\left(2a^2\right)\text{Subst}\left(\int \frac{1}{4a-x}\right.$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.$$

Mathematica [A] time = 0.14, size = 143, normalized size = 0.92

$$\frac{1}{144}\left(-48a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{c^{3/2}} + \frac{2\sqrt{a+bx^3+cx^6}\left(8c\left(4a\right.\right.}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c - 4
8*a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] - (3*b
*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])
/c^(3/2))/144
```

fricas [A] time = 1.50, size = 727, normalized size = 4.69

$$\left[\frac{48\,a^{\frac{3}{2}}c^2\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-3\left(b^3-12\,abc\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\right.}{288\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*c^2*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a
^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^
2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(48*sqrt(-a
)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6
+ a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x
^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 +
14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

3.207 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=150

$$\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{}{2\sqrt{a}\,\sqrt{}}\right.$$

```
[Out] -1/3*(c*x^6+b*x^3+a)^(3/2)/x^3-1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6
+b*x^3+a)^(1/2))*a^(1/2)+1/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/4*(2*c*x^3+3*b)*(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 732, 814, 843, 621, 206, 724}

$$\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{}{2\sqrt{a}\,\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*
x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])
])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(8*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{2}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^3\right)\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}-\frac{\text{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx,x,x\right.}{8c}\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{2}(ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}-(ab)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{}{\sqrt{a}}\right.\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+}}\right.
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 134, normalized size = 0.89

$$
\frac{1}{24}\left(\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}+\frac{2\sqrt{a+bx^3+cx^6}\left(-4a+5bx^3+2cx^6\right)}{x^3}-12\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/x^3 - 12*Sqrt[a]*b*
ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] + (3*(b^2 + 4*a*
c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[c])/24
```

fricas [A] time = 1.14, size = 713, normalized size = 4.75

$$\left[\frac{12\,\sqrt{a}\,bcx^3\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+3\left(b^2+4\,ac\right)\sqrt{c}\,x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2\right.}{48\,cx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^
6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 + b*x
^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqr
t(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*
x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

3.208 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{a}}\right.$$

```
[Out] -1/6*(c*x^6+b*x^3+a)^(3/2)/x^6-1/8*(4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1
/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/4*(-2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/x^3
```

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 732, 812, 843, 621, 206, 724}

$$-\frac{\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c*x^6)^(3/2
)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^7}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} - \frac{1}{8}\operatorname{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\,dx,\right.$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} + \frac{1}{2}(bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,d\right.$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} + (bc)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b}{\sqrt{a+}}\right.$$

$$= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6} - \frac{\left(b^2+4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right.}{8\sqrt{a}}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.89

$$\frac{1}{24}\left(-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}} - \frac{2\sqrt{a+bx^3+cx^6}\left(2a+5bx^3-4cx^6\right)}{x^6} + 12b\sqrt{c}\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] ((-2*(2*a + 5*b*x^3 - 4*c*x^6)*Sqrt[a + b*x^3 + c*x^6])/x^6 - (3*(b^2 + 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[a] + 1
2*b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/24
```

fricas [A] time = 1.10, size = 713, normalized size = 4.72

$$\left[\frac{12\,ab\sqrt{c}\,x^6\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+3\left(b^2+4\,ac\right)\sqrt{a}\,x^6\log\left(-\frac{(b}{}\right.}{48\,ax^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*lo
g(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 + b*x
^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sq
rt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

3.209 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+}}\right.$$

```
[Out] -1/9*(c*x^6+b*x^3+a)^(3/2)/x^9+1/48*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)
/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+1/3*c^(3/2)*arctanh(1/2*(2*c*x^3+b)
/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))-1/24*(2*a*b+(8*a*c+b^2)*x^3)*(c*x^6+b*x^3+a
)^(1/2)/a/x^6
```

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 732, 810, 843, 621, 206, 724}

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6) - (a + b*
x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{1}{6}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b\left(b^2-12ac\right)-8a}{x\sqrt{a+bx+cx^2}}\right.}{24a}$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{1}{3}c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{1}{3}\left(2c^2\right)\text{Subst}\left(\int \frac{1}{4c-}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9} + \frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\right.}{48a^{3/2}}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.91

$$\frac{1}{144}\left(\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}-\frac{2\sqrt{a+bx^3+cx^6}\left(8a^2+14abx^3+32acx^6+3b^2x^6\right)}{ax^9}+48c^{3/2}\text{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] ((-2*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 14*a*b*x^3 + 3*b^2*x^6 + 32*a*c*x^6))
/(a*x^9) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*
x^3 + c*x^6])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/144
```

fricas [A] time = 1.46, size = 771, normalized size = 4.73

$$\left[\frac{48\,a^2c^{\frac{3}{2}}x^9\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-3\left(b^3-12\,abc\right)\sqrt{a}\,x^9\log\left(-\right.}{288\,a^2x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8
*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^9*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*
b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x
^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-
a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt
(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9*arctan(1/2*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3
*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14
*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

3.210 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}} \, dx$

Optimal. Leaf size=133

$$-\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}+\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

```
[Out] -1/24*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a/x^12-1/128*(-4*a*c+b^2)^2*arctanh
(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/64*(-4*a*c+b^2)*(
b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6
```

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 720, 724, 206}

$$\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a
+ b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^3\right)$$

$$= -\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{16a}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} + \frac{(b^2-4ac)^2\,\text{S}}{}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2\,\text{S}}{}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2\,\text{t}}{}$$

Mathematica [A] time = 0.16, size = 138, normalized size = 1.04

$$-\frac{\frac{3(b^2-4ac)\left(x^6(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{a}(2a+bx^3)\sqrt{a+bx^3+cx^6}\right)}{8a^{3/2}x^6}+\frac{2(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{x^{12}}}{48a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/48*((2*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/x^12 + (3*(b^2 - 4*a*c)*
(-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*x^6*ArcTa
nh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6)])]))/(8*a^(3/2)*x^6))/a
```

fricas [A] time = 1.11, size = 319, normalized size = 2.40

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{a}\,x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\left(\left(3\,ab^3-20\,a^2bc\right)x^9-24\,a\right.}{768\,a^3x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*x
^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)
*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a*b^2
*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^9 -
24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a)
)/(a^3*x^12)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

3.211 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=162

$$\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-$$

```
[Out] 1/48*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1/15*(c*x^6+b*x^3+a)^(5/2
)/a/x^15+1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^
3+a)^(1/2))/a^(7/2)-1/128*b*(-4*a*c+b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/
a^3/x^6
```

Rubi [A] time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3*x^6) + (b
*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*x^
6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
```

```
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}} - \frac{b\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}} + \frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+\cdots}}{\cdots}\right.}{32a^2}$$

$$= -\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{(a+\cdots}{\cdots}$$

$$= -\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{(a+\cdots}{\cdots}$$

$$= -\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{(a+\cdots}{\cdots}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 1.03

$$\frac{b\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2} - 3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6} - x^6\left(b^2-4ac\right)\tanh\cdots\right.\right.}{768a^{7/2}x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/15*(a + b*x^3 + c*x^6)^(5/2)/(a*x^15) + (b*(16*a^(3/2)*(2*a + b*x^3)*(a
+ b*x^3 + c*x^6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[
a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqr
t[a + b*x^3 + c*x^6])])))/(768*a^(7/2)*x^12)
```

fricas [A] time = 1.08, size = 383, normalized size = 2.36

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{a}\,x^{15}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right) - 4\left(\left(15ab^4-100a^2b^2c\cdots\right.\right.}{7680a^4x^{15}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 -
 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sq
rt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*
x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4
*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

3.212 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\, dx$

Optimal. Leaf size=216

$$-\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}+\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{\left(7b^2-4ac\right.}{}$$

```
[Out] -1/576*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^12-1/18*(c*x^
6+b*x^3+a)^(5/2)/a/x^18+7/180*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^15-1/3072*(-4*a
*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1
/2))/a^(9/2)+1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)
^(1/2)/a^4/x^6
```

Rubi [A] time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}}+\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{\left(b^2-4ac\right)^2\left(7b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536
*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*
a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6
)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} - \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx,x,x^3\right)}{18a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} + \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180a^2x^{15}} + \frac{\left(7b^2-4ac\right)\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx\right.}{72a^2}$$

$$= -\frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} + \frac{7b\left(a+bx^3+c\right.}{180a^2x^{15}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+b\right.}{576a^3x^{12}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+b\right.}{576a^3x^{12}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+b\right.}{576a^3x^{12}}$$

Mathematica [A] time = 0.21, size = 206, normalized size = 0.95

$$-\frac{\frac{\left(\frac{7b^2}{2}-2ac\right)\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)\right)}{256a^{7/2}x^{12}}+\frac{\left(a+bx^3+cx^6\right.}{x^{18}}}{18a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] -1/18*((a + b*x^3 + c*x^6)^(5/2)/x^18 - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(10
*a*x^15) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^
6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*
x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])))/(256*a^(7/2)*x^12))/a
```

fricas [A] time = 1.83, size = 473, normalized size = 2.19

$$\left[-\frac{15\left(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)\sqrt{a}\,x^{18}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{6}+8\,abx^{3}+4\,\sqrt{cx^{6}+bx^{3}+a}\left(bx^{3}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{6}}\right)-4\left(\left(105\,ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
+ 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3
 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280
*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c
+ 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3
 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 7
60*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a
^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*
b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

3.213 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b\left(3b^2-\right.}{}$$

```
[Out] 1/384*b*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^4/x^12-1/21*(c*x
^6+b*x^3+a)^(5/2)/a/x^21+1/28*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^18-1/840*(-16*a
*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)/a^3/x^15+1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3
*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(11/2)-1/102
4*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^5/x^6
```

Rubi [A] time = 0.31, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}}+\frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx\right.}{1024a^5x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1
024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/
(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 + c*
x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(
840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2
*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} - \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{9b}{2}+2cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^3\right)}{21a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} + \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{3}{4}\left(21b^2-16ac\right)+\frac{9bcx}{2}\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,d\ldots\right.}{126a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} - \frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}} - \frac{\left(b\left(3\ldots\right.\right.}{\ldots}$$

$$= \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} \ldots$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+\ldots\right.}{384a^4x^{12}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+\ldots\right.}{384a^4x^{12}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+\ldots\right.}{384a^4x^{12}}$$

Mathematica [A] time = 0.49, size = 243, normalized size = 0.95

$$-\frac{\frac{\left(21abc-\frac{63b^3}{4}\right)\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)\right)}{1536a^{9/2}x^{12}} + \frac{\left(21b^2-16ac\right)\left(\ldots\right.}{40\ldots}}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -1/21*((a + b*x^3 + c*x^6)^(5/2)/x^21 - (3*b*(a + b*x^3 + c*x^6)^(5/2))/(4*
a*x^18) + ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(40*a^2*x^15) + (((
-63*b^3)/4 + 21*a*b*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2)
- 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] - (b
^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
))/(1536*a^(9/2)*x^12))/a
```

fricas [A] time = 2.03, size = 557, normalized size = 2.18

$$\left[-\frac{105\left(3b^7-28ab^5c+80a^2b^3c^2-64a^3bc^3\right)\sqrt{a}\,x^{21}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(315\ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(a
)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x
```

```
^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3
*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b
*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b
*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c
)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 - 28*a*b^
5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((315*a*b^
6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5
 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 1
28*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*
a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^6 + b\,x^3 + a\right)}^{3/2}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

3.214 $\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=141

$$\frac{ax^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/4*a*x^4*AppellF1(4/3,-3/2,-3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{ax^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx&=\frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\&=\frac{ax^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.87, size = 453, normalized size = 3.21

$$x\left(27x^3\left(640a^2c^2-404ab^2c+55b^4\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+8\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(-297*b^4*x^3 - 81*b^3*c*x^6 + 3464*b^2*c^2*x^9 + 5488*b*c^3*x^12 + 2
240*c^4*x^15 + 4*a^2*c*(459*b + 1280*c*x^3) + a*(-297*b^3 + 2052*b^2*c*x^3
+ 10204*b*c^2*x^6 + 7360*c^3*x^9)) + 216*a*b*(11*b^2 - 68*a*c)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 27*(55
*b^4 - 404*a*b^2*c + 640*a^2*c^2)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(232960*c^2*Sqrt[a + b*x^3 + c*
x^6])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^9+bx^6+ax^3\right)\sqrt{cx^6+bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(c x^6 + b x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^3*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.215 $\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=141

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/2*a*x^2*AppellF1(2/3,-3/2,-3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx &= \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{ax^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [B] time = 0.70, size = 410, normalized size = 2.91

$$\frac{x^2\left(10\left(448a^2c+27ab^2+698abcx^3+608ac^2x^6+27b^3x^3+277b^2cx^6+410bc^2x^9+160c^3x^{12}\right)-27bx^3\left(7b^2-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(10*(27*a*b^2 + 448*a^2*c + 27*b^3*x^3 + 698*a*b*c*x^3 + 277*b^2*c*x^6
 + 608*a*c^2*x^6 + 410*b*c^2*x^9 + 160*c^3*x^12) - 270*a*(b^2 - 16*a*c)*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 - 27*b*(7*b^2 - 52*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(17600*c*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^7+bx^4+ax\right)\sqrt{cx^6+bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.216 $\int \left(a+bx^3+cx^6\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{ax\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] a*x*AppellF1(1/3,-3/2,-3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{ax\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^3+cx^6\right)^{3/2}\,dx &= \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{ax\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [B] time = 0.67, size = 408, normalized size = 3.00

$$\frac{x\left(8\left(364a^2c+27ab^2+548abcx^3+476ac^2x^6+27b^3x^3+211b^2cx^6+296bc^2x^9+112c^3x^{12}\right)-27bx^3\left(5b^2-44ac\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(27*a*b^2 + 364*a^2*c + 27*b^3*x^3 + 548*a*b*c*x^3 + 211*b^2*c*x^6 +
476*a*c^2*x^6 + 296*b*c^2*x^9 + 112*c^3*x^12) - 216*a*(b^2 - 28*a*c)*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] -
27*b*(5*b^2 - 44*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(8960*c*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6+bx^3+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2), x)
```

3.217 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -a*AppellF1(-1/3,-3/2,-3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.48, size = 379, normalized size = 2.73

$$\frac{10\left(-80a^2-61abx^3-70acx^6+19b^2x^6+29bcx^9+10c^2x^{12}\right)+810abx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\right.}{80\!\!}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] (10*(-80*a^2 - 61*a*b*x^3 + 19*b^2*x^6 - 70*a*c*x^6 + 29*b*c*x^9 + 10*c^2*x
^12) + 810*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(800*x*Sqrt[a + b*x^3 +
 c*x^6])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^2,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

3.218 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -1/2*a*AppellF1(-2/3,-3/2,-3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.48, size = 379, normalized size = 2.69

$$\frac{8\left(-28a^2-11abx^3-20acx^6+17b^2x^6+25bcx^9+8c^2x^{12}\right)+648abx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{3};\frac{1}{2},\right.}{448x^2\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] (8*(-28*a^2 - 11*a*b*x^3 + 17*b^2*x^6 - 20*a*c*x^6 + 25*b*c*x^9 + 8*c^2*x^1
2) + 648*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 8*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(448*x^2*Sqrt[a + b*x^3 +
c*x^6])
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\, x^6 + b\, x^3 + a\right)^{3/2}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

3.219 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=171

$$\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx^3+cx^6}}{576c^4}-7$$

```
[Out] 1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^
6+b*x^3+a)^(1/2))/c^(9/2)-7/72*b*x^6*(c*x^6+b*x^3+a)^(1/2)/c^2+1/12*x^9*(c*
x^6+b*x^3+a)^(1/2)/c-1/576*(5*b*(-44*a*c+21*b^2)-2*c*(-36*a*c+35*b^2)*x^3)*
(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 832, 779, 621, 206}

$$\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx^3+cx^6}}{576c^4}-7$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])
/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x
^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x^2\left(-3a-\frac{7bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12c}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x\left(7ab+\frac{1}{4}\left(35b^2-36ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36c^2}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x\right.}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x\right.}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x\right.}{576c^4}$$

Mathematica [A] time = 0.11, size = 137, normalized size = 0.80

$$\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(4bc\left(55a-14cx^6\right)+24c^2x^3\left(2cx\right.\right.}{1152c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 70*b^2*c*x^3 + 4*b*c*(55*a -
 14*c*x^6) + 24*c^2*x^3*(-3*a + 2*c*x^6)) + 3*(35*b^4 - 120*a*b^2*c + 48*a^
2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^
(9/2))
```

fricas [A] time = 1.03, size = 303, normalized size = 1.77

$$\left[\frac{3\left(35\,b^{4}-120\,ab^{2}c+48\,a^{2}c^{2}\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\left(2\,cx^{3}+b\right)\sqrt{c}-4\,ac\right)+4\left(48\,c^{4}\right.}{2304\,c^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b
*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*
a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c
 + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^
5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{cx^{6}+bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

3.220 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=121

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

```
[Out] -1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/
2))/c^(7/2)+1/9*x^6*(c*x^6+b*x^3+a)^(1/2)/c+1/72*(-10*b*c*x^3-16*a*c+15*b^2
)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 742, 779, 621, 206}

$$\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9c}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Sub}}{}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Sub}}{}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b\left(5b^2-12ac\right)\tanh^{-}}{48c^{7}}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 0.86

$$\frac{\left(36abc-15b^3\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(8c\left(cx^6-2a\right)+15b^2-10bcx^3\right)}{144c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 10*b*c*x^3 + 8*c*(-2*a + c*x^6
)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 +
 c*x^6])])/(144*c^(7/2))
```

fricas [A] time = 1.20, size = 241, normalized size = 1.99

$$\left[-\frac{3\left(5\,b^3-12\,abc\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\left(8\,c^3x^6-10\,bc^2\right.}{288\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*
sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 10*
b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*(5*
b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c
 - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{cx^6 + bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

3.221 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

```
[Out] 1/24*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
c^(5/2)-1/4*b*(c*x^6+b*x^3+a)^(1/2)/c^2+1/6*x^3*(c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 742, 640, 621, 206}

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c)
+ ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/(24*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\operatorname{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}\\ &= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c^2}\\ &= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c^2}\\ &= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.85

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\left(2cx^3-3b\right)\sqrt{a+bx^3+cx^6}}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (3*b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(5/2))
```

fricas [A] time = 0.64, size = 203, normalized size = 1.95

$$\left[-\frac{\left(3b^2-4ac\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3\right.}{48c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c
*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*s
qrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

3.222 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[Out] -1/6*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+1/3*(
c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 640, 621, 206}

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.00

$$\frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

fricas [A] time = 1.00, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c}\,\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 + 4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c} - 4\,ac\right) + 4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c^2}, \frac{b\sqrt{-c}\,\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6*(
b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x
^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]
```

giac [A] time = 0.67, size = 61, normalized size = 0.90

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^3 - \sqrt{cx^6+bx^3+a}\right)\sqrt{c} - b\right|\right)}{6\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^6+bx^3+a}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3
/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.49, size = 55, normalized size = 0.81

$$\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,c}-\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 +
 c*x^3)/c^(1/2)))/(6*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

3.223 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

```
[Out] 1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1352, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

fricas [A] time = 1.07, size = 118, normalized size = 2.74

$$\left[\frac{\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 - 4\,\sqrt{cx^6 + bx^3 + a}\left(2\,cx^3 + b\right)\sqrt{c} - 4\,ac\right)}{6\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)}{3\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

giac [A] time = 0.68, size = 40, normalized size = 0.93

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/sqrt(
c)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.58, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{c\,x^6 + b\,x^3 + a} + \frac{c\,x^3 + \frac{b}{2}}{\sqrt{c}}\right)}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

3.224 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
[Out] -1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]
```

fricas [A] time = 0.90, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{6\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6 + b\,x^3 + a}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.57, size = 36, normalized size = 0.82

$$-\frac{\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] -log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

3.225 $\int \frac{1}{x^4 \sqrt{a+bx^3+cx^6}} \, dx$

Optimal. Leaf size=72

$$\frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

```
[Out] 1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)-1/3*(c
*x^6+b*x^3+a)^(1/2)/a/x^3
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 730, 724, 206}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

fricas [A] time = 1.17, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a}\,bx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right) - 4\sqrt{cx^6+bx^3+a}\,a}{12a^2x^3}, -\frac{\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3\cdots}{2(acx^6+abx\cdots}\right)\cdots}{6a\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*a)/
(a^2*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}\;x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.56, size = 56, normalized size = 0.78

$$\frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^3}{2}+a}{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}\right)}{6\,a^{3/2}}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^3)/2)/(a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))))/(6*a^(3/2))
- (a + b*x^3 + c*x^6)^(1/2)/(3*a*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\sqrt{a + bx^{3} + cx^{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

3.226 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

```
[Out] -1/24*(-4*a*c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))
/a^(5/2)-1/6*(c*x^6+b*x^3+a)^(1/2)/a/x^6+1/4*b*(c*x^6+b*x^3+a)^(1/2)/a^2/x^
3
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 744, 806, 724, 206}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x^3
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^
6])])/(24*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} + \frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.85

$$\frac{\left(4ac-3b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{2\sqrt{a}\left(3bx^3-2a\right)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((2*Sqrt[a]*(-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (-3*b^2 + 4*a*c
)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))
```

fricas [A] time = 1.07, size = 221, normalized size = 2.05

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}\left(3\,abx^3-2\,a^2\right)}{48\,a^3x^6}, \;(3 \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4
*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6
 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}\, x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6 + b\,x^3 + a}\; x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^6 + b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

3.227 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=145

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

```
[Out] 1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/a^(7/2)-1/9*(c*x^6+b*x^3+a)^(1/2)/a/x^9+5/36*b*(c*x^6+b*x^3+a)^(1/2)/a^2
/x^6-1/72*(-16*a*c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^3
```

Rubi [A] time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^2*
x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*
a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
```

```
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2-16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{18a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3} - \frac{\left(b\left(5b^2-\right.\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{\left(b\left(5b^2-\right.\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b\left(5b^2-1\right.}{}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.77

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}} + \frac{\sqrt{a+bx^3+cx^6}\left(-8a^2+2a\left(5bx^3+8cx^6\right)-15b^2x^6\right)}{72a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 15*b^2*x^6 + 2*a*(5*b*x^3 + 8*c*x^6)))/(
72*a^3*x^9) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6])])/(48*a^(7/2))
```

fricas [A] time = 1.22, size = 263, normalized size = 1.81

$$\left[-\frac{3\left(5\,b^{3}-12\,abc\right)\sqrt{a}\,x^{9}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\left(bx^{3}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{6}}\right)+4\left(\left(15\,ab^{2}-16\,a^{2}c\right)x^{6}-10\,a^{2}bx^{3}}{288\,a^{4}x^{9}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((15
*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^
4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^2
- 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{6}+bx^{3}+a}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^{6}+b\,x^{3}+a}\;x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\,\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

3.228 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=192

$$\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{\left(48a^2c^2-120ab^2c+3\right.}{}$$

```
[Out] -1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/a^(9/2)-1/12*(c*x^6+b*x^3+a)^(1/2)/a/x^12+7/72*b*(c*x^6+
b*x^3+a)^(1/2)/a^2/x^9-1/288*(-36*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^6
+5/576*b*(-44*a*c+21*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^3
```

Rubi [A] time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}+\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+}}{288a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^6])/(72*a^
2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b*(
21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*a*
b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6
])])/(384*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
```

```
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\frac{7b}{2}+3cx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(35b^2-36ac\right)+7bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b\left(21b^2\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b\left(21b^2\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b\left(21b^2\right.}{}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.73

$$\frac{\sqrt{a+bx^3+cx^6}\left(-48a^3+8a^2\left(7bx^3+9cx^6\right)-10abx^6\left(7b+22cx^3\right)+105b^3x^9\right)}{576a^4x^{12}} - \frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 105*b^3*x^9 - 10*a*b*x^6*(7*b + 22*c*x^
3) + 8*a^2*(7*b*x^3 + 9*c*x^6)))/(576*a^4*x^12) - ((35*b^4 - 120*a*b^2*c +
48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(38
4*a^(9/2))
```

fricas [A] time = 1.29, size = 327, normalized size = 1.70

$$\left[\frac{3\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)\sqrt{a}\,x^{12}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\left(5\left(21\,ab^3-44\,a^2b\right.\right.}{2304\,a^5x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*
a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2
 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*(3*(
35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21*a*b^3
 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)
*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}\;x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

3.229 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 168, normalized size = 1.20

$$\frac{x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2
, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^3/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

3.230 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 168, normalized size = 1.20

$$\frac{x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2
, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**3 + c*x**6), x)
```

3.231 $\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[Out] x*AppellF1(1/3,1/2,1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 163, normalized size = 1.21

$$\frac{x\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2,
1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*
a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(1/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^3 + cx^6}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

3.232 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

```
[Out] -AppellF1(-1/3,1/2,1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/x/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] time = 0.35, size = 343, normalized size = 2.49

$$\frac{5bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+8cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}\cdots}{20ax\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 5*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*x*Sqrt[a + b*x^
3 + c*x^6])
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{cx^8+bx^5+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^8 + b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

3.233 $\int \frac{1}{x^3 \sqrt{a+bx^3+cx^6}} \, dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -1/2*AppellF1(-2/3,1/2,1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}$$
$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] time = 0.30, size = 342, normalized size = 2.44

$$\frac{-2bx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+cx^6\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}\cdots}{\sqrt{b^2-4\cdots}}}}{8ax^2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) - 2*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{cx^9+bx^6+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^9 + b*x^6 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^6+bx^3+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{c\,x^6+b\,x^3+a}\;x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^6+bx^3+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

3.234 $\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}-\frac{2bx^6\sqrt{a+bx^3+cx}}{3c\left(b^2-4ac\right)}$$

```
[Out] 1/8*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c
^(7/2)+2/3*x^9*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*x^6*(c*
x^6+b*x^3+a)^(1/2)/c/(-4*a*c+b^2)-1/12*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b
^2)*x^3)*(c*x^6+b*x^3+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A] time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 738, 832, 779, 621, 206}

$$-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}+\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}+\frac{2x^9\left(2a+b\right.}{3\left(b^2-4ac\right)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*
Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*
(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^3*(b^2 - 4*a*c)) + ((5
*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{2\,\mathrm{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}\left(5b^2-12ac\right)\right.}{\sqrt{a+bx+cx^2}}\right.}{9c\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-\right.\right.}{12c^3\left(b^2\right.}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-\right.\right.}{12c^3\left(b^2\right.}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-\right.\right.}{12c^3\left(b^2\right.}$$

Mathematica [A] time = 0.18, size = 181, normalized size = 0.93

$$\frac{\frac{2\sqrt{c}\left(4a^2c\left(6cx^3-13b\right)+a\left(15b^3-62b^2cx^3-20bc^2x^6+8c^3x^9\right)+b^2x^3\left(15b^2+5bcx^3-2c^2x^6\right)\right)}{\sqrt{a+bx^3+cx^6}}-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{24c^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^
2*x^6) + a*(15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9)))/Sqrt[a + b*
x^3 + c*x^6] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(7/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.41, size = 591, normalized size = 3.03

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^3\right)\sqrt{c}\,\log\left(-8\,c^2x^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2
*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8*c
^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*
a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*
c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6
+ 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c
^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*
b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.235 $\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\left(-8ac+3b^2-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}+\frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)+2/3*x
^6*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)+1/3*(-2*b*c*x^3-8*a*c+3*b
^2)*(c*x^6+b*x^3+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 738, 779, 621, 206}

$$\frac{\left(-8ac+3b^2-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}+\frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 -
 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*Arc
Tanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\operatorname{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\tanh^{-1}\left(\frac{}{2}\right.}{2}$$

Mathematica [A] time = 0.12, size = 137, normalized size = 1.00

$$\frac{\frac{2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx^3+4c^2x^6\right)-b^2x^3\left(3b+cx^3\right)\right)}{\sqrt{a+bx^3+cx^6}}+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(8*a^2*c - b^2*x^3*(3*b + c*x^3) + a*(-3*b^2 + 10*b*c*x^3 + 4*c
^2*x^6)))/Sqrt[a + b*x^3 + c*x^6] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(5/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 0.90, size = 459, normalized size = 3.35

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^6+ab^3-4\,a^2bc+\left(b^4-4\,ab^2c\right)x^3\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\,\left(2\,cx^3\right.\right.}{12\left(\left(b^2c^4-4\,ac^5\right)x^6+ab^2c^3-4\,a^2c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x
^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2
```

```
*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a
^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4
*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*(3*((
b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(-c
)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
 - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b
^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.236 $\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}} dx$

Optimal. Leaf size=120

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

```
[Out] 1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+2/3*x^3*
(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*(c*x^6+b*x^3+a)^(1/2)/
c/(-4*a*c+b^2)
```

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 738, 640, 621, 206}

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt
[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}$$

$$= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3c}$$

$$= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx^3+}}\right.}{3c}$$

$$= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} + \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Mathematica [A] time = 0.10, size = 107, normalized size = 0.89

$$\frac{\frac{2\sqrt{c}\left(a\left(b-2cx^3\right)+b^2x^3\right)}{\sqrt{a+bx^3+cx^6}} - \left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c^(3/2
)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.05, size = 387, normalized size = 3.22

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\right.}{6\left(\left(b^2c^3-4\,ac^4\right)x^6+ab^2c^2-4\,a^2c^3+\left(b^3c^2-4\,abc^3\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*
b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c
^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^
```

```
2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3
+ a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.66, size = 84, normalized size = 0.70

$$\frac{\ln\left(\sqrt{c\,x^6 + b\,x^3 + a} + \frac{c\,x^3 + \frac{b}{2}}{\sqrt{c}}\right)}{3\,c^{3/2}} + \frac{\frac{a\,b}{2} - x^3\,\left(a\,c - \frac{b^2}{2}\right)}{3\,c\,\left(a\,c - \frac{b^2}{4}\right)\,\sqrt{c\,x^6 + b\,x^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(3/2)) + ((a*b)
/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.237 $\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] 2/3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1357, 636}

$$\frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 1.05

$$-\frac{2\left(2a+bx^3\right)}{3\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(2*a + b*x^3))/(3*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

fricas [A] time = 0.97, size = 68, normalized size = 1.74

$$\frac{2\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)}{3\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4
*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

giac [A] time = 1.32, size = 45, normalized size = 1.15

$$\frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

maple [A] time = 0.01, size = 38, normalized size = 0.97

$$-\frac{2\left(b\,x^3+2a\right)}{3\sqrt{c\,x^6+b\,x^3+a}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] -2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.43, size = 38, normalized size = 0.97

$$-\frac{2\,b\,x^3+4\,a}{\left(12\,a\,c-3\,b^2\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] -(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.238 $\int \frac{x^2}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] -2/3*(2*c*x^3+b)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1352, 613}

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, 
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)\\ &= -\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{2\left(b+2cx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

fricas [**A**] time = 0.87, size = 67, normalized size = 1.76

$$-\frac{2\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)}{3\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

giac [**A**] time = 1.41, size = 45, normalized size = 1.18

$$-\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

maple [**A**] time = 0.01, size = 37, normalized size = 0.97

$$\frac{\frac{4c\,x^3}{3}+\frac{2b}{3}}{\sqrt{c\,x^6+b\,x^3+a}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [**B**] time = 1.37, size = 37, normalized size = 0.97

$$\frac{4\,c\,x^3+2\,b}{\left(12\,a\,c-3\,b^2\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.239 $\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+2/3*(b*
c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 92, normalized size = 1.00

$$
\frac{1}{3}\left(\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - Ar
cTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/a^(3/2))/3
```

fricas [B] time = 0.85, size = 389, normalized size = 4.23

$$
\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^6+\left(b^3-4\,abc\right)x^3+ab^2-4\,a^2c\right)\sqrt{a}\,\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6}\cdots}{6\left(\left(a^2b^2c-4\,a^3c^2\right)x^6+a^3b^2-4\,a^4c+\left(a^2b^3-4\,a^3bc\right)x^3\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
- 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4
*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
```

```
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 -
4*a^3*b*c)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c\,x^6 + b\,x^3 + a\right)}^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.240 $\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+2/3*(b
*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^6+b*x^3+a)^(1/2)-1/3*(-8*a*c+3*b^
2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^3
```

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 740, 806, 724, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b
*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
```

```
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} - \frac{b\,\text{Subst}\left(\int \frac{\cdots}{x\sqrt{\cdots}}\cdots\right)}{\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-\cdots}\cdots\right)}{\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\tanh^{-1}\left(\frac{\cdots}{2\sqrt{a}\cdots}\right)}{2a^{5/\cdots}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.96

$$\frac{\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^3-8c^2x^6\right)+3b^2x^3\left(b+cx^3\right)\right)}{x^3\sqrt{a+bx^3+cx^6}} - 3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2
*x^6)))/(x^3*Sqrt[a + b*x^3 + c*x^6]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*a^(5/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.09, size = 485, normalized size = 3.42

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^9+\left(b^4-4\,ab^2c\right)x^6+\left(ab^3-4\,a^2bc\right)x^3\right)\sqrt{a}\,\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2\cdots}{x^6}\cdots\right)}{12\left(\left(a^3b^2c-4\,a^4c^2\right)x^9+\left(a^3b^3-4\,a^4bc\right)x^6+\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*
b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^6
 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))
/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^
5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^
3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*
a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a
^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a
^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)
*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.241 $\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2x^6\left(b^2-4ac\right)}+\frac{2\left(\cdots\right.}{3ax^6\left(b^2\cdots\right.}$$

```
[Out] -1/8*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
a^(7/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^6/(c*x^6+b*x^3+a)^(1/2)-1/
6*(-12*a*c+5*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^6+1/12*b*(-52*a*
c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^3
```

Rubi [A] time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 740, 834, 806, 724, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2x^6\left(b^2-4ac\right)}-\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{2\left(-\cdots\right.}{3ax^6\left(b^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (
b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - (
(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
/(8*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}b \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52a \cdots\right.}{12a^3 \cdots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52a \cdots\right.}{12a^3 \cdots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52a \cdots\right.}{12a^3 \cdots}$$

Mathematica [A] time = 0.13, size = 179, normalized size = 0.90

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{2\sqrt{a}\left(-8a^3c+2a^2\left(b^2+10bcx^3-12c^2x^6\right)+abx^3\left(-5b^2+62bcx^3+52c^2x^6\right)-15b^3x \cdots\right.}{x^6\sqrt{a+bx^3+cx^6}}}{24a^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 -
12*c^2*x^6) + a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6)))/(x^6*Sqrt[a + b*
x^3 + c*x^6]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(
2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]])/(24*a^(7/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.27, size = 615, normalized size = 3.11

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^{12}+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^9+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^6\right)\sqrt{a}\,\log\left(-\right.}{48\left(\left(a^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)*l
og(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a
)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 -
62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b
*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3
- 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*a*b^2*c
^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b^4
- 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 +
a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3*c - 52*
a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c
- 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7 \left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.242 $\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}-\frac{(256a\cdots}{}$$

```
[Out] 5/48*b*(-12*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/a^(9/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^9/(c*x^6+b*x^3+a)^(1/2)
-1/9*(-16*a*c+7*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^9+1/36*b*(-11
6*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^6-1/72*(256*a^2*c^2-
460*a*b^2*c+105*b^4)*(c*x^6+b*x^3+a)^(1/2)/a^4/(-4*a*c+b^2)/x^3
```

Rubi [A] time = 0.29, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 740, 834, 806, 724, 206}

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (
b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) -
((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^
2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(48*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-7b^2+16ac\right)-3bcx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-11\right.}{36a^3}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-11\right.}{36a^3}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-11\right.}{36a^3}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-11\right.}{36a^3}$$

Mathematica [A] time = 0.18, size = 223, normalized size = 0.87

$$\frac{\frac{2\sqrt{a}\left(-32a^4c+8a^3\left(b^2+7bcx^3+16c^2x^6\right)+2a^2x^3\left(-7b^3-86b^2cx^3+244bc^2x^6+128c^3x^9\right)+5ab^2x^6\left(7b^2-106bcx^3-92c^2x^6\right)+105b^4x^9\left(b+cx^3\right)\right)}{x^9\sqrt{a+bx^3+cx^6}}-15b\left(48a^2c\right.}{144a^{9/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106
*b*c*x^3 - 92*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(
-7*b^3 - 86*b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9)))/(x^9*Sqrt[a + b*x^3
+ c*x^6]) - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2
*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(144*a^(9/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.50, size = 705, normalized size = 2.75

$$\left[-\frac{15\left(\left(7\,b^5c-40\,ab^3c^2+48\,a^2bc^3\right)x^{15}+\left(7\,b^6-40\,ab^4c+48\,a^2b^2c^2\right)x^{12}+\left(7\,ab^5-40\,a^2b^3c+48\,a^3bc^2\right)x^9\right)\sqrt{a}\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b
^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*
x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b
^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3
- 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 +
(a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*((7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2
)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) +
2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^
2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x
^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b
^2 - 4*a^7*c)*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10}\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.243 $\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,3/2,3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\
&= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}
\end{aligned}$$

Mathematica [B] time = 0.32, size = 340, normalized size = 2.38

$$\frac{x\left(cx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+2b\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4a\ldots}}{\sqrt{b^2\ldots}}}\right.}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(-2*(b + 2*c*x^3) + 2*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^
3 + c*x^6])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^6+bx^3+a}\,x^3}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*
x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^3/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.244 $\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,3/2,3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$=\frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] time = 0.48, size = 362, normalized size = 2.53

$$\frac{x^2\left(8bcx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+5\left(4ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2c\ldots}{b-\sqrt{b^2-4ac}}}\right.}{30a\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (x^2*(-20*(b^2 - 2*a*c + b*c*x^3) + 5*(b^2 + 4*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*b*c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(30*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}\,x}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x/(c*x^6+b*x^3+a)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.245 $\int \frac{1}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

```
[Out] x*AppellF1(1/3,3/2,3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\ &= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] time = 0.47, size = 359, normalized size = 2.60

$$\frac{x\left(bcx^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-2\left(b^2-8ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2c}{b-\sqrt{b^2-4ac}}}\right.}{6a\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*(-4*(b^2 - 2*a*c + b*c*x^3) - 2*(b^2 - 8*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^3*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(6*a*(
-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6
+ 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(1/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(-3/2), x)
```

3.246 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

```
[Out] -AppellF1(-1/3,3/2,3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/a/x/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\ &= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] time = 0.71, size = 407, normalized size = 2.89

$$-\frac{5bx^3\left(12ac-5b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)-4\left(60a^2c+2cx^6\left(5\right.\right.}{60a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -1/60*(5*b*(-5*b^2 + 12*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(60*a^2*c - 25*b^2*x^3*(b + c*x^3)
 + 5*a*(-3*b^2 + 18*b*c*x^3 + 16*c^2*x^6) + 2*c*(5*b^2 - 16*a*c)*x^6*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/
(a^2*(b^2 - 4*a*c)*x*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{c^2x^{14}+2\,bcx^{11}+\left(b^2+2\,ac\right)x^8+2\,abx^5+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^14 + 2*b*c*x^11 + (b^2 + 2*a*c)*x^8
 + 2*a*b*x^5 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(c\,x^6 + b\,x^3 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.247 $\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -1/2*AppellF1(-2/3,3/2,3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*x^2*Sqrt[a + b*x^3 + c*x^6]
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\
&= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}
\end{aligned}$$

Mathematica [B] time = 0.64, size = 405, normalized size = 2.83

$$\frac{-48a^2c+2bx^3\left(7b^2-36ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+cx^6\left(20\right.}{24a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-48*a^2*c + 28*b^2*x^3*(b + c*x^3) + 4*a*(3*b^2 - 24*b*c*x^3 - 20*c^2*x^6)
 + 2*b*(7*b^2 - 36*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*(-7*b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(24*a^2*(-b^
2 + 4*a*c)*x^2*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}}{c^2x^{15}+2\,bcx^{12}+\left(b^2+2\,ac\right)x^9+2\,abx^6+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^15 + 2*b*c*x^12 + (b^2 + 2*a*c)*x^9
 + 2*a*b*x^6 + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.248 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(4+m)/d^4/(4+m)+(2*a*c+b^2)*(d*x)^(7+m)
/d^7/(7+m)+2*b*c*(d*x)^(10+m)/d^10/(10+m)+c^2*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1353}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b
^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{3+m}}{d^3}+\frac{\left(b^2+2ac\right)(dx)^{6+m}}{d^6}+\frac{2bc(dx)^{9+m}}{d^9}+\frac{c^2(dx)^{12+m}}{d^{12}}\right)\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.69

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^6\left(2ac+b^2\right)}{m+7}+\frac{2abx^3}{m+4}+\frac{2bcx^9}{m+10}+\frac{c^2x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m)
+ (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))
```

fricas [B] time = 0.84, size = 241, normalized size = 2.39

$$\left(\left(c^2m^4+22\,c^2m^3+159\,c^2m^2+418\,c^2m+280\,c^2\right)x^{13}+2\left(bcm^4+25\,bcm^3+195\,bcm^2+535\,bcm+364\,bc\right)x^1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*m
^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c + 7
42*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*
m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*
a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

giac [B] time = 0.47, size = 449, normalized size = 4.45

$$\frac{(dx)^m c^2m^4x^{13} + 22\,(dx)^m c^2m^3x^{13} + 159\,(dx)^m c^2m^2x^{13} + 2\,(dx)^m bcm^4x^{10} + 418\,(dx)^m c^2mx^{13} + 50\,(dx)^m bcm^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
+ 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^10
 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7 +
2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^7 +
56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^7 + 4
98*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^7 + 14
84*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7 + 1040*
(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 2402*(d*x)^
m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d*x)^m*a^2
*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4 + 445*m^3
 + 2485*m^2 + 5714*m + 3640)
```

maple [B] time = 0.01, size = 301, normalized size = 2.98

$$\frac{\left(c^2m^4x^{12} + 22c^2m^3x^{12} + 159c^2m^2x^{12} + 2bc\,m^4x^9 + 418c^2m\,x^{12} + 50bc\,m^3x^9 + 280c^2x^{12} + 390bc\,m^2x^9 + 2ac\,m^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x^
12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+10
70*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+249*
b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1040*
a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+1820*
a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(m+13)/(m+10)/(m+7)/(m+4)/
(m+1)
```

maxima [A] time = 1.10, size = 110, normalized size = 1.09

$$\frac{c^2d^mx^{13}x^m}{m+13} + \frac{2\,bcd^mx^{10}x^m}{m+10} + \frac{b^2d^mx^7x^m}{m+7} + \frac{2\,acd^mx^7x^m}{m+7} + \frac{2\,abd^mx^4x^m}{m+4} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^13*x^m/(m + 13) + 2*b*c*d^m*x^10*x^m/(m + 10) + b^2*d^m*x^7*x^m/(
m + 7) + 2*a*c*d^m*x^7*x^m/(m + 7) + 2*a*b*d^m*x^4*x^m/(m + 4) + (d*x)^(m +
 1)*a^2/(d*(m + 1))
```

mupad [B] time = 1.52, size = 260, normalized size = 2.57

$$(d\,x)^m\,\left(\frac{c^2\,x^{13}\,\left(m^4+22\,m^3+159\,m^2+418\,m+280\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{x^7\,\left(b^2+2\,a\,c\right)\,\left(m^4+28\,m^3+249\,m^2+742\,m+\right.}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^13*(418*m + 159*m^2 + 22*m^3 + m^4 + 280))/(5714*m + 2485*m
^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (x^7*(2*a*c + b^2)*(742*m + 249*m^2 +
 28*m^3 + m^4 + 520))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) +
 (a^2*x*(2074*m + 411*m^2 + 34*m^3 + m^4 + 3640))/(5714*m + 2485*m^2 + 445*
m^3 + 35*m^4 + m^5 + 3640) + (2*a*b*x^4*(1201*m + 321*m^2 + 31*m^3 + m^4 +
910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^10*(53
5*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4
+ m^5 + 3640))
```

sympy [A] time = 5.82, size = 1510, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**6
) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) - a*
b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3)/d
**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*
log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2
*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4,
Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 + 2*b*c
*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*d**m*m**4*x*x**m/(m**5 + 35*m
**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*d**m*m**3*x*x**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*d**m*m**2*x*
x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*d*
*m*m*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*
a**2*d**m*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
2*a*b*d**m*m**4*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
 3640) + 62*a*b*d**m*m**3*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 642*a*b*d**m*m**2*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 2402*a*b*d**m*m*x**4*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + 1820*a*b*d**m*x**4*x**m/(m**5 + 35*m
**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*d**m*m**4*x**7*x**m/(m*
*5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*d**m*m**3*x**
7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 498*a*c*d*
*m*m**2*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
 1484*a*c*d**m*m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 1040*a*c*d**m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
5714*m + 3640) + b**2*d**m*m**4*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485
*m**2 + 5714*m + 3640) + 28*b**2*d**m*m**3*x**7*x**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 249*b**2*d**m*m**2*x**7*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 742*b**2*d**m*m*x**7*x**m/(
m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 520*b**2*d**m*x**7
*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*b*c*d**m*
m**4*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 5
0*b*c*d**m*m**3*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 390*b*c*d**m*m**2*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
*2 + 5714*m + 3640) + 1070*b*c*d**m*m*x**10*x**m/(m**5 + 35*m**4 + 445*m**3
```

```
 + 2485*m**2 + 5714*m + 3640) + 728*b*c*d**m*x**10*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + c**2*d**m*m**4*x**13*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 22*c**2*d**m*m**3*x**13*x**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*d**m*m
**2*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 41
8*c**2*d**m*m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 280*c**2*d**m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5
714*m + 3640), True))
```

3.249 $\int (dx)^m \left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(4+m)/d^4/(4+m)+c*(d*x)^(7+m)/d^7/(7+m)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^
(7 + m))/(d^7*(7 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right)\,dx &= \int \left(a(dx)^m+\frac{b(dx)^{3+m}}{d^3}+\frac{c(dx)^{6+m}}{d^6}\right)dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.67

$$x(dx)^m\left(\frac{a}{m+1}+\frac{bx^3}{m+4}+\frac{cx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))
```

fricas [A] time = 0.85, size = 71, normalized size = 1.37

$$\frac{\left(\left(cm^2+5\,cm+4\,c\right)x^7+\left(bm^2+8\,bm+7\,b\right)x^4+\left(am^2+11\,am+28\,a\right)x\right)(dx)^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

giac [B] time = 0.35, size = 119, normalized size = 2.29

$$\frac{(dx)^m cm^2x^7 + 5\,(dx)^m cmx^7 + 4\,(dx)^m cx^7 + (dx)^m bm^2x^4 + 8\,(dx)^m bmx^4 + 7\,(dx)^m bx^4 + (dx)^m am^2x + 11\,(dx)}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x^
4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*m*
x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

maple [A] time = 0.00, size = 78, normalized size = 1.50

$$\frac{\left(c\,m^2x^6 + 5cm\,x^6 + 4c\,x^6 + b\,m^2x^3 + 8bm\,x^3 + 7b\,x^3 + a\,m^2 + 11am + 28a\right)x\,(dx)^m}{(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a),x)
```

```
[Out] x*(c*m^2*x^6+5*c*m*x^6+4*c*x^6+b*m^2*x^3+8*b*m*x^3+7*b*x^3+a*m^2+11*a*m+28*
a)*(d*x)^m/(m+7)/(m+4)/(m+1)
```

maxima [A] time = 1.16, size = 50, normalized size = 0.96

$$\frac{cd^m x^7 x^m}{m+7} + \frac{bd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] c*d^m*x^7*x^m/(m + 7) + b*d^m*x^4*x^m/(m + 4) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [B] time = 1.36, size = 89, normalized size = 1.71

$$(d\,x)^m\left(\frac{b\,x^4\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}+\frac{c\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28}+\frac{a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6),x)
```

```
[Out] (d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m +
 m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 12*
m^2 + m^3 + 28))
```

sympy [A] time = 1.47, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{6x^6}-\frac{b}{3x^3}+c\log(x)}{d^7} \\ \frac{-\frac{a}{3x^3}+b\log(x)+\frac{cx^3}{3}}{d^4} \\ \frac{a\log(x)+\frac{bx^3}{3}+\frac{cx^6}{6}}{d} \\ \frac{ad^m m^2 x x^m}{m^3+12m^2+39m+28}+\frac{11ad^m m x x^m}{m^3+12m^2+39m+28}+\frac{28ad^m x x^m}{m^3+12m^2+39m+28}+\frac{bd^m m^2 x^4 x^m}{m^3+12m^2+39m+28}+\frac{8bd^m m x^4 x^m}{m^3+12m^2+39m+28}+\frac{7bd^m x^4 x^m}{m^3+12m^2+39m+28}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3*
x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*x*
*6/6)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*
a*d**m*m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*d**m*x*x**m/(m**3 + 12*
m**2 + 39*m + 28) + b*d**m*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*
b*d**m*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*d**m*x**4*x**m/(m**3
+ 12*m**2 + 39*m + 28) + c*d**m*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28)
 + 5*c*d**m*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*d**m*x**7*x**m/(
m**3 + 12*m**2 + 39*m + 28), True))
```

3.250 $\int \frac{(dx)^m}{a+bx^3+cx^6} dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m
)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(
1+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1375, 364}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx&=\frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}\\&=\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}\end{aligned}$$

Mathematica [C] time = 0.06, size = 84, normalized size = 0.49

$$\frac{(dx)^m\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^5c+\#1^2b}\&\right]}{3m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^2 + 2*c*#1^5)) & ])/(3*m)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{cx^6+bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(d\,x)^m}{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d*x)^m/(a + b*x^3 + c*x^6), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^2} dx$

Optimal. Leaf size=315

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right){}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\right.}{}$$

```
[Out] 1/3*(d*x)^(1+m)*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^6+b*x^3+a)-1/3*c*
(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(
1/2)))*(b^2*(2-m)-4*a*c*(5-m)-b*(2-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3
/2)/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/3*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*
m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(2-m)-4*a*c*(5-m)+b*(2
-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.71, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1366, 1510, 364}

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right){}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 + c
*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
```

```
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} - \frac{\int \frac{(dx)^m\left(-b^2(2-m)+2ac(5-m)-bc(2-m)x^3\right)}{a+bx^3+cx^6}\,dx}{3a\left(b^2-4ac\right)}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} - \frac{\left(c\left(b^2(2-m)-b\sqrt{b^2-4ac}\,(2-m)-4ac(5-m)\right)\right)\int}{6a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}\,(2-m)-4ac(5-m)\right)(dx}{3a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right.}$$

Mathematica [C] time = 0.08, size = 78, normalized size = 0.25

$$\frac{x(dx)^m F_1\left(\frac{m+1}{3};2,2;\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{a^2(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (x*(d*x)^m*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(cx^6+bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(c\,x^6+b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(cx^6+bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{\left(c\,x^6+b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^2,x)
```

```
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.252 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1(1/3+1/3*m,-3/2,-3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1
+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int (dx)^m \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1+m}{3};-\frac{3}{2},-\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.27, size = 357, normalized size = 2.26

$$\frac{x(dx)^m\sqrt{a+bx^3+cx^6}\left(a\left(m^2+11m+28\right)F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)+(m+1)x^3\left(c(m+4\right.\right.}{(m+1)(m+4)(m+7)\sqrt{\frac{-\sqrt{b^2\ldots}}{b-\ldots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*(a*(28 + 11*m + m^2)*AppellF1[(1 + m)/3,
 -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + m)*x^3*(b*(7 + m)*AppellF1[(4 + m)/3, -1/2, -1/
2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])] + c*(4 + m)*x^3*AppellF1[(7 + m)/3, -1/2, -1/2, (10 + m)/3, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))/((1
+ m)*(4 + m)*(7 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6+bx^3+a\right)^{\frac{3}{2}}(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(c\,x^6 + b\,x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.253 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,-1/2,-1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1+2
*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 +
 m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int (dx)^m\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^6+bx^3+a}\,(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^6+bx^3+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\sqrt{c\,x^6+b\,x^3+a}\,\left(dx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^6+bx^3+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,x\right)}^m\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

3.254 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,1/2,1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{1}{2},\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{cx^6+bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(d\,x)^m}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

3.255 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,3/2,3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(c*x^6+b*x^3+a)^(1/
2)
```

Rubi [A] time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\ &= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{3}{2},\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 221, normalized size = 1.38

$$\frac{x(dx)^m\left(\sqrt{b^2-4ac}-b-2cx^3\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{3/2}F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(a+bx^3+cx^6\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^3 + c*x^6)^(3/2))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^6+bx^3+a}\,(dx)^m}{c^2x^{12}+2\,bcx^9+\left(b^2+2\,ac\right)x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a
*c)*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.256 $\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=155

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] (d*x)^(1+m)*(c*x^6+b*x^3+a)^p*AppellF1(1/3+1/3*m,-p,-p,4/3+1/3*m,-2*c*x^3/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^3/
(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3,
 (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int (dx)^m \left(1+\frac{}{b}\right.\\ &= \frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1+m}{3};-p,-p;\frac{4+n}{3}\right.}{d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.24, size = 179, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(c\,x^6 + b\,x^3 + a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.257 $\int x^8 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=224

$$\frac{2^p\left(2ac-b^2(p+2)\right)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}}-\frac{b(p+2)}{6c^2}$$

```
[Out] -1/6*b*(2+p)*(c*x^6+b*x^3+a)^(1+p)/c^2/(2*p^2+5*p+3)+1/3*x^3*(c*x^6+b*x^3+a
)^(1+p)/c/(3+2*p)+1/3*2^p*(2*a*c-b^2*(2+p))*(c*x^6+b*x^3+a)^(1+p)*hypergeom
([-p, 1+p],[2+p],1/2*(2*c*x^3+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-
2*c*x^3+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c^2/(1+p)/(3+2*p)/
(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1357, 742, 640, 624}

$$\frac{2^p\left(2ac-b^2(p+2)\right)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}}-\frac{b(p+2)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(6*c^2*(1 + p)*(3 + 2*p)) + (x^3*(
a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a*c - b^2*(2 + p))*(-
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3
+ c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2
*p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a+bx^3+cx^6\right)^p dx &= \frac{1}{3}\text{Subst}\left(\int x^2 \left(a+bx+cx^2\right)^p dx, x, x^3\right)\\ &= \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{\text{Subst}\left(\int(-a-b(2+p)x)\left(a+bx+cx^2\right)^p dx, x, x^3\right)}{3c(3+2p)}\\ &= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)} + \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} - \frac{\left(2ac-b^2(2+p)\right)\text{Subst}\left(\int \left(\right.\right.}{6c^2(3+}\\ &= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)} + \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{2^p\left(2ac-b^2(2+p)\right)\left(-\frac{b-\sqrt{b^2}}{\sqrt{}}\right.}{}\end{aligned}$$

Mathematica [C] time = 0.20, size = 162, normalized size = 0.72

$$\frac{1}{9}x^9\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(3;-p,-p;4;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{}{\sqrt{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^p*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6+bx^3+a\right)^p x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6+bx^3+a\right)^p x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^8\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^p,x)
[Out] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^p,x)
[Out] int(x^8*(a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

3.258 $\int x^5 \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=161

$$\frac{b2^p\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{6c(p+1)}$$

```
[Out] 1/6*(c*x^6+b*x^3+a)^(1+p)/c/(1+p)+1/3*2^p*b*(c*x^6+b*x^3+a)^(1+p)*hypergeom
([-p, 1+p],[2+p],1/2*(2*c*x^3+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-
2*c*x^3+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c/(1+p)/(-4*a*c+b^
2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1357, 640, 624}

$$\frac{b2^p\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{6c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hyper
geometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^
2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{3}\operatorname{Subst}\left(\int x\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)$$
$$=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}-\frac{b\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{6c}$$
$$=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}{}_2F_1\left(-p,1+p\right.}{3c\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [C] time = 0.21, size = 162, normalized size = 1.01

$$\frac{1}{6}x^6\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(2;-p,-p;3;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^p*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(cx^6+bx^3+a\right)^px^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^px^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^5\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^6+bx^3+a\right)^px^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^5*(a + b*x^3 + c*x^6)^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.259 $\int x^2 \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=130

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -1/3*2^(1+p)*(c*x^6+b*x^3+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(2*c*x^3+(
-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-2*c*x^3+(-4*a*c+b^2)^(1/2)-b)/(
-4*a*c+b^2)^(1/2))^(-1-p)/(1+p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1352, 624}

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 -
p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(1 +
 p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3+cx^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)\\ &= -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2c}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\,(1+p)}\end{aligned}$$

Mathematica [A] time = 0.11, size = 138, normalized size = 1.06

$$\frac{2^{p-1}\left(-\sqrt{b^2-4ac}+b+2cx^3\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-2cx^3-b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2
 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a
*c])^p)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \left(c\,x^6 + b\,x^3 + a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^2*(a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.260 $\int x^4 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.$$

```
[Out] 1/5*x^5*(c*x^6+b*x^3+a)^p*AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^3+cx^6\right)^p\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^4\left(1+\frac{2c}{b-\sqrt{b^2}}\right.\\ &= \frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3}\right.\end{aligned}$$

Mathematica [A] time = 0.16, size = 166, normalized size = 1.20

$$\frac{1}{5}x^5\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^4 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^4*(a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.261 $\int x^3 \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2}}\right.$$

```
[Out] 1/4*x^4*(c*x^6+b*x^3+a)^p*AppellF1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3+cx^6\right)^p\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^3\left(1+\frac{2c}{b-\sqrt{b^2}}\right.\\ &= \frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3}\right.\end{aligned}$$

Mathematica [A] time = 0.18, size = 166, normalized size = 1.20

$$\frac{1}{4}x^4\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^3 \left(c\,x^6 + b\,x^3 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^3*(a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.262 $\int x\left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.$$

```
[Out] 1/2*x^2*(c*x^6+b*x^3+a)^p*AppellF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 510}

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3+cx^6\right)^p\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-}}\right.\\ &= \frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};\right.\end{aligned}$$

Mathematica [A] time = 0.18, size = 166, normalized size = 1.20

$$\frac{1}{2}x^2\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(c\,x^6 + b\,x^3 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(c\,x^6 + b\,x^3 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x*(a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.263 $\int \left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-\ldots}}\right.$$

```
[Out] x*(c*x^6+b*x^3+a)^p*AppellF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1
+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1348, 429}

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-\ldots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3+cx^6\right)^p\,dx &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \left(1+\frac{2cx^3}{b-\sqrt{b^2-4a\ldots}}\right.\\ &= x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{\ldots}{b-\ldots}\right.\end{aligned}$$

Mathematica [A] time = 0.15, size = 161, normalized size = 1.21

$$x\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{\ldots}{\sqrt{b^2\ldots}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c\,x^6 + b\,x^3 + a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^6 + b\,x^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.264 $\int \frac{\left(a+bx^3+cx^6\right)^p}{x}\,dx$

Optimal. Leaf size=157

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

```
[Out] 1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(-b-(-4*a*c+
b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/p/(((2*c*x^3-(-4*a*c+b
^2)^(1/2)+b)/c/x^3)^p)/(((2*c*x^3+(-4*a*c+b^2)^(1/2)+b)/c/x^3)^p)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1357, 758, 133}

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b - S
qrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x}\,dx, x, x^3\right)$$
$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx\right.\right.$$
$$= \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2\right.}{3p}$$

Mathematica [A] time = 0.20, size = 157, normalized size = 1.00

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)}{3p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
 + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(c*x^3))^p)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^6+bx^3+a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x,x)
[Out] int((a + b*x^3 + c*x^6)^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x,x)
[Out] Timed out
```

3.265 $\int \frac{(a+bx^3+cx^6)^p}{x^2} dx$

Optimal. Leaf size=136

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(c*x^6+b*x^3+a)^p*AppellF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^2} dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.}{}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.}{x}$$

Mathematica [A] time = 0.16, size = 164, normalized size = 1.21

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^2,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p/x^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**2,x)
```

```
[Out] Timed out
```

3.266 $\int \frac{\left(a+bx^3+cx^6\right)^p}{x^3} dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

```
[Out] -1/2*(c*x^6+b*x^3+a)^p*AppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^3} dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p \left(\right.}{x^3}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{2x^2}$$

Mathematica [A] time = 0.18, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^2*((b - Sqrt[b^2 -
 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6 + bx^3 + a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^3,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**3,x)
```

```
[Out] Timed out
```

3.267 $\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx$

Optimal. Leaf size=164

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

```
[Out] -1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,1/2*(-b-(-4*a*c+b^2)^
(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-2*p)/x^3/(((2*c*x^3-(-4*
a*c+b^2)^(1/2)+b)/c/x^3)^p)/(((2*c*x^3+(-4*a*c+b^2)^(1/2)+b)/c/x^3)^p)
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{1357, 758, 133\}$

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] -(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -(b - Sqrt
[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(1 - 2*p)
*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(c*x^3))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^2}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right.\right.$$

$$= -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2(1-p);\right.}{3(1-2p)x^3}$$

Mathematica [A] time = 0.20, size = 162, normalized size = 0.99

$$\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right)}{3(2p-1)x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] (4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqr
t[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)]])/(3*(-1 + 2*p)
*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(c*x^3))^p)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^4,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**4,x)
[Out] Timed out
```

3.268 $\int \frac{\left(a+bx^3+cx^6\right)^p}{x^5}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

```
[Out] -1/4*(c*x^6+b*x^3+a)^p*AppellF1(-4/3,-p,-p,-1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^4/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*x^4*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^5}\,dx=\left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(\right.}{x^5}$$
$$=-\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{4x^4}$$

Mathematica [A] time = 0.20, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^4*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6 + bx^3 + a\right)^p}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^5,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^5,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**5,x)
```

```
[Out] Timed out
```

3.269 $\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

```
[Out] -1/5*(c*x^6+b*x^3+a)^p*AppellF1(-5/3,-p,-p,-2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^5/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1385, 510}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*x^5*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^p}{x^6}\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.}{}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.}{5x^5}$$

Mathematica [A] time = 0.16, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{\sqrt{b^2-4ac}-b}\right)}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^5*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^{6}+bx^{3}+a\right)^{p}}{x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{6}+bx^{3}+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^6,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{6}+bx^{3}+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^6,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p/x^6, x)
```

<u>**sympy**</u> [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**6,x)
```

```
[Out] Timed out
```

3.270 $\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$

Optimal. Leaf size=168

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

```
[Out] -1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,1/2*(-b-(-4*a*
c+b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-p)/x^6/(((2*c*x^3
-(-4*a*c+b^2)^(1/2)+b)/c/x^3)^p)/(((2*c*x^3+(-4*a*c+b^2)^(1/2)+b)/c/x^3)^p)
```

Rubi [A] time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{1357, 758, 133\}$

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] -(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -
(b - Sqrt[b^2 - 4*a*c])/(2*c*x^3), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*
(1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^3}\,dx, x, x^3\right)$$
$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx\right.\right.\right.$$
$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(1-p);-p,-p;\right.}{3(1-p)x^6}$$

Mathematica [A] time = 0.22, size = 164, normalized size = 0.98

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{\sqrt{b^2-4ac}-b}{2cx^3}\right.}{3(p-1)x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/2
*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(
-1 + p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^6+bx^3+a)^p}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^7,x)
```

[Out] int((c*x^6+b*x^3+a)^p/x^7,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^p/x^7,x)

[Out] int((a + b*x^3 + c*x^6)^p/x^7, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**p/x**7,x)

[Out] Timed out

3.271 $\int \frac{x^m}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=32

$$\frac{x^{m+1} {}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

```
[Out] x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],-x^4)/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 364}

$$\frac{x^{m+1} {}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^m}{1+2x^4+x^8} \, dx &= \int \frac{x^m}{\left(1+x^4\right)^2} \, dx \\ &= \frac{x^{1+m} {}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};-x^4\right)}{1+m} \end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.06

$$\frac{x^{m+1} {}_2F_1\left(2,\frac{m+1}{4};\frac{m+1}{4}+1;-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/(1 + m)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{x^8+2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 + 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 + 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+2*x^4+1),x)
```

```
[Out] int(x^m/(x^8+2*x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 + 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{x^8 + 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2*x^4 + x^8 + 1),x)
```

```
[Out] int(x^m/(2*x^4 + x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(x^4 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+2*x**4+1),x)
```

```
[Out] Integral(x**m/(x**4 + 1)**2, x)
```

3.272 $\int \frac{x^9}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{3x^2}{4} - \frac{3}{4}\tan^{-1}\left(x^2\right) - \frac{x^6}{4\left(x^4+1\right)}$$

```
[Out] 3/4*x^2-1/4*x^6/(x^4+1)-3/4*arctan(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 288, 321, 203}

$$-\frac{x^6}{4\left(x^4+1\right)} + \frac{3x^2}{4} - \frac{3}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule sreps

Rubi steps

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \int \frac{x^9}{\left(1+x^4\right)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{x^6}{4\left(1+x^4\right)} + \frac{3}{4}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, x^2\right)$$
$$= \frac{3x^2}{4} - \frac{x^6}{4\left(1+x^4\right)} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)$$
$$= \frac{3x^2}{4} - \frac{x^6}{4\left(1+x^4\right)} - \frac{3}{4}\tan^{-1}\left(x^2\right)$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 0.80

$$\frac{1}{4}\left(x^2\left(\frac{1}{x^4+1}+2\right) - 3\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 2*x^4 + x^8),x]
[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

fricas [**A**] time = 0.86, size = 31, normalized size = 1.03

$$\frac{2\,x^6 + 3\,x^2 - 3\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

giac [**A**] time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + \frac{x^2}{4\left(x^4+1\right)} - \frac{3}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

maple [**A**] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + \frac{x^2}{4x^4+4} - \frac{3\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+2*x^4+1),x)
[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

maxima [A] time = 2.16, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + \frac{x^2}{4\left(x^4+1\right)} - \frac{3}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.83

$$\frac{x^2}{4\left(x^4+1\right)} - \frac{3\,\mathrm{atan}\left(x^2\right)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(2*x^4 + x^8 + 1),x)
```

```
[Out] x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2
```

sympy [A] time = 0.13, size = 22, normalized size = 0.73

$$\frac{x^2}{2} + \frac{x^2}{4x^4+4} - \frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+2*x**4+1),x)
```

```
[Out] x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

3.273 $\int \frac{x^7}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=22

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

```
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1+2x^4+x^8}\,dx &= \int \frac{x^7}{\left(1+x^4\right)^2}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx,x,x^4\right)\\ &= \frac{1}{4\left(1+x^4\right)}+\frac{1}{4}\log\left(1+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.82

$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 2*x^4 + x^8),x]
[Out] ((1 + x^4)^(-1) + Log[1 + x^4])/4
```

fricas [A] time = 0.79, size = 23, normalized size = 1.05

$$\frac{(x^4+1)\log(x^4+1)+1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

giac [A] time = 0.28, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{\ln(x^4+1)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+2*x^4+1),x)
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

maxima [A] time = 0.98, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

mupad [B] time = 1.32, size = 18, normalized size = 0.82

$$\frac{\ln(x^4+1)}{4}+\frac{1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(2*x^4 + x^8 + 1),x)
[Out] log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

sympy [A] time = 0.11, size = 15, normalized size = 0.68

$$\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+2*x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

3.274 $\int \frac{x^5}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

```
[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 275, 288, 203}

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{1+2x^4+x^8}\,dx &= \int \frac{x^5}{\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{4}\tan^{-1}\left(x^2\right) - \frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 2*x^4 + x^8),x]
[Out] -1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

fricas [**A**] time = 0.79, size = 24, normalized size = 1.04

$$-\frac{x^2 - \left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

giac [**A**] time = 0.37, size = 19, normalized size = 0.83

$$-\frac{x^2}{4\left(x^4+1\right)} + \frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

maple [**A**] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{x^2}{4\left(x^4+1\right)} + \frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+2*x^4+1),x)
[Out] -1/4/(x^4+1)*x^2+1/4*arctan(x^2)
```

maxima [A] time = 2.02, size = 19, normalized size = 0.83

$$-\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

mupad [B] time = 1.37, size = 21, normalized size = 0.91

$$\frac{\operatorname{atan}\left(x^2\right)}{4}-\frac{x^2}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(2*x^4 + x^8 + 1),x)
[Out] atan(x^2)/4 - x^2/(4*(x^4 + 1))
```

sympy [A] time = 0.12, size = 15, normalized size = 0.65

$$-\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+2*x**4+1),x)
[Out] -x**2/(4*x**4 + 4) + atan(x**2)/4
```

3.275 $\int \frac{x^3}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{4\left(x^4+1\right)}$$

```
[Out] -1/4/(x^4+1)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 261}

$$-\frac{1}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x^4 + x^8),x]
[Out] -1/(4*(1 + x^4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1+2x^4+x^8} \, dx &= \int \frac{x^3}{\left(1+x^4\right)^2} \, dx \\ &= -\frac{1}{4\left(1+x^4\right)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^4 + x^8),x]
[Out] -1/4*1/(1 + x^4)
```

fricas [A] time = 0.79, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4/(x^4 + 1)
```

giac [A] time = 0.36, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+2*x^4+1),x)
[Out] -1/4/(x^4+1)
```

maxima [A] time = 0.89, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 + 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{4\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^4 + x^8 + 1),x)
[Out] -1/(4*(x^4 + 1))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+2*x**4+1),x)
[Out] -1/(4*x**4 + 4)
```

3.276 $\int \frac{x}{1+2x^4+x^8} dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{x^2}{4\left(x^4+1\right)}$$

```
[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 275, 199, 203}

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+2x^4+x^8}\,dx &= \int \frac{x}{\left(1+x^4\right)^2}\,dx \\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx,x,x^2\right) \\ &= \frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right) \\ &= \frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{4}\left(\tan^{-1}\left(x^2\right)+\frac{x^2}{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^4 + x^8),x]
[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4
```

fricas [A] time = 0.84, size = 23, normalized size = 1.00

$$\frac{x^2+\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{x^2}{4x^4+4}+\frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+2*x^4+1),x)
[Out] 1/4/(x^4+1)*x^2+1/4*arctan(x^2)
```

maxima [A] time = 2.17, size = 19, normalized size = 0.83

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.87

$$\frac{\operatorname{atan}\left(x^2\right)}{4}+\frac{x^2}{4\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^4 + x^8 + 1),x)
```

```
[Out] atan(x^2)/4 + x^2/(4*(x^4 + 1))
```

sympy [A] time = 0.12, size = 15, normalized size = 0.65

$$\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+2*x**4+1),x)
```

```
[Out] x**2/(4*x**4 + 4) + atan(x**2)/4
```

3.277 $\int \frac{1}{x(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{4(x^4+1)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

```
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4(x^4+1)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)dx,x,x^4\right)\\
&= \frac{1}{4\left(1+x^4\right)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{4(x^4+1)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

fricas [A] time = 0.81, size = 32, normalized size = 1.33

$$-\frac{\left(x^4+1\right)\log\left(x^4+1\right)-4\left(x^4+1\right)\log(x)-1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

giac [A] time = 0.36, size = 29, normalized size = 1.21

$$\frac{x^4+2}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+2*x^4+1),x)
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

maxima [A] time = 0.97, size = 24, normalized size = 1.00

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$\ln(x)-\frac{\ln\left(x^4+1\right)}{4}+\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*x^4 + x^8 + 1)),x)
[Out] log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

sympy [A] time = 0.13, size = 19, normalized size = 0.79

$$\log(x) - \frac{\log(x^4 + 1)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

3.278 $\int \frac{1}{x^3(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=30

$$-\frac{3}{4x^2}-\frac{3}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^2\left(x^4+1\right)}$$

```
[Out] -3/4/x^2+1/4/x^2/(x^4+1)-3/4*arctan(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^2\left(x^4+1\right)}-\frac{3}{4x^2}-\frac{3}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^3\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{1}{4x^2\left(1+x^4\right)} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{1}{4x^2\left(1+x^4\right)} - \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{1}{4x^2\left(1+x^4\right)} - \frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 30, normalized size = 1.00

$$
-\frac{1}{2x^2} + \frac{3}{4}\tan^{-1}\left(\frac{1}{x^2}\right) - \frac{x^2}{4\left(x^4+1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
[Out] -1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

fricas [**A**] time = 0.82, size = 31, normalized size = 1.03

$$
-\frac{3\,x^4 + 3\left(x^6+x^2\right)\arctan\left(x^2\right) + 2}{4\left(x^6+x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

giac [**A**] time = 0.34, size = 25, normalized size = 0.83

$$
-\frac{3\,x^4+2}{4\left(x^6+x^2\right)} - \frac{3}{4}\arctan\left(x^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

maple [**A**] time = 0.01, size = 25, normalized size = 0.83

$$
-\frac{x^2}{4\left(x^4+1\right)} - \frac{3\arctan\left(x^2\right)}{4} - \frac{1}{2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+2*x^4+1),x)
```

```
[Out] -1/2/x^2-1/4/(x^4+1)*x^2-3/4*arctan(x^2)
```

maxima [A] time = 1.98, size = 25, normalized size = 0.83

$$-\frac{3\,x^4+2}{4\left(x^6+x^2\right)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.83

$$-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}-\frac{\frac{3\,x^4}{4}+\frac{1}{2}}{x^6+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(2*x^4 + x^8 + 1)),x)
[Out] - (3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)
```

sympy [A] time = 0.15, size = 26, normalized size = 0.87

$$\frac{-3x^4-2}{4x^6+4x^2}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+2*x**4+1),x)
[Out] (-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

3.279 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

```
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)^2}\,dx,x,x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
[Out] -1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

fricas [A] time = 0.82, size = 44, normalized size = 1.33

$$-\frac{2x^4 - 2(x^8 + x^4)\log(x^4 + 1) + 8(x^8 + x^4)\log(x) + 1}{4(x^8 + x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8 +
 x^4)
```

giac [A] time = 0.35, size = 33, normalized size = 1.00

$$-\frac{2x^4 + 1}{4(x^8 + x^4)} + \frac{1}{2}\log(x^4 + 1) - \frac{1}{2}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

maple [A] time = 0.02, size = 28, normalized size = 0.85

$$-2\ln(x) + \frac{\ln(x^4 + 1)}{2} - \frac{1}{4x^4} - \frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+2*x^4+1),x)
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

maxima [A] time = 0.88, size = 33, normalized size = 1.00

$$-\frac{2x^4 + 1}{4(x^8 + x^4)} + \frac{1}{2}\log(x^4 + 1) - \frac{1}{2}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

mupad [B] time = 0.05, size = 31, normalized size = 0.94

$$\frac{\ln(x^4 + 1)}{2} - 2\ln(x) - \frac{\frac{x^4}{2} + \frac{1}{4}}{x^8 + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(2*x^4 + x^8 + 1)),x)
[Out] log(x^4 + 1)/2 - 2*log(x) - (x^4/2 + 1/4)/(x^4 + x^8)
```

sympy [A] time = 0.15, size = 31, normalized size = 0.94

$$\frac{-2x^4-1}{4x^8+4x^4}-2\log(x)+\frac{\log(x^4+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+2*x**4+1),x)
[Out] (-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

3.280 $\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx$

Optimal. Leaf size=37

$$-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{5}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^6\left(x^4+1\right)}$$

```
[Out] -5/12/x^6+5/4/x^2+1/4/x^6/(x^4+1)+5/4*arctan(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^6\left(x^4+1\right)}+\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{1}{4x^6\left(1+x^4\right)} + \frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6\left(1+x^4\right)} - \frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6\left(1+x^4\right)} + \frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6\left(1+x^4\right)} + \frac{5}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 33, normalized size = 0.89

$$
-\frac{1}{6x^6} + \frac{1}{x^2} - \frac{5}{4}\tan^{-1}\left(\frac{1}{x^2}\right) + \frac{x^2}{4\left(x^4+1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
[Out] -1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

fricas [**A**] time = 0.97, size = 36, normalized size = 0.97

$$
\frac{15\,x^8 + 10\,x^4 + 15\left(x^{10} + x^6\right)\arctan\left(x^2\right) - 2}{12\left(x^{10} + x^6\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

giac [**A**] time = 0.30, size = 31, normalized size = 0.84

$$
\frac{x^2}{4\left(x^4+1\right)} + \frac{6\,x^4 - 1}{6\,x^6} + \frac{5}{4}\arctan\left(x^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

maple [**A**] time = 0.01, size = 28, normalized size = 0.76

$$
\frac{x^2}{4x^4+4} + \frac{5\arctan\left(x^2\right)}{4} + \frac{1}{x^2} - \frac{1}{6x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+2*x^4+1),x)
[Out] -1/6/x^6+1/x^2+1/4/(x^4+1)*x^2+5/4*arctan(x^2)
```

maxima [A] time = 2.49, size = 30, normalized size = 0.81

$$\frac{15\,x^8+10\,x^4-2}{12\left(x^{10}+x^6\right)}+\frac{5}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

mupad [B] time = 0.05, size = 30, normalized size = 0.81

$$\frac{5\,\mathrm{atan}\left(x^2\right)}{4}+\frac{\frac{5\,x^8}{4}+\frac{5\,x^4}{6}-\frac{1}{6}}{x^6\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(2*x^4 + x^8 + 1)),x)
[Out] (5*atan(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))
```

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{5\,\mathrm{atan}\left(x^2\right)}{4}+\frac{15x^8+10x^4-2}{12x^{10}+12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+2*x**4+1),x)
[Out] 5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

3.281 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=104

$$\frac{5\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{x^5}{4\left(x^4+1\right)}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

```
[Out] 5/4*x-1/4*x^5/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(
1/2))*2^(1/2)+5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-5/32*ln(1+x^2+x*2^(1/2))*2^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 288, 321, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^5}{4\left(x^4+1\right)}+\frac{5\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*Ar
cTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]
) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+2x^4+x^8}\,dx &= \int \frac{x^8}{\left(1+x^4\right)^2}\,dx\\
&= -\frac{x^5}{4\left(1+x^4\right)}+\frac{5}{4}\int \frac{x^4}{1+x^4}\,dx\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{4}\int \frac{1}{1+x^4}\,dx\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{5}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}+
\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}+\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1\right.}{8\sqrt{2}}\\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\log}{}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.90

$$\frac{1}{32}\left(\frac{8x}{x^4+1}+5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+32x+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-10\sqrt{2}\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1 +
 Sqrt[2]*x + x^2])/32
```

fricas [A] time = 0.80, size = 132, normalized size = 1.27

$$\frac{32\,x^5 + 20\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}\,-1\right) + 20\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2 -}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^5 + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) +
 5*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) + 40*x)/(x^4 + 1)
```

giac [A] time = 0.34, size = 83, normalized size = 0.80

$$-\frac{5}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)-\frac{5}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{5}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{5}{32}\,\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

maple [A] time = 0.01, size = 69, normalized size = 0.66

$$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}-\frac{5\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+2*x^4+1),x)
```

```
[Out] x+1/4*x/(x^4+1)-5/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))-5/16*a
rctan(-1+2^(1/2)*x)*2^(1/2)-5/16*arctan(1+2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 2.02, size = 83, normalized size = 0.80

$$-\frac{5}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)-\frac{5}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{5}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{5}{32}\,\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

mupad [B] time = 1.37, size = 45, normalized size = 0.43

$$x+\frac{x}{4\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(2*x^4 + x^8 + 1),x)
```

```
[Out] x - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^(1/2)*atan(2^(1
/2)*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))
```

sympy [A] time = 0.18, size = 90, normalized size = 0.87

$$x+\frac{x}{4x^4+4}+\frac{5\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{5\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)}{32}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{16}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

3.282 $\int \frac{x^6}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^3}{4\left(x^4+1\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -1/4*x^3/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))
*2^(1/2)+3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 288, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 +
Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Lo
g[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{1+2x^4+x^8}\,dx &= \int \frac{x^6}{\left(1+x^4\right)^2}\,dx \\
&= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= -\frac{x^3}{4\left(1+x^4\right)} - \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{3\int
\\
&= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-
}{8\sqrt{2}} \\
&= -\frac{x^3}{4\left(1+x^4\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1-
}{1
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.94

$$
\frac{1}{32}\left(3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - 3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{8x^3}{x^4+1} - 6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right) + 6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[
2]*x + x^2])/32
```

fricas [A] time = 0.90, size = 129, normalized size = 1.30

$$-\frac{8\,x^3+12\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}\,-1\right)+12\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*x^3 + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) + 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) -
 3*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

giac [A] time = 0.29, size = 84, normalized size = 0.85

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{3}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.01, size = 70, normalized size = 0.71

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}+\frac{3\sqrt{2}\,\ln\left(\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^3/(x^4+1)+3/16*2^(1/2)*arctan(2^(1/2)*x-1)+3/32*2^(1/2)*ln((x^2-2^(1
/2)*x+1)/(x^2+2^(1/2)*x+1))+3/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.07, size = 84, normalized size = 0.85

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{3}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 1.33, size = 47, normalized size = 0.47

$$-\frac{x^3}{4\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))
```

sympy [A] time = 0.18, size = 90, normalized size = 0.91

$$-\frac{x^3}{4x^4+4}+\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

3.283 $\int \frac{x^4}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=97

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

```
[Out] -1/4*x/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*2
^(1/2)-1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x +
 x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{1+2x^4+x^8}\,dx &= \int \frac{x^4}{\left(1+x^4\right)^2}\,dx\\
&= -\frac{x}{4\left(1+x^4\right)}+\frac{1}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{x}{4\left(1+x^4\right)}+\frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{x}{4\left(1+x^4\right)}+\frac{1}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{16\sqrt{2}}-\frac{\int \frac{\sqrt{\ }}{-1+}}{1\ }\\
&= -\frac{x}{4\left(1+x^4\right)}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\,x\right)}{8\sqrt{2}}\\
&= -\frac{x}{4\left(1+x^4\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x\right.}{16\sqrt{2}}\end{aligned}$$

Mathematica [**A**] time = 0.06, size = 90, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{8x}{x^4+1}-\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
 x^2])/32
```

fricas [A] time = 0.96, size = 126, normalized size = 1.30

$$-\frac{4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}-1\right)+4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x+\cdots}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x
 + 1) - 1) + 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqr
t(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^
4 + 1)*log(x^2 - sqrt(2)*x + 1) + 8*x)/(x^4 + 1)
```

giac [A] time = 0.36, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$-\frac{x}{4\left(x^4+1\right)}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}+\frac{\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/4/(x^4+1)*x+1/32*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))+1/16*2^
(1/2)*arctan(2^(1/2)*x-1)+1/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 1.99, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

mupad [B] time = 0.08, size = 45, normalized size = 0.46

$$-\frac{x}{4\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))
```

sympy [A] time = 0.17, size = 82, normalized size = 0.85

$$-\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2 +
 sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2)*
x + 1)/16
```

3.284 $\int \frac{x^2}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{x^3}{4\left(x^4+1\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

```
[Out] 1/4*x^3/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*
2^(1/2)+1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 290, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{4\left(x^4+1\right)}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+2x^4+x^8}\,dx &= \int \frac{x^2}{\left(1+x^4\right)^2}\,dx\\
&= \frac{x^3}{4\left(1+x^4\right)} + \frac{1}{4}\int \frac{x^2}{1+x^4}\,dx\\
&= \frac{x^3}{4\left(1+x^4\right)} - \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{x^3}{4\left(1+x^4\right)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{\int \frac{\sqrt{2}}{-1+\sqrt{}}}{16}\\
&= \frac{x^3}{4\left(1+x^4\right)} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} -\\
&= \frac{x^3}{4\left(1+x^4\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+\right.}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 0.93

$$\frac{1}{32}\left(\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{8x^3}{x^4+1}-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x
+ x^2])/32
```

fricas [A] time = 0.92, size = 128, normalized size = 1.29

$$\frac{8\,x^3 - 4\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1} - 1\right) - 4\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sq
rt(2)*x + 1) - 1) - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^
2 - sqrt(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt
(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

giac [A] time = 0.35, size = 84, normalized size = 0.85

$$\frac{x^3}{4\left(x^4+1\right)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.01, size = 70, normalized size = 0.71

$$\frac{x^3}{4x^4+4} + \frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16} + \frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16} + \frac{\sqrt{2}\,\ln\left(\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)*x^3+1/16*2^(1/2)*arctan(2^(1/2)*x-1)+1/32*2^(1/2)*ln((x^2-2^(1/
2)*x+1)/(x^2+2^(1/2)*x+1))+1/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.06, size = 84, normalized size = 0.85

$$\frac{x^3}{4\left(x^4+1\right)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.05, size = 46, normalized size = 0.46

$$\frac{x^3}{4\left(x^4+1\right)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 - 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))
```

sympy **[A]** time = 0.17, size = 83, normalized size = 0.84

$$\frac{x^3}{4x^4+4}+\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+2*x**4+1),x)
```

```
[Out] x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**2
 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

3.285 $\int \frac{1}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{x}{4\left(x^4+1\right)}-\frac{3\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

```
[Out] 1/4*x/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))*2^
(1/2)-3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {28, 199, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{4\left(x^4+1\right)}-\frac{3\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqr
t[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1
 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{1+2x^4+x^8}\,dx = \int \frac{1}{\left(1+x^4\right)^2}\,dx$$

$$= \frac{x}{4\left(1+x^4\right)} + \frac{3}{4}\int \frac{1}{1+x^4}\,dx$$

$$= \frac{x}{4\left(1+x^4\right)} + \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= \frac{x}{4\left(1+x^4\right)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} - \frac{3\int \frac{}{-1}}{1}$$

$$= \frac{x}{4\left(1+x^4\right)} - \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{}\right.}{8\sqrt{2}}$$

$$= \frac{x}{4\left(1+x^4\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\log(1+}{16}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.94

$$\frac{1}{32}\left(\frac{8x}{x^4+1} - 3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + 3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - 6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right) + 6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*
x + x^2])/32
```

fricas [A] time = 0.85, size = 127, normalized size = 1.31

$$-\frac{12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}-1\right)+12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x}\right.}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*
x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - s
qrt(2)*x + 1) + 1) - 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + 3*sqrt(
2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) - 8*x)/(x^4 + 1)
```

giac [A] time = 0.34, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{3}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

maple [A] time = 0.00, size = 68, normalized size = 0.70

$$\frac{x}{4x^4+4}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}+\frac{3\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)*x+3/16*2^(1/2)*arctan(2^(1/2)*x-1)+3/32*2^(1/2)*ln((x^2+2^(1/2)
*x+1)/(x^2-2^(1/2)*x+1))+3/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.00, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{3}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

mupad [B] time = 1.31, size = 44, normalized size = 0.45

$$\frac{x}{4\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))
```

sympy [A] time = 0.20, size = 88, normalized size = 0.91

$$\frac{x}{4x^4+4}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+2*x**4+1),x)
```

```
[Out] x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan(s
qrt(2)*x + 1)/16
```

3.286 $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$\frac{1}{4x(x^4+1)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -5/4/x+1/4/x/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(1
/2))*2^(1/2)-5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+5/32*ln(1+x^2+x*2^(1/2))*2^(1
/2)
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x(x^4+1)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2
]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x\left(1+x^4\right)}+\frac{5}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5}{4}\int \frac{x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}+\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{5}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x}}{16\sqrt{2}}\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x}\right.}{8}\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.92

$$\frac{1}{32}\left(-5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{8x^3}{x^4+1}-\frac{32}{x}+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)-10\sqrt{2}\ \text{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*
ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2])/32
```

fricas [A] time = 0.89, size = 130, normalized size = 1.23

$$-\frac{40\,x^4-20\,\sqrt{2}\left(x^5+x\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right)-20\,\sqrt{2}\left(x^5+x\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2}\right.}{32\left(x^5+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(40*x^4 - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
 sqrt(2)*x + 1) - 1) - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqr
t(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^5 + x)*log(x^2 + sqrt(2)*x + 1)
+ 5*sqrt(2)*(x^5 + x)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^5 + x)
```

giac [A] time = 0.36, size = 88, normalized size = 0.83

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\ \text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

maple [A] time = 0.01, size = 75, normalized size = 0.71

$$-\frac{x^3}{4\left(x^4+1\right)}-\frac{5\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{16}-\frac{5\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{16}-\frac{5\sqrt{2}\ \ln\left(\frac{x^2-\sqrt{2}\ x+1}{x^2+\sqrt{2}\ x+1}\right)}{32}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+2*x^4+1),x)
```

```
[Out] -1/x-1/4/(x^4+1)*x^3-5/16*2^(1/2)*arctan(2^(1/2)*x-1)-5/32*2^(1/2)*ln((x^2-
2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))-5/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.03, size = 88, normalized size = 0.83

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\ \text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

mupad [B] time = 1.31, size = 49, normalized size = 0.46

$$-\frac{\frac{5\,x^4}{4}+1}{x^5+x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - ((5*x^4)/4 + 1)/(x + x^5) - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 -
5i/16) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(5/16 + 5i/16)
```

sympy [A] time = 0.21, size = 97, normalized size = 0.92

$$\frac{-5x^4-4}{4x^5+4x}-\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}+\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{16}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] (-5*x**4 - 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*
sqrt(2)*atan(sqrt(2)*x + 1)/16
```

3.287 $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^3\left(x^4+1\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

```
[Out] -7/12/x^3+1/4/x^3/(x^4+1)-7/16*arctan(-1+x*2^(1/2))*2^(1/2)-7/16*arctan(1+x
*2^(1/2))*2^(1/2)+7/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-7/32*ln(1+x^2+x*2^(1/2))
*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^3\left(x^4+1\right)}-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) -
 (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*S
qrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^4\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^3\left(1+x^4\right)}+\frac{7}{4}\int \frac{1}{x^4\left(1+x^4\right)}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{7}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx-\frac{7}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{7\int \frac{\sqrt{2}+}{-1-\sqrt{2}}}{16\sqrt{}}\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}+\frac{7\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{7\,\mathrm{Subst}\left(\int \right.}{}\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}\,x+\right.}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.91

$$
\frac{1}{96}\left(-\frac{24x}{x^4+1}-\frac{32}{x^3}+21\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)-21\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+42\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,x\right)-42\sqrt{2}\,\mathrm{ta}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]*L
og[1 + Sqrt[2]*x + x^2])/96
```

fricas [A] time = 0.62, size = 140, normalized size = 1.32

$$-\frac{56\,x^4 - 84\,\sqrt{2}\left(x^7 + x^3\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2 + \sqrt{2}\,x + 1} - 1\right) - 84\,\sqrt{2}\left(x^7 + x^3\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{\right.}{96\left(x^7 + x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(56*x^4 - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2
 + sqrt(2)*x + 1) - 1) - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)
*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 21*sqrt(2)*(x^7 + x^3)*log(x^2 + sqrt(2)*
x + 1) - 21*sqrt(2)*(x^7 + x^3)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^7 + x^3)
```

giac [A] time = 0.44, size = 87, normalized size = 0.82

$$-\frac{7}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) - \frac{7}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{7}{32}\,\sqrt{2}\,\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{7}{32}\,\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

maple [A] time = 0.01, size = 73, normalized size = 0.69

$$-\frac{x}{4\left(x^4 + 1\right)} - \frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,x - 1\right)}{16} - \frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,x + 1\right)}{16} - \frac{7\sqrt{2}\,\ln\left(\frac{x^2 + \sqrt{2}\,x + 1}{x^2 - \sqrt{2}\,x + 1}\right)}{32} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/3/x^3-1/4/(x^4+1)*x-7/32*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))
-7/16*2^(1/2)*arctan(2^(1/2)*x+1)-7/16*2^(1/2)*arctan(2^(1/2)*x-1)
```

maxima [A] time = 2.20, size = 90, normalized size = 0.85

$$-\frac{7}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) - \frac{7}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{7}{32}\,\sqrt{2}\,\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{7}{32}\,\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

mupad [B] time = 1.36, size = 51, normalized size = 0.48

$$-\frac{\frac{7\,x^4}{12}+\frac{1}{3}}{x^7+x^3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)
```

sympy [A] time = 0.22, size = 99, normalized size = 0.93

$$\frac{-7x^4-4}{12x^7+12x^3}+\frac{7\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{7\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)}{32}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{16}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] (-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
- 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16
 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

3.288 $\int \frac{1}{x^6(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^5\left(x^4+1\right)}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}x+\right.}{8\sqrt{2}}$$

```
[Out] -9/20/x^5+9/4/x+1/4/x^5/(x^4+1)+9/16*arctan(-1+x*2^(1/2))*2^(1/2)+9/16*arct
an(1+x*2^(1/2))*2^(1/2)+9/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-9/32*ln(1+x^2+x*2^
(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x^5\left(x^4+1\right)}-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}x+1\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x
^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^6\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^5\left(1+x^4\right)}+\frac{9}{4}\int \frac{1}{x^6\left(1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}+\frac{1}{4x^5\left(1+x^4\right)}-\frac{9}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5\left(1+x^4\right)}+\frac{9}{4}\int \frac{x^2}{1+x^4}\,dx\\
&= -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5\left(1+x^4\right)}-\frac{9}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{9}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5\left(1+x^4\right)}+\frac{9}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{9}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{9\int
\\
&= -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5\left(1+x^4\right)}+\frac{9\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{9\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{9\,\mathrm{Subs}
\\
&= -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5\left(1+x^4\right)}-\frac{9\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\log\left(1-\sqrt{
}{16\sqrt{
\end{aligned}$$

Mathematica [A] time = 0.08, size = 103, normalized size = 0.91

$$\frac{1}{160}\left(-\frac{32}{x^5}+45\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-45\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{40x^3}{x^4+1}+\frac{320}{x}-90\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] +
90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 45
*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

fricas [A] time = 0.94, size = 145, normalized size = 1.28

$$\frac{360\,x^8+288\,x^4-180\,\sqrt{2}\left(x^9+x^5\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}\,-1\right)-180\,\sqrt{2}\left(x^9+x^5\right)\arctan\left(-\sqrt{\right.}{160\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/160*(360*x^8 + 288*x^4 - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2)*x + sqrt
(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2
)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 45*sqrt(2)*(x^9 + x^5)*log(x
^2 + sqrt(2)*x + 1) + 45*sqrt(2)*(x^9 + x^5)*log(x^2 - sqrt(2)*x + 1) - 32)
/(x^9 + x^5)
```

giac [A] time = 0.30, size = 96, normalized size = 0.85

$$\frac{x^3}{4\left(x^4+1\right)}+\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{9}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5
```

maple [A] time = 0.01, size = 80, normalized size = 0.71

$$\frac{x^3}{4x^4+4}+\frac{9\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}+\frac{9\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}+\frac{9\sqrt{2}\,\ln\left(\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right)}{32}+\frac{2}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/5/x^5+2/x+1/4/(x^4+1)*x^3+9/32*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)
*x+1))+9/16*2^(1/2)*arctan(2^(1/2)*x+1)+9/16*2^(1/2)*arctan(2^(1/2)*x-1)
```

maxima [A] time = 2.01, size = 95, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{9}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{9}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8 + 36*x^4 - 4)/(x^9 + x^5)
```

mupad [B] time = 0.09, size = 55, normalized size = 0.49

$$\frac{\frac{9\,x^8}{4}+\frac{9\,x^4}{5}-\frac{1}{5}}{x^9+x^5}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{16}-\frac{9}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{16}+\frac{9}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(9/16 - 9i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(9/16 + 9i/16) + ((9*x^4)/5 + (9*x^8)/4 - 1/5)/(x^5 + x^9)
```

sympy [A] time = 0.23, size = 102, normalized size = 0.90

$$\frac{9\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{9\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{16}+\frac{45x^8+36x^4}{20x^9+20x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqrt(2)*x + 1
)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*atan(sqrt(2)*x + 1)/16
+ (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

3.289 $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{11}{28x^7}+\frac{11}{12x^3}-\frac{11\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{1}{4x^7\left(x^4+1\right)}-\frac{11\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\right.}{8\sqrt{}}$$

```
[Out] -11/28/x^7+11/12/x^3+1/4/x^7/(x^4+1)+11/16*arctan(-1+x*2^(1/2))*2^(1/2)+11/
16*arctan(1+x*2^(1/2))*2^(1/2)-11/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+11/32*ln(1
+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^7\left(x^4+1\right)}+\frac{11}{12x^3}-\frac{11}{28x^7}-\frac{11\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{11\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{}\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x
])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[
2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^8\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{4}\int \frac{1}{x^8\left(1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}+\frac{1}{4x^7\left(1+x^4\right)}-\frac{11}{4}\int \frac{1}{x^4\left(1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{11}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{11}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx-\frac{11}{}\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}-\frac{11\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{11\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{11}{}\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}-\frac{11\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{11\log(1}{1}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.89

$$\frac{1}{672}\left(-\frac{96}{x^7}+\frac{168x}{x^4+1}+\frac{448}{x^3}-231\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+231\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-462\sqrt{2}\tan^{-1}\left(1-\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x]
+ 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
+ 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

fricas [A] time = 0.95, size = 145, normalized size = 1.28

$$\frac{616\,x^8+352\,x^4-924\,\sqrt{2}\left(x^{11}+x^7\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right)-924\,\sqrt{2}\left(x^{11}+x^7\right)\arctan\left(-\right.}{672}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(616*x^8 + 352*x^4 - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt(2)*x + sqr
t(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt
(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 231*sqrt(2)*(x^11 + x^7)*l
og(x^2 + sqrt(2)*x + 1) - 231*sqrt(2)*(x^11 + x^7)*log(x^2 - sqrt(2)*x + 1)
 - 96)/(x^11 + x^7)
```

giac [A] time = 0.36, size = 94, normalized size = 0.83

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{11}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1) + 1/21*(14*x^4 - 3)/x^7
```

maple [A] time = 0.01, size = 78, normalized size = 0.69

$$\frac{x}{4x^4+4}+\frac{11\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}+\frac{11\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}+\frac{11\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{32}+\frac{2}{3x^3}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+2*x^4+1),x)
```

```
[Out] -1/7/x^7+2/3/x^3+1/4/(x^4+1)*x+11/16*2^(1/2)*arctan(2^(1/2)*x-1)+11/32*2^(1
/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))+11/16*2^(1/2)*arctan(2^(1/2)*x+
1)
```

maxima [A] time = 1.93, size = 95, normalized size = 0.84

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{11}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 + x^7
)
```

mupad [B] time = 0.10, size = 55, normalized size = 0.49

$$\frac{\frac{11\,x^8}{12}+\frac{11\,x^4}{21}-\frac{1}{7}}{x^{11}+x^7}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{11}{16}+\frac{11}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{11}{16}-\frac{11}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(11/16 + 11i/16) + 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(11/16 - 11i/16) + ((11*x^4)/21 + (11*x^8)/12 - 1/7)/(x^7
 + x^11)
```

sympy [A] time = 0.23, size = 102, normalized size = 0.90

$$-\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{77x^8+}{84x^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x
+ 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x + 1
)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)
```

3.290 $\int \frac{x^m}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

```
[Out] x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],x^4)/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1-2x^4+x^8}\,dx &= \int \frac{x^m}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.07

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+1}{4}+1;x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(1 + m)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{x^8-2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-2*x^4+1),x)
```

```
[Out] int(x^m/(x^8-2*x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{x^8 - 2\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8 - 2*x^4 + 1),x)
```

```
[Out] int(x^m/(x^8 - 2*x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(x-1)^2\,(x+1)^2\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-2*x**4+1),x)
```

```
[Out] Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

3.291 $\int \frac{x^9}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=32

$$\frac{3x^2}{4}-\frac{3}{4}\tanh^{-1}\left(x^2\right)+\frac{x^6}{4\left(1-x^4\right)}$$

```
[Out] 3/4*x^2+1/4*x^6/(-x^4+1)-3/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 288, 321, 207}

$$\frac{x^6}{4\left(1-x^4\right)}+\frac{3x^2}{4}-\frac{3}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-2x^4+x^8}\,dx &= \int \frac{x^9}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx,x,x^2\right)\\ &= \frac{x^6}{4\left(1-x^4\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,x^2\right)\\ &= \frac{3x^2}{4}+\frac{x^6}{4\left(1-x^4\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\ &= \frac{3x^2}{4}+\frac{x^6}{4\left(1-x^4\right)}-\frac{3}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.22

$$\frac{1}{8}\left(3\log\left(1-x^2\right)-3\log\left(x^2+1\right)+2\left(\frac{1}{1-x^4}+2\right)x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

fricas [A] time = 0.76, size = 46, normalized size = 1.44

$$\frac{4\,x^6-6\,x^2-3\left(x^4-1\right)\log\left(x^2+1\right)+3\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/(
x^4 - 1)
```

giac [A] time = 0.33, size = 35, normalized size = 1.09

$$\frac{1}{2}\,x^2-\frac{x^2}{4\left(x^4-1\right)}-\frac{3}{8}\,\log\left(x^2+1\right)+\frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 41, normalized size = 1.28

$$\frac{x^2}{2}+\frac{3\ln\left(x^2-1\right)}{8}-\frac{3\ln\left(x^2+1\right)}{8}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-2*x^4+1),x)
```

```
[Out] 1/2*x^2-1/8/(x^2+1)-3/8*ln(x^2+1)-1/8/(x^2-1)+3/8*ln(x^2-1)
```

maxima [A] time = 1.03, size = 34, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{x^2}{4\left(x^4-1\right)} - \frac{3}{8}\log\left(x^2+1\right) + \frac{3}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.81

$$\frac{x^2}{2} - \frac{x^2}{4\left(x^4-1\right)} - \frac{3\operatorname{atanh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - 2*x^4 + 1),x)
[Out] x^2/2 - x^2/(4*(x^4 - 1)) - (3*atanh(x^2))/4
```

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{x^2}{2} - \frac{x^2}{4x^4-4} + \frac{3\log\left(x^2-1\right)}{8} - \frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-2*x**4+1),x)
[Out] x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

3.292 $\int \frac{x^7}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=26

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

```
[Out] 1/4/(-x^4+1)+1/4*ln(-x^4+1)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-2x^4+x^8} \, dx &= \int \frac{x^7}{\left(-1+x^4\right)^2} \, dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(-1+x)^2} \, dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right) dx, x, x^4\right) \\
&= \frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{1}{4}\log\left(x^4-1\right)-\frac{1}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 2*x^4 + x^8),x]
[Out] -1/4*1/(-1 + x^4) + Log[-1 + x^4]/4
```

fricas [A] time = 0.77, size = 23, normalized size = 0.88

$$\frac{\left(x^4-1\right)\log\left(x^4-1\right)-1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

giac [A] time = 0.41, size = 19, normalized size = 0.73

$$-\frac{1}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

maple [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{\ln\left(x^4-1\right)}{4}-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-2*x^4+1),x)
[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)
```

maxima [A] time = 1.13, size = 18, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

mupad [B] time = 0.05, size = 20, normalized size = 0.77

$$\frac{\ln\left(x^4-1\right)}{4}-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - 2*x^4 + 1),x)
[Out] log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

sympy [A] time = 0.10, size = 15, normalized size = 0.58

$$\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-2*x**4+1),x)
```

```
[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

3.293 $\int \frac{x^5}{1-2x^4+x^8}\, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] 1/4*x^2/(-x^4+1)-1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 275, 288, 207}

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = \int \frac{x^5}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{4\left(1-x^4\right)} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{4\left(1-x^4\right)} - \frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Mathematica [**A**] time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(\log\left(1-x^2\right) - \log\left(x^2+1\right) - \frac{2x^2}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

fricas [**B**] time = 0.80, size = 40, normalized size = 1.60

$$-\frac{2\,x^2 + \left(x^4-1\right)\log\left(x^2+1\right) - \left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

giac [**A**] time = 0.50, size = 30, normalized size = 1.20

$$-\frac{x^2}{4\left(x^4-1\right)} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

maple [**A**] time = 0.01, size = 36, normalized size = 1.44

$$\frac{\ln\left(x^2-1\right)}{8} - \frac{\ln\left(x^2+1\right)}{8} - \frac{1}{8\left(x^2+1\right)} - \frac{1}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-2*x^4+1),x)
[Out] -1/8/(x^2+1)-1/8*ln(x^2+1)-1/8/(x^2-1)+1/8*ln(x^2-1)
```

maxima [A] time = 0.91, size = 29, normalized size = 1.16

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

mupad [B] time = 1.27, size = 21, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{x^2}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - 2*x^4 + 1),x)
[Out] - atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}+\frac{\log\left(x^2-1\right)}{8}-\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-2*x**4+1),x)
[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

3.294 $\int \frac{x^3}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=13

$$\frac{1}{4(1-x^4)}$$

```
[Out] 1/4/(-x^4+1)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 261}

$$\frac{1}{4(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-2x^4+x^8}\,dx &= \int \frac{x^3}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4(1-x^4)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] -1/4*1/(-1 + x^4)
```

fricas [A] time = 0.85, size = 9, normalized size = 0.69

$$-\frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4/(x^4 - 1)
```

giac [A] time = 0.50, size = 9, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 - 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-2*x^4+1),x)
[Out] -1/4/(x^4-1)
```

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$-\frac{1}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - 2*x^4 + 1),x)
[Out] -1/(4*(x^4 - 1))
```

sympy [A] time = 0.10, size = 8, normalized size = 0.62

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-2*x**4+1),x)
[Out] -1/(4*x**4 - 4)
```

3.295 $\int \frac{x}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)+\frac{x^2}{4\left(1-x^4\right)}$$

```
[Out] 1/4*x^2/(-x^4+1)+1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 275, 199, 207}

$$\frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{1-2x^4+x^8}\,dx = \int \frac{x}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{4\left(1-x^4\right)} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{4\left(1-x^4\right)} + \frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\log\left(1-x^2\right)+\log\left(x^2+1\right)-\frac{2x^2}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

fricas [B] time = 0.87, size = 40, normalized size = 1.60

$$-\frac{2\,x^2-\left(x^4-1\right)\log\left(x^2+1\right)+\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

giac [A] time = 0.47, size = 30, normalized size = 1.20

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 36, normalized size = 1.44

$$-\frac{\ln\left(x^2-1\right)}{8}+\frac{\ln\left(x^2+1\right)}{8}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-2*x^4+1),x)
[Out] -1/8/(x^2+1)+1/8*ln(x^2+1)-1/8/(x^2-1)-1/8*ln(x^2-1)
```

maxima [A] time = 0.92, size = 29, normalized size = 1.16

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{x^2}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

3.296 $\int \frac{1}{x(1-2x^4+x^8)} dx$

Optimal. Leaf size=28

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

```
[Out] 1/4/(-x^4+1)+ln(x)-1/4*ln(-x^4+1)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-2x^4+x^8)} dx &= \int \frac{1}{x(-1+x^4)^2} dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2 x} dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{1-x} + \frac{1}{(-1+x)^2} + \frac{1}{x}\right) dx, x, x^4\right) \\
&= \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log(1-x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{1}{4(x^4-1)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4
```

fricas [A] time = 0.81, size = 32, normalized size = 1.14

$$-\frac{\left(x^4-1\right)\log\left(x^4-1\right)-4\left(x^4-1\right)\log(x)+1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

giac [A] time = 0.38, size = 30, normalized size = 1.07

$$\frac{x^4-2}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4\right)-\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

maple [A] time = 0.02, size = 47, normalized size = 1.68

$$\ln(x)-\frac{\ln(x-1)}{4}-\frac{\ln(x+1)}{4}-\frac{\ln\left(x^2+1\right)}{4}+\frac{1}{16x+16}+\frac{1}{8x^2+8}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-2*x^4+1),x)
```

```
[Out] ln(x)+1/16/(x+1)-1/4*ln(x+1)-1/4*ln(x^2+1)+1/8/(x^2+1)-1/16/(x-1)-1/4*ln(x-
1)
```

maxima [A] time = 0.94, size = 24, normalized size = 0.86

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

mupad [B] time = 0.06, size = 22, normalized size = 0.79

$$\ln(x)-\frac{\ln\left(x^4-1\right)}{4}-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

sympy [A] time = 0.13, size = 19, normalized size = 0.68

$$\log(x) - \frac{\log(x^4 - 1)}{4} - \frac{1}{4x^4 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-2*x**4+1),x)
[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

3.297 $\int \frac{1}{x^3(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=32

$$-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^2\left(1-x^4\right)}$$

```
[Out] -3/4/x^2+1/4/x^2/(-x^4+1)+3/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^2\left(1-x^4\right)}-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = \int \frac{1}{x^3(-1+x^4)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+x^2)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{4x^2(1-x^4)} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{x^2(-1+x^2)}\,dx, x, x^2\right)$$
$$= -\frac{3}{4x^2} + \frac{1}{4x^2(1-x^4)} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)$$
$$= -\frac{3}{4x^2} + \frac{1}{4x^2(1-x^4)} + \frac{3}{4}\tanh^{-1}(x^2)$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.28

$$\frac{1}{8}\left(-3\log(1-x^2) + 3\log(x^2+1) + \frac{4-6x^4}{x^2(x^4-1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

fricas [B] time = 0.67, size = 54, normalized size = 1.69

$$-\frac{6\,x^4 - 3\,(x^6 - x^2)\log(x^2+1) + 3\,(x^6 - x^2)\log(x^2-1) - 4}{8\,(x^6 - x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)/
(x^6 - x^2)
```

giac [A] time = 0.41, size = 38, normalized size = 1.19

$$-\frac{3\,x^4 - 2}{4\,(x^6 - x^2)} + \frac{3}{8}\log(x^2+1) - \frac{3}{8}\log(|x^2-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))
```

maple [A] time = 0.02, size = 50, normalized size = 1.56

$$-\frac{3\ln(x-1)}{8} - \frac{3\ln(x+1)}{8} + \frac{3\ln(x^2+1)}{8} - \frac{1}{2x^2} + \frac{1}{16x+16} - \frac{1}{8(x^2+1)} - \frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-2*x^4+1),x)
```

[Out] -1/2/x^2+1/16/(x+1)-3/8*ln(x+1)+3/8*ln(x^2+1)-1/8/(x^2+1)-1/16/(x-1)-3/8*ln
(x-1)

maxima [A] time = 0.95, size = 37, normalized size = 1.16

$$-\frac{3\,x^4-2}{4\left(x^6-x^2\right)}+\frac{3}{8}\,\log\left(x^2+1\right)-\frac{3}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.81

$$\frac{3\,\mathrm{atanh}\left(x^2\right)}{4}+\frac{\frac{3\,x^4}{4}-\frac{1}{2}}{x^2-x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)
```

sympy [A] time = 0.15, size = 36, normalized size = 1.12

$$\frac{2-3x^4}{4x^6-4x^2}-\frac{3\log\left(x^2-1\right)}{8}+\frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-2*x**4+1),x)
```

```
[Out] (2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8
```

3.298 $\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4\left(1-x^4\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

```
[Out] -1/4/x^4+1/4/(-x^4+1)+2*ln(x)-1/2*ln(-x^4+1)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4\left(1-x^4\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2x^2}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4\left(1-x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

fricas [A] time = 0.66, size = 50, normalized size = 1.35

$$-\frac{2 x^4+2 \left(x^8-x^4\right) \log \left(x^4-1\right)-8 \left(x^8-x^4\right) \log(x)-1}{4 \left(x^8-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8 -
 x^4)
```

giac [A] time = 0.45, size = 36, normalized size = 0.97

$$-\frac{2 x^4-1}{4 \left(x^8-x^4\right)}+\frac{1}{2} \log \left(x^4\right)-\frac{1}{2} \log \left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

maple [A] time = 0.02, size = 54, normalized size = 1.46

$$2 \ln(x)-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\frac{1}{4x^4}+\frac{1}{16x+16}+\frac{1}{8x^2+8}-\frac{1}{16\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-2*x^4+1),x)
```

```
[Out] -1/4/x^4+2*ln(x)+1/16/(x+1)-1/2*ln(x+1)-1/2*ln(x^2+1)+1/8/(x^2+1)-1/16/(x-1
)-1/2*ln(x-1)
```

maxima [A] time = 0.83, size = 35, normalized size = 0.95

$$-\frac{2 x^4-1}{4 \left(x^8-x^4\right)}-\frac{1}{2} \log \left(x^4-1\right)+\frac{1}{2} \log \left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.86

$$2 \ln(x)-\frac{\ln\left(x^4-1\right)}{2}+\frac{\frac{x^4}{2}-\frac{1}{4}}{x^4-x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] 2*log(x) - log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)
```

sympy [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{1-2x^4}{4x^8-4x^4}+2\log(x)-\frac{\log(x^4-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-2*x**4+1),x)
```

```
[Out] (1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

3.299 $\int \frac{1}{x^7\left(1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=39

$$-\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{5}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^6\left(1-x^4\right)}$$

[Out] -5/12/x^6-5/4/x^2+1/4/x^6/(-x^4+1)+5/4*arctanh(x^2)

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 - 2*x^4 + x^8)),x]

[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(-1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4}\text{Subst}\left(\int \frac{1}{x^4\left(-1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4}\text{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{1}{4x^6\left(1-x^4\right)}+\frac{5}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.26

$$-\frac{1}{6x^6}-\frac{1}{x^2}-\frac{5}{8}\log\left(1-x^2\right)+\frac{5}{8}\log\left(x^2+1\right)-\frac{x^2}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
[Out] -1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 +
x^2])/8
```

fricas [B] time = 0.91, size = 59, normalized size = 1.51

$$-\frac{30\,x^8-20\,x^4-15\left(x^{10}-x^6\right)\log\left(x^2+1\right)+15\left(x^{10}-x^6\right)\log\left(x^2-1\right)-4}{24\left(x^{10}-x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*log
(x^2 - 1) - 4)/(x^10 - x^6)
```

giac [A] time = 0.33, size = 42, normalized size = 1.08

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{6\,x^4+1}{6\,x^6}+\frac{5}{8}\,\log\left(x^2+1\right)-\frac{5}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(x
^2 - 1))
```

maple [A] time = 0.02, size = 55, normalized size = 1.41

$$-\frac{5\ln\left(x-1\right)}{8}-\frac{5\ln\left(x+1\right)}{8}+\frac{5\ln\left(x^2+1\right)}{8}-\frac{1}{x^2}-\frac{1}{6x^6}+\frac{1}{16x+16}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-2*x^4+1),x)
```

```
[Out] -1/6/x^6-1/x^2+1/16/(x+1)-5/8*ln(x+1)+5/8*ln(x^2+1)-1/8/(x^2+1)-1/16/(x-1)-
5/8*ln(x-1)
```

maxima [A] time = 0.94, size = 42, normalized size = 1.08

$$-\frac{15\,x^8-10\,x^4-2}{12\,\left(x^{10}-x^6\right)}+\frac{5}{8}\,\log\left(x^2+1\right)-\frac{5}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2 -
 1)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.82

$$\frac{5\,\mathrm{atanh}\left(x^2\right)}{4}-\frac{-\frac{5\,x^8}{4}+\frac{5\,x^4}{6}+\frac{1}{6}}{x^6-x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)
```

sympy [A] time = 0.18, size = 41, normalized size = 1.05

$$-\frac{5\log\left(x^2-1\right)}{8}+\frac{5\log\left(x^2+1\right)}{8}+\frac{-15x^8+10x^4+2}{12x^{10}-12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-2*x**4+1),x)
```

```
[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**10
 - 12*x**6)
```

3.300 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=34

$$\frac{x^5}{4\left(1-x^4\right)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

[Out] 5/4*x+1/4*x^5/(-x^4+1)-5/8*arctan(x)-5/8*arctanh(x)

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 288, 321, 212, 206, 203}

$$\frac{x^5}{4\left(1-x^4\right)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 - 2*x^4 + x^8),x]

[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1-2x^4+x^8}\,dx &= \int \frac{x^8}{\left(-1+x^4\right)^2}\,dx\\
&= \frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{x^4}{-1+x^4}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{1}{-1+x^4}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.12

$$-\frac{x}{4\left(x^4-1\right)}+x+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(x+1)-\frac{5}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x])
/16
```

fricas [B] time = 0.80, size = 49, normalized size = 1.44

$$\frac{16\,x^5-10\left(x^4-1\right)\arctan(x)-5\left(x^4-1\right)\log\left(x+1\right)+5\left(x^4-1\right)\log\left(x-1\right)-20\,x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 - 1
)*log(x - 1) - 20*x)/(x^4 - 1)
```

giac [A] time = 0.28, size = 30, normalized size = 0.88

$$x-\frac{x}{4\left(x^4-1\right)}-\frac{5}{8}\arctan(x)-\frac{5}{16}\log\left(|x+1|\right)+\frac{5}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(x
 - 1))
```

maple [A] time = 0.01, size = 43, normalized size = 1.26

$$x+\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}+\frac{5\ln(x-1)}{16}-\frac{5\ln(x+1)}{16}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-2*x^4+1),x)
```

```
[Out] x-1/16/(x+1)-5/16*ln(x+1)+1/8/(x^2+1)*x-5/8*arctan(x)-1/16/(x-1)+5/16*ln(x-
1)
```

maxima [A] time = 2.13, size = 28, normalized size = 0.82

$$x-\frac{x}{4\left(x^4-1\right)}-\frac{5}{8}\arctan(x)-\frac{5}{16}\log(x+1)+\frac{5}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

mupad [B] time = 1.29, size = 26, normalized size = 0.76

$$x-\frac{5\operatorname{atan}(x)}{8}-\frac{x}{4\left(x^4-1\right)}+\frac{\operatorname{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - 2*x^4 + 1),x)
```

```
[Out] x + (atan(x*1i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 32, normalized size = 0.94

$$x-\frac{x}{4x^4-4}+\frac{5\log(x-1)}{16}-\frac{5\log(x+1)}{16}-\frac{5\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-2*x**4+1),x)
```

```
[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

3.301 $\int \frac{x^6}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=29

$$\frac{x^3}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x^3/(-x^4+1)+3/8*arctan(x)-3/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 288, 298, 203, 206}

$$\frac{x^3}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = \int \frac{x^6}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} + \frac{3}{4}\int \frac{x^2}{-1+x^4}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} - \frac{3}{8}\int \frac{1}{1-x^2}\,dx + \frac{3}{8}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} + \frac{3}{8}\tan^{-1}(x) - \frac{3}{8}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.21

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1}+3\log(1-x)-3\log(x+1)+6\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

fricas [B] time = 0.82, size = 46, normalized size = 1.59

$$-\frac{4\,x^3-6\left(x^4-1\right)\arctan(x)+3\left(x^4-1\right)\log\left(x+1\right)-3\left(x^4-1\right)\log\left(x-1\right)}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)
*log(x - 1))/(x^4 - 1)
```

giac [A] time = 0.40, size = 31, normalized size = 1.07

$$-\frac{x^3}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log\left(|x+1|\right)+\frac{3}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
- 1))
```

maple [A] time = 0.02, size = 42, normalized size = 1.45

$$-\frac{x}{8\left(x^2+1\right)}+\frac{3\arctan(x)}{8}+\frac{3\ln\left(x-1\right)}{16}-\frac{3\ln\left(x+1\right)}{16}-\frac{1}{16\left(x+1\right)}-\frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)-3/16*ln(x+1)-1/8/(x^2+1)*x+3/8*arctan(x)-1/16/(x-1)+3/16*ln(x-1
)
```

maxima [A] time = 2.03, size = 29, normalized size = 1.00

$$-\frac{x^3}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log(x+1)+\frac{3}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\frac{3\,\mathrm{atan}(x)}{8}-\frac{3\,\mathrm{atanh}(x)}{8}-\frac{x^3}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - 2*x^4 + 1),x)
[Out] (3*atan(x))/8 - (3*atanh(x))/8 - x^3/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 32, normalized size = 1.10

$$-\frac{x^3}{4x^4-4}+\frac{3\log(x-1)}{16}-\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-2*x**4+1),x)
[Out] -x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

3.302 $\int \frac{x^4}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=27

$$\frac{x}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x/(-x^4+1)-1/8*arctan(x)-1/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 288, 212, 206, 203}

$$\frac{x}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = \int \frac{x^4}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} + \frac{1}{4}\int \frac{1}{-1+x^4}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} - \frac{1}{8}\int \frac{1}{1-x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) - \frac{1}{8}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{16}\left(-\frac{4x}{x^4-1} + \log(1-x) - \log(x+1) - 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

fricas [B] time = 0.86, size = 43, normalized size = 1.59

$$-\frac{2\left(x^4-1\right)\arctan(x) + \left(x^4-1\right)\log\left(x+1\right) - \left(x^4-1\right)\log\left(x-1\right) + 4\,x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
+ 4*x)/(x^4 - 1)
```

giac [A] time = 0.51, size = 29, normalized size = 1.07

$$-\frac{x}{4\left(x^4-1\right)} - \frac{1}{8}\arctan(x) - \frac{1}{16}\log\left(|x+1|\right) + \frac{1}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$\frac{x}{8x^2+8} - \frac{\arctan(x)}{8} + \frac{\ln\left(x-1\right)}{16} - \frac{\ln\left(x+1\right)}{16} - \frac{1}{16\left(x+1\right)} - \frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)-1/16*ln(x+1)+1/8/(x^2+1)*x-1/8*arctan(x)-1/16/(x-1)+1/16*ln(x-1
)
```

maxima [A] time = 1.92, size = 27, normalized size = 1.00

$$-\frac{x}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\log(x+1)+\frac{1}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$-\frac{\operatorname{atan}(x)}{8}-\frac{\operatorname{atanh}(x)}{8}-\frac{x}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - 2*x^4 + 1),x)
[Out] - atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 26, normalized size = 0.96

$$-\frac{x}{4x^4-4}+\frac{\log(x-1)}{16}-\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-2*x**4+1),x)
[Out] -x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

3.303 $\int \frac{x^2}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x^3/(-x^4+1)-1/8*arctan(x)+1/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 290, 298, 203, 206}

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = \int \frac{x^2}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} - \frac{1}{4}\int \frac{x^2}{-1+x^4}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} + \frac{1}{8}\int \frac{1}{1-x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.14

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1} - \log(1-x) + \log(x+1) - 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

fricas [B] time = 0.83, size = 45, normalized size = 1.55

$$-\frac{4\,x^3 + 2\left(x^4-1\right)\arctan(x) - \left(x^4-1\right)\log\left(x+1\right) + \left(x^4-1\right)\log\left(x-1\right)}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*log
(x - 1))/(x^4 - 1)
```

giac [A] time = 0.34, size = 31, normalized size = 1.07

$$-\frac{x^3}{4\left(x^4-1\right)} - \frac{1}{8}\arctan(x) + \frac{1}{16}\log\left(|x+1|\right) - \frac{1}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
- 1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.45

$$-\frac{x}{8\left(x^2+1\right)} - \frac{\arctan(x)}{8} - \frac{\ln\left(x-1\right)}{16} + \frac{\ln\left(x+1\right)}{16} - \frac{1}{16\left(x+1\right)} - \frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)+1/16*ln(x+1)-1/8/(x^2+1)*x-1/8*arctan(x)-1/16/(x-1)-1/16*ln(x-1
)
```

maxima [A] time = 1.71, size = 29, normalized size = 1.00

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)+\frac{1}{16}\log\left(x+1\right)-\frac{1}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\frac{\operatorname{atanh}(x)}{8}-\frac{\operatorname{atan}(x)}{8}-\frac{x^3}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - 2*x^4 + 1),x)
[Out] atanh(x)/8 - atan(x)/8 - x^3/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 27, normalized size = 0.93

$$-\frac{x^3}{4x^4-4}-\frac{\log\left(x-1\right)}{16}+\frac{\log\left(x+1\right)}{16}-\frac{\operatorname{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-2*x**4+1),x)
[Out] -x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

3.304 $\int \frac{1}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=27

$$\frac{x}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x/(-x^4+1)+3/8*arctan(x)+3/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {28, 199, 212, 206, 203}

$$\frac{x}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-2x^4+x^8}\,dx &= \int \frac{1}{\left(-1+x^4\right)^2}\,dx \\
&= \frac{x}{4\left(1-x^4\right)} - \frac{3}{4}\int \frac{1}{-1+x^4}\,dx \\
&= \frac{x}{4\left(1-x^4\right)} + \frac{3}{8}\int \frac{1}{1-x^2}\,dx + \frac{3}{8}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{4\left(1-x^4\right)} + \frac{3}{8}\tan^{-1}(x) + \frac{3}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.22

$$
\frac{1}{16}\left(-\frac{4x}{x^4-1} - 3\log(1-x) + 3\log(x+1) + 6\tan^{-1}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

fricas [B] time = 0.85, size = 44, normalized size = 1.63

$$
\frac{6\left(x^4-1\right)\arctan(x) + 3\left(x^4-1\right)\log\left(x+1\right) - 3\left(x^4-1\right)\log\left(x-1\right) - 4\,x}{16\left(x^4-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
1) - 4*x)/(x^4 - 1)
```

giac [A] time = 0.37, size = 29, normalized size = 1.07

$$
-\frac{x}{4\left(x^4-1\right)} + \frac{3}{8}\arctan(x) + \frac{3}{16}\log\left(|x+1|\right) - \frac{3}{16}\log\left(|x-1|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$
\frac{x}{8x^2+8} + \frac{3\arctan(x)}{8} - \frac{3\ln\left(x-1\right)}{16} + \frac{3\ln\left(x+1\right)}{16} - \frac{1}{16\left(x+1\right)} - \frac{1}{16\left(x-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)+3/16*ln(x+1)+1/8/(x^2+1)*x+3/8*arctan(x)-1/16/(x-1)-3/16*ln(x-1
)
```

maxima [A] time = 2.13, size = 27, normalized size = 1.00

$$-\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)+\frac{3}{16}\log\left(x+1\right)-\frac{3}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$\frac{3\,\mathrm{atan}(x)}{8}+\frac{3\,\mathrm{atanh}(x)}{8}-\frac{x}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 2*x^4 + 1),x)
[Out] (3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))
```

sympy [A] time = 0.16, size = 31, normalized size = 1.15

$$-\frac{x}{4x^4-4}-\frac{3\log\left(x-1\right)}{16}+\frac{3\log\left(x+1\right)}{16}+\frac{3\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**4+1),x)
[Out] -x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

3.305 $\int \frac{1}{x^2(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{4x(1-x^4)}-\frac{5}{4x}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

```
[Out] -5/4/x+1/4/x/(-x^4+1)-5/8*arctan(x)+5/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x(1-x^4)}-\frac{5}{4x}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}+\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.11

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1}-\frac{16}{x}-5\log(1-x)+5\log(x+1)-10\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/1
6
```

fricas [B] time = 0.52, size = 55, normalized size = 1.53

$$-\frac{20\,x^4+10\left(x^5-x\right)\arctan(x)-5\left(x^5-x\right)\log\left(x+1\right)+5\left(x^5-x\right)\log\left(x-1\right)-16}{16\left(x^5-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
x)*log(x - 1) - 16)/(x^5 - x)
```

giac [A] time = 0.37, size = 37, normalized size = 1.03

$$-\frac{5\,x^4-4}{4\left(x^5-x\right)}-\frac{5}{8}\arctan(x)+\frac{5}{16}\log\left(|x+1|\right)-\frac{5}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 47, normalized size = 1.31

$$-\frac{x}{8\left(x^2+1\right)}-\frac{5\arctan(x)}{8}-\frac{5\ln\left(x-1\right)}{16}+\frac{5\ln\left(x+1\right)}{16}-\frac{1}{x}-\frac{1}{16\left(x+1\right)}-\frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/x-1/16/(x+1)+5/16*ln(x+1)-1/8/(x^2+1)*x-5/8*arctan(x)-1/16/(x-1)-5/16*ln
(x-1)
```

maxima [A] time = 2.09, size = 35, normalized size = 0.97

$$-\frac{5\,x^4-4}{4\left(x^5-x\right)}-\frac{5}{8}\,\arctan(x)+\frac{5}{16}\,\log\left(x+1\right)-\frac{5}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x -
 1)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{5\,\mathrm{atanh}(x)}{8}-\frac{5\,\mathrm{atan}(x)}{8}+\frac{\frac{5\,x^4}{4}-1}{x-x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (5*atanh(x))/8 - (5*atan(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)
```

sympy [A] time = 0.18, size = 37, normalized size = 1.03

$$\frac{4-5x^4}{4x^5-4x}-\frac{5\log\left(x-1\right)}{16}+\frac{5\log\left(x+1\right)}{16}-\frac{5\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 5*x**4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x)
/8
```

3.306 $\int \frac{1}{x^4(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=36

$$-\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

```
[Out] -7/12/x^3+1/4/x^3/(-x^4+1)+7/8*arctan(x)+7/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^3\left(1-x^4\right)}-\frac{7}{12x^3}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^4\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^3\left(1-x^4\right)}-\frac{7}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}-\frac{7}{4}\int \frac{1}{-1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}+\frac{7}{8}\int \frac{1}{1-x^2}\,dx+\frac{7}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.06

$$\frac{1}{48}\left(-\frac{12x}{x^4-1}-\frac{16}{x^3}-21\log(1-x)+21\log(x+1)+42\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x]
)/48
```

fricas [B] time = 0.73, size = 63, normalized size = 1.75

$$-\frac{28\,x^4-42\left(x^7-x^3\right)\arctan(x)-21\left(x^7-x^3\right)\log\left(x+1\right)+21\left(x^7-x^3\right)\log\left(x-1\right)-16}{48\left(x^7-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*(
x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

giac [A] time = 0.46, size = 34, normalized size = 0.94

$$-\frac{x}{4\left(x^4-1\right)}-\frac{1}{3\,x^3}+\frac{7}{8}\arctan(x)+\frac{7}{16}\log\left(|x+1|\right)-\frac{7}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 47, normalized size = 1.31

$$\frac{x}{8x^2+8}+\frac{7\arctan(x)}{8}-\frac{7\ln(x-1)}{16}+\frac{7\ln(x+1)}{16}-\frac{1}{3x^3}-\frac{1}{16\,(x+1)}-\frac{1}{16\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-2*x^4+1),x)
```

```
[Out] -1/3/x^3-1/16/(x+1)+7/16*ln(x+1)+1/8/(x^2+1)*x+7/8*arctan(x)-1/16/(x-1)-7/1
6*ln(x-1)
```

maxima [A] time = 1.98, size = 37, normalized size = 1.03

$$-\frac{7\,x^4-4}{12\,\left(x^7-x^3\right)}+\frac{7}{8}\,\arctan(x)+\frac{7}{16}\,\log\left(x+1\right)-\frac{7}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log(
x - 1)
```

mupad [B] time = 1.29, size = 28, normalized size = 0.78

$$\frac{7\,\mathrm{atan}(x)}{8}+\frac{7\,\mathrm{atanh}(x)}{8}+\frac{\frac{7\,x^4}{12}-\frac{1}{3}}{x^3-x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)
```

sympy [A] time = 0.18, size = 39, normalized size = 1.08

$$\frac{4-7x^4}{12x^7-12x^3}-\frac{7\log\left(x-1\right)}{16}+\frac{7\log\left(x+1\right)}{16}+\frac{7\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*at
an(x)/8
```

3.307 $\int \frac{1}{x^6(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{9}{20x^5}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

[Out] -9/20/x^5-9/4/x+1/4/x^5/(-x^4+1)-9/8*arctan(x)+9/8*arctanh(x)

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{20x^5}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 - 2*x^4 + x^8)),x]

[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[x])/8

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^6\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^6\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}+\frac{9}{8}\int \frac{1}{1-x^2}\,dx-\frac{9}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.19

$$-\frac{1}{5x^5}-\frac{x^3}{4\left(x^4-1\right)}-\frac{2}{x}-\frac{9}{16}\log(1-x)+\frac{9}{16}\log(x+1)-\frac{9}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/16
 + (9*Log[1 + x])/16
```

fricas [B] time = 0.85, size = 68, normalized size = 1.58

$$-\frac{180\,x^8-144\,x^4+90\left(x^9-x^5\right)\arctan(x)-45\left(x^9-x^5\right)\log\left(x+1\right)+45\left(x^9-x^5\right)\log\left(x-1\right)-16}{80\left(x^9-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x
+ 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

giac [A] time = 0.45, size = 43, normalized size = 1.00

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{10\,x^4+1}{5\,x^5}-\frac{9}{8}\arctan(x)+\frac{9}{16}\log\left(|x+1|\right)-\frac{9}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x
+ 1)) - 9/16*log(abs(x - 1))
```

maple [A] time = 0.02, size = 52, normalized size = 1.21

$$-\frac{x}{8\left(x^2+1\right)}-\frac{9\arctan(x)}{8}-\frac{9\ln(x-1)}{16}+\frac{9\ln(x+1)}{16}-\frac{2}{x}-\frac{1}{5x^5}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/5/x^5-2/x-1/16/(x+1)+9/16*ln(x+1)-1/8/(x^2+1)*x-9/8*arctan(x)-1/16/(x-1)
-9/16*ln(x-1)
```

maxima [A] time = 2.43, size = 42, normalized size = 0.98

$$-\frac{45\,x^8-36\,x^4-4}{20\left(x^9-x^5\right)}-\frac{9}{8}\arctan(x)+\frac{9}{16}\log(x+1)-\frac{9}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*log(x + 1) -
 9/16*log(x - 1)
```

mupad [B] time = 0.04, size = 34, normalized size = 0.79

$$\frac{9\,\mathrm{atanh}(x)}{8}-\frac{9\,\mathrm{atan}(x)}{8}-\frac{-\frac{9\,x^8}{4}+\frac{9\,x^4}{5}+\frac{1}{5}}{x^5-x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (9*atanh(x))/8 - (9*atan(x))/8 - ((9*x^4)/5 - (9*x^8)/4 + 1/5)/(x^5 - x^9)
```

sympy [A] time = 0.20, size = 44, normalized size = 1.02

$$-\frac{9\log(x-1)}{16}+\frac{9\log(x+1)}{16}-\frac{9\,\mathrm{atan}(x)}{8}+\frac{-45x^8+36x^4+4}{20x^9-20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-2*x**4+1),x)
```

```
[Out] -9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 + (-45*x**8 + 36*x**4 + 4)
/(20*x**9 - 20*x**5)
```

3.308 $\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

```
[Out] -11/28/x^7-11/12/x^3+1/4/x^7/(-x^4+1)+11/8*arctan(x)+11/8*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{12x^3}-\frac{11}{28x^7}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*A
rcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^8\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{x^8\left(-1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}+\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{-1+x^4}\,dx\\
&= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\int \frac{1}{1-x^2}\,dx+\frac{11}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{1}{336}\left(-\frac{48}{x^7}-\frac{84x}{x^4-1}-\frac{224}{x^3}-231\log(1-x)+231\log(x+1)+462\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] + 2
31*Log[1 + x])/336
```

fricas [B] time = 0.90, size = 68, normalized size = 1.58

$$-\frac{308\,x^8-176\,x^4-462\left(x^{11}-x^7\right)\arctan(x)-231\left(x^{11}-x^7\right)\log\left(x+1\right)+231\left(x^{11}-x^7\right)\log\left(x-1\right)-48}{336\left(x^{11}-x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*l
og(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)
```

giac [A] time = 0.45, size = 41, normalized size = 0.95

$$-\frac{x}{4\left(x^4-1\right)}-\frac{14\,x^4+3}{21\,x^7}+\frac{11}{8}\arctan(x)+\frac{11}{16}\log\left(|x+1|\right)-\frac{11}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(x
 + 1)) - 11/16*log(abs(x - 1))
```

maple [A] time = 0.02, size = 52, normalized size = 1.21

$$\frac{x}{8x^2+8}+\frac{11\arctan(x)}{8}-\frac{11\ln(x-1)}{16}+\frac{11\ln(x+1)}{16}-\frac{2}{3x^3}-\frac{1}{7x^7}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-2*x^4+1),x)
```

```
[Out] -1/7/x^7-2/3/x^3-1/16/(x+1)+11/16*ln(x+1)+1/8/(x^2+1)*x+11/8*arctan(x)-1/16
/(x-1)-11/16*ln(x-1)
```

maxima [A] time = 1.94, size = 42, normalized size = 0.98

$$-\frac{77\,x^8-44\,x^4-12}{84\left(x^{11}-x^7\right)}+\frac{11}{8}\arctan(x)+\frac{11}{16}\log(x+1)-\frac{11}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/16*log(x +
1) - 11/16*log(x - 1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.79

$$\frac{11\,\mathrm{atan}(x)}{8}+\frac{11\,\mathrm{atanh}(x)}{8}-\frac{-\frac{11\,x^8}{12}+\frac{11\,x^4}{21}+\frac{1}{7}}{x^7-x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (11*atan(x))/8 + (11*atanh(x))/8 - ((11*x^4)/21 - (11*x^8)/12 + 1/7)/(x^7 -
 x^11)
```

sympy [A] time = 0.21, size = 44, normalized size = 1.02

$$-\frac{11\log(x-1)}{16}+\frac{11\log(x+1)}{16}+\frac{11\,\mathrm{atan}(x)}{8}+\frac{-77x^8+44x^4+12}{84x^{11}-84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-2*x**4+1),x)
```

```
[Out] -11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 + (-77*x**8 + 44*x**4 +
 12)/(84*x**11 - 84*x**7)
```

3.309 $\int \frac{x^m}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=163

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] 2*c*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b-(-4*a*c+b^2)^(
1/2)))/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^(1+m)*hypergeo
m([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(-4*a*c
+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1375, 364}

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) - (
2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b + Sq
rt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{a+bx^4+cx^8}\,dx&=\frac{c\int \frac{x^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{x^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}\\&=\frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)}-\frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.06, size = 82, normalized size = 0.50

$$\frac{x^m \text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^7c+\#1^3b}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (x^m*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]/((x/(x - #1))^m*(b*#1^3 + 2*c*#1^7)) & ])/(4*m)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{cx^8+bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(c*x^8 + b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c*x^8+b*x^4+a),x)
```

```
[Out] int(x^m/(c*x^8+b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^4 + c*x^8),x)
```

```
[Out] int(x^m/(a + b*x^4 + c*x^8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.310 $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/4*(-2*a*c+b^2)*arctanh((2*c*x^4+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^4\right)}{4c}\\
&= \frac{x^4}{4c}-\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c^2}\\
&= \frac{x^4}{4c}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c^2}\\
&= \frac{x^4}{4c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^4+cx^8\right)+2cx^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

fricas [A] time = 0.99, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^4-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^8+bx^4+\right.}{8\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b
*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3), 1
/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 17.07, size = 75, normalized size = 0.93

$$\frac{x^4}{4\,c}-\frac{b\log\left(cx^8+bx^4+a\right)}{8\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2*
c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 111, normalized size = 1.37

$$\frac{x^4}{4c}-\frac{a\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{b^2\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;c^2}-\frac{b\ln\left(c\,x^8+b\,x^4+a\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2))*a+1/4/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*
a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.69, size = 3916, normalized size = 48.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4 + c*x^8),x)
```

```
[Out] x^4/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*b
^2*c^2)) - (atan((8*c^4*x^4*(((a*c - b^2)*(((((2*a*c - b^2)*((((448*b^4*c^6
 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2
*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16
*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c
^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*
c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2
)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*c
^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c
- b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c
 - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*
b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^
4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(
4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/
(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((8*a^3*c^5 -
20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144
*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^
4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 1
6*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*
a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b^3*(4*b^3 - 16*a*b*c)*(2*a*c -
b^2)^3)/(2*c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*a^3*c^2) -
```

((b^3 - 3*a*b*c)*(((4*b^3 - 16*a*b*c)*((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - (b^7 - a^3*b*c^3 + 3*a^2*b^3*c^2 - 3*a*b^5*c)/c^4 + ((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b^3*(2*a*c - b^2)^4)/(8*c^4*(4*a*c - b^2)^2)))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6*c) - (c^2*(a*c - b^2)*(4*a*c - b^2)^2*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2*b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - (a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2*c^2))))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6*c)) + (c^2*(4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*((a*b^6 - 2*a^2*b^4*c + a^3*b^2*c^2)/c^4 + ((4*b^3 - 16*a*b*c)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2*b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - ((4*b^3 - 16*a*b*c)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 -

```
512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*
c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/
2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*b^2*(2*a*c - b^2)^4)/(4*c^4*(4*a*c -
 b^2)^2)))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b
^6*c)))*(2*a*c - b^2))/(4*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 4.11, size = 316, normalized size = 3.90

$$\left(-\frac{b}{8c^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)\log\left(x^4 + \frac{-ab - 16ac^2\left(-\frac{b}{8c^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right) + 4b^2c\left(-\frac{b}{8c^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-}{8c^2(4ac-b^2)}\right.}{2ac-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*
log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 + (-a*b
 - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a
*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

3.311 $\int \frac{x^9}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

```
[Out] 1/2*x^2/c-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1359, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

```
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2c} - \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2c}$$
$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b}\right.}{4c}$$
$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.12, size = 210, normalized size = 1.09

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}x^2}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

fricas [B] time = 0.64, size = 1071, normalized size = 5.58

$$-\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3abc+\left(b^2c^3-4ac^4\right)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\log\left(-\left(ab^2-a^2c\right)x^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^4-5ab^2c+4a^2c^2-\left(b^3c^3-4abc^4\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 -
 a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^
3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^
3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
- 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2
```

```
*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqr
t(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqr
t(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4
*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a
*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 -
 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b
^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x^2)/c
```

giac [B] time = 18.17, size = 2043, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b
^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 -
 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 -
 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 -
4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*a^2*b^2*c^2 - 4*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^3 - 2*(b^2 - 4*a*c)*a*b^
2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + s
qrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*
a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2) + 1/8*(2*a*b^3*c^3 - 8*
a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3
*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 -
 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
```

```
)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*a*b^
4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2*c^2)*ab
s(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3
 - 4*a^2*c^4)*c^2)
```

maple [B] time = 0.03, size = 360, normalized size = 1.88

$$\frac{\sqrt{2}\, a \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c\, x^{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c\, x^{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\, b^{2} \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c\, x^{2}}{\sqrt{\left(-b+\sqrt{-4ac-\cdots}\right.}}\right)}{4\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2/c*x^2+1/4/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*a-1/4/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-1/4/c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(-
4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^{2}}{2c}-\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,b^{4}-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,ab^{2}c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,b^{3}c-2\,b^{4}c+16\,\sqrt{2}\,\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\,a^{2}c^{2}+8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^{2}-4\,ac}\,c}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 3.07, size = 5659, normalized size = 29.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*
c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3
*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^
5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^
4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^
```

2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5
*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c +
20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^
6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^
4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3
- 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^
2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4
x^2(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^
8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16
*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 -
((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*
b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^
4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2) + (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16
*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4
+ 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2
c^8)(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
c - b^2)^3)^(1/2))/(32(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x
^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^
5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i + atan(((((16*(a*b^8 + 4*a^5*c^4 -
8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 -
256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6
- 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3

```
*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*
b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3)
)/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3
*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^
(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*
c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3
*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*
a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 -
256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^
7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8
*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(
-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*
a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2
*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*(a*b^8 + 4*a
^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*
b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a
*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3
 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c
^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c
+ a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^
(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*2i + x^2/(2*c)
```

sympy [A] time = 4.42, size = 134, normalized size = 0.70

$$\operatorname{RootSum}\left(t^4\left(4096a^2c^5 - 2048ab^2c^4 + 256b^4c^3\right) + t^2\left(192a^2bc^2 - 112ab^3c + 16b^5\right) + a^3, \left(t \mapsto t\log\left(x^2 + \frac{256}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2 +
(256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2*c
- 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)
```

3.312 $\int \frac{x^7}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(8*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c}$$
$$= \frac{\log\left(a+bx^4+cx^8\right)}{8c} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^4+cx^8\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^4 + c*x^8])/(8*c)
```

fricas [A] time = 1.07, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c-4\,ac^2\right)}, \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 17.12, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\sqrt{-b^2+4\,ac}\,c} + \frac{\log\left(cx^8+bx^4+a\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
8*log(c*x^8 + b*x^4 + a)/c
```

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;c} + \frac{\ln\left(c\,x^8+b\,x^4+a\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.61, size = 2654, normalized size = 42.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4 + c*x^8),x)
```

```
[Out] (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*at
an((8*x^4*(((a*c - b^2)*(((((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*c^
4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32
*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2
*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(1
6*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^
2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2
- 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((6
4*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (4*b
^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*
c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c
^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2
- 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b
^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)*(4*a*c - b^2
)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c - b^2)^2) + b^3 -
((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))
/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*
c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4
*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*
a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b
^2)^(1/2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b
^2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(448*b^3*c^3 -
 (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)
^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (25
6*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64
*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2))
))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/b^4 + ((4*a*c - b^2)^(
3/2)*(b^3 - 3*a*b*c)*(a*b^2 + (((b*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*
a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3
*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(
4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)
```

```
*(4*a*c - b^2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) + (a*b^6)/(4*(
4*a*c - b^2)^2) - ((16*a*c - 4*b^2)*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (
512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2
*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*
b^2*c))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(768*a*b^2*c^
3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c -
b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((768*a*b^2*c^3 - (512*a*b
^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*
c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c
- b^2)^(1/2))))/(a^3*b^4*c^2) + ((a*c - b^2)*(4*a*c - b^2)^2*(((((16*a*c -
4*b^2)*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16
*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*
a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((
768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16
*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^
2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*((b*((b*(768*a*
b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*
a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*
(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4
*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c - b^2)^(1/2)) +
(b*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(
64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b
^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c))/(8*c*(4*a*c - b^2)^(1/2))
 + (a*b^5*c*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2))))
/(a^3*b^4*c^2)))/(4*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.18, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c
*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(-b
*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c + b**2
)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

3.313 $\int \frac{x^5}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=159

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/4*arctan(x^2*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c
^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1359, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 -
4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)+\frac{1}{4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subs}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.09, size = 171, normalized size = 1.08

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[
b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.92, size = 567, normalized size = 3.57

$$\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) +
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

giac [B] time = 18.74, size = 1036, normalized size = 6.52

$$\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}c}\,b^4-8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}c}\,ab^2c-2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}c}\,b^3c-2b^4c+16\sqrt{2}\sqrt{bc+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4
- 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^
3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(
b^2 - 4*a*c)*b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))
/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c
^2 - 4*a^2*c^3)*abs(c))
```

maple [A] time = 0.02, size = 216, normalized size = 1.36

$$\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)
*b+1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 2.81, size = 1220, normalized size = 7.67

$$\operatorname{atan}\left(\frac{}{8\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{512\,a^2\,c^3-256\,a\,b^2\,c^2+32\,b^4\,c}}+128\,b^5\,c\,{\left(\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{512\,a^2\,c^3-256\,a\,b^2\,c^2+32\,b^4\,c}\right)}^{3/2}+64\,a^2\,c^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + b^3*x
^2*1i - a*b*c*x^2*4i)/(8*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)
 + 128*b^5*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3
 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*((
(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32
*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(((b^6 - 64*a
^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 51
2*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 2048*a^2*b*c^3*(((b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 -
 256*a*b^2*c^2))^(3/2) - 48*a*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2)
)^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i - atan((x^2*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - b^3*x^2*1i + a*b*c
*x^2*4i)/(8*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) + 128*b^5*c
*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)
/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*(-(b^3 + (b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 5
12*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 48*a*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 2
56*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c
^2))^(3/2) + 2048*a^2*b*c^3*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2)
))*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*
c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i
```

sympy [A] time = 2.51, size = 76, normalized size = 0.48

$$\operatorname{RootSum}\left(t^{4}\left(4096a^{2}c^{3}-2048ab^{2}c^{2}+256b^{4}c\right)+t^{2}\left(-64abc+16b^{3}\right)+a,\left(t\mapsto t\log\left(512t^{3}ac^{2}-128t^{3}b^{2}c-4tb\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-64
*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**2*
c - 4*_t*b + x**2)))
```

3.314 $\int \frac{x^3}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

[Out] -1/2*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^4 + c*x^8),x]

[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

fricas [A] time = 0.92, size = 129, normalized size = 3.39

$$\left[\frac{\log\left(\frac{2\,c^{2}x^{8}+2\,bcx^{4}+b^{2}-2\,ac-\left(2\,cx^{4}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{8}+bx^{4}+a}\right)}{4\,\sqrt{b^{2}-4\,ac}}, -\frac{\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^{4}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\,\left(b^{2}-4\,ac\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*
a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 17.34, size = 36, normalized size = 0.95

$$\frac{\arctan\left(\frac{2\,cx^{4}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\sqrt{-b^{2}+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 37, normalized size = 0.97

$$\frac{\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.37, size = 260, normalized size = 6.84

$$-\frac{\operatorname{atan}\left(\frac{\left(4ac-b^2\right)^2\left(\frac{\left(\frac{4ac^4}{4ac-b^2}-\frac{4ab^2c^4}{\left(4ac-b^2\right)^2}\right)\left(b^3-3abc\right)}{8a^3c^2\sqrt{4ac-b^2}}-x^4\left(\frac{\left(\frac{2c^4}{\sqrt{4ac-b^2}}-\frac{6b^2c^4}{\left(4ac-b^2\right)^{3/2}}\right)\left(ac-b^2\right)}{8a^3c^2}-\frac{\left(b^3-3abc\right)\left(\frac{6bc^4}{4ac-b^2}-\frac{2b^3c^4}{\left(4ac-b^2\right)^2}\right)}{8a^3c^2\sqrt{4ac-b^2}}\right)+\frac{bc^2\left(ac-b^2\right)}{a^2\left(4ac-b^2\right)^{3/2}}\right)}{2c^4}\right)}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4 + c*x^8),x)
```

```
[Out] -atan(((4*a*c - b^2)^2*((((4*a*c^4)/(4*a*c - b^2) - (4*a*b^2*c^4)/(4*a*c -
b^2)^2)*(b^3 - 3*a*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)) - x^4*((((2*c^4)/(
4*a*c - b^2)^(1/2) - (6*b^2*c^4)/(4*a*c - b^2)^(3/2))*(a*c - b^2))/(8*a^3*c
^2) - ((b^3 - 3*a*b*c)*((6*b*c^4)/(4*a*c - b^2) - (2*b^3*c^4)/(4*a*c - b^2)
^2))/(8*a^3*c^2*(4*a*c - b^2)^(1/2))) + (b*c^2*(a*c - b^2))/(a^2*(4*a*c - b
^2)^(3/2))))/(2*c^4))/(2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.77, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4
```

3.315 $\int \frac{x}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=154

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/2*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2
)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x^2*2^(1/2)*c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1093, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{a+bx^4+cx^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}} - \frac{c\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.08, size = 133, normalized size = 0.86

$$\frac{\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
- Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 0.68, size = 619, normalized size = 4.02

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/s
qrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c
))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c
 - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(-(b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 + 1/
2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqr
t(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/
4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt
(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c)))
```

giac [B] time = 19.50, size = 1030, normalized size = 6.69

$$\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4 - 8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c - 2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c - 2b^4c + 16\sqrt{2}\sqrt{bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*
a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*a
bs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
- 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*abs(c))
```

maple [A] time = 0.01, size = 120, normalized size = 0.78

$$-\frac{\sqrt{2}\, c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c\, x^{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, c\, x^{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)-1/2*c/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^{8}+bx^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 2.27, size = 1105, normalized size = 7.18

$$\operatorname{atan}\left(\frac{b^4\,x^2\,1\mathrm{i}+}{128\,a^2\,b^5\left(-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{512\,a^3\,c^2-256\,a^2\,b^2\,c+32\,a\,b^4}\right)^{3/2}-64\,a^3\,c^2\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{512\,a^3\,c^2-256\,a^2\,b^2\,c+32\,a\,b^4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3
*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2
*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/
2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) + 2048*
a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b^3 + (b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512
*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^
2*6i)/(128*a^2*b^5*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*
c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*
c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(((b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4
+ 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3*c*(((b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c
^2 - 256*a^2*b^2*c))^(3/2) + 2048*a^4*b*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^
2*b^2*c))^(3/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
- b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 3.36, size = 88, normalized size = 0.57

$$\operatorname{RootSum}\left(t^4\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-64t^3a}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-64
*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*_t
**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

3.316 $\int \frac{1}{x(a+bx^4+cx^8)} \, dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/8*ln(c*x^8+b*x^4+a)/a+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a}+\frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\operatorname{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

fricas [A] time = 1.04, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^8+bx^4+a\right)+8\left(b^2-4\,ac\right)\log(x)}{8\left(ab^2-4\,a^2c\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 16.28, size = 68, normalized size = 0.99

$$-\frac{b \arctan\left(\frac{2 c x^{4}+b}{\sqrt{-b^{2}+4 a c}}\right)}{4 \sqrt{-b^{2}+4 a c}\, a}-\frac{\log\left(c x^{8}+b x^{4}+a\right)}{8 a}+\frac{\log\left(x^{4}\right)}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2 c x^{4}+b}{\sqrt{4 a c-b^{2}}}\right)}{4 \sqrt{4 a c-b^{2}}\, a}+\frac{\ln(x)}{a}-\frac{\ln\left(c x^{8}+b x^{4}+a\right)}{8 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/a*ln(x)-1/8*ln(c*x^8+b*x^4+a)/a-1/4/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4
+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.18, size = 1690, normalized size = 24.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] log(x)/a + (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*
c)) - (b*atan((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*
a*b^4*c)*((b^9*c^4)/(128*a^4*(4*a*c - b^2)^(5/2)) + (2*b^5*c^4*(16*a*c - 4*
b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(16*a*c - 4*b^2)
^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/
(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(16*a*c - 4*b^2)*
(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(25
6*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)) - (3*b^7*c^4*(16*a*c - 4*b
^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2))))/(b^4*c^8*(81*a
*c - 20*b^2)) + (128*a^5*x^4*(((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c)*(((576*b
^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 -
64*a^2*c)))*(16*a*c - 4*b^2)^4)/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(576*b^
3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 6
4*a^2*c))))/(4096*a^4*(4*a*c - b^2)^2) + (b^2*(1280*b^5*c^4 - 4608*a*b^3*c^
5)*(16*a*c - 4*b^2)^3)/(128*a^2*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)) - (3
*b^2*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(
```

```
16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)^2
*(4*a*c - b^2)) - (b^4*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2
048*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2)))/(32*a^5*c^4*(81*a*c - 20*b
^2)) + ((5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*b^4*c)*((b^5*(1280*b^5*
c^4 - 4608*a*b^3*c^5))/(32768*a^5*(4*a*c - b^2)^(5/2)) - (3*b^3*(1280*b^5*c
^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^2)/(1024*a^3*(16*a*b^2 - 64*a^2*c)^2*
(4*a*c - b^2)^(3/2)) + (b*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^
4)/(128*a*(16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(576*b^3*c^5 -
((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)
))*(16*a*c - 4*b^2)^3)/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) +
 (b^3*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*
(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(3/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(81*a*c - 20*b^2)))*(4
*a*c - b^2)^(5/2))/(b^4*c^4) + (4*(4*a*c - b^2)^(5/2)*(5*b^5 + 23*a^2*b*c^2
 - 24*a*b^3*c)*(((16*a*c - 4*b^2)^4*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c -
 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(256*b
^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(4096*a^4
*(4*a*c - b^2)^2) - (b^8*c^4*(16*a*c - 4*b^2))/(8*a^3*(16*a*b^2 - 64*a^2*c)
*(4*a*c - b^2)^2) + (2*b^6*c^4*(16*a*c - 4*b^2)^3)/(a*(16*a*b^2 - 64*a^2*c)
^3*(4*a*c - b^2)) - (3*b^2*(16*a*c - 4*b^2)^2*(256*b^4*c^4 - (128*a*b^4*c^4
*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(128*a^2*(16*a*b^2 - 64*a^2*c)^2
*(4*a*c - b^2))))/(b^4*c^8*(81*a*c - 20*b^2))))/(4*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 14.47, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+}}{8a\left(4ac-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(b*s
qrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

3.317 $\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*ar
ctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c
+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1359, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
```

```
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{S}$$
$$= -\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2}}}\right.}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^6c+\#1^2b}\&\right]}{4a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

fricas [B] time = 0.97, size = 1134, normalized size = 6.16

$$-\frac{\sqrt{\frac{1}{2}}ax^2\sqrt{-\frac{b^3-3abc+\left(a^3b^2-4a^4c\right)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{a^6b^2-4a^7c}}}{a^3b^2-4a^4c}}\log\left(-\left(b^2c^2-ac^3\right)x^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^5-5ab^3c+4a^2bc^2-\left(a^3b^4-6a^4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2
*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4
 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a
^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sq
rt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c +
 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
```

```
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
 - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^
2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x^2)
```

giac [B] time = 15.89, size = 2055, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 -
 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*
b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*a*b*c^3)*x^4 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2
- 4*a*c)*a*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 -
 4*a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a
^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) + 1/8*(2*a*b^4*c^2 - 8*a
^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 -
4*a*c)*a*b^2*c^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^
2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 - (sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5
*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
```

```
c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(a
))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^
2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2
- 4*a^3*c^3)*abs(a)*abs(c)) - 1/2/(a*x^2)
```

maple [A] time = 0.02, size = 240, normalized size = 1.30

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\, \sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\, \sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}\right.}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x^2)*b-1/4/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x^2)*b-1/2/a/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

mupad [B] time = 2.42, size = 5451, normalized size = 29.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2
*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 245
76*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x
^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a
^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
```

6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan((((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*

```
b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6
) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a
^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c
)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(
512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
 b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b
^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^
5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 6553
6*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5
*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^
3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^
2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1
/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4
*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 92
16*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3
072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 -
896*a^10*b^2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(2*a*x^2)
```

sympy [A] time = 15.34, size = 153, normalized size = 0.83

$$\operatorname{RootSum}\left(t^4\left(4096a^5c^2 - 2048a^4b^2c + 256a^3b^4\right) + t^2\left(192a^2bc^2 - 112ab^3c + 16b^5\right) + c^3, \left(t \mapsto t\log\left(x^2 + \frac{-51}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2 +
(-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*_t*
a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*
x**2)
```

3.318 $\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4-b*ln(x)/a^2+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/4*(-2*a*c+b^2)*arctanh
((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^4\right)}{4a}\\
&= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^4\right)}{4a}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^4\right)}{4a^2}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,d\right.}{8a^2}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{4a^2}\\
&= -\frac{1}{4ax^4}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 92, normalized size = 1.03

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a^2
)
```

fricas [A] time = 1.00, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x^4\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^3-4\,abc\right)x^4\log\left(cx^8+bx^4+a\right)+8\left(b^3-\right.}{8\left(a^2b^2-4\,a^3c\right)x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2
 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

giac [A] time = 14.50, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^8+bx^4+a\right)}{8\,a^2}-\frac{b\log\left(x^4\right)}{4\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^4-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*x
^4 - a)/(a^2*x^4)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a}+\frac{b^2\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\,a^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c\,x^8+b\,x^4+a\right)}{8a^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/x^4/a-1/a^2*b*ln(x)+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/2/a/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*c+1/4/a^2/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^4+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.79, size = 8817, normalized size = 99.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (atan((4*a^5*(4*a*c - b^2)^2*(5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a*
b^5*c)*(((4*b^3 - 16*a*b*c)*((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5
- (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2)
)/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2
))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(
4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(
4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) - (b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^
(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 16
*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 - 256
*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b
^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a
*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(
64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^
3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*
b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*
a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((
16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5
- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3
*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2
*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*
a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((
4*b^3 - 16*a*b*c)*((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4
*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*
a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c
 - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)
^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)
*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*
b*c)*((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 1
6*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2
)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(
64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5
- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3
*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2
*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a
*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + (((a
^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*a^2*b^
3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 +
((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c
^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)
)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2
))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*c - b^2)^5)/(128*a^9*(4*a*c
- b^2)^(5/2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4
- 8*a*b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)) - (128*a^10*x^4*(((5*b^6
- a^3*c^3 + 26*a^2*b^2*c^2 - 25*a*b^4*c)*(c^9/a^5 + ((4*b^3 - 16*a*b*c)*((2
0*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((
4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b
*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a
^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^
3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^
2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c
)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1
6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b
^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a
*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2
)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b
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*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c
- b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b
^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*
b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*
c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^
2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^
4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 34
56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*
c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
 - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64
*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a
*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(12
80*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
 - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1
280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c
- 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b
^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5
 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^
3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*
a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b
^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 -
((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1
6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b
^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c -
b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((((2*a*c
 - b^2)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b
^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16
*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c
)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2
)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 -
16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(
4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b
^3 - 16*a*b*c)*(2*a*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024
*a^11*(4*a*c - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(
4*a*c - b^2)^(1/2)) - ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^4*(1280*a^5*b^5*c^4
 - 4608*a^6*b^3*c^5))/(8192*a^13*(4*a*c - b^2)^2*(64*a^3*c - 16*a^2*b^2))))
/(32*a^5*c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)) + ((5*b^7 - 23*a^3*b*c^3 + 66
*a^2*b^3*c^2 - 35*a*b^5*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*(((2*a*c -
b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(128
0*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
- b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(12
80*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c -
 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c -
b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^
3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a
*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024*a^11*(4*a*c - b^2)
^(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 1
6*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 34
56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*
c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3
*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c
- 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b
*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a
^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^
3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3
*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b
*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*
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b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4
608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b
^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/
2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((20*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*
((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 +
 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^
2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*
a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c -
 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
 b^2)^(1/2)) + ((((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 -
3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^
3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2
)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b
^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a
*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 -
4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64
*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4
*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608
*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*
c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 460
8*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(
64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3
 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c
)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/
(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(
8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((
2*a*c - b^2)^5*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(32768*a^15*(4*a*c -
b^2)^(5/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 20*b^4 + 80*a*b^2*
c)))*(4*a*c - b^2)^(5/2))/(16*a^4*c^8 + b^8*c^4 - 8*a*b^6*c^5 + 24*a^2*b^4*
c^6 - 32*a^3*b^2*c^7) + (4*a^5*(4*a*c - b^2)^(5/2)*(5*b^6 - a^3*c^3 + 26*a^
2*b^2*c^2 - 25*a*b^4*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((((256
*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(6
4*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4
*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3
 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))
/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 -
256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^
2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 1
6*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(
2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4
*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*
(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*
(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (b*c^8)/a^5 + ((4*b^3 - 16*a*b*
c)*((a^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*
a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/
a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a
*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^
2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(6
4*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256
*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(6
4*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4
*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 1
6*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2
*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 -
(128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c -
 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c -
16*a^2*b^2)) - (((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*
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((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5
*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c
- 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*
(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2)) - ((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*
c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a
*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c
- b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*
(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (b^4
*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^(3/2)*(64*a^3
*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4
*b^3 - 16*a*b*c)*(2*a*c - b^2)^4)/(32*a^7*(4*a*c - b^2)^2*(64*a^3*c - 16*a^
2*b^2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4 - 8*a*
b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)))*(2*a*c - b^2))/(4*a^2*(4*a*c -
 b^2)^(1/2)) - (b*log(x))/a^2 - (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))
/(2*(64*a^3*c - 16*a^2*b^2)) - 1/(4*a*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.319 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=381

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right.}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] 1/3*x^3/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-
2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4
)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*a
rctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a
*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctanh(2^
(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^
(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 0.65, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1367, 1510, 298, 205, 208}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right.}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx}{3c}$$
$$= \frac{x^3}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$
$$= \frac{x^3}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} -$$
$$= \frac{x^3}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2}{\sqrt{b^2}}\right.}{2\,2^3}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.18

$$\frac{4x^3 - 3\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] (4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

fricas [B] time = 3.73, size = 6296, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^3 + 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7
```

- 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(-1/2*((b^6*c^7 - 10*a*b^4*c^8 + 32*a^
2*b^2*c^9 - 32*a^3*c^10)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^
4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^
2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4)*x + sqrt(1/2)*(b^9 - 9*a*b^7*c + 26*a^2*b
^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^
2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6
*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15
+ 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((2*(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*
b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c + 53*a^2*b^7*c^2 - 10
3*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 - (b^8*c^7 - 13*a*b^6*c^8 + 6
0*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^4*c^11)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c
^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^
4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c
+ 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt(
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^
3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^2*b^6 - 5*a^3*b^4*c + 6
*a^4*b^2*c^2 - a^5*c^3)) - 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*
a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*
b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^1
7)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(1/2*(sqrt(1/2)*(b^9 - 9*
a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (b^6*c^7 - 10*a*b
^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^
14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-
(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 +
16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2
*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt((2*(
a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12
*a*b^9*c + 53*a^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5
+ (b^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^4*c^1
1)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16
- 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (
b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14
- 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 +
16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)) - ((b^6*c
^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*x*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)) + (b^9 -
9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4)*x)*sqrt(sqrt(1/2
)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*
c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15
+ 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))))/
(a^2*b^6 - 5*a^3*b^4*c + 6*a^4*b^2*c^2 - a^5*c^3)) - 3*c*sqrt(sqrt(1/2)*sqr
t(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8
+ 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*
a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/

```
2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*
a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 -
17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*
a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2
*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b
^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10
*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c
^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/
(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^
4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2
 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c
^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 1
02*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152
*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 36
4*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqr
t(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8
*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3
*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*
c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9
)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^1
5 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))
- (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x) - 3*c*sqrt(sqrt(1/2)
*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 +
 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*lo
g(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 +
553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^
7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 -
320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48
*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a
^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17
)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*
b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 1
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))
/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2
*c^2 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c
 + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 +
 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9
- 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b
```

```
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 -
8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^
9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 19.53Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 63, normalized size = 0.17

$$\frac{x^3}{3c}-\frac{\left(\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^6 b+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^2 a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)+x\right)}{4c\left(2\,\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/3/c*x^3-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_
Z^8*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.49, size = 12709, normalized size = 33.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3
*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96
*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 +
2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
```

```
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(
a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((
8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3
 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*
b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 -
 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((8192*a^6*
b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(
-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^
3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((8192*a^6*b*c^6 - 256*
a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^1
0)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^
2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*
b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b
^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
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6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^
2*c^10)))^(1/4)*2i + atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b
^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8
 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b
^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*
c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a
^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(
512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c
^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b
^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*
c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a
^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(
512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c
^10)))^(1/4)*1i - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 -
8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*
b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^
3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3
/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1
/4)*1i)/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*
b^3*c^5)/c^3 - (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^
4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 +
 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5
 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11
 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*
x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11
 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((
8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3
 + (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*
b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
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(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 -
 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7
*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + 2*atan(((((8192*a^6*b*c^6 - 256*a
^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-
(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10))
)^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c
^7)*4i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a
^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*
a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (((8192*a^6*b*c^6 - 256*a^3*b^7*c
^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*
(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/
c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^
2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9
 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2
))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 256
0*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^
6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(
b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a
^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*
(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2
31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*
b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 25
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6*a^3*b^2*c^10)))^(1/4)*1i + (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4
*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8
 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b
^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^
2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^
11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^
3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c
 + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3
*b^2*c^10)))^(1/4)*1i + (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)
 + 2*atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^
5*b^3*c^5)/c^3 - (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
+ 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^
4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 +
 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5
 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i
 - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)
 - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^
5)/c^3 + (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2
*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(
1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*
b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*
a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 +
 a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8
*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*
(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 +
b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((81
92*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 -
 (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9
- 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*
```

```
c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 1
6*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5
 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7
- 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (((8192*a^
6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(
-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^
5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4
*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
+ 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*
a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*
a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (2*(a^8*c - a^7
*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + x^3/(3*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.320 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(
2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*
c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))
*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.57, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$
$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} +$$
$$= \frac{x}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2-}{\sqrt{b^2}}\right.}{2\sqrt[4]{2}}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.19

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

fricas [B] time = 1.80, size = 5082, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
```

2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(2*sqrt(1/2)*((b^10*c^
5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128
*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^11 - 1
3*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c
^5)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*
c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^
6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^2*b^4 - 3*a^3
*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^
3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b
*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^4*b^
4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/
4*(2*sqrt(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8
+ 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*
a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c
^4 - 32*a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4
*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^
2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^
4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c
^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr

```
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))/(a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3
*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*
a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*
c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^
5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*x)/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.73Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 59, normalized size = 0.16

$$\frac{x}{c}+\frac{\left(-\operatorname{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^4 b-a\right)\ln\left(-\operatorname{RootOf}\left(\_Z^8c+b\_Z^4+a\right)+x\right)}{4c\left(2\operatorname{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^7 c+\operatorname{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/c*x+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\int\frac{bx^4+a}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 3.97, size = 10382, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(
a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5
))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a
^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a
^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
```

12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)
(-(b^9 + b^4(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16
*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c
^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4
*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*
b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4
*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^
9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*
a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
c^2(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(

```
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 +
 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b
^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c +
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((16*(a^3*b^6
 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*
b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3
 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4
)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4
*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*
b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
```

```
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 1
6*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 2
56*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/
4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((16*(a^3
*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)
/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*
a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6
*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^
(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 -
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4
*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*
c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c +
13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
```

```
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6
 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + x/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.321 $\int \frac{x^6}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=325

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh}{2\,2^{3/4}c^{3/4}\sqrt{b^2}}$$

```
[Out] -1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^
2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(1
/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/
c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2
)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+
b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1374, 298, 205, 208}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh}{2\,2^{3/4}c^{3/4}\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b + Sqrt[b^2 -
 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*Arc
Tanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4
)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*
c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
```

```
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}-$$

$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

fricas [B] time = 1.32, size = 3912, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^5*c^3
 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)) + (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 1
6*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*
a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((2*(a*b^2 - a^2*c)*x^2 - sqrt(1/2)*(b^5 -
5*a*b^3*c + 4*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3
*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2
*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)
))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)) +
 sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*(sqrt(1/2
)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
```

```
16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt((2*(
a*b^2 - a^2*c)*x^2 - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a
*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b
^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)) + ((b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*
b*c^5)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x)*sqrt(sqrt(1/2)*sq
rt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))))/(a*b^2 - a^2*c)) + 1/4*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c
+ 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5
- 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 -
 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2
 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7
 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*lo
g(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^
3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^
2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3
*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2
*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^
3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b
^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c
^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c
)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*s
qrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a
*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*s
qrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c -
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.13

$$\frac{\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^6 \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 3.51, size = 8033, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 +
256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b
^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x
*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))
)^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5
*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 25
6*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^
5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^
2*c^6)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
```

```
40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4
096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4
*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5
)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4
)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384
*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^
2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 -
 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/
2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))^(1/4)*2i + atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^
(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(
-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 1
6384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*1i - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*
c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*
(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*
b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a
^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
+ 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) + ((-(b^7 - b^2*(-(4*a*c - b^2)
^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 2
56*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4
- x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 1
6*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 20
```

```
48*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b
^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^
5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*
c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 - b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*2i - 2*atan((((-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^
2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c
^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^
6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*
b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3
 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - ((-(b^7 + b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2
048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32
768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c
 - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40
*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/(((-
(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*
b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b
^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^
3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))
^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4
*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(
1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^
5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*
1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(1/4)*1i + 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c
^6)))^(1/4) - 2*atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*
(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a
*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4
*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a
^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
```

```
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4) - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6
)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 +
96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4
 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*
(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*
b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a
^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-
(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*
b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + ((-(b^7 - b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^
4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4
*b^2*c^4 + x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^
8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5
*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a
^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + 2*a^4*b
*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 -
16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)
```

sympy [A] time = 61.58, size = 230, normalized size = 0.71

$$\operatorname{RootSum}\left(t^8\left(16777216a^4c^7-16777216a^3b^2c^6+6291456a^2b^4c^5-1048576ab^6c^4+65536b^8c^3\right)+t^4\left(-12288a^3b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**7 - 16777216*a**3*b**2*c**6 + 6291456*a**2*
b**4*c**5 - 1048576*a*b**6*c**4 + 65536*b**8*c**3) + _t**4*(-12288*a**3*b*c
**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + a**3, Lambda(_t, _
t*log(x + (2097152*_t**7*a**4*c**7 - 2621440*_t**7*a**3*b**2*c**6 + 1179648
*_t**7*a**2*b**4*c**5 - 229376*_t**7*a*b**6*c**4 + 16384*_t**7*b**8*c**3 -
1280*_t**3*a**3*b*c**3 + 1600*_t**3*a**2*b**3*c**2 - 576*_t**3*a*b**5*c + 6
4*_t**3*b**7)/(a**3*c - a**2*b**2))))
```

3.322 $\int \frac{x^4}{a+bx^4+cx^8} dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\right.}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4a}}$$

```
[Out] 1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2
)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(1/
4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c
^(1/4)/(-4*a*c+b^2)^(1/2)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b
^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\right.}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 -
4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
```

```
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}}+\frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}}{2\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\sqrt[4]{-}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

fricas [B] time = 1.13, size = 2479, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2
*c + 16*a^2*c^2)*x - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt
(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + sqrt(s
qrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))
)*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
```

```
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b - (b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*x + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b -
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/4*sqrt(sqr
t(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*
log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sq
rt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(sqr
t(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*
log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.13

$$\frac{\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 3.63, size = 8169, normalized size = 25.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^
2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5
+ (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^
4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b
*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((-(b^5 + (-(4*a*
c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 1
6*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c
 - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 40
96*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c
^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) +
x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(1/4)*1i)/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*
c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^
3*b^4*c^5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8
192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^
3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*
c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c
^6) - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c
^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^
4 - 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^
6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((-(b^5 + (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
- 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7
```

- 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*
a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b
^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4
+ 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^
2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a
^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*
a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5
 - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^
3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*
c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)*1i + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^
2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256
*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64
*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c
^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 +
 (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - atan(((((
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26
2144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) +
 x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 1
6*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c
^4)))^(1/4) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((((-(b^5 - (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b
^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c^6 + 102
4*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c

^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + x*(8*a^3*c^
4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*
b^2*c^4)))^(1/4)*1i)/(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^
5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*
b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))
^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^
5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + ((((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26214
4*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*
(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c
- b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*
a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a
^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(1/4) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*
a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((((-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b
^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a
^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^
6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*
a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i
+ x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4) - ((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*
a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^
4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a
^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c -
b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - x*(8*a^3*c^4 - 4*
a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))^(1/4))/(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*
c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 19660
8*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^
5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8
*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)
*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16
*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^
5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (((

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(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26
2144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1
i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c
^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))^(1/4)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b
^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 - (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)
```

sympy [A] time = 2.92, size = 126, normalized size = 0.39

$$\mathrm{RootSum}\left(t^8\left(16777216a^4c^5-16777216a^3b^2c^4+6291456a^2b^4c^3-1048576ab^6c^2+65536b^8c\right)+t^4\left(4096a^2bc\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2*
b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3 +
16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

3.323 $\int \frac{x^2}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\,\mathrm{ta}}{2^{3/4}\sqrt{b^2}}$$

```
[Out] -1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4
)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctanh(2^(1
/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b-(-4*a*c+b^2)^(1/2)
)^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c
+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4
)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1
/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 0.29, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1375, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\,\mathrm{ta}}{2^{3/4}\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (c^(1/4)*ArcTan[(
2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a
*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
```

```
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\ldots}}{\sqrt{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tanh^{-}\ldots}{2^{3/4}\sqrt{b^2-4a\ldots}}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log(x-\#1)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

fricas [B] time = 1.08, size = 2746, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)))*arctan(-(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x*sqrt(sqrt(1/2)*sqrt(-(
b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) + sqrt(1/2)*(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2)))*sqrt((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c + (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2)))/c)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)) - sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*arctan(-((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x/sqrt(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - sqrt(1/2)*(a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)*sqrt((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c - (a*b^6 -
 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2
```

```
*b^2*c + 16*a^3*c^2)))/c)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*
b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a
^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b +
(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2
)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-
1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^
3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)) + c*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2
*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*
sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/
2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*
b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)) + c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^8+b*x^4+a),x)
[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^8 + bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 2.34, size = 6067, normalized size = 19.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4 + c*x^8),x)
```

```
[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^
(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*
a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10
240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*
x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)
- ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*
(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^
2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4))/(((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a
*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
- 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*
c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*x)*(-(b^5 -
(-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*
c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^
5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5
 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4
 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 2*a*c^5))*
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 +
 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - a
tan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a
*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/
4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240
*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(
```

b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a
*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*
c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 -
16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(((-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4
 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4
+ 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4
*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 327
68*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)
^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^
6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)
^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^
6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b
c^6 - x(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^
(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c
^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + 2*a*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*2i - atan((((-(b^5 + (-(4*a*c
 - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*
a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c
 - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*
a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024
*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*
a^3*b*c^6 + 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c +
96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5 + (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^
4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) + 256*a*b^5*c^4 + 4096*a^3*b*c^6
 - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240
*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a
^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5
 - 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5)
- 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
^3)))^(1/4) + 2*a*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*

```
a^4*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2
*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^
2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c
^5 - 32768*a^3*b^2*c^6)*1i + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a^2*b^3*
c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 3
2768*a^3*b^2*c^6)*1i - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5)*1
i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2
*c^3)))^(1/4)*1i - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*
b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*
b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 3276
8*a^3*b^2*c^6)*1i + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a^2*b^3*c^5)*1i +
 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^
3)))^(1/4)*1i + 2*a*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 25
6*a^4*b^2*c^3)))^(1/4)
```

sympy [A] time = 4.10, size = 172, normalized size = 0.55

$$\operatorname{RootSum}\left(t^8\left(16777216a^5c^4-16777216a^4b^2c^3+6291456a^3b^4c^2-1048576a^2b^6c+65536ab^8\right)+t^4\left(4096a^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3*
b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**4*b
*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*_t**
7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))
```

3.324 $\int \frac{1}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=315

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-$$

```
[Out] 1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)
/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(3/4)*arctanh(2^(1/
4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))
^(3/4)/(-4*a*c+b^2)^(1/2)-1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
-1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/
4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.30, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1347, 212, 208, 205}

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
```

```
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{c\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{\cdot}}}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2}}}$$

$$= \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\,\mathrm{ta}}{\sqrt[4]{2}\sqrt{b^2-}}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

fricas [B] time = 1.31, size = 3929, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/4*(2*sqrt(1
/2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c
^4)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*x)*
sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 -
 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3
+ 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*sqrt((2
*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c + 14*a^2*b^2*c^2 - 8*a^
3*c^3 - (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*a^6*b*c^3)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)))*sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
```

```
 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)) + sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/4*(2*sqrt(1/2)*((a^3*b^
8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*x*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*x)*sqrt(sqrt(1/
2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (a^3*b^8 - 14*a^4
*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2))*sqrt((2*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c +
14*a^2*b^2*c^2 - 8*a^3*c^3 + (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*
a^6*b*c^3)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 -
 a*c^3)))/(b^2*c^2 - a*c^3)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
+ (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqr
t(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*
b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2
*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(
1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
```

```
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^8 + bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 40, normalized size = 0.13

$$\frac{\ln\left(-\operatorname{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right) + x\right)}{8\operatorname{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^7 c + 4\operatorname{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^8+b*x^4+a),x)
[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^8 + bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 3.42, size = 10337, normalized size = 32.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4 + c*x^8),x)
[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((
-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4
*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144
*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11
264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^
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(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -

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48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*
c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^
2*c^3)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8
+ 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(6
4*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*
c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*
b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 -
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
 a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
 + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 - b^2*(-(4*a*c - b^2
)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^
2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196
608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40
960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40
*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 2
56*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*
b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 +
 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(
-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4
*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^7 + b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)
^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*
a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5
 + 196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b
*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b
*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3
/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 25
6*a^6*b^2*c^3)))^(1/4) - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/
4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 -
262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7
*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96
*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*
```

c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - 2*atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6* c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)

sympy [A] time = 19.79, size = 177, normalized size = 0.56

$$\mathrm{RootSum}\left(t^8\left(16777216a^7c^4-16777216a^6b^2c^3+6291456a^5b^4c^2-1048576a^4b^6c+65536a^3b^8\right)+t^4\left(-12288\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**8*(16777216*a**7*c**4 - 16777216*a**6*b**2*c**3 + 6291456*a**5*b**4*c**2 - 1048576*a**4*b**6*c + 65536*a**3*b**8) + _t**4*(-12288*a**3*b*c**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + c**3, Lambda(_t, _t*log(x + (16384*_t**5*a**5*b*c**2 - 8192*_t**5*a**4*b**3*c + 1024*_t**5*a**3*b**5 + 8*_t*a**2*c**2 - 16*_t*a*b**2*c + 4*_t*b**4)/(a*c**2 - b**2*c))))

3.325 $\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right.}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-\ldots}}$$

```
[Out] -1/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(1-b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4
)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c+b^2
)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1/4)*arctan(2^(1/4)
*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/
a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(
-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^
2)^(1/2))^(1/4)
```

Rubi [A] time = 0.41, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right.}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^
(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(
1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{x^2\left(-b-cx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}$$

$$= -\frac{1}{ax}+\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\,a}-\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{-b}}}{2\sqrt{2}\,a}$$

$$= -\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}}{\sqrt[4]{-b+\sqrt{b^2}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.04, size = 71, normalized size = 0.20

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

fricas [B] time = 2.42, size = 5125, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*((a^5*b^5 - 8*a^6*b
^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
```

^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x - sqrt(1/2)*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a^3*b^3*c^3 + 12*a^4*b*c^4 + (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*(sqrt(1/2)*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt((2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a^3*b^3*c^3 + 12*a^4*b*c^4 - (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)) + ((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -

```
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a
^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98
*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*s
qrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) - a*x*sqrt(sqrt(1/2)*
sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 13
8*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c +
 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))
)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^
2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2
- 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8
*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^
7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + 4)/(a*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.78Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 63, normalized size = 0.17

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^6c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^2b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)+x\right)}{4a\left(2\,\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^3b\right)}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/a/x-1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.81, size = 10509, normalized size = 28.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 -
256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b
^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2
816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c
^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(
-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
 a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*
c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816
*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*
```

x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2
c^3)))^(3/4)(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 25
6*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768
*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 8
1920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*
c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6
*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
- 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13
*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 28
16*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x
)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a
^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8
+ 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15
*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 143
36*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5

)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^

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8*b^2*c^3)))^(1/4) + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 -
x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a
^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 -
 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b
^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a
^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*
b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c -
 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b
^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c
^3)))^(1/4)*2i + 2*atan((((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^
8 - x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c
^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256
*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7
)*1i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^
4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 104
96*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4))/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 -
 x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4
- 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^
11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1
i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 1
6*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
```

```
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^
4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 104
96*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - 1/(a*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.326 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=365

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-\cdots\right.}$$

```
[Out] -1/3/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c
+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctan(2^(
1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3
/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(
-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*
c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.40, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(3*a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (
c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx &= -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx}{3a}\\
&= -\frac{1}{3ax^3}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}
\\
&= -\frac{1}{3ax^3}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{-b-\sqrt{b^2-}}}}{2a\sqrt{-b-\sqrt{b^2-4}}}\\
&= -\frac{1}{3ax^3}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}}{\sqrt[4]{-b+\sqrt{b^2}}}\right.}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4a}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

fricas [B] time = 2.70, size = 6324, normalized size = 17.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a
^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
```

```
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8
*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 - 17*a^8*b^9
*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5
)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4
*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^
2 - 64*a^17*c^3)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^
3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7)*x)*sq
rt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48
*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) - (b^1
4 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 45
7*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 1
13*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4
 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^
15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9
*c^2))*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*x^2 + sqrt
(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c^2 - 147*a^3*b^6*c^3 + 156*a^4*b^4*
c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 + (a^7*b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2
 - 160*a^10*b^3*c^3 + 128*a^11*b*c^4)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c
 + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 +
6*a^2*b^2*c^6 - a^3*c^7)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b
^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10
*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c
^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/
(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^5 - 5*a*b^4*c^6 + 6*a^2*b^2*c
^7 - a^3*c^8)) - 12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^
3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(
a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 -
 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320
*a^12*b*c^5)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*
a^16*b^2*c^2 - 64*a^17*c^3)) - (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328
*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7
*c^7)*x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^1
4*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b
^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
+ (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3
 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (a^7*
b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4
 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c +
 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 1
```

4*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*x^2 + sqrt(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c^2 - 147*a^3*b^6*c^3 + 156*a^4*b^4*c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 - (a^7*b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2 - 160*a^10*b^3*c^3 + 128*a^11*b*c^4)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)))/(b^6*c^5 - 5*a*b^4*c^6 + 6*a^2*b^2*c^7 - a^3*c^8)) - 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x - 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3

```
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x
- 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a
^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c
^2)))) - 4)/(a*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 21.84Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 62, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^4c-b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)+x\right)}{4a\left(2\,\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^3b\right)}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/3/a/x^3+1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.57, size = 16497, normalized size = 45.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan(-((((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4
```

- 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)) /(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^

4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - atan(-(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6

- 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c
 - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*
a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b
^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3
)))^(1/4)*1i)/(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*
a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b
^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (
-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256
*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^
6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8
 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)
 - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*
a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 +
 a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11
*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920
*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 -
 122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c
 - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*
a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^1
3*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)
)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 -
 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*
a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c
^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c
^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 -
b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5
*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*
a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(1/4)*2i - atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*

a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c

^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^
2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 1
6*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*
c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*
a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*
a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^
4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^1
1 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3
*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c
 + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10
*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*
(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 + b^6*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 2
80*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(1/4)*2i + 2*atan(-(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
 + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a
^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7
) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^
14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(
-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)
))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*
a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^
5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4
*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8
*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*
b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*
a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)

```
^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*
c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752
*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5
*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(1/4))/(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9
*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(
b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*
c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a
^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^
8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4
)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^
10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
+ 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 - b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8
+ 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*
((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13
*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 -
16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8
+ 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^1
6*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*
a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5
)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a
^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7
 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))
^(1/4)*1i))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2
*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(
1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4
```

```
*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - 1/(3*a*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.327 $\int \frac{x^m}{1+x^4+x^8} \, dx$

Optimal. Leaf size=127

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[Out] -2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(1+I*3^(1/2)))/(1+
m)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*
x^4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 - I*Sqr
t[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1
 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1+x^4+x^8}\,dx &= -\frac{i\int \frac{x^m}{\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}+\frac{i\int \frac{x^m}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 1.13, size = 488, normalized size = 3.84

$$x^m\left(-\frac{\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{\#1^2m^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+3\#1^2m\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+2\#1^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+\#1^2}{2\#1^3-\#1}\right.}{m^2+3m+2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (x^m*(((-I)*(Hypergeometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1/3) - x)]/
(x/(-(-1)^(1/3) + x))^m + Hypergeometric2F1[-m, -m, 1 - m, (-1)^(2/3)/((-1)
^(2/3) - x)]/(x/(-(-1)^(2/3) + x))^m - Hypergeometric2F1[-m, -m, 1 - m, (-1
)^(1/3)/((-1)^(1/3) + x)]/(x/((-1)^(1/3) + x))^m - Hypergeometric2F1[-m, -m
, 1 - m, (-1)^(2/3)/((-1)^(2/3) + x)]/(x/((-1)^(2/3) + x))^m))/Sqrt[3] + Ro
otSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/
((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] - RootSum[1 - #1^2 + #1^4 & , (m*x^2 +
m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
 - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#
1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*#1^3) & ]/(
2 + 3*m + m^2)))/(4*m)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{x^8+x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+x^4+1),x)
```

```
[Out] int(x^m/(x^8+x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8 + x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^4 + x^8 + 1),x)
```

```
[Out] int(x^m/(x^4 + x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)\left(x^4 - x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+x**4+1),x)
```

```
[Out] Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

3.328 $\int \frac{x^{11}}{1+x^4+x^8} dx$

Optimal. Leaf size=44

$$\frac{x^4}{4} - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8+x^4+1\right)$$

```
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4} - \frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{1+x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\ &= \frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + x^4 + x^8),x]
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

fricas [A] time = 0.80, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

giac [A] time = 0.34, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

maple [A] time = 0.00, size = 36, normalized size = 0.82

$$\frac{x^4}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+x^4+1),x)
```

```
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 2.42, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

mupad [B] time = 0.05, size = 37, normalized size = 0.84

$$\frac{x^4}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4 + x^8 + 1),x)
```

```
[Out] x^4/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12 - log(x^4 + x^8 +
1)/8
```

sympy [A] time = 0.14, size = 42, normalized size = 0.95

$$\frac{x^4}{4} - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

3.329 $\int \frac{x^9}{1+x^4+x^8} \, dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/2*x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1)
*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1359, 1122, 1161, 618, 204}

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3
]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

```
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\
&= \frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 98, normalized size = 1.81

$$\frac{x^2}{2}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}}-\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*Sq
rt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I)*
Sqrt[3]])
```

fricas [A] time = 1.05, size = 40, normalized size = 0.74

$$\frac{1}{2}x^2-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x^2\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^6+2x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(x^6 + 2*x^2))
```

giac [A] time = 0.32, size = 42, normalized size = 0.78

$$\frac{1}{2}x^2-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

maple [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{x^2}{2} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+x^4+1),x)
```

```
[Out] 1/2*x^2-1/6*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2
*x^2-1)*3^(1/2))
```

maxima [A] time = 2.39, size = 42, normalized size = 0.78

$$\frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\ \arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right) - \frac{1}{6}\,\sqrt{3}\ \arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

mupad [B] time = 0.04, size = 43, normalized size = 0.80

$$\frac{x^2}{2} - \frac{\sqrt{3}\ \left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^4 + x^8 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(
1/2)*x^2)/3)))/12
```

sympy [A] time = 0.14, size = 51, normalized size = 0.94

$$\frac{x^2}{2} + \frac{\sqrt{3}\ \left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(
3)*x**2/3))/12
```

3.330 $\int \frac{x^7}{1+x^4+x^8} dx$

Optimal. Leaf size=37

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx, x, x^4\right)$$
$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{8}\log\left(1+x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^4 + x^8),x]
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

fricas [A] time = 0.92, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) + \frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

giac [A] time = 0.39, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) + \frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)}{12} + \frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+x^4+1),x)
[Out] 1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4+1)*3^(1/2))
```

maxima [A] time = 2.43, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) + \frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12
```

sympy [A] time = 0.13, size = 37, normalized size = 1.00

$$\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

3.331 $\int \frac{x^5}{1+x^4+x^8} dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[Out] 1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(
1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1+x^4+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x}\right.\\
&= \frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\,\text{Subst}\left(\right.\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [C] time = 0.12, size = 94, normalized size = 1.25

$$\frac{\sqrt{1-i\sqrt{3}}\,\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)+\sqrt{1+i\sqrt{3}}\,\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(1 + x^4 + x^8),x]
```

```
[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[1
 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

fricas [A] time = 0.87, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

giac [A] time = 0.39, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^4-x^2+1\right)}{8}-\frac{\ln\left(x^4+x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))+1/8*ln(x^4-x^
2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

maxima [A] time = 2.49, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

mupad [B] time = 0.09, size = 51, normalized size = 0.68

$$\mathrm{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + x^8 + 1),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

sympy [A] time = 0.21, size = 76, normalized size = 1.01

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

3.332 $\int \frac{x^3}{1+x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = $\{1352, 618, 204\}$

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+x^4+x^8} \, dx &= \frac{1}{4} \text{Subst}\left(\int \frac{1}{1+x+x^2} \, dx, x, x^4\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+2x^4\right)\right) \\
&= \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4 + x^8),x]
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

fricas [A] time = 0.78, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

giac [A] time = 0.36, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+x^4+1),x)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4+1)*3^(1/2))
```

maxima [A] time = 2.47, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

mupad [B] time = 1.30, size = 17, normalized size = 0.74

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + x^8 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6
```

sympy [A] time = 0.12, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

3.333 $\int \frac{x}{1+x^4+x^8} dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/8*ln(x^4-x^2+1)+1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^
(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\operatorname{Subs}\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 1.05

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)-\sqrt{1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 + x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sqr
t[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

fricas [A] time = 0.91, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

giac [A] time = 0.31, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^{2}-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^{2}+1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^{4}-x^{2}+1\right)}{8}+\frac{\ln\left(x^{4}+x^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))-1/8*ln(x^4-x^2
+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

maxima [A] time = 2.59, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{2}+1\right)\right)+\frac{1}{12}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{2}-1\right)\right)+\frac{1}{8}\log\left(x^{4}+x^{2}+1\right)-\frac{1}{8}\log\left(x^{4}-x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

mupad [B] time = 1.28, size = 51, normalized size = 0.68

$$\mathrm{atan}\left(\frac{\sqrt{3}\,x^{2}}{2}-\frac{x^{2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x^{2}}{2}+\frac{x^{2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + x^8 + 1),x)
```

```
[Out] atan((3^(1/2)*x^2)/2 - (x^2*1i)/2)*(3^(1/2)/12 + 1i/4) + atan((3^(1/2)*x^2)
/2 + (x^2*1i)/2)*(3^(1/2)/12 - 1i/4)
```

sympy [A] time = 0.20, size = 76, normalized size = 1.01

$$-\frac{\log\left(x^{4}-x^{2}+1\right)}{8}+\frac{\log\left(x^{4}+x^{2}+1\right)}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^{2}}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^{2}}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

3.334 $\int \frac{1}{x\left(1+x^4+x^8\right)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

```
[Out] ln(x)-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+x^4+x^8)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(1+x+x^2)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.08, size = 138, normalized size = 3.54

$$\frac{1}{24}\left(-\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)-\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)-3\log\left(x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 + 24*Log[x] - Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2] - Sqr
t[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2] - 3*
Log[1 + x + x^2])/24
```

fricas [A] time = 0.79, size = 32, normalized size = 0.82

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + lo
g(x)
```

giac [A] time = 0.38, size = 36, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

maple [B] time = 0.01, size = 87, normalized size = 2.23

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/8*ln(x^4-x
^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)+1/12*3^(1/
2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 3.05, size = 36, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

mupad [B] time = 1.30, size = 34, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12
```

sympy [A] time = 0.15, size = 41, normalized size = 1.05

$$\log(x)-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

3.335 $\int \frac{1}{x^3(1+x^4+x^8)} dx$

Optimal. Leaf size=54

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/2/x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1
)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1359, 1123, 1161, 618, 204}

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/S
qrt[3]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
```

```
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{-1-x^2}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right)-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)\\
&= -\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 100, normalized size = 1.85

$$\frac{1}{12}\left(-\frac{6}{x^2}+i\sqrt{3}\log\left(2x^2-i\sqrt{3}-1\right)-i\sqrt{3}\log\left(2x^2+i\sqrt{3}-1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] + I*Sqrt[3]*Log[-1 - I*Sqrt[3] + 2*x^2] - I*Sqrt[3]*Log[-1 + I*Sq
rt[3] + 2*x^2])/12
```

fricas [A] time = 0.80, size = 45, normalized size = 0.83

$$-\frac{\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,x^2\right)+\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\left(x^6+2\,x^2\right)\right)+3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)*
(x^6 + 2*x^2)) + 3)/x^2
```

giac [A] time = 0.38, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

maple [A] time = 0.01, size = 57, normalized size = 1.06

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^
(1/2))-1/2/x^2-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.39, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

mupad [B] time = 0.04, size = 43, normalized size = 0.80

$$-\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^4 + x^8 + 1)),x)
```

```
[Out] - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*x
^2)/3)))/12 - 1/(2*x^2)
```

sympy [A] time = 0.16, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3
))/12 - 1/(2*x**2)
```

3.336 $\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\log(x)$$

```
[Out] -1/4/x^4-ln(x)+1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4
 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)+\frac{1}{4}\,\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\
&= -\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 141, normalized size = 2.94

$$\frac{1}{24}\left(-\frac{6}{x^4}+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)+3\log\left(x^2-x+1\right)+3\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^
2] + Sqrt[3]*(-I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] + 3*Log[1 - x +
x^2] + 3*Log[1 + x + x^2])/24
```

fricas [A] time = 0.76, size = 49, normalized size = 1.02

$$-\frac{2\sqrt{3}\,x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4+1\right)\right)-3\,x^4\log\left(x^8+x^4+1\right)+24\,x^4\log(x)+6}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
+ 1) + 24*x^4*log(x) + 6)/x^4
```

giac [A] time = 0.31, size = 46, normalized size = 0.96

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{x^4-1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*log
(x^8 + x^4 + 1) - 1/4*log(x^4)
```

maple [B] time = 0.01, size = 94, normalized size = 1.96

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\ln(x)+\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+x^4+1),x)
```

```
[Out] -1/4/x^4-ln(x)+1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/8
*ln(x^4-x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^2-x+1)+1
/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.79, size = 41, normalized size = 0.85

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^4
 + 1) - 1/4*log(x^4)
```

mupad [B] time = 0.06, size = 41, normalized size = 0.85

$$\frac{\ln\left(x^8+x^4+1\right)}{8}-\ln(x)-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 + x^8 + 1)/8 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

sympy [A] time = 0.18, size = 48, normalized size = 1.00

$$-\log(x)+\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+x**4+1),x)
```

```
[Out] -log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12 - 1/(4*x**4)
```

3.337 $\int \frac{1}{x^7 \left(1+x^4+x^8\right)} \, dx$

Optimal. Leaf size=89

$$-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/6/x^6+1/2/x^2+1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^
2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{2x^2}-\frac{1}{6x^6}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(
1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]
/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(1+x^2+x^4)}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{-3-3x^2}{x^2(1+x^2+x^4)}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int -\frac{3x^2}{1+x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right.$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x\right.$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.$$
$$= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+\right.$$

Mathematica [C] time = 0.11, size = 142, normalized size = 1.60

$$\frac{1}{24}\left(-\frac{4}{x^6}+\frac{12}{x^2}+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] + Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2]
 + Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2
] - 3*Log[1 + x + x^2])/24
```

fricas [A] time = 0.79, size = 84, normalized size = 0.94

$$\frac{2\sqrt{3}\,x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+2\sqrt{3}\,x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-3x^6\log\left(x^4+x^2+1\right)+3x^6\log\left(x^4-x\right.}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan(
1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2 +
 1) + 12*x^4 - 4)/x^6
```

giac [A] time = 0.27, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

maple [A] time = 0.01, size = 95, normalized size = 1.07

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+x^4+1),x)
```

```
[Out] -1/6/x^6+1/2/x^2-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1
/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)
+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.37, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

mupad [B] time = 0.04, size = 62, normalized size = 0.70

$$\mathrm{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^4 + x^8 + 1)),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6
```

sympy [A] time = 0.25, size = 88, normalized size = 0.99

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3*x
**4 - 1)/(6*x**6)
```

3.338 $\int \frac{x^8}{1+x^4+x^8} \, dx$

Optimal. Leaf size=141

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}($$

```
[Out] x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^
2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1367, 1419, 1094, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^4 + x^8),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^
2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 +
Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
```

```
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+x^4+x^8}\,dx &= x-\int \frac{1+x^4}{1+x^4+x^8}\,dx\\
&= x-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= x-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= x-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{8}\int\\
&= x+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{1}{4}\\
&= x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+\right.
\end{aligned}$$

Mathematica [C] time = 0.28, size = 139, normalized size = 0.99

$$\frac{1}{24}\left(3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24x-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{i\tan^{-1}\left(\frac{1}{2}\left(1-i\right.\right.}{\sqrt{-6+6i}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/(1 + x^4 + x^8),x]
```

```
[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[(
(1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTan[
(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x
^2] - 3*Log[1 + x + x^2])/24
```

fricas [A] time = 0.87, size = 212, normalized size = 1.50

$$\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/12*sqrt(6)
*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3
)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*lo
g(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2
)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 -
x + 1)
```

giac [A] time = 0.39, size = 109, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(
3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.04, size = 110, normalized size = 0.78

$$x-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+x^4+1),x)
```

```
[Out] x-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/24*ln(1+x^2-x*
3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)-1
/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x +
 1) + 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.10, size = 100, normalized size = 0.71

$$x-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^4 + x^8 + 1),x)
```

```
[Out] x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

sympy [C] time = 0.71, size = 192, normalized size = 1.36

$$x+\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-\frac{\sqrt{3}\,i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\left(-\frac{1}{8}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+x**4+1),x)
```

```
[Out] x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24
)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sq
rt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*lo
g(-9216*_t**5 - 8*_t + x)))
```

3.339 $\int \frac{x^6}{1+x^4+x^8} dx$

Optimal. Leaf size=88

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1372, 1164, 628, 1161, 618, 204}

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
```

```
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{1+x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\ &= \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\operatorname{Subst}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.77

$$\frac{\log\left(-x^2+\sqrt{3}x-1\right)-\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^4 + x^8),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqrt
[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

fricas [A] time = 0.82, size = 70, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2-2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

giac [A] time = 0.33, size = 66, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

maple [A] time = 0.01, size = 67, normalized size = 0.76

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/
12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{2}\int\frac{x^{2}-1}{x^{4}-x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

mupad [B] time = 1.31, size = 38, normalized size = 0.43

$$-\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^{2}}{3}-\frac{2}{3}\right)}\right)+\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^{2}}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) + atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

sympy [A] time = 0.18, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^{2}-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^{2}+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

3.340 $\int \frac{x^4}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}\right.$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2+
x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8
 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx\\ &= -\left(\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,d\\ &= -\left(\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx\right)-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx\\ &= -\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+\right.\end{aligned}$$

Mathematica [C] time = 0.16, size = 135, normalized size = 0.96

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqrt
[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] +
 3*Log[1 + x + x^2])/24
```

fricas [A] time = 0.88, size = 211, normalized size = 1.51

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x +
 1)
```

giac [A] time = 0.40, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
+ 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.02, size = 109, normalized size = 0.78

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}\,x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/24*3^(1/2)*ln(x^
2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))-1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/4
*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1
) - 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.07, size = 99, normalized size = 0.71

$$-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{}{1+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 + x^8 + 1),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)
```

sympy [C] time = 0.72, size = 197, normalized size = 1.41

$$\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}\,i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}-\frac{\sqrt{3}\,i}{6}\right)+\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x)))
```

3.341 $\int \frac{x^2}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-\right.$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^2+
x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8
 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
```

```
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx$$
$$= -\left(\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx\right) - \frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}$$
$$= -\left(\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx\right) + \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx - \frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx +$$
$$= \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}$$
$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{1}{8}\log\left(1-x+\right.$$

Mathematica [C] time = 0.15, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(6\log\left(x^2-x+1\right) - 6\log\left(x^2+x+1\right) + 4i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) - 4i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2] -
6*Log[1 + x + x^2])/48
```

fricas [A] time = 0.95, size = 211, normalized size = 1.51

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2} - \sqrt{3}\right) - \frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sqrt(2)*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(
```

```
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x +
 1)
```

giac [A] time = 0.34, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
- 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 109, normalized size = 0.78

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/24*3^(1/2)*ln(x
^2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))+1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/
4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1)
+ 1/8*log(x^2 - x + 1)
```

mupad [B] time = 1.31, size = 97, normalized size = 0.69

$$\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x}{1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

sympy [C] time = 0.71, size = 214, normalized size = 1.53

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/24
)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4423
68*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*
_t**3 + x)))
```

3.342 $\int \frac{1}{1+x^4+x^8}\,dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1346, 1164, 628, 1161, 618, 204}

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
```

```
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\,\text{S}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.77

$$\frac{-\log\left(-x^2+\sqrt{3}\,x-1\right)+\log\left(x^2+\sqrt{3}\,x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqrt
[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

fricas [A] time = 0.94, size = 70, normalized size = 0.80

$$\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(x^3+2\,x\right)\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,x\right)+\frac{1}{12}\,\sqrt{3}\,\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

giac [A] time = 0.39, size = 66, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{12}\,\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right)-\frac{1}{12}\,\sqrt{3}\,\log\left(x^2-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

maple [A] time = 0.01, size = 67, normalized size = 0.76

$$\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{12}+\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/
12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

mupad [B] time = 0.04, size = 40, normalized size = 0.45

$$-\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right)-\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) - atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

sympy [A] time = 0.18, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 -
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

3.343 $\int \frac{1}{x^2(1+x^4+x^8)} dx$

Optimal. Leaf size=145

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}$$

```
[Out] -1/x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln
(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*
3^(1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*
3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1368, 1506, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 -
x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Lo
g[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^2\left(-1-x^4\right)}{1+x^4+x^8}\,dx\\
&= -\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{x}+\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^2+x^{}}\\
&= -\frac{1}{x}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\\
&= -\frac{1}{x}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x\right.}{8\sqrt{3}}\\
&= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(\right.
\end{aligned}$$

Mathematica [C] time = 0.21, size = 140, normalized size = 0.97

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2i\sqrt{-6-6i\sqrt{3}}\,\tan\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] (-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2*
I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x +
 x^2] + 3*Log[1 + x + x^2])/24
```

fricas [A] time = 0.98, size = 224, normalized size = 1.54

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x\arctan\left(-\frac{1}{3}\sqrt{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1
/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqrt(
6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sq
rt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + sqrt(6)*sqrt(2)*x*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(6)*sqrt(2)*x*log(-sqrt(6)*sqrt(2)*
x + 2*x^2 + 2) - 4*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*x*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 6*x*log(x^2 + x + 1) - 6*x*log(x^2 - x + 1) -
 48)/x
```

giac [A] time = 0.33, size = 113, normalized size = 0.78

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqr
t(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 114, normalized size = 0.79

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/x-1/24*3^(1/2)*l
n(x^2-3^(1/2)*x+1)-1/4*arctan(2*x-3^(1/2))+1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)
-1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}-\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)

mupad [B] time = 0.05, size = 102, normalized size = 0.70

$$\mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x}{1+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(x^4 + x^8 + 1)),x)

[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4) - 1/x

sympy [C] time = 0.74, size = 218, normalized size = 1.50

$$\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{7}-384\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{3}-442\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(x**8+x**4+1),x)

[Out] (-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8 - sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/24)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 442368*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24)**7) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7 - 384*_t**3 + x))) - 1/x

3.344 $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan$$

```
[Out] -1/3/x^3-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/
8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2
*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/
2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1419, 1094, 634, 618, 204, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/
4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1
 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) -
 Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
```

```
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-3-3x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= -\frac{1}{3x^3}-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\\
&= -\frac{1}{3x^3}+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x\right.}{8\sqrt{3}}\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\mathrm{lo}
\end{aligned}$$

Mathematica [C] time = 0.29, size = 148, normalized size = 1.01

$$\frac{1}{24}\left(-\frac{8}{x^3}+3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)-\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(\sqrt{3}+i\right)}}+\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(\sqrt{3}-i\right)}}-2\sqrt{3}\,\mathrm{t}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] (-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])] +
 ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*Sq
rt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*
Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

fricas [B] time = 0.94, size = 240, normalized size = 1.63

$$4\sqrt{6}\sqrt{3}\sqrt{2}x^3\arctan\left(-\tfrac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\tfrac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^3\arctan\left(-\tfrac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqr
t(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6
)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - sqrt(6)*sqrt(2)
*x^3*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(6)*sqrt(2)*x^3*log(-sqrt(6)*
sqrt(2)*x + 2*x^2 + 2) - 4*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sq
rt(3)*x^3*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^3*log(x^2 + x + 1) + 6*x^3*lo
g(x^2 - x + 1) - 16)/x^3
```

giac [A] time = 0.38, size = 113, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x -
 sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 114, normalized size = 0.78

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/3/x^3+1/24*3^(1
/2)*ln(x^2-3^(1/2)*x+1)-1/4*arctan(2*x-3^(1/2))-1/24*3^(1/2)*ln(x^2+3^(1/2)
*x+1)-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-
1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{3x^3}-\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2
 + x + 1) + 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 104, normalized size = 0.71

$$-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1i}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1i}{12}\right)-\mathrm{atan}\left(\frac{x\,2i}{-1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right)-\mathrm{atan}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + x^8 + 1)),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4) - 1/(3*x^3)
```

sympy [C] time = 0.75, size = 197, normalized size = 1.34

$$\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-\frac{\sqrt{3}\,i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**5
) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(3)
*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3
)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9
216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

3.345 $\int \frac{1}{x^6(1+x^4+x^8)}\,dx$

Optimal. Leaf size=98

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/5/x^5+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))
*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1368, 1504, 12, 1372, 1164, 628, 1161, 618, 204}

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{-5-5x^4}{x^2\left(1+x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{5}\int -\frac{5x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\int \frac{x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,d}{4\sqrt{3}}\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1\right.\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.97

$$\frac{1}{60}\left(-\frac{12}{x^5}+5\sqrt{3}\log\left(-x^2+\sqrt{3}x-1\right)-5\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{60}{x}+10\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+10\sqrt{3}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log[1
 + Sqrt[3]*x + x^2])/60
```

fricas [A] time = 1.12, size = 90, normalized size = 0.92

$$\frac{10\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+10\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}x\right)+5\sqrt{3}x^5\log\left(\frac{x^4+5x^2-2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)+60x^4-12}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arcta
n(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1
)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

giac [A] time = 0.40, size = 100, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*arc
tan(2*x - sqrt(3))
```

maple [A] time = 0.01, size = 75, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{1}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+x^4+1),x)
```

```
[Out] -1/5/x^5+1/x+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*3^(1/2)*ln(x^2-3^
(1/2)*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*
3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5x^4-1}{5x^5}+\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1),
 x)
```

mupad [B] time = 0.04, size = 52, normalized size = 0.53

$$\frac{x^4-\frac{1}{5}}{x^5}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/6 -
(3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))))/6
```

sympy [A] time = 0.22, size = 94, normalized size = 0.96

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{12}+\frac{\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + (5*x**4 - 1)/(5*x**5)
```

3.346 $\int \frac{1}{x^8(1+x^4+x^8)} dx$

Optimal. Leaf size=154

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-$$

```
[Out] -1/7/x^7+1/3/x^3+1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2
-x+1)+1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(
1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2
+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{3x^3}-\frac{1}{7x^7}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqr
t[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*
x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(
8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{1}{7x^7}+\frac{1}{7}\int \frac{-7-7x^4}{x^4\left(1+x^4+x^8\right)}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{21}\int -\frac{21x^4}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\int \frac{x^4}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}-\frac{\log(}{}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2\right.$$

Mathematica [C] time = 0.34, size = 171, normalized size = 1.11

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}}+\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\right.\right.}{2\sqrt{-6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*S
qrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(
2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcT
an[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8
```

fricas [B] time = 0.89, size = 246, normalized size = 1.60

$$-\frac{28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(28*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*
x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 28
*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 7*sqrt(6)*s
qrt(2)*x^7*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 7*sqrt(6)*sqrt(2)*x^7*log(-
sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x + 1
)) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x - 1)) - 42*x^7*log(x^2 + x + 1)
 + 42*x^7*log(x^2 - x + 1) - 112*x^4 + 48)/x^7
```

giac [A] time = 0.43, size = 120, normalized size = 0.78

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*arctan(2*x + sqrt(3)) + 1/4
*arctan(2*x - sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 119, normalized size = 0.77

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+x^4+1),x)
```

```
[Out] -1/7/x^7+1/3/x^3+1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1
/24*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))-1/24*3^(1/2)*ln(x^2
+3^(1/2)*x+1)+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1
/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{7x^4-3}{21x^7}+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1), x
) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 110, normalized size = 0.71

$$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1i}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{12}\right)-\mathrm{atan}\left(\frac{x\,2i}{-1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}i\right)-\mathrm{atan}\left(\frac{x\,2i}{1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right)-\mathrm{atan}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4/3 - 1/7)/x^7 - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) -
atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i
 + 1))*(3^(1/2)/12 + 1i/4) - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12
+ 1/4)
```

sympy [C] time = 0.76, size = 209, normalized size = 1.36

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
```

```
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

3.347 $\int \frac{x^m}{1-x^4+x^8}\,dx$

Optimal. Leaf size=127

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[Out] -2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(1+I*3^(1/2)))/(1+m
)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^
4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 - I*Sqrt
[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1 +
 m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1-x^4+x^8}\,dx &= -\frac{i\int \frac{x^m}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}+\frac{i\int \frac{x^m}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.62

$$\frac{x^m\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^7-\#1^3}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (x^m*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x -
 #1))]/((x/(x - #1))^m*(-#1^3 + 2*#1^7)) & ])/(4*m)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{x^8 - x^4 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-x^4+1),x)
```

```
[Out] int(x^m/(x^8-x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8 - x^4 + 1),x)
```

```
[Out] int(x^m/(x^8 - x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-x**4+1),x)
```

```
[Out] Integral(x**m/(x**8 - x**4 + 1), x)
```

3.348 $\int \frac{x^{11}}{1-x^4+x^8}\,dx$

Optimal. Leaf size=46

$$\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

```
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4}+\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{1-x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= \frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - x^4 + x^8),x]
[Out] x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

fricas [A] time = 0.82, size = 37, normalized size = 0.80

$$\frac{1}{4}x^4-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

giac [A] time = 0.34, size = 37, normalized size = 0.80

$$\frac{1}{4}x^4-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

maple [A] time = 0.01, size = 38, normalized size = 0.83

$$\frac{x^4}{4}-\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-x^4+1),x)
```

```
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 2.11, size = 37, normalized size = 0.80

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) + \frac{1}{8}\log\left(x^8 - x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

mupad [B] time = 0.05, size = 39, normalized size = 0.85

$$\frac{\ln\left(x^8 - x^4 + 1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^4}{3}\right)}{12} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 - x^4 + 1),x)
```

```
[Out] log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 + x
^4/4
```

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$\frac{x^4}{4} + \frac{\log\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

3.349 $\int \frac{x^9}{1-x^4+x^8} \, dx$

Optimal. Leaf size=57

$$\frac{x^2}{2}+\frac{\log \left(x^4-\sqrt{3}\, x^2+1\right)}{4 \sqrt{3}}-\frac{\log \left(x^4+\sqrt{3}\, x^2+1\right)}{4 \sqrt{3}}$$

```
[Out] 1/2*x^2+1/12*ln(1+x^4-3^(1/2)*x^2)*3^(1/2)-1/12*ln(1+x^4+3^(1/2)*x^2)*3^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1122, 1164, 628}

$$\frac{x^2}{2}+\frac{\log \left(x^4-\sqrt{3}\, x^2+1\right)}{4 \sqrt{3}}-\frac{\log \left(x^4+\sqrt{3}\, x^2+1\right)}{4 \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - x^4 + x^8),x]
```

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(4*Sqrt[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-x^4+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{1-x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}+\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}+\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\ &= \frac{x^2}{2}+\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(6x^2+\sqrt{3}\log\left(-x^4+\sqrt{3}x^2-1\right)-\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - x^4 + x^8),x]
```

```
[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
+ x^4])/12
```

fricas [A] time = 0.88, size = 47, normalized size = 0.82

$$\frac{1}{2}x^2+\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5x^4-2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 -
 x^4 + 1))
```

giac [B] time = 0.32, size = 99, normalized size = 1.74

$$\frac{1}{2}x^2+\frac{1}{4}\left(x^4-1\right)\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}\left(x^4-1\right)\arctan\left(2x^2-\sqrt{3}\right)+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x^
2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) - 1/
24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$\frac{x^2}{2}+\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-x^4+1),x)
```

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 + \int \frac{(x^4-1)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.31, size = 29, normalized size = 0.51

$$\frac{x^2}{2} - \frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2x^4}{9}+\frac{2}{9}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - x^4 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6
```

sympy [A] time = 0.13, size = 48, normalized size = 0.84

$$\frac{x^2}{2} + \frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{12} - \frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt(
3)*x**2 + 1)/12
```

3.350 $\int \frac{x^7}{1-x^4+x^8} \, dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] 1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{8}\log\left(1-x^4+x^8\right) - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

fricas [A] time = 0.86, size = 32, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

giac [A] time = 0.42, size = 32, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3}\;\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12} + \frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-x^4+1),x)
[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.97, size = 32, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

mupad [B] time = 1.28, size = 34, normalized size = 0.87

$$\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - x^4 + 1),x)
```

```
[Out] log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-x**4+1),x)
```

```
[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

3.351 $\int \frac{x^5}{1-x^4+x^8}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*ln(1+x^4-3^(1/2)*x
^2)*3^(1/2)-1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1-x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\ &= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)+\frac{\operatorname{Subst}\left(\int \right.}{}\\ &= \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right)-\\ &= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x\right.}{8\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.13, size = 98, normalized size = 1.20

$$\frac{\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt[
-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

fricas [B] time = 0.91, size = 171, normalized size = 2.09

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sq
rt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(2*x^4
 - sqrt(6)*sqrt(2)*x^2 + 2)
```

giac [A] time = 0.35, size = 76, normalized size = 0.93

$$\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right)+\frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}x^4\arctan\left(2x^2-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*x^4*log(x^4 - sq
rt(3)*x^2 + 1) + 1/4*x^4*arctan(2*x^2 + sqrt(3)) + 1/4*x^4*arctan(2*x^2 - s
qrt(3))
```

maple [A] time = 0.01, size = 65, normalized size = 0.79

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}-\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*3^(1/2)*ln(x^4-3^(
1/2)*x^2+1)-1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.05, size = 53, normalized size = 0.65

$$-\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

sympy [A] time = 0.21, size = 70, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\mathrm{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\mathrm{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2 +
 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

3.352 $\int \frac{x^3}{1-x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

fricas [A] time = 0.93, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

giac [A] time = 0.41, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-x^4+1),x)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.94, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

mupad [B] time = 1.29, size = 17, normalized size = 0.74

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6
```

sympy [A] time = 0.12, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

3.353 $\int \frac{x}{1-x^4+x^8} \, dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-3^(1/2)*x
^2)*3^(1/2)+1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

```
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right) - \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}\\
&= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right)\\
&= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x\right.}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 1.01

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)-\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 - x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I*
Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

fricas [B] time = 1.00, size = 171, normalized size = 2.09

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sq
rt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(2*x^4
 - sqrt(6)*sqrt(2)*x^2 + 2)
```

giac [A] time = 0.41, size = 64, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)+\frac{1}{4}\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x^2-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x^
2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt(3))
```

maple [A] time = 0.01, size = 65, normalized size = 0.79

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*ln(x^4-3^(
1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.04, size = 53, normalized size = 0.65

$$-\mathrm{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)*
x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)
```

sympy [A] time = 0.21, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\mathrm{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\mathrm{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

3.354 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
+ 2*#1^4) & ]/4
```

fricas [A] time = 0.88, size = 34, normalized size = 0.83

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log
(x)
```

giac [A] time = 0.35, size = 38, normalized size = 0.93

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.95, size = 38, normalized size = 0.93

$$\frac{1}{12}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)+\frac{1}{4}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

mupad [B] time = 1.29, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

sympy [A] time = 0.16, size = 41, normalized size = 1.00

$$\log{\left(x \right)}-\frac{\log{\left(x^8-x^4+1 \right)}}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

3.355 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

```
[Out] -1/2/x^2-1/12*ln(1+x^4-3^(1/2)*x^2)*3^(1/2)+1/12*ln(1+x^4+3^(1/2)*x^2)*3^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1123, 1164, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
 x^4]/(4*Sqrt[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)$$
$$=-\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)$$
$$=-\frac{1}{2x^2}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}$$
$$=-\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-x^4+\sqrt{3}\,x^2-1\right)+\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

fricas [A] time = 0.80, size = 50, normalized size = 0.88

$$\frac{\sqrt{3}\,x^2\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)-6}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
+ 1)) - 6)/x^2
```

giac [B] time = 0.31, size = 99, normalized size = 1.74

$$-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right)-\frac{1}{24}\left(\sqrt{3}\,x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}\,x^2+1\right)+\frac{1}{24}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt(
3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqrt(
3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{\sqrt{3}\;\ln\left(x^4-\sqrt{3}\;x^2+1\right)}{12}+\frac{\sqrt{3}\;\ln\left(x^4+\sqrt{3}\;x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-x^4+1),x)
```

[Out] -1/2/x^2-1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^2}-\int \frac{\left(x^4-1\right)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)

mupad [B] time = 1.27, size = 29, normalized size = 0.51

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\,\sqrt{3}\,x^2}{9\,\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x^8 - x^4 + 1)),x)

[Out] (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)

sympy [A] time = 0.15, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**8-x**4+1),x)

[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2 + 1)/12 - 1/(2*x**2)

3.356 $\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4
 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x+x^2\right)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{4x^4}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
2*#1^4) & ]/4
```

fricas [A] time = 1.00, size = 51, normalized size = 1.06

$$-\frac{2\sqrt{3}\,x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+3x^4\log\left(x^8-x^4+1\right)-24x^4\log(x)+6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
+ 1) - 24*x^4*log(x) + 6)/x^4
```

giac [A] time = 0.40, size = 48, normalized size = 1.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{x^4+1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*log
(x^8 - x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-x^4+1),x)
```

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.97, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^4
 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.07, size = 41, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

sympy [A] time = 0.19, size = 48, normalized size = 1.00

$$\log(x)-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12 - 1/(4*x**4)
```

3.357 $\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}$$

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*l
n(1+x^4-3^(1/2)*x^2)*3^(1/2)+1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2}-\frac{1}{6x^6}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2
]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(
8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1-x^2+x^4\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} + \frac{1}{6}\text{Subst}\left(\int \frac{3-3x^2}{x^2\left(1-x^2+x^4\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{6}\text{Subst}\left(\int \frac{3x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx, x, x$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x$$
$$= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{\log}{}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.58

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&, \frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right] - \frac{1}{6x^6} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1
 + 2*#1^4) & ]/4
```

fricas [B] time = 0.90, size = 193, normalized size = 2.01

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}\,x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\,x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2\,x^4+\sqrt{6}\sqrt{2}\,x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}\,x^6\arctan\left(-\frac{1}{3}\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^6*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^6*log(
2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24*x^4 - 8)/x^6
```

giac [A] time = 0.42, size = 56, normalized size = 0.58

$$-\frac{1}{12}\sqrt{3}\,x^4\log\left(x^4+\sqrt{3}\,x^2+1\right)+\frac{1}{12}\sqrt{3}\,x^4\log\left(x^4-\sqrt{3}\,x^2+1\right)-\frac{3\,x^4+1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6
```

maple [A] time = 0.01, size = 75, normalized size = 0.78

$$-\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ \ln\left(x^4-\sqrt{3}\ x^2+1\right)}{24}+\frac{\sqrt{3}\ \ln\left(x^4+\sqrt{3}\ x^2+1\right)}{24}-\frac{1}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-x^4+1),x)
```

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*3
^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,x^4+1}{6\,x^6}-\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.06, size = 63, normalized size = 0.66

$$\mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{\frac{x^4}{2}+\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6
```

sympy [A] time = 0.26, size = 83, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{\mathrm{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\mathrm{atan}\left(2x^2+\sqrt{3}\right)}{4}+\frac{-3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4 -
1)/(6*x**6)
```

3.358 $\int \frac{x^8}{1-x^4+x^8}\,dx$

Optimal. Leaf size=356

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(\right.$$

```
[Out] x-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*l
n(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((
-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6
^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))
/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2
^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+
1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(
1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.33, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1367, 1421, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt
[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 +
 Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*S
qrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x
+ x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 +
(Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + S
qrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x-\int \frac{1-x^4}{1-x^4+x^8}\,dx$$

$$= x+\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= x-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\cdots$$

$$= x+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+\cdots}$$

$$= x-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)\cdots$$

$$= x+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\cdots$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
2*#1^7) & ]/4
```

fricas [B] time = 0.94, size = 716, normalized size = 2.01

$$-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12\right)+\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 +
2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/4
8*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sq
rt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/96*sq
rt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/96*sq
rt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sq
rt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*
(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2
) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqr
t(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3
) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*
sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(
3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
 + sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqr
t(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt
(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2
) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x
^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)
*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3
)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) + x
```

giac [A] time = 0.35, size = 254, normalized size = 0.71

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
```

```
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) + x
```

maple [C] time = 0.02, size = 44, normalized size = 0.12

$$x+\frac{\left(\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{4}-1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-4\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-x^4+1),x)
```

```
[Out] x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+\int\frac{x^{4}-1}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.16, size = 209, normalized size = 0.59

$$x+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8i\right)^{1/4}}+\frac{\sqrt{3}\,x\,1i}{\left(8-\sqrt{3}\,8i\right)^{1/4}}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1i}{\left(8-\sqrt{3}\,8i\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8i\right)^{1/4}}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}}{12}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - x^4 + 1),x)
```

```
[Out] x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i
)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*
8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3
^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2
^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^
(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.20, size = 26, normalized size = 0.07

$$x+\mathrm{RootSum}\left(5308416t^{8}-2304t^{4}+1,\left(t\mapsto t\log\left(9216t^{5}-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
8*_t + x)))
```

3.359 $\int \frac{x^6}{1-x^4+x^8} dx$

Optimal. Leaf size=275

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)-1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1372, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{1-x^4+x^8}\,dx = -\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \ldots$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \ldots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \ldots$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^4 + x^8),x]

[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

fricas [A] time = 0.84, size = 215, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) - 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) + 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

giac [A] time = 0.40, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 32, normalized size = 0.12

$$\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(9\,\mathrm{RootOf}\left(9\_Z^4+1\right)^3 x-3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
```

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

3.360 $\int \frac{x^4}{1-x^4+x^8} \, dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] -1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*
2^(1/2)-1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-
1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(
1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/
(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^
(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1
/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x
^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/8*ln(1+x^2+x*(1
/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.21, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int}{}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} -$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log}{}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

fricas [B] time = 0.95, size = 567, normalized size = 1.63

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^2+12\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(2*sqrt(6)*
sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - 1/48*sqrt(6)*(sqrt(3)*
sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 12*x^2 + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*
sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1
2*x^2 + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) +
8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 1/1
2*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*s
qrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/12*sqrt(
6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt
(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) + 1/24*sqrt(6)*sqr
t(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqr
t(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqr
t(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) + 1/24*sqrt(6)
*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4
*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-
4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

giac [A] time = 0.48, size = 253, normalized size = 0.73

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.12

$$\frac{\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.33, size = 474, normalized size = 1.37

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\cdots\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1
/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2
^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8
 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3
^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*
8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8
i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2
)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)
^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 +
(2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3
^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) +
 (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4))/12
```

sympy [A] time = 3.19, size = 24, normalized size = 0.07

$$\mathrm{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*
_t + x)))
```

3.361 $\int \frac{x^2}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+$$

```
[Out] 1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2
^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1
/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1
/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(
1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/
2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^
2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/
2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[
3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{\int \frac{1}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \int$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \mathrm{l}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \mathrm{log}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

fricas [B] time = 1.02, size = 567, normalized size = 1.60

$$-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^2+12\right)+\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(2*sqrt(6)
*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 1/48*sqrt(6)*(sqrt(3)
*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sq
rt(sqrt(3) + 2) + 12*x^2 + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))
*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) +
12*x^2 + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) +
 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 1/
12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*
sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sq
rt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/12*sqrt
(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sq
rt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) + 1/24*sqrt(6)*sq
rt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sq
rt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sq
rt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) + 1/24*sqrt(6
)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-
4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(
-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

giac [A] time = 0.46, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8 - x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.08, size = 286, normalized size = 0.81

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/12
 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4)*
1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/
2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i
- 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^
(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*
1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.28, size = 26, normalized size = 0.07

$$\mathrm{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 - 192t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 - 1
92*_t**3 + x)))
```

3.362 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(-1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(-1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+}}{1+\sqrt{}}}{4\sqrt{3}}$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \int$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} +$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \text{lo}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

fricas [A] time = 0.86, size = 215, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

giac [A] time = 0.40, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 30, normalized size = 0.11

$$\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+3\,\mathrm{RootOf}\left(9\_Z^4+1\right)x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.04, size = 53, normalized size = 0.19

$$\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)
```

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

3.363 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=360

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2\right.$$

```
[Out] -1/x+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/
8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arcta
n((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/
2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2
)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/
2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/
2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*
2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2)
)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.24, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[
(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(1-x^4\right)}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{-2\sqrt{2+\sqrt{3}}-\left(-2-\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\int \frac{-2\sqrt{}}{1}}{4}$$

$$= -\frac{1}{x}+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}$$

$$= -\frac{1}{x}+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}\,x+\right.$$

$$= -\frac{1}{x}+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right.$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#
1 + 2*#1^5) & ]/4
```

fricas [B] time = 0.95, size = 732, normalized size = 2.03

$$-\frac{8\sqrt{6}\sqrt{2}x\sqrt{\sqrt{3}+2}\arctan\left(\frac{1}{6}\sqrt{6}\sqrt{12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(8*sqrt(6)*sqrt(2)*x*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2
 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(s
qrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt
(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 8*sqrt(6)*sqrt(2)*x
*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sq
rt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2)
)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(
sqrt(3) + 2) + sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arct
an(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*s
```

```
qrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8
) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) -
sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)
*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3)
+ 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) -
2*sqrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(12*x^2 +
2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 2*s
qrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*s
qrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - sqrt(6
)*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + sqrt(6)
*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 96)/x
```

giac [A] time = 0.46, size = 258, normalized size = 0.72

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/x
```

maple [C] time = 0.02, size = 52, normalized size = 0.14

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^6-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/x-1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^6-x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.29, size = 253, normalized size = 0.70

$$-\frac{1}{x}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(-1+\sqrt{3}\,1i\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{2\left(-1+\sqrt{3}\,1i\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{2\left(-1+\sqrt{3}\,1i\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(-1+\sqrt{3}\,1i\right)}\right)\left(\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - 1/x - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)
) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8
i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4))
 - (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4
)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4))
+ (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/1
2
```

sympy [A] time = 3.29, size = 29, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 + 384t^3 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 + 3
84*_t**3 + x))) - 1/x
```

3.364 $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right.}$$

```
[Out] -1/3/x^3+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*
6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)
-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^
(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))
/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6
^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/
8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+
x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A] time = 0.24, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1368, 1421, 1169, 634, 618, 204, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{3-3x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{3x^3}-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{}}}$$

$$= -\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x\right.$$

$$= -\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-}{\sqrt{2-\sqrt{3}}}\right.$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.18

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-#1^3 + 2*#1^7) & ]/4
```

fricas [B] time = 0.74, size = 756, normalized size = 2.04

$$\frac{8\sqrt{6}\sqrt{2}x^3\sqrt{\sqrt{3}+2}\arctan\left(\frac{1}{6}\sqrt{6}\sqrt{12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^
2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(
sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqr
t(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 8*sqrt(6)*sqrt(2)*
x^3*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt
(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sq
rt(sqrt(3) + 2) + sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)
*arctan(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)
*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3
) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) - sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(1/12*
sqrt(6)*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*s
qrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*
sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3)
+ 2) + 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*lo
g(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ 12) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*l
og(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
 + 12) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)
*log(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
 8) + 12) - sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) +
 8)*log(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3
) + 8) + 12) - 32)/x^3
```

giac [A] time = 0.35, size = 258, normalized size = 0.70

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
```

```
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1) - 1/3/x^3
```

maple [C] time = 0.01, size = 50, normalized size = 0.14

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/3/x^3+1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^3}-\int\frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.29, size = 213, normalized size = 0.58

$$-\frac{1}{3\,x^3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8i\right)^{1/4}}+\frac{\sqrt{3}\,x\,1i}{\left(8-\sqrt{3}\,8i\right)^{1/4}}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1i}{\left(8-\sqrt{3}\,8i\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8i\right)^{1/4}}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}}{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))
*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4
) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1/(3*x
^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^
(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.25, size = 31, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-9216t^5+8t+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*_
t + x))) - 1/(3*x**3)
```

3.365 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

Optimal. Leaf size=287

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{}\right.}{4\sqrt{6}}$$

```
[Out] -1/5/x^5-1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^
(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*
2^(1/2)))*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1
/2*2^(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)
-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1
/24*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1
/2)+1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2
)
```

Rubi [A] time = 0.24, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1504, 12, 1372, 1169, 634, 618, 204, 628}

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{}\right.}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]
) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTa
n[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2
 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sq
rt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2]/(4*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{5-5x^4}{x^2\left(1-x^4+x^8\right)}\,dx \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{1}{5}\int \frac{5x^6}{1-x^4+x^8}\,dx \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx \\
&= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+ \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+ \\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}- \\
&= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.19

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{5x^5}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 
2*#1^4) & ]/4
```

fricas [A] time = 0.99, size = 238, normalized size = 0.83

$$20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)+20\sqrt{3}\sqrt{2}x^5\arctan\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sq
rt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3
*x^2 - 2)) + 20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^
2 - sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 
2))/(3*x^2 - 2)) + 5*sqrt(3)*sqrt(2)*x^5*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x
) + 3*x^2 + 1) - 5*sqrt(3)*sqrt(2)*x^5*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) 
+ 3*x^2 + 1) - 120*x^4 - 24)/x^5
```

giac [A] time = 0.38, size = 217, normalized size = 0.76

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x^5
```

maple [C] time = 0.01, size = 43, normalized size = 0.15

$$-\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(9\,\mathrm{RootOf}\left(9\_Z^4+1\right)^3x-3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+x^2\right)}{4}-\frac{1}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-x^4+1),x)
```

```
[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5x^4+1}{5x^5}-\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.30, size = 63, normalized size = 0.22

$$-\frac{x^4+\frac{1}{5}}{x^5}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
(x^4 + 1/5)/x^5
```

sympy [A] time = 0.27, size = 182, normalized size = 0.63

$$\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
- sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x
+ 1)/24 + (-5*x**4 - 1)/(5*x**5)
```

3.366 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

Optimal. Leaf size=377

$$-\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}}\left(2-\right.$$

```
[Out] -1/7/x^7-1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2
*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2
))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2
*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*
2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2
*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(
1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1
/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}-\frac{1}{7x^7}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}}\left(2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] -
 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[
3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 -
 Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x
^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx = -\frac{1}{7x^7}+\frac{1}{7}\int \frac{7-7x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{21}\int \frac{21x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{}{1+\sqrt{2+}}}{8}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\sqrt{2+}\right.}{8\sqrt{3\left(2-\right.}}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\right.}{4\sqrt{3\left(\right.}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{7x^7}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 +
 2*#1^4) & ]/4
```

fricas [B] time = 0.97, size = 614, normalized size = 1.63

$$\frac{56\sqrt{6}\sqrt{2}x^7\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^2+12}\sqrt{\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2)
+ 56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt
(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 28
*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(
```

```
2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2
) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3
)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt
(3) + 2) - 224*x^4 - 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(
sqrt(3) + 2)*log(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 1
2) + 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log
(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - 7*sqrt(6)*
(sqrt(3)*sqrt(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt
(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) + 7*sqrt(6)*(sqrt(3)*sqrt
(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sqrt(3) + 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 96)/x^7
```

giac [A] time = 0.38, size = 265, normalized size = 0.70

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

maple [C] time = 0.02, size = 51, normalized size = 0.14

$$-\frac{\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{4}\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}\right)}-\frac{1}{3x^{3}}-\frac{1}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-x^4+1),x)
```

```
[Out] -1/7/x^7-1/3/x^3-1/4*sum(1/(2*_R^7-_R^3)*_R^4*ln(-_R+x),_R=RootOf(_Z^8-_Z^4
+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7\,x^{4}+3}{21\,x^{7}}-\int\frac{x^{4}}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.06, size = 486, normalized size = 1.29

$$-\frac{\frac{x^4}{3}+\frac{1}{7}}{x^7}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8i\right)}^{1/4}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8i\right)}^{1/4}\,1i}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\,{\left(8-\sqrt{3}\,8i\right)}^{1/4}\,1i}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,(8}{2\left(\frac{\sqrt{3}\,\sqrt{8-}}{4}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (x^4/3 + 1/7)/x^7 + (3^(1/2)*atan((x*(8 - 3^
(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2
)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1
/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/1
2 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(
3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2))
- (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i +
1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i
+ 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)
*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i
+ 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2
)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2
)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.37, size = 37, normalized size = 0.10

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(18432t^5 - 4t + x\right)\right)\right) + \frac{-7x^4 - 3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*_
t + x))) + (-7*x**4 - 3)/(21*x**7)
```

3.367 $\int \frac{x^m}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=117

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[Out] 2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(3-5^(1/2)))/(1+m)/
(3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4
/(3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (
1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m
))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1+3x^4+x^8} \, dx &= \frac{\int \frac{x^m}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} \, dx}{\sqrt{5}}-\frac{\int \frac{x^m}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} \, dx}{\sqrt{5}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}\end{aligned}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.68

$$\frac{x^m \text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^7+3\#1^3}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (x^m*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
 - #1))]/((x/(x - #1))^m*(3*#1^3 + 2*#1^7)) & ])/(4*m)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{x^8+3\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 + 3*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+3*x^4+1),x)
```

```
[Out] int(x^m/(x^8+3*x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(3*x^4 + x^8 + 1),x)
```

```
[Out] int(x^m/(3*x^4 + x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+3*x**4+1),x)
```

```
[Out] Integral(x**m/(x**8 + 3*x**4 + 1), x)
```

3.368 $\int \frac{x^{11}}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[Out] 1/4*x^4-1/40*ln(2*x^4-5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(2*x^4+5^(1/2)+3)*(1
5+7*5^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{4}\,\text{Subst}\left(\int \frac{-1-3x}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{}{\frac{3}{2}+}\right.$$
$$= \frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.92

$$\frac{1}{40}\left(10x^4+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}-3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*Lo
g[3 + Sqrt[5] + 2*x^4])/40
```

fricas [A] time = 0.84, size = 62, normalized size = 1.00

$$\frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}\left(2\,x^4+3\right)+7}{x^8+3\,x^4+1}\right) - \frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 +
 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)
```

giac [A] time = 0.50, size = 50, normalized size = 0.81

$$\frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right) - \frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

maple [A] time = 0.00, size = 38, normalized size = 0.61

$$\frac{x^4}{4} - \frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20} - \frac{3\ln\left(x^8+3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.28, size = 50, normalized size = 0.81

$$\frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4 - \sqrt{5}\,+3}{2\,x^4 + \sqrt{5}\,+3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

mupad [B] time = 0.13, size = 64, normalized size = 1.03

$$\frac{7\sqrt{5}\,\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} - \frac{3\,\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{3\,\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{7\sqrt{5}\,\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(3*x^4 + x^8 + 1),x)
```

```
[Out] (7*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 - (3*log(5^(1/2)/2 + x^4 + 3/2))/
8 - (3*log(x^4 - 5^(1/2)/2 + 3/2))/8 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2
))/40 + x^4/4
```

sympy [A] time = 0.14, size = 60, normalized size = 0.97

$$\frac{x^4}{4} + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+1),x)
```

```
[Out] x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/40
 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

3.369 $\int \frac{x^9}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] 1/2*x^2+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctan(x^2*2^
(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1122, 1166, 203}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sq
rt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1+3x^2}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) - \frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}\ldots}\right.$$
$$= \frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) + \frac{1}{20}\sqrt{180-80\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Mathematica [A] time = 0.15, size = 97, normalized size = 1.08

$$\frac{1}{40}\left(20x^2 - \sqrt{6-2\sqrt{5}}\,\left(15+7\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) + \sqrt{2\left(3+\sqrt{5}\right)}\,\left(7\sqrt{5}-15\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]
*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2
]*x^2])/40
```

fricas [B] time = 0.89, size = 154, normalized size = 1.71

$$\frac{1}{2}\,x^2 - \frac{1}{5}\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}+9}\,\arctan\left(\frac{1}{4}\,\sqrt{2\,x^4-\sqrt{5}+3}\left(3\,\sqrt{5}\,\sqrt{2}+7\,\sqrt{2}\right)\sqrt{-4\,\sqrt{5}+9} - \frac{1}{2}\left(3\,\sqrt{5}\,x^2+7\,x^2\right)\sqrt{-4\,\sqrt{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^4 - sqrt(5)
+ 3)*(3*sqrt(5)*sqrt(2) + 7*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(3*sqrt(5)*
x^2 + 7*x^2)*sqrt(-4*sqrt(5) + 9)) - 1/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan
(-1/4*(6*sqrt(5)*x^2 - 14*x^2 - sqrt(2*x^4 + sqrt(5) + 3)*(3*sqrt(5)*sqrt(2
) - 7*sqrt(2)))*sqrt(4*sqrt(5) + 9))
```

giac [A] time = 0.56, size = 66, normalized size = 0.73

$$\frac{1}{2}\,x^2 - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))
```

maple [B] time = 0.05, size = 117, normalized size = 1.30

$$\frac{x^2}{2} + \frac{7\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)} - \frac{3\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}} - \frac{7\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)} - \frac{3\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+3*x^4+1),x)
```

```
[Out] 1/2*x^2-7/5*5^(1/2)/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))-3/(2+2*5^(1/2
))*arctan(4*x^2/(2+2*5^(1/2)))+7/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+
2*5^(1/2)))-3/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 - \int \frac{(3x^4+1)x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.34, size = 130, normalized size = 1.44

$$2\,\mathrm{atanh}\left(\frac{1280\,x^2\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}-192}+\frac{768\sqrt{5}\,x^2\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}-192}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}-2\,\mathrm{atanh}\left(\frac{1280\,x^2\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}+192}-\frac{768\sqrt{5}}{64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(1
/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80)^
(1/2) - 2*atanh((1280*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192) -
 (768*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(- 5^(1/
2)/20 - 9/80)^(1/2) + x^2/2
```

sympy [A] time = 0.21, size = 54, normalized size = 0.60

$$\frac{x^2}{2}+2\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10 +
 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

3.370 $\int \frac{x^7}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[Out] 1/40*ln(2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(2*x^4+5^(1/2)+3)*(5+3*5^(1/2
))
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+3x+x^2}\,dx, x, x^4\right)\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x\right.\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + S
qrt[5] + 2*x^4])/40
```

fricas [A] time = 0.90, size = 56, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4+\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)+\frac{1}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) + 1/8*log(x^8 + 3*x^4 + 1)
```

giac [A] time = 0.50, size = 45, normalized size = 0.82

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.60

$$\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^8+3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.17, size = 45, normalized size = 0.82

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

mupad [B] time = 1.36, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40
```

sympy [**A**] time = 0.14, size = 53, normalized size = 0.96

$$\left(\frac{1}{8} - \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+3*x**4+1),x)
```

```
[Out] (1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*log
(x**4 + sqrt(5)/2 + 3/2)
```

3.371 $\int \frac{x^5}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))+1/2*arctan(x^2*2^(1/2
)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1130, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - S
qrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1+3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+3x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\right.\\
&= \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.93

$$\frac{2\sqrt{5}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\left(5-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt[
(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

fricas [B] time = 1.06, size = 165, normalized size = 2.04

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+3}-\frac{1}{20}\sqrt{10}\left(3\sqrt{5}x^2-5x^2\right)\sqrt{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(1/40*sqrt(10)*sqrt(2*x^4 + sqrt(5)
+ 3)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 3) - 1/20*sqrt(10)*(3*s
qrt(5)*x^2 - 5*x^2)*sqrt(sqrt(5) + 3)) + 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*a
rctan(1/40*sqrt(10)*sqrt(2*x^4 - sqrt(5) + 3)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2
))*sqrt(-sqrt(5) + 3) - 1/20*sqrt(10)*(3*sqrt(5)*x^2 + 5*x^2)*sqrt(-sqrt(5)
 + 3))
```

giac [A] time = 0.55, size = 47, normalized size = 0.58

$$\frac{1}{20}x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5)
*arctan(2*x^2/(sqrt(5) - 1))
```

maple [B] time = 0.02, size = 110, normalized size = 1.36

$$\frac{\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{3\sqrt{5}\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}+\frac{\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+3*x^4+1),x)
```

```
[Out] 1/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/5*5^(1/2)/(2+2*5^(1/2))*arcta
n(4/(2+2*5^(1/2))*x^2)+1/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-3/5*5^
(1/2)/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.12, size = 117, normalized size = 1.44

$$2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}+\frac{28\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}-\frac{28\,\sqrt{5}\,x^2}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((60*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3) + (28*5^(1/2)*x^
2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3))*(5^(1/2)/160 - 3/160)^(1/2) -
 2*atanh((60*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3) - (28*5^(1/2)
*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3))*(- 5^(1/2)/160 - 3/160)^
(1/2)
```

sympy [A] time = 0.20, size = 49, normalized size = 0.60

$$-2\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+3*x**4+1),x)
```

```
[Out] -2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*at
an(2*x**2/(1 + sqrt(5)))
```

3.372 $\int \frac{x^3}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+3x+x^2}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 3+2x^4\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}-3\right)-\log\left(2x^4+\sqrt{5}+3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

fricas [B] time = 0.84, size = 43, normalized size = 1.87

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
))
```

giac [A] time = 0.60, size = 31, normalized size = 1.35

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.48, size = 31, normalized size = 1.35

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

mupad [B] time = 1.33, size = 30, normalized size = 1.30

$$\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{8\sqrt{5}\,x^4+3\sqrt{5}}{18x^4+7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10
```

sympy [A] time = 0.12, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/2
)/20
```

3.373 $\int \frac{x}{1+3x^4+x^8} dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[Out] 1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctan(x^2*2^(1/2)/(3+
5^(1/2))^(1/2))/(5+5^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1359, 1093, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sq
rt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.99

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(3
 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

fricas [B] time = 0.95, size = 128, normalized size = 1.71

$$\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(-\frac{1}{10}\sqrt{10}\sqrt{5}\,x^2\sqrt{-\sqrt{5}+3} + \frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2\,x^4+\sqrt{5}+3}\sqrt{-\sqrt{5}+3}\right) - \frac{1}{10}\sqrt{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*arctan(-1/10*sqrt(10)*sqrt(5)*x^2*sqrt(-sq
rt(5) + 3) + 1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 + sqrt(5) + 3)*sqrt(-
sqrt(5) + 3)) - 1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(-1/20*(2*sqrt(10)*sq
rt(5)*x^2 - sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 - sqrt(5) + 3))*sqrt(sqrt(5
) + 3))
```

giac [A] time = 0.42, size = 41, normalized size = 0.55

$$\frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) + \frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan(
2*x^2/(sqrt(5) - 1))
```

maple [A] time = 0.02, size = 60, normalized size = 0.80

$$\frac{2\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)} - \frac{2\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+3*x^4+1),x)
```

```
[Out] -2/5*5^(1/2)/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+2/5*5^(1/2)/(-2+2*5^
(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.05, size = 125, normalized size = 1.67

$$2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}-\frac{72\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}+\frac{72\,\sqrt{5}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((160*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18) - (72*5^(1/2
)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18))*(5^(1/2)/160 - 3/160)^
(1/2) - 2*atanh((160*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18) +
(72*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18))*(- 5^(1/2)
/160 - 3/160)^(1/2)
```

sympy [A] time = 0.20, size = 49, normalized size = 0.65

$$2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+3*x**4+1),x)
```

```
[Out] 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*ata
n(2*x**2/(1 + sqrt(5)))
```

3.374 $\int \frac{1}{x(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[Out] ln(x)-1/40*ln(2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(2*x^4-5^(1/2)+3)*(5+3*
5^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1+3x+x^2\right)}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \log(x) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) - \frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int\right.$$
$$= \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right) + \frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(2x^4+\sqrt{5}+3\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5]
)*Log[3 + Sqrt[5] + 2*x^4])/40
```

fricas [A] time = 0.70, size = 58, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4+\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

giac [A] time = 0.48, size = 51, normalized size = 0.89

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20} + \ln(x) - \frac{\ln\left(x^8+3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+3*x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.50, size = 51, normalized size = 0.89

$$-\frac{3}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)-\frac{1}{8}\,\log\left(x^8+3\,x^4+1\right)+\frac{1}{4}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 1.41, size = 42, normalized size = 0.74

$$\ln(x)-\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\,\sqrt{5}}{40}+\frac{1}{8}\right)+\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\,\sqrt{5}}{40}-\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2
+ x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)
```

sympy [A] time = 0.16, size = 58, normalized size = 1.02

$$\log\left(x\right)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+3*x**4+1),x)
```

```
[Out] log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqrt
(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

3.375 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/2/x^2-1/40*arctan(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)+1/2*
arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1123, 1166, 203}

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
- ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{-3-x^2}{1+3x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{\frac{3}{2}}\right.$$
$$= -\frac{1}{2x^2}+\frac{1}{10}\sqrt{45-20\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.73

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^2 + 2*#1^6) & ]/4
```

fricas [B] time = 0.93, size = 158, normalized size = 1.78

$$-\frac{2\sqrt{5}\,x^2\sqrt{-4\sqrt{5}+9}\,\arctan\left(\frac{1}{4}\sqrt{2x^4+\sqrt{5}+3}\left(\sqrt{5}\sqrt{2}+3\sqrt{2}\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}x^2+3x^2\right)\sqrt{-4\sqrt{5}+9}\right.}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(5)*x^2*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^4 + sqrt(5) +
 3)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x^2 +
 3*x^2)*sqrt(-4*sqrt(5) + 9)) + 2*sqrt(5)*x^2*sqrt(4*sqrt(5) + 9)*arctan(-1
/4*(2*sqrt(5)*x^2 - 6*x^2 - sqrt(2*x^4 - sqrt(5) + 3)*(sqrt(5)*sqrt(2) - 3*
sqrt(2)))*sqrt(4*sqrt(5) + 9)) + 5)/x^2
```

giac [A] time = 0.46, size = 68, normalized size = 0.76

$$-\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1/
20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2/x
^2
```

maple [B] time = 0.03, size = 117, normalized size = 1.31

$$-\frac{\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{3\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/5*5^(1/2)/(2+2*5^(1/2))*arct
an(4/(2+2*5^(1/2))*x^2)-1/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-3/5*5
^(1/2)/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-1/2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^{2}}-\int \frac{\left(x^{4}+3\right)x}{x^{8}+3\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.30, size = 130, normalized size = 1.46

$$2\,\mathrm{atanh}\left(\frac{26880\,x^{2}\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}+\frac{12032\,\sqrt{5}\,x^{2}\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}-2\,\mathrm{atanh}\left(\frac{26880\,x^{2}\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (12
032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^(1
/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*
5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/
2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)
```

sympy [A] time = 0.24, size = 56, normalized size = 0.63

$$-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\mathrm{atan}\left(\frac{2x^{2}}{-1+\sqrt{5}}\right)+2\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\mathrm{atan}\left(\frac{2x^{2}}{1+\sqrt{5}}\right)-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+3*x**4+1),x)
```

```
[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*at
an(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

3.376 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

```
[Out] -1/4/x^4-3*ln(x)+1/40*ln(2*x^4+5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(2*x^4-5^(1
/2)+3)*(15+7*5^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1+3x+x^2\right)}\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{x\left(1+3x+x^2\right)}\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(-\frac{3}{x}+\frac{8+3x}{1+3x+x^2}\right)dx,x,x^4\right)\\ &= -\frac{1}{4x^4}-3\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{8+3x}{1+3x+x^2}\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right.\\ &= -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{}\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

fricas [A] time = 0.82, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}\,x^4\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}\left(2\,x^4+3\right)+7}{x^8+3\,x^4+1}\right)+15\,x^4\log\left(x^8+3\,x^4+1\right)-120\,x^4\log(x)-10}{40\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*
x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

giac [A] time = 0.55, size = 63, normalized size = 0.95

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{3\,x^4-1}{4\,x^4}+\frac{3}{8}\log\left(x^8+3\,x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
- 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

maple [A] time = 0.01, size = 42, normalized size = 0.64

$$-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20}-3\ln(x)+\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+3*x^4+1),x)
```

```
[Out] -1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^
(1/2)
```

maxima [A] time = 1.29, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{4x^4}+\frac{3}{8}\log\left(x^8+3x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 + 3
/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

mupad [B] time = 1.36, size = 49, normalized size = 0.74

$$\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)-\frac{1}{4x^4}-3\ln(x)-\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/40 + 3/8) - 1/(4*x^4) - 3*log(x) -
log(5^(1/2)/2 + x^4 + 3/2)*((7*5^(1/2))/40 - 3/8)
```

sympy [A] time = 0.19, size = 65, normalized size = 0.98

$$-3\log(x)+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

3.377 $\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] -1/6/x^6+3/2/x^2-1/2*arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1/
2))+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1359, 1123, 1281, 1166, 203}

$$\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sq
rt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{-9-3x^2}{x^2\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int \frac{-24-9x^2}{1+3x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Su}\\
&= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{1230+550\sqrt{5}}\,\tan^{}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 73, normalized size = 0.75

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+8\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{6x^6}+\frac{3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*L
og[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

fricas [B] time = 0.88, size = 180, normalized size = 1.86

$$\frac{3\sqrt{10}\,x^6\sqrt{-55\sqrt{5}+123}\,\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3}\left(7\sqrt{5}\sqrt{2}+15\sqrt{2}\right)\sqrt{-55\sqrt{5}+123}-\frac{1}{20}\sqrt{10}\left(7\sqrt{5}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/30*(3*sqrt(10)*x^6*sqrt(-55*sqrt(5) + 123)*arctan(1/40*sqrt(10)*sqrt(2*x^
4 + sqrt(5) + 3)*(7*sqrt(5)*sqrt(2) + 15*sqrt(2))*sqrt(-55*sqrt(5) + 123) -
 1/20*sqrt(10)*(7*sqrt(5)*x^2 + 15*x^2)*sqrt(-55*sqrt(5) + 123)) - 3*sqrt(1
0)*x^6*sqrt(55*sqrt(5) + 123)*arctan(1/40*(sqrt(10)*sqrt(2*x^4 - sqrt(5) +
3)*(7*sqrt(5)*sqrt(2) - 15*sqrt(2)) - 2*sqrt(10)*(7*sqrt(5)*x^2 - 15*x^2))*
sqrt(55*sqrt(5) + 123)) + 45*x^4 - 5)/x^6
```

giac [A] time = 0.51, size = 77, normalized size = 0.79

$$\frac{1}{20}\left(3x^4\left(\sqrt{5}-5\right)+8\sqrt{5}-40\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(3x^4\left(\sqrt{5}+5\right)+8\sqrt{5}+40\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) + 1
/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) + 1/
6*(9*x^4 - 1)/x^6
```

maple [B] time = 0.03, size = 122, normalized size = 1.26

$$\frac{7\sqrt{5}\ \arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}+\frac{3\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{7\sqrt{5}\ \arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}+\frac{3\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+3*x^4+1),x)
```

```
[Out] -1/6/x^6+3/2/x^2-7/5*5^(1/2)/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/(2
+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+7/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4
/(-2+2*5^(1/2))*x^2)+3/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,x^4-1}{6\,x^6}+\int\frac{\left(3\,x^4+8\right)x}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.12, size = 136, normalized size = 1.40

$$2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}\,-2550075}-\frac{1488300\,\sqrt{5}\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}\,-2550075}\right)\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}-2\,\mathrm{atanh}\left(\frac{3327500\,x^2}{1140425\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((3327500*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) -
2550075) - (1488300*5^(1/2)*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425
*5^(1/2) - 2550075))*((11*5^(1/2))/32 - 123/160)^(1/2) - 2*atanh((3327500*x
^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 2550075) + (1488
300*5^(1/2)*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 255
0075))*(- (11*5^(1/2))/32 - 123/160)^(1/2) + ((3*x^4)/2 - 1/6)/x^6
```

sympy [A] time = 0.27, size = 65, normalized size = 0.67

$$2\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] 2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/4
0)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

3.378 $\int \frac{x^8}{1+3x^4+x^8} dx$

Optimal. Leaf size=460

$$-\frac{\sqrt[4]{123-55\sqrt{5}}\ \log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\ \log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\ldots}\right.}{4\ 2^{3/4}\sqrt{5}}$$

```
[Out] x+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)*2^(1/4
)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)
*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(12
3-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(
1/4)+5^(1/2)-1)*(123-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/
4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(
1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*
ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+55*5^(1/2))^(1/4)*2^
(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+5
5*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 440, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1367, 1422, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{984-440\sqrt{5}}\ \log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\ \log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\ldots\right.}{8\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt
[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x
)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
+ 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*Sqrt[10]) + ((123 + 55*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x - \int \frac{1+3x^4}{1+3x^4+x^8}\,dx$$

$$= x - \frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx - \frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= x + \frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}$$

$$= x + \frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+x^2}\,dx + \frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int$$

$$= x - \frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+2x^2\right) + \frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2}\right.$$

$$= x - \frac{\sqrt[4]{123-55\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123-55\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123+55}}{}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.13

$$x - \frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]/4
```

fricas [B] time = 1.05, size = 1012, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*(55*sqrt(5)
- 123)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5
*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5
) - 7))*(1292*sqrt(5) - 2889)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 1
23) + 1/40*sqrt(10)*(2889*sqrt(5)*x - 6460*x)*(110*sqrt(5) + 246)^(5/4)*sqr
t(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(
5) + 246)*sqrt(55*sqrt(5) + 123)) + 1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)
*sqrt(55*sqrt(5) + 123)*(55*sqrt(5) - 123)*arctan(1/80*sqrt(10)*sqrt(20*x^2
 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) -
 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7))*(1292*sqrt(5) - 2889)*(110*sqrt
(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(10)*(2889*sqrt(5)*x - 6
460*x)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*s
qrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) - 1/8
0*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^
```

```
(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*
sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5)
 + 246))*(1292*sqrt(5) + 2889)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)
^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*x)*(-110*sqrt(5) + 246)^(5/4
) + 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55
*sqrt(5) + 123)) - 1/80*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)
*(-110*sqrt(5) + 246)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(3*
sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5)
+ 7)*sqrt(-110*sqrt(5) + 246))*(1292*sqrt(5) + 2889)*sqrt(-55*sqrt(5) + 123
)*(-110*sqrt(5) + 246)^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*x)*(-1
10*sqrt(5) + 246)^(5/4) - 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sq
rt(5) + 246))*sqrt(-55*sqrt(5) + 123)) - 1/80*sqrt(10)*sqrt(2)*(110*sqrt(5)
 + 246)^(1/4)*log(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(11
0*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7)) + 1/80*
sqrt(10)*sqrt(2)*(110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)
*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) +
246)*(3*sqrt(5) - 7)) + 1/80*sqrt(10)*sqrt(2)*(-110*sqrt(5) + 246)^(1/4)*lo
g(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246
)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5) + 246)) - 1/80*sqrt(10)*sqrt(
2)*(-110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x +
5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(
5) + 246)) + x
```

giac [A] time = 0.73, size = 240, normalized size = 0.52

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sqrt(25
*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sq
rt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*x^2) -
1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 + 2992900*
x^2) + x
```

maple [C] time = 0.01, size = 46, normalized size = 0.10

$$x+\frac{\left(-3\,\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{4}-1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{7}+12\,\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+3*x^4+1),x)
```

```
[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x-\int\frac{3\,x^{4}+1}{x^{8}+3\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.44, size = 216, normalized size = 0.47

$$x-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}}{2\left(55\sqrt{5}-12\right.}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(3*x^4 + x^8 + 1),x)
```

```
[Out] x - (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)) + (2
^(1/4)*5^(1/2)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123)^(1/4
))/20 + (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(55*5^(1/2) - 123)^(1/4)) -
(2^(1/4)*5^(1/2)*x)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2) - 123)^(1/4))
/20 + (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(- 55*5^(1/2) - 123)^(1/4)) +
 (2^(1/4)*5^(1/2)*x*1i)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123
)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(55*5^(1/2) - 123)
^(1/4)) - (2^(1/4)*5^(1/2)*x*1i)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2)
- 123)^(1/4)*1i)/20
```

sympy [A] time = 1.59, size = 29, normalized size = 0.06

$$x+\mathrm{RootSum}\left(40960000t^8+787200t^4+1,\left(t\mapsto t\log\left(\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t**
5/11 + 1288*_t/55 + x)))
```

3.379 $\int \frac{x^6}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=431

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\, \log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\, \log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{4\sqrt{10}}$$

```
[Out] -1/40*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^(1
/2)-1/40*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^
(1/2)-1/80*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))^(3
/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(3-5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4
))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2)
)^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)-1/80*ln(2*x^2+2*2^(1/4
)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\, \log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\, \log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqrt
[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9
- 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(
3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2
^(1/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}} +$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\sqrt{1}}$$

$$= \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{36-16\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\right.}{4\sqrt[4]{2}\sqrt{5}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

fricas [B] time = 0.98, size = 725, normalized size = 1.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 + (3*sqrt(
5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sq
rt(5) - 3))*(21*sqrt(5) - 47)*(4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt(5)*sqrt(
2)*x - 47*sqrt(2)*x)*(4*sqrt(5) + 9)^(5/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(4*s
qrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3))*(21*sqrt(5) -
 47)*(4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(4*s
qrt(5) + 9)^(5/4) + 1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan
(1/2*sqrt(2*x^2 + (3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4
) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9))*(21*sqrt(5) + 47)*(-4*sqrt(5) + 9)^
(5/4) - 1/2*(21*sqrt(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) -
1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - (3
*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sq
rt(-4*sqrt(5) + 9))*(21*sqrt(5) + 47)*(-4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt
(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) + 1) + 1/40*sqrt(5)*sq
rt(2)*(4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)
*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) - 1/40*sqrt(5)*
sqrt(2)*(4*sqrt(5) + 9)^(1/4)*log(2*x^2 - (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) + 1/40*sqrt(5
)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5)*sqrt(2)*x + 7*sqrt(
2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9)) - 1/40*s
qrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(2*x^2 - (3*sqrt(5)*sqrt(2)*x + 7*
sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9))
```

giac [A] time = 0.66, size = 239, normalized size = 0.55

$$\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{80}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) + 20)*log(400*(x + sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5
) + 20)*log(400*(x - sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5)
 - 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(10*sqrt
(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)
```

maple [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^6 \ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.46, size = 149, normalized size = 0.35

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\left(-4\sqrt{5}-9\right)^{1/4}}{8\sqrt{5}+24}\right)\left(-4\sqrt{5}-9\right)^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\left(4\sqrt{5}-9\right)^{1/4}}{8\sqrt{5}-24}\right)\left(4\sqrt{5}-9\right)^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\left(-4\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\sqrt{5}+24}\right)\left(\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(- 4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(- 4*5^(1/2)
 - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) - 2
4))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4)*16
i)/(8*5^(1/2) + 24))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*(4*5
^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10
```

sympy [A] time = 1.53, size = 26, normalized size = 0.06

$$\mathrm{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-1792000t^7-4920t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**7
 - 4920*_t**3 + x)))
```

3.380 $\int \frac{x^4}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=451

$$\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1
/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^
(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))^(1
/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4
))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2)
)^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4
)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4
)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*
Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[
5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx +$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+\right.}{4\,2^{3/4}\sqrt{5}}$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\right.}{2\,2^{3/4}\sqrt{5}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

fricas [B] time = 0.84, size = 843, normalized size = 1.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)
 + 1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*
sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) +
3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3
)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*ar
ctan(-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5)
 + 3) + 1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^
2 + 5*sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt
(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(
5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^
(3/4)*arctan(1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) +
10*x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sq
rt(5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/
4) + 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) +
 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4
)*arctan(1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sqrt(
5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/4)
- 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)
) + 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2
)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)
*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) +
 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqr
t(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4
)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(
-2*sqrt(5) + 6))
```

giac [A] time = 0.77, size = 239, normalized size = 0.53

$$\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5}-\frac{1}{80}\left(\pi+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) + 5)*l
```

```
og(625*(x + sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*log(
625*(x - sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*log(422
5*(x + sqrt(sqrt(5) - 1))^2 + 4225*x^2) + 1/40*sqrt(5*sqrt(5) - 5)*log(4225
*(x - sqrt(sqrt(5) - 1))^2 + 4225*x^2)
```

maple [C] time = 0.02, size = 40, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+3*x^4+1),x)
[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.20, size = 454, normalized size = 1.01

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}}{2\left(\frac{3\,\sqrt{2}\,\sqrt{-}}{2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4 + x^8 + 1),x)
[Out] (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(-
 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) - (2^(
3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(- 5^(1/2) - 3)^(1/2))
/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1/4))/20
 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2
)*(- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) -
(2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(- 5^(1/2) - 3)
^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1
/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*((3
*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2))
 + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/
2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4))/20
+ (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2)*(
5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)) + (2^(3/4
)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 +
 (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4)*1i)/20
```

sympy [A] time = 1.48, size = 24, normalized size = 0.05

$$\mathrm{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-51200t^5-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
12*_t + x)))
```

3.381 $\int \frac{x^2}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=427

$$\frac{\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{5}\,\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{5}\,\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3\right.}\right.}{4\sqrt{5}}$$

```
[Out] 1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(1/
2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(
1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2^(3/4)/(3-5^(1
/2))^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2
^(3/4)/(3-5^(1/2))^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4)
)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/
4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))
^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)+1/40*ln(2*x^2+2*2^(1/4)
*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)
```

Rubi [A] time = 0.26, antiderivative size = 431, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1375, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*S
qrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4
)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3
 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) -
((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*
x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqr
t[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*(3
 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt
[5]))^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= -\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} - \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x}}{4\sqrt{5}}$$

$$= \frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\,2^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5+3\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

fricas [B] time = 0.97, size = 955, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)
+ 1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3
/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(3*sqrt(5) - 5)*(2*sqr
t(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(
2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqr
t(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*s
qrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/80*sqrt(-sqrt(10)*(3*sqrt(5)*sqrt(2
)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*
(sqrt(5) - 3))*(3*sqrt(5) - 5)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) - 1/
8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/
80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(
1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/
4) + 40*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(3*sqrt(5) + 5)*sqrt(-
sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt(10)*(3*sqrt(5)*x + 5*x)*(-
2*sqrt(5) + 6)^(3/4) + 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6)
)*sqrt(-sqrt(5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*
sqrt(5) + 6)^(3/4)*arctan(1/80*sqrt(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt
(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) +
 6))*(3*sqrt(5) + 5)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt
(10)*(3*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^(3/4) - 5*(sqrt(5)*sqrt(2) + 3*sq
rt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)*sqrt(2)*(2
*sqrt(5) + 6)^(1/4)*log(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqr
t(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80*sqr
t(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*
sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5)
 - 3)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*(3*sqrt(
5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 10*(sqrt(5) +
 3)*sqrt(-2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*lo
g(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40
*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))
```

giac [A] time = 0.59, size = 239, normalized size = 0.56

$$\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5}-\frac{1}{80}\left(\pi+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
```

```
arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) + 5)
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

maple [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.09, size = 275, normalized size = 0.64

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right.}{2\,\left(3\,\sqrt{5}-7\right)}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7))
 - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2)
- 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*7i)/(
2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/
2) - 7)))*(5^(1/2) - 3)^(1/4)*1i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(
- 5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)) + (3*2^(3/4)*5^(1/2)*x*(- 5^(1/2)
 - 3)^(1/4))/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1
/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) + 7)) + (2^(3/4
)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^
(1/4)*1i)/20
```

sympy [A] time = 1.51, size = 26, normalized size = 0.06

$$\mathrm{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-6144000t^7-2240t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
- 2240*_t**3 + x)))
```

3.382 $\int \frac{1}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=414

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\sqrt{10}}$$

```
[Out] -1/20*arctan(-1+x*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/20*arctan(1+x
*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/40*ln(1+2*x^2+5^(1/2)-2*x*(5^(
1/2)+1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/40*ln(1+2*x^2+5^(1/2)+2*x*(5^(1/2)+
1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/20*arctan(-1+x*(5^(1/2)+1)^(1/2))*(20+10
*5^(1/2))^(1/2)+1/20*arctan(1+x*(5^(1/2)+1)^(1/2))*(20+10*5^(1/2))^(1/2)-1/
40*ln(-1+2*x^2+5^(1/2)-2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)+1/40*ln
(-1+2*x^2+5^(1/2)+2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1347, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] -((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqr
t[10]) + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3
 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*
(2*(3 + Sqrt[5]))^(3/4)) - ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sq
rt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}}}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)}$$

$$= -\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\right.}\right.}{4}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.10

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

fricas [B] time = 0.99, size = 733, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - sqrt(4*s
qrt(5) + 9)*(3*sqrt(5) - 7) + (sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5)
+ 9)^(1/4))*(4*sqrt(5) + 9)^(3/4)*(3*sqrt(5) - 7) - 1/2*(3*sqrt(5)*sqrt(2)*
x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(4*sqrt(
5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) -
 (sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4))*(4*sqrt(5) + 9)^(
3/4)*(3*sqrt(5) - 7) - 1/2*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) +
 9)^(3/4) + 1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqr
t(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) + (sqrt(5)*sqrt(2)*x + 3*sqr
t(2)*x)*(-4*sqrt(5) + 9)^(1/4))*(3*sqrt(5) + 7)*(-4*sqrt(5) + 9)^(3/4) - 1/
2*(3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sq
rt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 + (3*sqrt(5) + 7
)*sqrt(-4*sqrt(5) + 9) - (sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)
^(1/4))*(3*sqrt(5) + 7)*(-4*sqrt(5) + 9)^(3/4) - 1/2*(3*sqrt(5)*sqrt(2)*x +
 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 1) - 1/40*sqrt(5)*sqrt(2)*(4*sqrt(5)
 + 9)^(1/4)*log(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) + (sqrt(5)*sqrt
(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)*(4*sqrt(
5) + 9)^(1/4)*log(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) - (sqrt(5)*sq
rt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) - 1/40*sqrt(5)*sqrt(2)*(-4*sq
rt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) + (sqrt(5
)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)*(
-4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) - (s
qrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4))
```

giac [A] time = 0.53, size = 239, normalized size = 0.58

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) - 20)*log(10000*(x + sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10*sq
rt(5) - 20)*log(10000*(x - sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10
*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*sqrt(10*
sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)
```

maple [C] time = 0.01, size = 37, normalized size = 0.09

$$\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.08, size = 403, normalized size = 0.97

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{64\,\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,(4\,\sqrt{}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((144*x*(- 4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9)
^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4
))/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*
5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(24*5^
(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*x*(4*
5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^
(1/2)))*(4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4
)*144i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) +
 (5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2
) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 - (5^(1/2)*
atan((x*(4*5^(1/2) - 9)^(1/4)*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*
(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*
(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(4*5^(1/2) - 9)^(1/4)*1i
)/10
```

sympy [A] time = 1.52, size = 26, normalized size = 0.06

$$\mathrm{RootSum}\left(40960000t^8 + 115200t^4 + 1, \left(t \mapsto t\log\left(-9600t^5 - \frac{47t}{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
47*_t/2 + x)))
```

3.383 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=416

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{8\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/x+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+
1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+1/40*l
n(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^(1/4)-
1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^
(1/4)-1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1/4)*5
^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(246+110*5^(1
/2))^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(
246+110*5^(1/2))^(1/4)*5^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1510, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{8\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqr
t[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 -
 (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan
[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5])
 + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/40 - ((6150 - 2750
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/40
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(-3-x^4\right)}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}\cdots}{\frac{3}{2}-\cdots}}{4\sqrt{10}}$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\,2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}\cdots}}{8\,2^{3/4}\sqrt{5}}$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{1}{x}+\frac{\sqrt[4]{246+110\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{\sqrt[4]{246+110\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{1}{20\cdots}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^5+3\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/
(3*#1 + 2*#1^5) & ]/4
```

fricas [B] time = 1.02, size = 1017, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sq
rt(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*x - 360*x)*(110*sqrt(5) + 2
46)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x
 - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5)
+ 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(110*sqrt(5) + 246)^(3/4)*sqrt(
55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5)
 + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt
(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*
x - 360*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(-sq
rt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 4
0*x^2 - 20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(11
0*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 1
23*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*
```

```
sqrt(5)*x + 123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4)*arcta
n(1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) +
 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246))*(161*sq
rt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4) - 1/40*(2*s
qrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) + 5*(55*sqrt(5)*
sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 123)) +
 sqrt(10)*(55*sqrt(5)*x + 123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 24
6)^(3/4)*arctan(1/40*sqrt(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*
(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5)
+ 246))*(161*sqrt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3
/4) - 1/40*(2*sqrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) -
 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sq
rt(5) + 123)) - sqrt(10)*sqrt(2)*x*(110*sqrt(5) + 246)^(1/4)*log(sqrt(10)*(
47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 -
20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9)) + sqrt(10)*sqrt(2)*x*(110*sqrt(
5) + 246)^(1/4)*log(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*s
qrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9))
+ sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^(1/4)*log(sqrt(10)*(47*sqrt(5)*sq
rt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5
) + 9)*sqrt(-110*sqrt(5) + 246)) - sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^
(1/4)*log(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) +
246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246)) + 80)/x
```

giac [A] time = 0.60, size = 244, normalized size = 0.59

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) - 55)*log(748225*(x + sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sqrt(25
*sqrt(5) - 55)*log(748225*(x - sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sq
rt(25*sqrt(5) + 55)*log(180625*(x + sqrt(sqrt(5) - 1))^2 + 180625*x^2) - 1/
40*sqrt(25*sqrt(5) + 55)*log(180625*(x - sqrt(sqrt(5) - 1))^2 + 180625*x^2)
 - 1/x
```

maple [C] time = 0.01, size = 52, normalized size = 0.12

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^6+3\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+3\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+3*x^4+1),x)
```

```
[Out] -1/4*sum((_R^6+3*_R^2)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))-
1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^6+3\,x^2}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.29, size = 292, normalized size = 0.70

$$-\frac{1}{x}-\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}{2\,\left(3025\,\sqrt{5}+6765\right)}+\frac{1155\,2^{3/4}\,\sqrt{5}\,x\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}{2\,\left(3025\,\sqrt{5}+6765\right)}\right)\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}{20}-\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}}{2\,(30}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^4 + x^8 + 1)),x)
```

```
[Out] - 1/x - (2^(3/4)*5^(1/2)*atan((2585*2^(3/4)*x*(- 55*5^(1/2) - 123)^(1/4))/(
2*(3025*5^(1/2) + 6765)) + (1155*2^(3/4)*5^(1/2)*x*(- 55*5^(1/2) - 123)^(1/
4))/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2585*2^(3/4)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) - 676
5)) - (1155*2^(3/4)*5^(1/2)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) -
6765)))*(55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(-
55*5^(1/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) + 6765)) + (2^(3/4)*5^(1/2)
*x*(- 55*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/
2) - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(55*5^(1/2) - 123
)^(1/4)*2585i)/(2*(3025*5^(1/2) - 6765)) - (2^(3/4)*5^(1/2)*x*(55*5^(1/2) -
 123)^(1/4)*1155i)/(2*(3025*5^(1/2) - 6765)))*(55*5^(1/2) - 123)^(1/4)*1i)/
20
```

sympy [A] time = 1.61, size = 32, normalized size = 0.08

$$\mathrm{RootSum}\left(40960000t^{8}+787200t^{4}+1,\left(t\mapsto t\log\left(\frac{19251200t^{7}}{11}+\frac{369792t^{3}}{11}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**7
/11 + 369792*_t**3/11 + x))) - 1/x
```

3.384 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=466

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/3/x^3+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/2))^(1/4
)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/
2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/
2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3+
5^(1/2))^(1/4)+5^(1/2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arct
an(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-
1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*
5^(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(843+377*5^(
1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(
1/2)-1)*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1422, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843
 - 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(
3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]
)/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-9-3x^4}{1+3x^4+x^8}\,dx$$
$$= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$
$$= -\frac{1}{3x^3}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}dx}{8\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}dx}{8\sqrt{5}}+\frac{\left(-5+3\right.}{}$$
$$= -\frac{1}{3x^3}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}dx}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{}{-\sqrt{\frac{1}{2}(3}}}{4\,2^{3/4}}$$
$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\mathrm{lo}}{}$$
$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{843}}{}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^3 + 2*#1^7) & ]/4
```

fricas [B] time = 0.86, size = 1057, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/240*(3*sqrt(10)*sqrt(2)*x^3*(754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt
(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sq
rt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(754*sqr
t(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*
x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47
)) - 3*sqrt(10)*sqrt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt(1
0)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21
*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686)) + 3*sqrt(10)*sqrt(2)*x^3*(-754*sq
rt(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)
*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1
686)) - 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*s
qrt(377*sqrt(5) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(
5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5
) + 1686)*(21*sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(
```

```
5/4)*sqrt(377*sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(
754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) - 1/8*(377*sqrt(5)*sqrt(2
) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) - 3*sqrt
(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*sqrt(377*sqrt(5
) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x -
 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*
sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*
sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) +
1686)^(5/4)*sqrt(377*sqrt(5) + 843) + 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2
))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt
(5)*x^3 + 843*x^3)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arc
tan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*
x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 16
86))*(23184*sqrt(5) + 51841)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)
^(5/4) - 1/40*(sqrt(10)*(51841*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^
(5/4) + 5*(377*sqrt(5)*sqrt(2) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sq
rt(-377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*
sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2
 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4
) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(23184*sqrt(5) + 51841)*
sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(10)*(5184
1*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) - 5*(377*sqrt(5)*sqrt(2
) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843)) + 80)
/x^3
```

giac [A] time = 0.70, size = 244, normalized size = 0.52

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145}+\frac{1}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt(65*
sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2) - 1/4
0*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 93122500*
x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) - 1))^2 +
 53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt(sqrt(5)
 - 1))^2 + 53728900*x^2) - 1/3/x^3
```

maple [C] time = 0.01, size = 50, normalized size = 0.11

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^4-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4-3)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))-1/3/x
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^3}-\int\frac{x^4+3}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.18, size = 492, normalized size = 1.06

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{20735\,2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)(3}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^4 + x^8 + 1)),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((46371*2^(3/4)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*
2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843
)^(1/2))) - (20735*2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*2^
(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^
(1/2))))*(377*5^(1/2) - 843)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((46371*2^(3/
4)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1
/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (20735*2^(3/4)*5
^(1/2)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843
)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2
) - 843)^(1/4))/20 - 1/(3*x^3) + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 377*5^
(1/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2) + 1
508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 3
77*5^(1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2
) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2) - 84
3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(377*5^(1/2) - 843)^(1/4
)*46371i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)
*(377*5^(1/2) - 843)^(1/2))) - (2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4)
*20735i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*
(377*5^(1/2) - 843)^(1/2))))*(377*5^(1/2) - 843)^(1/4)*1i)/20
```

sympy [A] time = 1.62, size = 34, normalized size = 0.07

$$\operatorname{RootSum}{\left(40960000 t^{8} + 5395200 t^{4} + 1, \left(t \mapsto t \log{\left(\frac{179200 t^{5}}{377} + \frac{23112 t}{377} + x \right)} \right)\right)} - \frac{1}{3 x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5/
377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

3.385 $\int \frac{x^m}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=117

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[Out] 2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(3-5^(1/2)))/(1+m)/(
3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(
3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1375, 364}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[5
])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{1-3x^4+x^8} \, dx &= \frac{\int \frac{x^m}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} \, dx}{\sqrt{5}}-\frac{\int \frac{x^m}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} \, dx}{\sqrt{5}}\\
&= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}
\end{aligned}$$

Mathematica [C] time = 0.57, size = 575, normalized size = 4.91

$$x^m\left(\frac{\text{RootSum}\left[\#1^4-\#1^2-1\&,\frac{\#1^2m^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+3\#1^2m\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+2\#1^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+\#1^2m(}{2\#1^3-\#1}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (x^m*(-RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(
x - #1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] + (RootSum[-1 - #1^2 + #1^4 &
 , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1
- m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m,
1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m
, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2
*#1^3) & ] - (2 + 3*m + m^2)*RootSum[-1 + #1^2 + #1^4 & , Hypergeometric2F1
[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(#1 + 2*#1^3)) & ] - RootSu
m[-1 + #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeo
metric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hyperg
eometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hype
rgeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2
)/(x/#1)^m)/(#1 + 2*#1^3) & ])/(2 + 3*m + m^2)))/(4*m)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{x^8-3\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - 3*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-3*x^4+1),x)
```

```
[Out] int(x^m/(x^8-3*x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8 - 3*x^4 + 1),x)
```

```
[Out] int(x^m/(x^8 - 3*x^4 + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(x^4 - x^2 - 1\right)\left(x^4 + x^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-3*x**4+1),x)
```

```
[Out] Integral(x**m/((x**4 - x**2 - 1)*(x**4 + x**2 - 1)), x)
```

3.386 $\int \frac{x^{11}}{1-3x^4+x^8} dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[Out] 1/4*x^4+1/40*ln(-2*x^4-5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*
(15+7*5^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{1-3x^4+x^8}\,dx &= \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{x^2}{1-3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{-1+3x}{1-3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{3}{2}-\ldots}\right.\\
&= \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.90

$$\frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log[
-3 + Sqrt[5] + 2*x^4])/40
```

fricas [A] time = 0.71, size = 62, normalized size = 1.00

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}\left(2\,x^4-3\right)+7}{x^8-3\,x^4+1}\right)+\frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 -
 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

giac [A] time = 0.42, size = 53, normalized size = 0.85

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^4-\sqrt{5}-3\right|}{\left|2\,x^4+\sqrt{5}-3\right|}\right)+\frac{3}{8}\,\log\left(\left|x^8-3\,x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3
)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.00, size = 38, normalized size = 0.61

$$\frac{x^4}{4}-\frac{7\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{20}+\frac{3\ln\left(x^8-3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

maxima [A] time = 1.39, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) + \frac{3}{8}\log\left(x^8 - 3x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 3
/8*log(x^8 - 3*x^4 + 1)
```

mupad [B] time = 0.12, size = 64, normalized size = 1.03

$$\frac{3\ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{8} + \frac{3\ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{8} + \frac{7\sqrt{5}\ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{40} - \frac{7\sqrt{5}\ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (3*log(x^4 - 5^(1/2)/2 - 3/2))/8 + (3*log(5^(1/2)/2 + x^4 - 3/2))/8 + (7*5^
(1/2)*log(x^4 - 5^(1/2)/2 - 3/2))/40 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2
))/40 + x^4/4
```

sympy [A] time = 0.14, size = 58, normalized size = 0.94

$$\frac{x^4}{4} + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-3*x**4+1),x)
```

```
[Out] x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(5
)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

3.387 $\int \frac{x^9}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=90

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\, x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\, x^2\right)$$

```
[Out] 1/2*x^2+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctanh(x^2*
2^(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1122, 1166, 207}

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\, x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)} \tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\, x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (S
qrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1-3x^2}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= \frac{x^2}{2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int -\right.$$
$$= \frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{180-80\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right.}\right.$$

Mathematica [A] time = 0.05, size = 103, normalized size = 1.14

$$\frac{1}{20}\left(10x^2+\left(2\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5-2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2*S
qrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

fricas [B] time = 0.88, size = 114, normalized size = 1.27

$$\frac{1}{2}x^2+\frac{1}{10}\sqrt{5}\log\left(\frac{2x^4+2x^2-\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+\frac{1}{10}\sqrt{5}\log\left(\frac{2x^4-2x^2-\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-\frac{1}{4}\log\left(x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

giac [A] time = 0.46, size = 97, normalized size = 1.08

$$\frac{1}{2}x^2+\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)+\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
+ 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/4
*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.74

$$\frac{x^2}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x^4-x^2-1\right)}{4}-\frac{\ln\left(x^4+x^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-3*x^4+1),x)
```

```
[Out] 1/2*x^2-1/4*ln(x^4+x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/4*ln
(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

maxima [A] time = 1.36, size = 92, normalized size = 1.02

$$\frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1
/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^4
+ x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.33, size = 90, normalized size = 1.00

$$\frac{x^2}{2}-\mathrm{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}+192}+\frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}+192}\right)\,\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)-\mathrm{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}-192}-\frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}-192}\right)\,\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x^2/2 - atanh((64*x^2)/(64*5^(1/2) + 192) + (64*5^(1/2)*x^2)/(64*5^(1/2) +
192))*(5^(1/2)/5 + 1/2) - atanh((64*x^2)/(64*5^(1/2) - 192) - (64*5^(1/2)*x
^2)/(64*5^(1/2) - 192))*(5^(1/2)/5 - 1/2)
```

sympy [B] time = 0.38, size = 170, normalized size = 1.89

$$\frac{x^2}{2}+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-\frac{47\sqrt{5}}{20}-120\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-120\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{47}{20}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(5)
/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 - 12
0*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sqrt(5
)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

3.388 $\int \frac{x^7}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[Out] 1/40*ln(-2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*(5+3*5^(1
/2))
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1-3x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1-3x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\right.\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 + S
qrt[5] + 2*x^4])/40
```

fricas [A] time = 0.79, size = 57, normalized size = 1.04

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) + 1/8*log(x^8 - 3*x^4 + 1)
```

giac [A] time = 0.42, size = 48, normalized size = 0.87

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*l
og(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.00, size = 33, normalized size = 0.60

$$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^8-3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

maxima [A] time = 1.48, size = 45, normalized size = 0.82

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^8
 - 3*x^4 + 1)
```

mupad [B] time = 0.10, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - 3*x^4 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 - 3/2)/8 + log(5^(1/2)/2 + x^4 - 3/2)/8 + (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 - 3/2))/40 - (3*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2))/40
```

sympy [**A**] time = 0.14, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-3*x**4+1),x)
```

```
[Out] (1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*log
(x**4 - 3/2 + sqrt(5)/2)
```

3.389 $\int \frac{x^5}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

```
[Out] 1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))-1/2*arctanh(x^2*2^(1/
2)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1130, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(3 -
 Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-3x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\right.\\
&= -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.12

$$\frac{1}{40}\left(\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}-1\right)-\left(5+\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5] -
 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1 +
Sqrt[5] + 2*x^2])/40
```

fricas [B] time = 0.62, size = 109, normalized size = 1.35

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4+2x^2-\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4-2x^2-\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

giac [B] time = 0.44, size = 92, normalized size = 1.14

$$\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqr
t(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(x
^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.00, size = 62, normalized size = 0.77

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\ln\left(x^4+x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x
^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

maxima [B] time = 1.92, size = 87, normalized size = 1.07

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5
)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1)
 + 1/8*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.38, size = 77, normalized size = 0.95

$$-\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}-3}\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)-\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}+3}\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - 3*x^4 + 1),x)
```

```
[Out] - atanh((4*x^2)/(5^(1/2) - 3) - (2*5^(1/2)*x^2)/(5^(1/2) - 3))*(5^(1/2)/20
+ 1/4) - atanh((4*x^2)/(5^(1/2) + 3) + (2*5^(1/2)*x^2)/(5^(1/2) + 3))*(5^(1
/2)/20 - 1/4)
```

sympy [B] time = 0.37, size = 165, normalized size = 2.04

$$\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right)+\left(\frac{1}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] (-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40)
**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 + 3*
sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sqrt(
5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)**3
+ 3*sqrt(5)/10 + 3/2)
```

3.390 $\int \frac{x^3}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] 1/10*arctanh(1/5*(-2*x^4+3)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1-3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-3x+x^2}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, -3+2x^4\right)\right)\\
&= \frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}+3\right)-\log\left(2x^4+\sqrt{5}-3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 3*x^4 + x^8),x]
[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

fricas [B] time = 0.58, size = 43, normalized size = 1.87

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] 1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
))
```

giac [A] time = 0.54, size = 33, normalized size = 1.43

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
[Out] 1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-3*x^4+1),x)
[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

maxima [A] time = 1.41, size = 31, normalized size = 1.35

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))
```

mupad [B] time = 1.57, size = 30, normalized size = 1.30

$$\frac{\sqrt{5}\ \operatorname{atanh}\left(\frac{3\sqrt{5}-8\sqrt{5}\,x^4}{18x^4-7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - 3*x^4 + 1),x)
[Out] (5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10
```

sympy [A] time = 0.12, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\,\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20}-\frac{\sqrt{5}\,\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/2
)/20
```

3.391 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[Out] 1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctanh(x^2*2^(1/2)/(
3+5^(1/2))^(1/2))/(5+5^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1359, 1093, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}$$
$$= -\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.21

$$\frac{1}{40}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)\right)-\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(\sqrt{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5
] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*x^2])/40
```

fricas [B] time = 0.87, size = 107, normalized size = 1.43

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4+2x^2+\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4-2x^2+\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-\frac{1}{8}\log\left(x^4+x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

giac [B] time = 0.43, size = 92, normalized size = 1.23

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(
x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\ln\left(x^4+x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x
^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

maxima [B] time = 1.45, size = 87, normalized size = 1.16

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1
) + 1/8*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.30, size = 83, normalized size = 1.11

$$\mathrm{atanh}\left(\frac{29x^2}{8\sqrt{5}-18}-\frac{13\sqrt{5}x^2}{8\sqrt{5}-18}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)+\mathrm{atanh}\left(\frac{29x^2}{8\sqrt{5}+18}+\frac{13\sqrt{5}x^2}{8\sqrt{5}+18}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - 3*x^4 + 1),x)
```

```
[Out] atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1/
2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1/2
) + 18))*(5^(1/2)/20 + 1/4)
```

sympy [B] time = 0.37, size = 165, normalized size = 2.20

$$\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{8}+\frac{\sqrt{}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)**
3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sqrt
(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt(5)
/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3
+ 7*sqrt(5)/10 + 7/2)
```

3.392 $\int \frac{1}{x(1-3x^4+x^8)} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[Out] ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(-2*x^4-5^(1/2)+3)*(5+
3*5^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] - 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1-3x+x^2\right)}\,dx,x,x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{1-3x+x^2}\,dx,x,x^4\right)$$
$$= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)-\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \right.$$
$$= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5])
*Log[-3 + Sqrt[5] + 2*x^4])/40
```

fricas [A] time = 0.86, size = 59, normalized size = 1.04

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

giac [A] time = 0.48, size = 54, normalized size = 0.95

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{4}\log\left(x^4\right)-\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*l
og(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.01, size = 64, normalized size = 1.12

$$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\ln(x)-\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\ln\left(x^4+x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-3*x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(
x^4+x^2-1)+3/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

maxima [A] time = 1.45, size = 51, normalized size = 0.89

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^8
 - 3*x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.43, size = 42, normalized size = 0.74

$$\ln(x)+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] log(x) + log(x^4 - 5^(1/2)/2 - 3/2)*((3*5^(1/2))/40 - 1/8) - log(5^(1/2)/2
+ x^4 - 3/2)*((3*5^(1/2))/40 + 1/8)
```

sympy [A] time = 0.16, size = 58, normalized size = 1.02

$$\log(x)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-3*x**4+1),x)
```

```
[Out] log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/40
 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

3.393 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/2/x^2+1/40*arctanh(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/2
*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1123, 1166, 207}

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-3x^2+x^4\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{3-x^2}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) - \frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\Bigg($$
$$= -\frac{1}{2x^2} - \frac{1}{10}\sqrt{45-20\sqrt{5}}\,\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) + \frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right.}\right.}{4\sqrt{10}}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{20}\left(-\frac{10}{x^2} - \left(\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)\right) + \left(5-2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right) + \left(5+2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + 2*
Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

fricas [B] time = 0.87, size = 125, normalized size = 1.40

$$\frac{2\sqrt{5}\,x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right) + 2\sqrt{5}\,x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right) - 5x^2\log\left(x^4+x^2-1\right) + 5x^2\log\left(x^4\right.}{20x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^
2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4
- x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/x^2
```

giac [A] time = 0.38, size = 97, normalized size = 1.09

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right) - \frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right) - \frac{1}{2x^2} - \frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right) + \frac{1}{4}\log\left(\left|x^4-x^2-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/
4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.75

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{5} + \frac{\ln\left(x^4-x^2-1\right)}{4} - \frac{\ln\left(x^4+x^2-1\right)}{4} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*ln(x^4+x^2
-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))-1/2/x^2
```

maxima [A] time = 1.46, size = 92, normalized size = 1.03

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x^4
 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

mupad [B] time = 0.06, size = 88, normalized size = 0.99

$$\operatorname{atanh}\left(\frac{12736\,x^2}{3520\sqrt{5}-7872}-\frac{5696\sqrt{5}\,x^2}{3520\sqrt{5}-7872}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)+\operatorname{atanh}\left(\frac{12736\,x^2}{3520\sqrt{5}+7872}+\frac{5696\sqrt{5}\,x^2}{3520\sqrt{5}+7872}\right)\left(\frac{\sqrt{5}}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((12736*x^2)/(3520*5^(1/2) - 7872) - (5696*5^(1/2)*x^2)/(3520*5^(1/2)
- 7872))*(5^(1/2)/5 - 1/2) + atanh((12736*x^2)/(3520*5^(1/2) + 7872) + (569
6*5^(1/2)*x^2)/(3520*5^(1/2) + 7872))*(5^(1/2)/5 + 1/2) - 1/(2*x^2)
```

sympy [B] time = 0.41, size = 172, normalized size = 1.93

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1/
4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3 +
123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-1/4
 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 - sqr
t(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

3.394 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

```
[Out] -1/4/x^4+3*ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(-2*x^4-5^
(1/2)+3)*(15+7*5^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-3x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{3-x}{x\left(1-3x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\,\text{Subst}\left(\int \left(\frac{3}{x}+\frac{8-3x}{1-3x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+3\log(x)+\frac{1}{4}\,\text{Subst}\left(\int \frac{8-3x}{1-3x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+3\log(x)+\frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right.\\
&= -\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-\right.
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.92

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}+3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

fricas [A] time = 0.82, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}\,x^4\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)-15x^4\log\left(x^8-3x^4+1\right)+120x^4\log(x)-10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*
x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

giac [A] time = 0.51, size = 66, normalized size = 1.00

$$\frac{7}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)-\frac{3x^4+1}{4x^4}+\frac{3}{4}\log\left(x^4\right)-\frac{3}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*(
3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.01, size = 71, normalized size = 1.08

$$-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}+\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+3\ln(x)-\frac{3\ln\left(x^4-x^2-1\right)}{8}-\frac{3\ln\left(x^4+x^2-1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-3*x^4+1),x)
```

```
[Out] -1/4/x^4+3*ln(x)-3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/
2))-3/8*ln(x^4+x^2-1)+7/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

maxima [A] time = 1.40, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4}-\frac{3}{8}\log\left(x^8-3x^4+1\right)+\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 - 3
/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

mupad [B] time = 1.35, size = 49, normalized size = 0.74

$$3\ln(x)-\frac{1}{4x^4}+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] 3*log(x) - 1/(4*x^4) + log(x^4 - 5^(1/2)/2 - 3/2)*((7*5^(1/2))/40 - 3/8) -
log(5^(1/2)/2 + x^4 - 3/2)*((7*5^(1/2))/40 + 3/8)
```

sympy [A] time = 0.19, size = 66, normalized size = 1.00

$$3\log(x)+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)/
40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

3.395 $\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] -1/6/x^6-3/2/x^2-1/2*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1
/2))+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1359, 1123, 1281, 1166, 207}

$$-\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + S
qrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/
2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{9-3x^2}{x^2\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{-24+9x^2}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(15+7\sqrt{5}\right)\text{S}$$
$$= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{1230+550\sqrt{5}}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 1.14

$$\frac{1}{120}\left(-\frac{20}{x^6}-\frac{180}{x^2}-3\left(25+11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)+3\left(25-11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+3\left(25+11\sqrt{5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
- 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[
5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

fricas [B] time = 0.85, size = 130, normalized size = 1.34

$$\frac{33\sqrt{5}\,x^6\log\left(\frac{2x^4+2x^2+\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+33\sqrt{5}\,x^6\log\left(\frac{2x^4-2x^2+\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-75\,x^6\log\left(x^4+x^2-1\right)+75\,x^6\log}{120\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x
^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) - 18
0*x^4 - 20)/x^6
```

giac [A] time = 0.41, size = 104, normalized size = 1.07

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\log\left(\left|x^4-x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*
sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4
 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.01, size = 72, normalized size = 0.74

$$\frac{11\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}+\frac{11\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\frac{5\ln\left(x^4-x^2-1\right)}{8}-\frac{5\ln\left(x^4+x^2-1\right)}{8}-\frac{3}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-3*x^4+1),x)
```

```
[Out] -1/6/x^6-3/2/x^2+5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1
/2))-5/8*ln(x^4+x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

maxima [A] time = 1.32, size = 99, normalized size = 1.02

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sqr
t(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x^6
 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.38, size = 95, normalized size = 0.98

$$\operatorname{atanh}\left(\frac{4126100\,x^2}{1140425\,\sqrt{5}-2550075}-\frac{1845250\,\sqrt{5}\,x^2}{1140425\,\sqrt{5}-2550075}\right)\left(\frac{11\,\sqrt{5}}{20}-\frac{5}{4}\right)+\operatorname{atanh}\left(\frac{4126100\,x^2}{1140425\,\sqrt{5}+2550075}+\frac{}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((4126100*x^2)/(1140425*5^(1/2) - 2550075) - (1845250*5^(1/2)*x^2)/(11
40425*5^(1/2) - 2550075))*((11*5^(1/2))/20 - 5/4) + atanh((4126100*x^2)/(11
40425*5^(1/2) + 2550075) + (1845250*5^(1/2)*x^2)/(1140425*5^(1/2) + 2550075
))*((11*5^(1/2))/20 + 5/4) - ((3*x^4)/2 + 1/6)/x^6
```

sympy [B] time = 0.44, size = 199, normalized size = 2.05

$$\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right)+\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] (11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt(
5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/8
- 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x**2
 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5/8 -
 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)
/50 + 2207/22) + (-9*x**4 - 1)/(6*x**6)
```

3.396 $\int \frac{x^8}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=170

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-}}{2\sqrt{5}}$$

```
[Out] x+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5^(1
/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/2))^(1/4)*
2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/
2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1367, 1422, 212, 206, 203}

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-}}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqr
t[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sq
rt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/
(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]
)/(4*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
```

```
&& IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-3x^4+x^8}\,dx &= x-\int \frac{1-3x^4}{1-3x^4+x^8}\,dx\\ &= x-\frac{1}{10}\left(-15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= x+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx\\ &= x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\end{aligned}$$

Mathematica [A] time = 0.27, size = 160, normalized size = 0.94

$$x+\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\right.}{\sqrt{10\left(1+\sqrt{5}\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
 + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])
]
```

fricas [B] time = 0.87, size = 304, normalized size = 1.79

$$-\frac{1}{10}\sqrt{10}\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(2\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x-5x\right)\right)\sqrt{5\sqrt{5}+11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(5*sqrt(5) + 11)*arctan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt
(5) + 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x - 5*x))*
sqrt(5*sqrt(5) + 11)) - 1/10*sqrt(10)*sqrt(5*sqrt(5) - 11)*arctan(1/20*(sqr
t(10)*sqrt(2*x^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt(10
)*(2*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) + 1/40*sqrt(10)*sqrt(5*sqrt(5)
```

```
 - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqr
t(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) +
5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5)
 + 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(-sq
rt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + x
```

giac [A] time = 0.68, size = 148, normalized size = 0.87

$$-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}+110}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
+ 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*
log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log(abs
(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x - sq
rt(1/2*sqrt(5) - 1/2))) + x
```

maple [A] time = 0.06, size = 205, normalized size = 1.21

$$x+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{2\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{2\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(\right.}{\sqrt{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-3*x^4+1),x)
```

```
[Out] x-2/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5
^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arcta
n(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2
+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
-2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(
1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2))^(1
/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+\frac{1}{2}\int\frac{2x^2+1}{x^4-x^2-1}\,dx-\frac{1}{2}\int\frac{2x^2-1}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
1)/(x^4 + x^2 - 1), x)
```

mupad [B] time = 1.44, size = 246, normalized size = 1.45

$$x-\frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20}-\frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,3}{2\left(275\sqrt{5}-605\right)}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(
1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(- 50*5^(1/
2) - 110)^(1/2)*1i)/20 - (atan((x*(110 - 50*5^(1/2))^(1/2)*55i)/(2*(275*5^(
1/2) - 605)) - (5^(1/2)*x*(110 - 50*5^(1/2))^(1/2)*33i)/(2*(275*5^(1/2) - 6
05)))*(110 - 50*5^(1/2))^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 110)^(1/2)*5
5i)/(2*(275*5^(1/2) - 605)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(1/2)*33i)/(2*(
275*5^(1/2) - 605)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2)
 + 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(1/2)*x*(50*5^(1/2) + 110)^
(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(50*5^(1/2) + 110)^(1/2)*1i)/20
```

sympy [**A**]   time = 1.24, size = 58, normalized size = 0.34

$$x+\text{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)+\text{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 +
 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(
-15360*_t**5/11 + 1288*_t/55 + x)))
```

3.397 $\int \frac{x^6}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=167

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}+\ldots$$

```
[Out] -1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1/2
)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1
/2)+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(1/4)*
5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(
1/4)*5^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1374, 298, 203, 206}

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
 - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
 - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt
[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqr
t[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}$$

$$= \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}}{2\,2^{3/4}\sqrt{}}$$

Mathematica [A] time = 0.15, size = 160, normalized size = 0.96

$$\frac{\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} + \frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}} - \frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((3
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + Sqr
t[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

fricas [B] time = 0.75, size = 255, normalized size = 1.53

$$\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{5}+2}-\frac{1}{2}\left(\sqrt{5}x-3x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 1)*(sqrt(5)
*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(5) + 2) - 1/2*(sqrt(5)*x - 3*x)*sqrt(sqrt(5
) + 2)) + 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) - 1
)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + 3*x)*s
qrt(sqrt(5) - 2)) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(s
qrt(5) - 1) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*
(sqrt(5) - 1) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 1)*sqr
t(sqrt(5) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 1)*s
qrt(sqrt(5) - 2) + 2*x)
```

giac [A] time = 0.75, size = 147, normalized size = 0.88

$$\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*s
qrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10)*
log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x
 - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sqrt(1
/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*sqrt(5
) - 1/2)))
```

maple [A] time = 0.03, size = 206, normalized size = 1.23

$$-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(-\right.}{2\sqrt{-2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2/(-2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2/
(-2+2*5^(1/2))^(1/2)*x)-1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(
1/2)*x)+3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)
+1/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)+3/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8 - 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.19, size = 147, normalized size = 0.88

$$\frac{\sqrt{5}\ \operatorname{atan}\left(\frac{16\,x\,\sqrt{2-\sqrt{5}}}{8\,\sqrt{5}-24}\right)\,\sqrt{\sqrt{5}-2}\ 1\mathrm{i}}{10}+\frac{\sqrt{5}\ \operatorname{atan}\left(\frac{16\,x\,\sqrt{-\sqrt{5}-2}}{8\,\sqrt{5}+24}\right)\,\sqrt{\sqrt{5}+2}\ 1\mathrm{i}}{10}+\frac{\sqrt{5}\ \operatorname{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\ 16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\sqrt{2-\sqrt{5}}\ 1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(2 - 5^(1/2))^(1/2))/(8*5^(1/2) - 24))*(5^(1/2) - 2)^(1
/2)*1i)/10 + (5^(1/2)*atan((16*x*(- 5^(1/2) - 2)^(1/2))/(8*5^(1/2) + 24))*(
5^(1/2) + 2)^(1/2)*1i)/10 + (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*16i)/(8*5^
(1/2) - 24))*(2 - 5^(1/2))^(1/2)*1i)/10 + (5^(1/2)*atan((x*(- 5^(1/2) - 2)^
(1/2)*16i)/(8*5^(1/2) + 24))*(- 5^(1/2) - 2)^(1/2)*1i)/10
```

sympy [A] time = 1.22, size = 53, normalized size = 0.32

$$\operatorname{RootSum}\left(6400t^4 - 320t^2 - 1, \left(t \mapsto t\log\left(-1792000t^7 + 4920t^3 + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 320t^2 - 1, \left(t \mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920
*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-1792
000*_t**7 + 4920*_t**3 + x)))
```

3.398 $\int \frac{x^4}{1-3x^4+x^8} dx$

Optimal. Leaf size=173

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+$$

```
[Out] 1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)*5^(1
/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)
*5^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4)*2^(
3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4
)*2^(3/4)*5^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1374, 212, 206, 203}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) +
(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (
((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx + \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx -$$

$$= -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\right.}}{}$$

Mathematica [A] time = 0.19, size = 132, normalized size = 0.76

$$\frac{\sqrt{\sqrt{5}-1}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\sqrt{5}-1}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-\sqrt{1+\sqrt{5}}\tanh^{-1}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*Ar
cTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10]
)
```

fricas [B] time = 0.89, size = 271, normalized size = 1.57

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{20}\sqrt{10}\left(\sqrt{5}x-5x\right)\sqrt{\sqrt{5}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/40*sqrt(10)*sqrt(2*x^2 + sqrt(5)
+ 1)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 1) - 1/20*sqrt(10)*(sqrt(
5)*x - 5*x)*sqrt(sqrt(5) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/4
0*sqrt(10)*sqrt(2*x^2 + sqrt(5) - 1)*(sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(sqr
t(5) - 1) - 1/20*sqrt(10)*(sqrt(5)*x + 5*x)*sqrt(sqrt(5) - 1)) - 1/40*sqrt(
10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x) + 1/40
*sqrt(10)*sqrt(sqrt(5) + 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x)
 + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) - 1) +
 10*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5)
 - 1) + 10*x)
```

giac [A] time = 0.61, size = 147, normalized size = 0.85

$$-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(1
0*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) +
```

```
10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(
abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x +
sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqrt(1/2
*sqrt(5) - 1/2)))
```

maple [A] time = 0.04, size = 206, normalized size = 1.19

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-3*x^4+1),x)
```

```
[Out] -1/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/2/(2+2
*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2/(-2+2*5^(1/2))^(1/2)*x)-1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2/
(-2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(
1/2)*x)-1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)
-1/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)-1/2/(2+2*
5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 1.47, size = 269, normalized size = 1.55

$$\frac{\sqrt{10}\ \mathrm{atan}\left(\frac{\sqrt{10}\ x\ \sqrt{-\sqrt{5}-1}\ 1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\ \sqrt{10}\ x\ \sqrt{-\sqrt{5}-1}\ 3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\sqrt{-\sqrt{5}-1}\ 1\mathrm{i}}{20}+\frac{\sqrt{10}\ \mathrm{atan}\left(\frac{\sqrt{10}\ x\ \sqrt{1-\sqrt{5}}\ 1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\ \sqrt{10}\ x\ \sqrt{1-\sqrt{5}}\ 3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (5
^(1/2)*10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) - 1)))*(- 5^(1/2)
- 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*1i)/(2*(
5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(10*(5^(1/2) +
1)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^
(1/2)*1i)/(2*(5^(1/2) - 1)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(
10*(5^(1/2) - 1)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*
(5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) -
1)^(1/2)*3i)/(10*(5^(1/2) + 1)))*(5^(1/2) - 1)^(1/2)*1i)/20
```

sympy [A] time = 1.20, size = 49, normalized size = 0.28

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(-512\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t +
 x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 +
 12*_t + x)))
```

3.399 $\int \frac{x^2}{1-3x^4+x^8}\, dx$

Optimal. Leaf size=145

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\tan^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}\,x\right)-\frac{1}{20}\sqrt{10+10\sqrt{5}}\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\,x\right)-\frac{1}{20}\sqrt{10\sqrt{5}-10}\tanh^{-1}\left(\frac{1}{2}\sqrt{2\right.$$

```
[Out] 1/20*arctan(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctanh
(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctan(1/2*x*(2+2*
5^(1/2))^(1/2))*(10+10*5^(1/2))^(1/2)+1/20*arctanh(1/2*x*(2+2*5^(1/2))^(1/2
))*(10+10*5^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1375, 298, 203, 206}

$$\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}+\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\right.\right.}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - Ar
cTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) + ((
(3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}} + \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}}$$

$$= \frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} + \frac{\sqrt[4]{\frac{1}{2}}\ldots}{}$$

Mathematica [A] time = 0.04, size = 131, normalized size = 0.90

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[2
/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*
x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 +
Sqrt[5])]
```

fricas [B] time = 0.96, size = 255, normalized size = 1.76

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1} - \frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right) - \frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2
*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1) - 1/10*sqrt(10)*sqrt(5)*x*sqrt(sqrt(5
) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt
(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) - 1) - 1/10*sqrt(10)*sqrt(5)*x*s
qrt(sqrt(5) - 1)) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) +
 5)*sqrt(sqrt(5) - 1) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(1
0)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1
)*log(sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt
(sqrt(5) + 1)*log(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x)
```

giac [A] time = 0.62, size = 147, normalized size = 1.01

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) - 1
```

```
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

maple [A] time = 0.03, size = 110, normalized size = 0.76

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-3*x^4+1),x)
```

```
[Out] -1/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/5*5^(1/
2)/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2)/(-2+2*
5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2)/(2+2*5^(1/2))^
(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.08, size = 269, normalized size = 1.86

$$\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\left(3\,\sqrt{5}-7\right)}\right)\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\left(3\,\sqrt{5}+7\right)}\right)\sqrt{\sqrt{}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) -
1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2)
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

sympy [A] time = 1.19, size = 53, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*_
t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(6144000*
_t**7 - 2240*_t**3 + x)))
```

3.400 $\int \frac{1}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=169

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(
3/4)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/
2))^(3/4)+1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/4)*
5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/4)*5
^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1347, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)))
+ ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5
]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4
)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sq
rt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$
$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}}$$
$$= -\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{(3+}{}$$

Mathematica [A] time = 0.16, size = 160, normalized size = 0.95

$$\frac{\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}} + \frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(\sqrt{5}-1\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sqrt
[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

fricas [B] time = 0.90, size = 251, normalized size = 1.49

$$-\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}-1}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+2}-\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) - 1)*(sqrt(5
)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 2) - 1/2*(sqrt(5)*x - x)*sqrt(sqrt(5) +
 2)) + 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 1)*(
sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + x)*sqrt(sqr
t(5) - 2)) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 3)*sqrt(sqrt(5)
- 2) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 3)*sqrt(sqrt(5
) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(
5) - 3) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqr
t(5) - 3) + 2*x)
```

giac [A] time = 0.48, size = 147, normalized size = 0.87

$$-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5*
sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 10)
*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(
x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + sqrt(
1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*sqrt(
5) - 1/2)))
```

maple [A] time = 0.03, size = 206, normalized size = 1.22

$$\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\ \arctan\left(\cdots\right.}{10\sqrt{-2+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(-2+2*5^(1/2)
)^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*arctan(2/
(-2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5
^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)
-1/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.08, size = 245, normalized size = 1.45

$$-\frac{\sqrt{5}\ \mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\ 144\mathrm{i}}{104\,\sqrt{5}-232}-\frac{\sqrt{5}\,x\,\sqrt{2-\sqrt{5}}\ 64\mathrm{i}}{104\,\sqrt{5}-232}\right)\sqrt{2-\sqrt{5}}\ 1\mathrm{i}}{10}+\frac{\sqrt{5}\ \mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\ 144\mathrm{i}}{104\,\sqrt{5}+232}+\frac{\sqrt{5}\,x\,\sqrt{-\sqrt{5}-2}\ 64\mathrm{i}}{104\,\sqrt{5}+232}\right)\sqrt{-\sqrt{5}-\cdots}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((x*(- 5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (5^(1/2)
*x*(- 5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(- 5^(1/2) - 2)^(1/2)*1i
)/10 - (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*144i)/(104*5^(1/2) - 232) - (5^
(1/2)*x*(2 - 5^(1/2))^(1/2)*64i)/(104*5^(1/2) - 232))*(2 - 5^(1/2))^(1/2)*1
i)/10 + (5^(1/2)*atan((x*(5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) - 232) - (5
^(1/2)*x*(5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) - 232))*(5^(1/2) - 2)^(1/2)*
1i)/10 - (5^(1/2)*atan((x*(5^(1/2) + 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (
5^(1/2)*x*(5^(1/2) + 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(5^(1/2) + 2)^(1/2)
*1i)/10
```

sympy [A] time = 1.21, size = 53, normalized size = 0.31

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
+ x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 -
 47*_t/2 + x)))
```

3.401 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+}}\right.}{4\sqrt{5}}$$

```
[Out] -1/x+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4
)*5^(1/2)-1/40*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/4)*2^
(3/4)*5^(1/2)+1/40*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/
4)*2^(3/4)*5^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1368, 1510, 298, 203, 206}

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+}}\right.}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*
Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/
4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]
)/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4
*2^(1/4)*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
```

```
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(3-x^4\right)}{1-3x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{}{\sqrt{3-\sqrt{5}}}}{2\sqrt{10}}$$

$$= -\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{9}}{}$$

Mathematica [A] time = 0.27, size = 174, normalized size = 1.01

$$-\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(\sqrt{5}-3\right)\tanh^{-1}}{2\sqrt{10\left(1+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 +
 Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(
1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[
10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*
Sqrt[10*(1 + Sqrt[5])])
```

fricas [B] time = 0.93, size = 313, normalized size = 1.82

$$4\sqrt{10}\,x\sqrt{5\sqrt{5}+11}\,\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(3\sqrt{5}x-5x\right)\right)\sqrt{5\sqrt{5}+11}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(4*sqrt(10)*x*sqrt(5*sqrt(5) + 11)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 +
sqrt(5) - 1)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(3*sqrt(5)*x - 5*
x))*sqrt(5*sqrt(5) + 11)) - 4*sqrt(10)*x*sqrt(5*sqrt(5) - 11)*arctan(1/40*(
sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt
(10)*(3*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) - sqrt(10)*x*sqrt(5*sqrt(5)
```

```
 - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)
*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5)
+ 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)
*(2*sqrt(5) - 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sq
rt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - 40)/x
```

giac [A] time = 0.54, size = 152, normalized size = 0.88

$$\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(5
0*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5) -
 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/x
```

maple [A] time = 0.03, size = 211, normalized size = 1.23

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{}{\sqrt{-}}\right.}{2\sqrt{-2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-3*x^4+1),x)
```

```
[Out] 1/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/2/(-2+2*5^(1/2))^(1/2)*ar
ctan(2/(-2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2/(
-2+2*5^(1/2))^(1/2)*x)-1/x+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2)
)^(1/2)*x)+3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)
*x)-1/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)+3/10*5^(1/2)/(2
+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\frac{1}{2}\int\frac{x^2+2}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{x^2-2}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 - 2
)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 1.34, size = 250, normalized size = 1.45

$$-\frac{1}{x}-\frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,1155i}{2\left(3025\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,517i}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{-50\sqrt{5}-110}\,1i}{20}+\frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,1155i}{2\left(3025\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\sqrt{110}}{2\left(3025\right.}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(110 - 50*5^(1/2))^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/
2)*x*(110 - 50*5^(1/2))^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(110 - 50*5^
(1/2))^(1/2)*1i)/20 - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*1155i)/(2*(3025*5
^(1/2) + 6765)) + (5^(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*517i)/(2*(3025*5^(1
/2) + 6765)))*(- 50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 11
0)^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(
1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 - (at
an((x*(50*5^(1/2) + 110)^(1/2)*1155i)/(2*(3025*5^(1/2) + 6765)) + (5^(1/2)*
x*(50*5^(1/2) + 110)^(1/2)*517i)/(2*(3025*5^(1/2) + 6765)))*(50*5^(1/2) + 1
10)^(1/2)*1i)/20 - 1/x
```

sympy [A] time = 1.25, size = 63, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 3
69792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*l
og(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

3.402 $\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx$

Optimal. Leaf size=182

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan}{2\sqrt{5}}$$

```
[Out] -1/3/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5^(1/2))^(1/
4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5
^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*
(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4
)*2^(3/4))*(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1368, 1422, 212, 206, 203}

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(
4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]
))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*2^(3/4)*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
```

```
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{9-3x^4}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{3x^3}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)^{3/2}}{}\\
&= -\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(843+377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\right.}\right.}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 166, normalized size = 0.91

$$-\frac{1}{3x^3}+\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tanh}{\sqrt{10\left(1\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
 Sqrt[5])]
```

fricas [B] time = 0.93, size = 327, normalized size = 1.80

$$\frac{12\sqrt{10}\,x^3\sqrt{13\sqrt{5}+29}\,\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(2\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x-5x\right)\right)\sqrt{13\sqrt{5}+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(12*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*arctan(1/20*(sqrt(10)*sqrt(2*x
^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x
 - 5*x))*sqrt(13*sqrt(5) + 29)) + 12*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*arc
tan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)
```

```
) - 2*sqrt(10)*(2*sqrt(5)*x + 5*x))*sqrt(13*sqrt(5) - 29)) - 3*sqrt(10)*x^3
*sqrt(13*sqrt(5) - 29)*log(sqrt(10)*sqrt(13*sqrt(5) - 29)*(7*sqrt(5) + 15)
+ 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(-sqrt(10)*sqrt(13*sqrt(5
) - 29)*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*log
(sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) + 20*x) - 3*sqrt(10)*x^3*s
qrt(13*sqrt(5) + 29)*log(-sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) +
 20*x) - 40)/x^3
```

giac [A] time = 0.67, size = 152, normalized size = 0.84

$$-\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{130\sqrt{5}-290}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqrt(
5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) - 2
90)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290)*lo
g(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log(abs(
x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

maple [A] time = 0.03, size = 209, normalized size = 1.15

$$\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}+\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{2\sqrt{5}\,\operatorname{arcta}}{5\sqrt{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-3*x^4+1),x)
```

```
[Out] 2/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/(2+2*5^(
1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+2/5*5^(1/2)/(-2+2*5^(1/2))^(1/
2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5
^(1/2))^(1/2)*x)+2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^
(1/2)*x)+1/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+2/5*5^(1/
2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)-1/(2+2*5^(1/2))^(1/2
)*arctan(2/(2+2*5^(1/2))^(1/2)*x)-1/3/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3x^3}-\frac{1}{2}\int\frac{2x^2+3}{x^4+x^2-1}\,dx+\frac{1}{2}\int\frac{2x^2-3}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((2
*x^2 - 3)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 0.20, size = 268, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-130\sqrt{5}-290}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,x\sqrt{-130\sqrt{5}-290}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{-130\sqrt{5}-290}\,1\mathrm{i}}{20}+\operatorname{atan}\left(\frac{x\sqrt{290-130\sqrt{5}}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,x\sqrt{}}{10\left(8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(- 130*5^(1/2) - 290)^(1/2)*20735i)/(2*(87841*5^(1/2) + 196417)) +
 (5^(1/2)*x*(- 130*5^(1/2) - 290)^(1/2)*46371i)/(10*(87841*5^(1/2) + 196417
)))*(- 130*5^(1/2) - 290)^(1/2)*1i)/20 + (atan((x*(290 - 130*5^(1/2))^(1/2)
*20735i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*x*(290 - 130*5^(1/2))^(1/2
)*46371i)/(10*(87841*5^(1/2) - 196417)))*(290 - 130*5^(1/2))^(1/2)*1i)/20 -
 1/(3*x^3) - (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*20735i)/(2*
(87841*5^(1/2) - 196417)) - (5^(1/2)*10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*463
71i)/(10*(87841*5^(1/2) - 196417)))*(13*5^(1/2) - 29)^(1/2)*1i)/20 - (10^(1
/2)*atan((10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*20735i)/(2*(87841*5^(1/2) + 19
6417)) + (5^(1/2)*10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*46371i)/(10*(87841*5^(
1/2) + 196417)))*(13*5^(1/2) + 29)^(1/2)*1i)/20
```

sympy [A] time = 1.28, size = 63, normalized size = 0.35

$$\operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 2
3112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*log
(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

3.403 $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=173

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292}}{}$$

```
[Out] -1/5/x^5-3/x+1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5^(1/2
))^(1/4)*5^(1/2)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5
^(1/2))^(1/4)*5^(1/2)-1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(2889+12
92*5^(1/2))^(1/4)*5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(28
89+1292*5^(1/2))^(1/4)*5^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 1504, 1510, 298, 203, 206}

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1
/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)
^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqr
t[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
```

```
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{15-5x^4}{x^2\left(1-3x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{5}\int \frac{x^2\left(-40+15x^4\right)}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7+3\sqrt{5}\right.}{}\\
&= -\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{46224-20672\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{46224+20672\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{}\right.}{4\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.27, size = 189, normalized size = 1.09

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(7-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*S
qrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5
])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(1 +
Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

fricas [B] time = 1.20, size = 300, normalized size = 1.73

$$-\frac{4\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\,\arctan\left(\frac{1}{4}\left(\sqrt{2x^2+\sqrt{5}-1}\left(3\sqrt{5}\sqrt{2}-7\sqrt{2}\right)-6\sqrt{5}x+14x\right)\sqrt{17\sqrt{5}+38}\right)+4\sqrt{5}x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*(4*sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*arctan(1/4*(sqrt(2*x^2 + sqrt(5)
 - 1)*(3*sqrt(5)*sqrt(2) - 7*sqrt(2)) - 6*sqrt(5)*x + 14*x)*sqrt(17*sqrt(5)
 + 38)) + 4*sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*arctan(1/4*(sqrt(2*x^2 + sqrt
(5) + 1)*(3*sqrt(5)*sqrt(2) + 7*sqrt(2)) - 6*sqrt(5)*x - 14*x)*sqrt(17*sqrt
(5) - 38)) + sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(sqrt(17*sqrt(5) - 38)*(5
*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(-sqrt(17*sqrt
(5) - 38)*(5*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(s
qrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + sqrt(5)*x^5*sqrt(17*sqrt(5)
+ 38)*log(-sqrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + 60*x^4 + 4)/x^5
```

giac [A] time = 0.54, size = 159, normalized size = 0.92

$$\frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(8
5*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5) -
 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

maple [A] time = 0.04, size = 216, normalized size = 1.25

$$\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{3\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{3\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{7\sqrt{5}\,\mathrm{ar}}{10\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-3*x^4+1),x)
```

```
[Out] -1/5/x^5-3/x-7/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)
*x)+3/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-7/10*5^(1/2)/(
-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)-3/2/(-2+2*5^(1/2))^(1/
2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+7/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arcta
nh(2/(-2+2*5^(1/2))^(1/2)*x)+3/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/
2))^(1/2)*x)+7/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*
x)-3/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{15\,x^4+1}{5\,x^5}-\frac{1}{2}\int\frac{3\,x^2+5}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{3\,x^2-5}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/2
*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 1.49, size = 257, normalized size = 1.49

$$-\frac{3\,x^4+\frac{1}{5}}{x^5}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-85\,\sqrt{5}-190}\,372096\mathrm{i}}{2550408\,\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\,\sqrt{-85\,\sqrt{5}-190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{-85\,\sqrt{5}-190}\,1\mathrm{i}}{10}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{190-85\,\sqrt{5}}\,372096\mathrm{i}}{2550408\,\sqrt{5}-5702888}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (5
^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888)))
*(190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(- 85*5^(1/2) - 190)^(1/2)*3720
96i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(- 85*5^(1/2) - 190)^(1/2)*83
2048i)/(5*(2550408*5^(1/2) + 5702888)))*(- 85*5^(1/2) - 190)^(1/2)*1i)/10 +
 (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (
5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888))
)*(85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/2)*37209
6i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1/2)*83204
8i)/(5*(2550408*5^(1/2) + 5702888)))*(85*5^(1/2) + 190)^(1/2)*1i)/10 - (3*x
^4 + 1/5)/x^5
```

sympy [A] time = 1.29, size = 73, normalized size = 0.42

$$\mathrm{RootSum}\left(6400t^4-6080t^2-1,\left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right)+\mathrm{RootSum}\left(6400t^4+6080t^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
 - 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambda(
_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4 - 1
)/(5*x**5)
```

3.404 $\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{}}{}$$

```
[Out] -1/7/x^7-1/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(39603-17711*5^
(1/2))^(1/4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*
(39603-17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(
1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*
x*(3+5^(1/2))^(1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 1504, 1422, 212, 206, 203}

$$-\frac{1}{x^3}-\frac{1}{7x^7}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqr
t[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((
3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*
ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])
/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
```

```
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{21-7x^4}{x^4\left(1-3x^4+x^8\right)}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{21}\int \frac{-168+63x^4}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,d\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{1}{2}\\
&= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\,\mathrm{ta}}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 189, normalized size = 1.00

$$-\frac{1}{7x^7}-\frac{1}{x^3}+\frac{\left(11+5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(11-5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-11-5\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{(5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(
2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]
*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[Sqrt[
2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

fricas [B] time = 1.04, size = 332, normalized size = 1.76

$$\frac{28\sqrt{10}\,x^7\sqrt{89\sqrt{5}+199}\,\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2\,x^2+\sqrt{5}-1}\left(11\sqrt{5}\sqrt{2}-25\sqrt{2}\right)-2\sqrt{10}\left(11\sqrt{5}\,x-25\,x\right)\right)\sqrt{8\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/280*(28*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*arctan(1/40*(sqrt(10)*sqrt(2*
x^2 + sqrt(5) - 1)*(11*sqrt(5)*sqrt(2) - 25*sqrt(2)) - 2*sqrt(10)*(11*sqrt(
5)*x - 25*x))*sqrt(89*sqrt(5) + 199)) + 28*sqrt(10)*x^7*sqrt(89*sqrt(5) - 1
99)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(11*sqrt(5)*sqrt(2) + 2
5*sqrt(2)) - 2*sqrt(10)*(11*sqrt(5)*x + 25*x))*sqrt(89*sqrt(5) - 199)) - 7*
sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(sqrt(10)*sqrt(89*sqrt(5) - 199)*(9*
sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(-sqrt(10)
*sqrt(89*sqrt(5) - 199)*(9*sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*s
qrt(5) + 199)*log(sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5) - 20) + 10*x)
- 7*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*log(-sqrt(10)*sqrt(89*sqrt(5) + 199
)*(9*sqrt(5) - 20) + 10*x) - 280*x^4 - 40)/x^7
```

giac [A] time = 0.56, size = 159, normalized size = 0.84

$$-\frac{1}{20}\sqrt{890\sqrt{5}-1990}\,\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{890\sqrt{5}+1990}\,\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}-19\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*sqr
t(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5)
- 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 1990)*lo
g(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

maple [A] time = 0.04, size = 216, normalized size = 1.14

$$\frac{11\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{5\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{11\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{5\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{11\sqrt{5}\ldots}{1\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-3*x^4+1),x)
```

```
[Out] -1/7/x^7-1/x^3+11/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1
/2)*x)-5/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+11/10*5^(1/
2)/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+5/2/(-2+2*5^(1/2))
^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+11/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*
arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+5/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*
5^(1/2))^(1/2)*x)+11/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^
(1/2)*x)-5/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7\,x^4+1}{7\,x^7}-\frac{1}{2}\int\frac{5\,x^2+8}{x^4+x^2-1}\,dx+\frac{1}{2}\int\frac{5\,x^2-8}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2*
integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 0.21, size = 291, normalized size = 1.54

$$-\frac{x^4+\frac{1}{7}}{x^7}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-89\,\sqrt{5}-199}\,6677047\mathrm{i}}{2\left(74049691\,\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-89\,\sqrt{5}-199}\,14930373\mathrm{i}}{10\left(74049691\,\sqrt{5}+165580139\right)}\right)\sqrt{-89\,\sqrt{5}-199}\,1\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*6677047i)/(2*(7404969
1*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*14
930373i)/(10*(74049691*5^(1/2) + 165580139)))*(- 89*5^(1/2) - 199)^(1/2)*1i
)/20 - (x^4 + 1/7)/x^7 + (10^(1/2)*atan((10^(1/2)*x*(199 - 89*5^(1/2))^(1/2
)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(199 -
 89*5^(1/2))^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(199 - 8
9*5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) - 199)^(1/
2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(89*5
^(1/2) - 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(89*5^(
1/2) - 199)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) + 199)^(1
/2)*6677047i)/(2*(74049691*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(89*
5^(1/2) + 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) + 165580139)))*(89*5^
(1/2) + 199)^(1/2)*1i)/20
```

sympy [A] time = 1.30, size = 70, normalized size = 0.37

$$\mathrm{RootSum}\left(6400t^4-15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)+\mathrm{RootSum}\left(6400t^4+15920t^2-1,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
- 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(_t
, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7*x*
*7)
```

3.405 $\int \frac{x^3}{2+3x^4+x^8}\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

```
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{2+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{2+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\ &= \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

fricas [A] time = 0.88, size = 17, normalized size = 0.81

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

giac [A] time = 0.28, size = 17, normalized size = 0.81

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln\left(x^4+1\right)}{4}-\frac{\ln\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+2),x)
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{256}{9\left(144x^4+160\right)}-\frac{7}{9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + x^8 + 2),x)
[Out] -atanh(256/(9*(144*x^4 + 160)) - 7/9)/2
```

sympy [A] time = 0.12, size = 15, normalized size = 0.71

$$\frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+2),x)
[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

3.406 $\int \frac{x^{11}}{2+3x^4+x^8} \, dx$

Optimal. Leaf size=26

$$\frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

```
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(2 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{2+3x^4+x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{2+3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{-2-3x}{2+3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(2 + 3*x^4 + x^8),x]
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

fricas [A] time = 0.92, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

giac [A] time = 0.36, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{x^4}{4}+\frac{\ln\left(x^4+1\right)}{4}-\ln\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+2),x)
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

maxima [A] time = 0.86, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

mupad [B] time = 1.32, size = 22, normalized size = 0.85

$$\frac{\ln\left(x^4+1\right)}{4}-\ln\left(x^4+2\right)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(3*x^4 + x^8 + 2),x)
```

```
[Out] log(x^4 + 1)/4 - log(x^4 + 2) + x^4/4
```

sympy [A] time = 0.13, size = 19, normalized size = 0.73

$$\frac{x^4}{4}+\frac{\log\left(x^4+1\right)}{4}-\log\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+2),x)
```

```
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

3.407 $\int \frac{x^9}{2+x^5+x^{10}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[x^9/(2 + x^5 + x^10),x]

[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{2+x^5+x^{10}}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x}{2+x+x^2}\,dx, x, x^5\right)\\ &= -\left(\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)\right)+\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{2+x+x^2}\,dx, x, x^5\right)\\ &= \frac{1}{10}\log\left(2+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + x^5 + x^10),x]
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

fricas [A] time = 1.08, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

giac [A] time = 2.72, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{7}\,\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)}{35}+\frac{\ln\left(x^{10}+x^5+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^10+x^5+2),x)
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 2.05, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

mupad [B] time = 1.35, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^5 + x^10 + 2),x)
```

```
[Out] log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

sympy [A] time = 0.14, size = 37, normalized size = 1.00

$$\frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**10+x**5+2),x)
```

```
[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

3.408 $\int \frac{x^4}{2+x^5+x^{10}}\,dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{2+x^5+x^{10}}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)\\
&= -\left(\frac{2}{5}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)\right)\\
&= \frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

fricas [A] time = 0.96, size = 18, normalized size = 0.78

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

giac [A] time = 2.96, size = 18, normalized size = 0.78

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{7}\,\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10+x^5+2),x)
```

```
[Out] 2/35*7^(1/2)*arctan(1/7*(2*x^5+1)*7^(1/2))
```

maxima [A] time = 2.09, size = 18, normalized size = 0.78

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
```

```
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

mupad [B] time = 1.33, size = 20, normalized size = 0.87

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^5 + x^10 + 2),x)
```

```
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

sympy [A] time = 0.13, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\,\operatorname{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10+x**5+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

3.409 $\int \frac{1}{x(1+x^5+x^{10})} dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\\
&= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)+\frac{1}{5}\,\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)-\frac{1}{10}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}$$

Mathematica [C] time = 0.04, size = 197, normalized size = 5.05

$$-\frac{1}{5}\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)}{8\#1^7-7\#}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

fricas [A] time = 0.87, size = 32, normalized size = 0.82

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

giac [A] time = 0.41, size = 33, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

maple [B] time = 0.04, size = 66, normalized size = 1.69

$$-\frac{\sqrt{3}\ \arctan\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}+\ln(x)-\frac{\ln\left(x^2+x+1\right)}{10}-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^10+x^5+1),x)
```

```
[Out] -1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)*arctan(2/3*3^(1/
2)*x^5+1/3*3^(1/2))+ln(x)-1/10*ln(x^2+x+1)
```

maxima [A] time = 2.03, size = 36, normalized size = 0.92

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
1/5*log(x^5)
```

mupad [B] time = 0.06, size = 34, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

sympy [A] time = 0.17, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**10+x**5+1),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

3.410 $\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log(x)$$

```
[Out] -1/5/x^5-ln(x)+1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5
 + x^10]/10
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}+\frac{1}{5}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right)dx, x, x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{x}{1+x+x^2}\,dx, x, x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^5\right)+\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^5
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^5\right)\\
&= -\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 208, normalized size = 4.33

$$
\frac{1}{30}\left(6\operatorname{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&, \frac{4\#1^7\log(x-\#1)-4\#1^6\log(x-\#1)+\#1^5\log(x-\#1)}{8\#}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] (-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x + x
^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #1]
 + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1]*#
1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#
1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

fricas [A] time = 0.80, size = 49, normalized size = 1.02

$$
-\frac{2\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-3x^5\log\left(x^{10}+x^5+1\right)+30x^5\log(x)+6}{30x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^5
 + 1) + 30*x^5*log(x) + 6)/x^5
```

giac [A] time = 0.25, size = 45, normalized size = 0.94

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)+\frac{x^5-1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*lo
g(x^10 + x^5 + 1) - log(abs(x))
```

maple [A] time = 0.02, size = 73, normalized size = 1.52

$$-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\ln(x)+\frac{\ln\left(x^2+x+1\right)}{10}+\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^10+x^5+1),x)
```

```
[Out] 1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)*arctan(2/3*3^(1/2
)*x^5+1/3*3^(1/2))-1/5/x^5-ln(x)+1/10*ln(x^2+x+1)
```

maxima [A] time = 2.07, size = 41, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 + x
^5 + 1) - 1/5*log(x^5)
```

mupad [B] time = 1.37, size = 41, normalized size = 0.85

$$\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\ln(x)-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15 - 1/(5*x^5)
```

sympy [A] time = 0.20, size = 48, normalized size = 1.00

$$-\log\left(x\right)+\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**10+x**5+1),x)
```

```
[Out] -log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15 - 1/(5*x**5)
```

3.411 $\int \frac{1}{x+x^6+x^{11}}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1594, 1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^6 + x^11)^(-1),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x+x^6+x^{11}}\,dx &= \int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx\\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx, x, x^5\right)\\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^5\right)\\
&= \log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^5\right)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 197, normalized size = 5.05

$$-\frac{1}{5}\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)}{8\#1^7-7\#}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^6 + x^11)^(-1),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

fricas [A] time = 0.90, size = 32, normalized size = 0.82

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

giac [A] time = 0.35, size = 33, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

maple [B] time = 0.02, size = 66, normalized size = 1.69

$$-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}+\ln(x)-\frac{\ln\left(x^2+x+1\right)}{10}-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11+x^6+x),x)
```

```
[Out] -1/15*3^(1/2)*arctan(2/3*3^(1/2)*x^5+1/3*3^(1/2))+ln(x)-1/10*ln(x^2+x+1)-1/
10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{5}\int\frac{4x^7-3x^6-x^5+3x^4-x^3+2x^2-x}{x^8-x^7+x^5-x^4+x^3-x+1}\,dx-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

```
[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6 -
 x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x) -
 1/10*log(x^2 + x + 1) + log(x)
```

mupad [B] time = 0.03, size = 34, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^6 + x^11),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

sympy [A] time = 0.17, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

3.412 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} dx$

Optimal. Leaf size=147

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}-\frac{bx(b^2-2ac)}{c^4}+\frac{x^2(b^2-ac)}{2c^3}$$

```
[Out] -b*(-2*a*c+b^2)*x/c^4+1/2*(-a*c+b^2)*x^2/c^3-1/3*b*x^3/c^2+1/4*x^4/c+1/2*(a
^2*c^2-3*a*b^2*c+b^4)*ln(c*x^2+b*x+a)/c^5+b*(5*a^2*c^2-5*a*b^2*c+b^4)*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx(b^2-2ac)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*
x + c*x^2])/(2*c^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \int \frac{x^5}{a+bx+cx^2}\,dx$$
$$= \int \left(-\frac{b\left(b^2-2ac\right)}{c^4}+\frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}+\frac{ab\left(b^2-2ac\right)+\left(b^4-3ab^2c+a^2c^2\right)x}{c^4\left(a+bx+cx^2\right)}\right)dx$$
$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\int \frac{ab\left(b^2-2ac\right)+\left(b^4-3ab^2c+a^2c^2\right)x}{a+bx+cx^2}\,dx}{c^4}$$
$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\left(b^4-3ab^2c+a^2c^2\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^5}-\frac{\left(b\left(b^4-5ab\right.\right.}{}$$
$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\left(b^4-3ab^2c+a^2c^2\right)\log\left(a+bx+cx^2\right)}{2c^5}+\frac{\left(b\left(b^4\right.\right.}{}$$
$$= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{b\left(b^4-5ab^2c+5a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{\left(b^4\right.}{}$$

Mathematica [A] time = 0.12, size = 140, normalized size = 0.95

$$\frac{6\left(a^2c^2-3ab^2c+b^4\right)\log(a+x(b+cx))-\frac{12b\left(5a^2c^2-5ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx\left(-4bc\left(cx^2-6a\right)+3c^2x\left(cx^2-2a\right)\right.}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
- (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x)])
/(12*c^5)
```

fricas [A] time = 0.90, size = 466, normalized size = 3.17

$$\left[\frac{3\left(b^2c^4-4\,ac^5\right)x^4-4\left(b^3c^3-4\,abc^4\right)x^3+6\left(b^4c^2-5\,ab^2c^3+4\,a^2c^4\right)x^2+6\left(b^5-5\,ab^3c+5\,a^2bc^2\right)\sqrt{b^2-4\,ac}\,\log}{12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2
 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
```

```
x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(
b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^
5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3
 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a
^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

giac [A] time = 0.29, size = 145, normalized size = 0.99

$$\frac{3\,c^3x^4-4\,bc^2x^3+6\,b^2cx^2-6\,ac^2x^2-12\,b^3x+24\,abcx}{12\,c^4}+\frac{\left(b^4-3\,ab^2c+a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\,c^5}-\frac{\left(b^5-5\,ab^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*a
*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 - (b
^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^5)
```

maple [A] time = 0.01, size = 236, normalized size = 1.61

$$\frac{x^4}{4c}-\frac{b\,x^3}{3c^2}-\frac{5a^2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3}+\frac{5a\,b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^4}-\frac{a\,x^2}{2c^2}-\frac{b^5\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^5}+\frac{b^2x^2}{2c^3}+\frac{a^2\ln\left(c\,x^2+bx+\right.}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+a/x^2+b/x),x)
```

```
[Out] 1/4/c*x^4-1/3*b*x^3/c^2-1/2/c^2*x^2*a+1/2/c^3*x^2*b^2+2/c^3*a*b*x-1/c^4*b^3
*x+1/2/c^3*ln(c*x^2+b*x+a)*a^2-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2+1/2/c^5*ln(c*x
^2+b*x+a)*b^4-5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
^2*b+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^
5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.16, size = 183, normalized size = 1.24

$$x\left(\frac{b\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)}{c}+\frac{a\,b}{c^3}\right)+\frac{x^4}{4\,c}-x^2\left(\frac{a}{2\,c^2}-\frac{b^2}{2\,c^3}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\left(-4\,a^3\,c^3+13\,a^2\,b^2\,c^2-7\,a\,b^4\,c+b^6\right)}{2\left(4\,a\,c^6-b^2\,c^5\right)}-\frac{b\,x^3}{3\,c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c + a/x^2 + b/x),x)
```

```
[Out] x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2/
(2*c^3)) - (log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*b^
4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*atan((b + 2*c*x)/(4*a*
c - b^2)^(1/2))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.25, size = 605, normalized size = 4.12

$$-\frac{bx^3}{3c^2}+x^2\left(-\frac{a}{2c^2}+\frac{b^2}{2c^3}\right)+x\left(\frac{2ab}{c^3}-\frac{b^3}{c^4}\right)+\left(-\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2-5ab^2c+b^4\right)}{2c^5\left(4ac-b^2\right)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x+\frac{2a^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b**
3/c**4) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5
*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**
3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a**2*
c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c
 + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b
**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*
c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b**2)*(5*a*
*2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**
2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c
**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c
 - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(b*sqrt(-
4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (
a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**
5)) + x**4/(4*c)
```

3.413 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=118

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

```
[Out] (-a*c+b^2)*x/c^3-1/2*b*x^2/c^2+1/3*x^3/c-1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a)
/c^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1354, 701, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (
b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \int \frac{x^4}{a+bx+cx^2}\,dx$$
$$= \int \left(\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x}{c^3\left(a+bx+cx^2\right)}\right)dx$$
$$= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\int \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{c^3}$$
$$= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^4}+\frac{\left(b^4-4ab^2c+2a^2c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^4}$$
$$= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\text{Subst}\left(\int \frac{}{b^2-}\right.}{c^4}$$
$$= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+\right.}{2c^4}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.95

$$\frac{\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-3\left(b^3-2abc\right)\log(a+x(b+cx))+cx\left(-6ac+6b^2-3bcx+2c^2x^2\right)}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^
2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 - 2*
a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

fricas [A] time = 0.95, size = 383, normalized size = 3.25

$$\left[\frac{2\left(b^2c^3-4\,ac^4\right)x^3-3\left(b^3c^2-4\,abc^3\right)x^2+3\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b}{cx^2+bx+a}\right.}{6\left(b^2c^4-4\,ac^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4*
a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*b^2
*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a
))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b
*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*
```

```
c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 -
 4*a*c^5)]
```

giac [A] time = 0.39, size = 113, normalized size = 0.96

$$\frac{2\,c^2x^3 - 3\,bcx^2 + 6\,b^2x - 6\,acx}{6\,c^3} - \frac{\left(b^3 - 2\,abc\right)\log\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*l
og(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

maple [A] time = 0.00, size = 190, normalized size = 1.61

$$\frac{x^3}{3c} + \frac{2a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{4a\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3} + \frac{b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^4} - \frac{b\,x^2}{2c^2} + \frac{ab\ln\left(c\,x^2 + bx + a\right)}{c^3} - \frac{ax}{c^2} - \frac{b^3\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+a/x^2+b/x),x)
```

```
[Out] 1/3/c*x^3-1/2*b*x^2/c^2-1/c^2*a*x+1/c^3*b^2*x+1/c^3*ln(c*x^2+b*x+a)*a*b-1/2
/c^4*ln(c*x^2+b*x+a)*b^3+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^2-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
*b^2+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.40, size = 151, normalized size = 1.28

$$\frac{x^3}{3\,c} - x\left(\frac{a}{c^2} - \frac{b^2}{c^3}\right) - \frac{b\,x^2}{2\,c^2} + \frac{\ln\left(c\,x^2 + b\,x + a\right)\,\left(8\,a^2\,b\,c^2 - 6\,a\,b^3\,c + b^5\right)}{2\,\left(4\,a\,c^5 - b^2\,c^4\right)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a^2\,c^2 - 4\,a\right.}{c^4\,\sqrt{4\,a\,c - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c + a/x^2 + b/x),x)
```

```
[Out] x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*(
b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c -
b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c
^4*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.01, size = 498, normalized size = 4.22

$$-\frac{bx^2}{2c^2} + x\left(-\frac{a}{c^2} + \frac{b^2}{c^3}\right) + \left(\frac{b\left(2ac - b^2\right)}{2c^4} - \frac{\sqrt{-4ac + b^2}\,\left(2a^2c^2 - 4ab^2c + b^4\right)}{2c^4\left(4ac - b^2\right)}\right)\log\left(x + \frac{-3a^2bc + ab^3 + 4ac^4\left(\frac{b(2ac-}{2c^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) - s
qrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)
))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) - sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))
) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2
 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c +
 b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4
*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a*b**3 +
4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*
c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*
c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

3.414 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[Out] -b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \int \frac{x^3}{a+bx+cx^2}\,dx$$
$$= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx$$
$$= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2}$$
$$= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}$$
$$= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^3}$$
$$= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.94

$$\frac{\left(b^2-ac\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

fricas [A] time = 0.88, size = 297, normalized size = 3.34

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)x^2-\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^3c-4\,abc^2\right)x+\left(b^4-5\,ab^2\right.}{2\left(b^2c^3-4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.26, size = 86, normalized size = 0.97

$$\frac{cx^2-2\,bx}{2\,c^2}+\frac{\left(b^2-ac\right)\log\left(cx^2+bx+a\right)}{2\,c^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [A] time = 0.00, size = 132, normalized size = 1.48

$$\frac{3ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2}-\frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3}+\frac{x^2}{2c}-\frac{a\ln\left(c\,x^2+bx+a\right)}{2c^2}+\frac{b^2\ln\left(c\,x^2+bx+a\right)}{2c^3}-\frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x),x)
```

```
[Out] 1/2/c*x^2-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.13, size = 112, normalized size = 1.26

$$\frac{x^2}{2\,c}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)}-\frac{b\,x}{c^2}+\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + a/x^2 + b/x),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.84, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\,\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\right.}{3abc-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

3.415 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=70

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1340, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx\\
&= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\
&= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\
&= \frac{x}{c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.04

$$\frac{\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

fricas [A] time = 0.79, size = 235, normalized size = 3.36

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2c-4\,ac^2\right)x+\left(b^3-4\,abc\right)\log\left(cx^2+bx+a\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.27, size = 67, normalized size = 0.96

$$\frac{x}{c}-\frac{b\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 101, normalized size = 1.44

$$-\frac{2a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{b\ln\left(cx^2+bx+a\right)}{2c^2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x),x)
```

```
[Out] 1/c*x-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.42, size = 172, normalized size = 2.46

$$\frac{x}{c}+\frac{b^3\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}+\frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}}-\frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x\right.}{4\,a\,c^3-b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

sympy [B] time = 0.60, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right.}{2ac-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
```

```
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

3.416 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2
)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \int \frac{x}{a+bx+cx^2}\,dx$$
$$= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}$$
$$= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx)) - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

fricas [A] time = 0.90, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)}, \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

giac [A] time = 0.29, size = 55, normalized size = 0.98

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c} + \frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

maple [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} + \frac{\ln\left(c\,x^2+bx+a\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x,x)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.17, size = 112, normalized size = 2.00

$$\frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

sympy [B] time = 0.32, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

3.417 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] 2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

fricas [A] time = 0.81, size = 120, normalized size = 3.33

$$\left[\frac{\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)}{\sqrt{b^{2}-4\,ac}}, -\frac{2\,\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)}{b^{2}-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.35, size = 34, normalized size = 0.94

$$\frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^2,x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.05, size = 46, normalized size = 1.28

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

sympy [B] time = 0.22, size = 124, normalized size = 3.44

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}}{2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

3.418 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
```

```
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

fricas [A] time = 0.89, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)+2\left(b^2-4\,ac\right)\log(x)}{2\left(ab^2-4\,a^2c\right)},\ \frac{2\sqrt{-b}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.30, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2 \, cx + b}{\sqrt{-b^{2} + 4 \, ac}}\right)}{\sqrt{-b^{2} + 4 \, ac} a} - \frac{\log\left(cx^{2} + bx + a\right)}{2 \, a} + \frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\, a} + \frac{\ln(x)}{a} - \frac{\ln\left(c\, x^{2}+bx+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^3,x)
```

```
[Out] 1/a*ln(x)-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.72, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{a} - \ln\left(b\,c - \left(x\,\left(6\,a\,c^2 - 2\,b^2\,c\right) - a\,b\,c\right)\,\left(\frac{1}{2\,a} - \frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right) + 3\,c^2\,x\right)\,\left(\frac{1}{2\,a} - \frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right) - \ln\left(\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

sympy [B] time = 4.08, size = 564, normalized size = 9.10

$$\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a\left(4ac-b^{2}\right)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^{4}c^{2}\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a\left(4ac-b^{2}\right)} - \frac{1}{2a}\right)^{2} - 14a^{3}b^{2}c\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a\left(4ac-b^{2}\right)} - \frac{1}{2a}\right)^{2} - 12a^{3}c^{2}\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a\left(4ac-b^{2}\right)}\right.}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**3,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

3.419 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx &= \int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{ax}+\frac{\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx}{a} \\
&= -\frac{1}{ax}+\frac{\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx}{a} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^2} \\
&= -\frac{1}{ax}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.95

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

fricas [A] time = 0.95, size = 269, normalized size = 3.32

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+a\right)}{2\left(a^2b^2-4\,a^3c\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

giac [A] time = 0.39, size = 79, normalized size = 0.98

$$\frac{b \log\left(cx^2 + bx + a\right)}{2 a^2} - \frac{b \log\left(|x|\right)}{a^2} + \frac{\left(b^2 - 2 ac\right) \arctan\left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{\sqrt{-b^2 + 4 ac}\, a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

maple [A] time = 0.01, size = 112, normalized size = 1.38

$$-\frac{2c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\; a} + \frac{b^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\; a^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln\left(c\, x^2 + bx + a\right)}{2a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^4,x)
```

```
[Out] -1/a/x-1/a^2*b*ln(x)+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.81, size = 339, normalized size = 4.19

$$\frac{\ln\Big(2\, a\, b^3 + 2\, b^4\, x - 2\, a\, b^2\, \sqrt{b^2 - 4\, a\, c} + a^2\, c\, \sqrt{b^2 - 4\, a\, c} - 2\, b^3\, x\, \sqrt{b^2 - 4\, a\, c} + 2\, a^2\, c^2\, x - 7\, a^2\, b\, c - 8\, a\, b^2\, c\, x +}{4\, a^3\, c - a^2\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
```

```
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] Timed out
```

3.420 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal. Leaf size=104

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3+
b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx &= \int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{2ax^2}+\frac{\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx}{a} \\
&= -\frac{1}{2ax^2}+\frac{\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx}{a} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^3}-\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\left(b\left(b^2-3ac\right)\right)\mathrm{Subst}\left(\ldots\right.}{\ldots} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx\ldots\right.}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+2\log(x)\left(b^2-ac\right)+\left(ac-b^2\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

fricas [A] time = 1.06, size = 358, normalized size = 3.44

$$\left[-\frac{\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+\left(b^4-5\,ab^2c+4\,a^2c^2\right)x^2\log\left(c\ldots\right.}{2\left(a^3b^2-4\,a^4c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

giac [A] time = 0.31, size = 105, normalized size = 1.01

$$-\frac{\left(b^{2}-ac\right)\log\left(cx^{2}+bx+a\right)}{2\,a^{3}}+\frac{\left(b^{2}-ac\right)\log\left(|x|\right)}{a^{3}}-\frac{\left(b^{3}-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}\,a^{3}}+\frac{2\,abx-a^{2}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

maple [A] time = 0.01, size = 150, normalized size = 1.44

$$\frac{3bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\;a^{2}}-\frac{b^{3}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\;a^{3}}-\frac{c\ln(x)}{a^{2}}+\frac{c\ln\left(c\,x^{2}+bx+a\right)}{2a^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln\left(c\,x^{2}+bx+a\right)}{2a^{3}}+\frac{b}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^5,x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+b/a^2/x+1/2/a^2*c*ln(c*x^2+b*x+a)-
1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.87, size = 447, normalized size = 4.30

$$\frac{\ln\left(2\,a\,b^{4}+2\,b^{5}\,x+6\,a^{3}\,c^{2}+2\,a\,b^{3}\,\sqrt{b^{2}-4\,a\,c}+2\,b^{4}\,x\,\sqrt{b^{2}-4\,a\,c}-9\,a^{2}\,b^{2}\,c-10\,a\,b^{3}\,c\,x-3\,a^{2}\,b\,c\,\sqrt{b^{2}-4\,a\,c}\right.}{4\,a^{4}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**5,x)
```

```
[Out] Timed out
```

3.421 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal. Leaf size=137

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2
*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{b}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
```

```
x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3\left(a+bx+cx^2\right)}\,dx}{a}$$

$$= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a^3\left(a+bx+cx^2\right)}\right)dx}{a}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{a^4}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4} + \frac{\left(b^4-4\right.}{}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4} - \frac{\left(b^4\right.}{}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4}$$

Mathematica [A] time = 0.10, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3} + \frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3a^2b}{x^2} - 6\log(x)\left(b^3-2abc\right) + 3\left(b^3-2abc\right)\log(a+x(b+cx)) + \frac{6a\left(ac-b^2\right)}{x}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

fricas [A] time = 1.04, size = 445, normalized size = 3.25

$$\left[\frac{3\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{b^2-4\,ac}\,x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\left(b^5-6\,ab^3c+\right.}{6\left(a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

giac [A] time = 0.38, size = 136, normalized size = 0.99

$$\frac{\left(b^3-2\,abc\right)\log\left(cx^2+bx+a\right)}{2\,a^4}-\frac{\left(b^3-2\,abc\right)\log\left(|x|\right)}{a^4}+\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^4}+\frac{3\,a^2bx-2\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

maple [A] time = 0.01, size = 214, normalized size = 1.56

$$\frac{2c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^2}-\frac{4b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^3}+\frac{b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^4}+\frac{2bc\ln(x)}{a^3}-\frac{bc\ln\left(c\,x^2+bx+a\right)}{a^3}-\frac{b^3\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^6,x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2*b/a^2/x^
2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.92, size = 524, normalized size = 3.82

$$\ln\Big(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}\,-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}\,+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}\,-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] Timed out
```

3.422 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}$$

```
[Out] -b*(-11*a*c+3*b^2)*x/c^3/(-4*a*c+b^2)+1/2*(-8*a*c+3*b^2)*x^2/c^2/(-4*a*c+b^
2)-b*x^3/c/(-4*a*c+b^2)+x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^2*
c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2
)^(3/2)+1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/c^4
```

Rubi [A] time = 0.20, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1354, 738, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^
2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Lo
g[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx=\int \frac{x^5}{\left(a+bx+cx^2\right)^2}\,dx$$

$$=\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{x^3(8a+3bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$=\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \left(\frac{b\left(3b^2-11ac\right)}{c^3}-\frac{\left(3b^2-8ac\right)x}{c^2}+\frac{3bx^2}{c}-\frac{ab\left(3b^2-11ac\right)+\left(b^2-4ac\right)\left(3b^2-2ac\right)x}{c^3\left(a+bx+cx^2\right)}\right.}{-b^2+4ac}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{ab\left(3b^2-\right.}{}}{}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(3b^2-2a\right.}{}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(3b^2-2a\right.}{}$$

$$=-\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)}+\frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(3b^4-\right.}{}$$

Mathematica [A] time = 0.22, size = 163, normalized size = 0.83

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{2\left(2a^3c^2+a^2bc(5cx-4b)+ab^3(b-5cx)+b^5x\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\left(3b^2-2ac\right)\log(a+x(b+cx))-4bcx+c^2x^2}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(
-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2
) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

fricas [B] time = 0.92, size = 1029, normalized size = 5.25

$$\left[\frac{2\,ab^6 - 16\,a^2b^4c + 36\,a^3b^2c^2 - 16\,a^4c^3 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^4 - 3\left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x^3 - \left(4\,b^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 -
 (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 - 20
*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (
3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2
*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2
*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 + 2*(3*a*b
^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*
x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11*a*b^5*c + 41*a^2
*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*
a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b
^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]
```

giac [A] time = 0.33, size = 188, normalized size = 0.96

$$-\frac{\left(3\,b^5 - 20\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^4 - 4\,ac^5\right)\sqrt{-b^2+4\,ac}} + \frac{\left(3\,b^2 - 2\,ac\right)\log\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{c^2x^2 - 4\,bcx}{2\,c^4} + \frac{ab^4 - 4\,a^2b^2c}{(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c
^4)
```

maple [B] time = 0.01, size = 434, normalized size = 2.21

$$-\frac{5a^2bx}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)c^2}+\frac{30a^2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2}+\frac{5a\,b^3x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)c^3}-\frac{20a\,b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x)^2,x)
```

```
[Out] 1/2/c^2*x^2-2/c^3*b*x-5/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b*x*a^2+5/c^3/(c*x^2+
b*x+a)/(4*a*c-b^2)*b^3*x*a-1/c^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^5*x-2/c^2/(c*x
^2+b*x+a)*a^3/(4*a*c-b^2)+4/c^3/(c*x^2+b*x+a)*a^2/(4*a*c-b^2)*b^2-1/c^4/(c*
x^2+b*x+a)*a/(4*a*c-b^2)*b^4-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a^2+7/c^3/(4
*a*c-b^2)*ln(c*x^2+b*x+a)*a*b^2-3/2/c^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/
c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b-20/c^3/(4*a
*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3+3/c^4/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.82, size = 382, normalized size = 1.95

$$\frac{x^2}{2\,c^2}-\frac{\frac{a\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^4\,x^2+b\,c^3\,x+a\,c^3}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(128\,a^4\,c^4-288\,a^3\,b^2\,c^3+168\,a^2\,b^4\,c^2-38\,a\,b^6\,c\right.}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*(
b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3*x
) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2
*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*
c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*c^2 -
20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30*a^2*b*
c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c - b^2)^(3
/2))
```

sympy [B] time = 2.37, size = 1012, normalized size = 5.16

$$-\frac{2bx}{c^3}+\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\,\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(-\frac{b\sqrt{-(4ac-}}{2c^4(64a^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*
b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2
*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c
**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c
**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b
**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-b*sqrt(-
(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))
)/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b**2)**3)*(3
0*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(b*s
qrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c
**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
 (2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (-2
*a**3*c**2 + 4*a**2*b**2*c - a*b**4 + x*(-5*a**2*b*c**2 + 5*a*b**3*c - b**5
))/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*c**6 - b**2*c**5) + x*(4*a*b*c**5
 - b**3*c**4)) + x**2/(2*c**2)
```

3.423 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx\right.}{c^3}$$

```
[Out] 2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a*
c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+
b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1340, 738, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}-\frac{b\log\left(a+bx\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx=\int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx$$

$$=\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$=\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right)dx}{-b^2+4ac}$$

$$=\frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2\int \frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{c^2\left(b^2-4ac\right)}$$

$$=\frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3}+\frac{\left(b^4-6ab\right.}{}$$

$$=\frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}-\frac{(2(}{}$$

$$=\frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh}{c^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.18, size = 132, normalized size = 0.88

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{\left(b^2-4ac\right)(a+x(b+cx))}-b\log(a+x(b+cx))+cx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

fricas [B] time = 0.91, size = 837, normalized size = 5.58

$$\left[-\frac{ab^5 - 7\,a^2b^3c + 12\,a^3bc^2 - \left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)x^3 - \left(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3\right)x^2 + \left(ab^4 - 6\,a^2b^2c + 6\,a^3c^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

giac [A] time = 0.37, size = 161, normalized size = 1.07

$$\frac{2\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2+4\,ac}} + \frac{x}{c^2} - \frac{b\log\left(cx^2+bx+a\right)}{c^3} - \frac{\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)x}{c} + \frac{ab^3-3\,a^2bc}{c}}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

maple [B] time = 0.01, size = 352, normalized size = 2.35

$$\frac{2a^2x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)c} - \frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c} - \frac{4a\,b^2x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)c^2} + \frac{12a\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2} + \frac{}{\left(c\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)*a^2*b/(4*a
```

```
*c-b^2)+1/c^3/(c*x^2+b*x+a)*a*b^3/(4*a*c-b^2)-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*
x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.80, size = 261, normalized size = 1.74

$$\frac{x}{c^2}+\frac{\frac{a\left(b^3-3abc\right)}{c\left(4ac-b^2\right)}+\frac{x\left(2a^2c^2-4ab^2c+b^4\right)}{c\left(4ac-b^2\right)}}{c^3x^2+bc^2x+ac^2}+\frac{\ln\left(cx^2+bx+a\right)\left(-128a^3bc^3+96a^2b^3c^2-24ab^5c+2b^7\right)}{2\left(64a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3\right)}-\frac{2\,\mathrm{atan}\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*
b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x
^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 -
 b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^
(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 -
 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 1.73, size = 842, normalized size = 5.61

$$\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
```

```
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

3.424 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} dx$

Optimal. Leaf size=114

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{bx}{c(b^2-4ac)}+\frac{\log(a+bx+cx^2)}{2c^2}$$

```
[Out] -b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)
*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^2+
b*x+a)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 738, 773, 634, 618, 206, 628}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{bx}{c(b^2-4ac)}+\frac{\log(a+bx+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
```

```
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx=\int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx$$

$$=\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$=-\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx}{c\left(b^2-4ac\right)}$$

$$=-\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}-\frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2\left(b^2-4ac\right)}$$

$$=-\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}+\frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\right.}{c^2\left(b\right.}$$

$$=-\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(a+b\right.}{2c^2}$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.96

$$\frac{\frac{2\left(-2a^2c+ab(b-3cx)+b^3x\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

fricas [B] time = 0.91, size = 635, normalized size = 5.57

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+\left(ab^3-6\,a^2bc+\left(b^3c-6\,abc^2\right)x^2+\left(b^4-6\,ab^2c\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2}{cx^2}\right.}{2\left(ab^4c^2-8\,a^2b^2c^3+16\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

giac [A] time = 0.36, size = 125, normalized size = 1.10

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{ab^2-2\,a^2c+\left(b^3-3\,abc\right)x}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 209, normalized size = 1.83

$$-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c}+\frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2}+\frac{2a\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)c}-\frac{b^2\ln\left(c\,x^2+bx+a\right)}{2\left(4ac-b^2\right)c^2}+\frac{\frac{\left(3ac-b^2\right)bx}{\left(4ac-b^2\right)c^2}+\frac{\left(2ac-b^2\right)}{\left(4ac-b^2\right)}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x,x)
```

```
[Out] ((3*a*c-b^2)/(4*a*c-b^2)*b/c^2*x+(2*a*c-b^2)/(4*a*c-b^2)*a/c^2)/(c*x^2+b*x+
a)+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b+1/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.86, size = 279, normalized size = 2.45

$$\frac{\frac{a\left(2ac-b^2\right)}{c^2\left(4ac-b^2\right)}+\frac{bx\left(3ac-b^2\right)}{c^2\left(4ac-b^2\right)}}{cx^2+bx+a}-\frac{\ln\left(cx^2+bx+a\right)\left(-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6\right)}{2\left(64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{c^2\left(4ac-b^2\right)^{5/2}\left(\frac{2bx\left(6ac-\right.}{c\left(4ac-b^2\right.}\right.}{b^3-}\right.}{c^2\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)^2),x)
```

```
[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^
2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c
 - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/
(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 1.32, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)+8a^2c+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

3.425 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} dx$

Optimal. Leaf size=71

$$\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)-4*a*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1352, 614, 618, 206}

$$\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\left(c+bx+ax^2\right)^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{b+\frac{2a}{x}}{\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$
$$= \frac{b+\frac{2a}{x}}{\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{(4a)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)}{b^2-4ac}$$
$$= \frac{b+\frac{2a}{x}}{\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{4a\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 1.14

$$\frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))} + \frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.94, size = 387, normalized size = 5.45

$$\left[-\frac{ab^3-4a^2bc+2\left(ac^2x^2+abcx+a^2c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+\left(b^4-6ab^2c+8a^2c^2\right)}{ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^2+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

giac [A] time = 0.38, size = 88, normalized size = 1.24

$$-\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^2-4ac\right)\sqrt{-b^2+4ac}} - \frac{b^2x-2acx+ab}{\left(b^2c-4ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

maple [A] time = 0.01, size = 97, normalized size = 1.37

$$\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{ab}{\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^2,x)
```

```
[Out] (-(2*a*c-b^2)/(4*a*c-b^2)/c*x+1/c*a*b/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.37, size = 135, normalized size = 1.90

$$-\frac{\frac{x\left(2\,a\,c-b^2\right)}{c\left(4\,a\,c-b^2\right)}-\frac{a\,b}{c\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.60, size = 280, normalized size = 3.94

$$-2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2ab}{4ac}\right)+2a\sqrt{-\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
```

```
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

3.426 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 638, 618, 206}

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$
$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 1.05

$$\frac{2a+bx}{\left(b^2-4ac\right)(a+x(b+cx))} - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.86, size = 338, normalized size = 5.12

$$\left[\frac{2\,ab^2-8\,a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}, \frac{2\,ab^2-8\,a^2c}{ab^4-8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

giac [A] time = 0.37, size = 76, normalized size = 1.15

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))

maple [A] time = 0.00, size = 70, normalized size = 1.06

$$-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-bx-2a}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^3,x)

[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.37, size = 110, normalized size = 1.67

$$-\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(c + a/x^2 + b/x)^2),x)

[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)

sympy [B] time = 0.56, size = 253, normalized size = 3.83

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**2/x**3,x)

[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
3) + b2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
2 + x2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))

3.427 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} \, dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 614, 618, 206}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} \, dx = \int \frac{1}{\left(a+bx+cx^2\right)^2} \, dx$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2c)\int \frac{1}{a+bx+cx^2} \, dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx, x, b+2cx\right)}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 1.06

$$-\frac{\frac{4c \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

fricas [B] time = 0.92, size = 341, normalized size = 5.17

$$\left[-\frac{b^3 - 4\,abc + 2\left(c^2x^2 + bcx + ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right) + 2\left(b^2c-4\,ac^2\right)x}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^2 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x}, -\frac{b^3 - \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

giac [A] time = 0.39, size = 76, normalized size = 1.15

$$-\frac{4\,c \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.00, size = 68, normalized size = 1.03

$$\frac{4c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^4,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.08, size = 119, normalized size = 1.80

$$\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.58, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\frac{1}{\left(4ac\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.428 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c*
x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*x+
a)/a^2
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}-\frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2\ldots}\right.}{a^2\left(b^2-\ldots\right.}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+c\ldots\right.}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 107, normalized size = 0.99

$$
\frac{\frac{2a\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}-\log(a+x(b+cx))+2\log(x)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

fricas [B] time = 1.08, size = 781, normalized size = 7.23

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + \left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2}{cx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

giac [A] time = 0.36, size = 126, normalized size = 1.17

$$-\frac{\left(b^3 - 6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2b^2 - 4\,a^3c\right)\sqrt{-b^2 + 4\,ac}} - \frac{\log\left(cx^2 + bx + a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx + ab^2 - 2\,a^2c}{\left(cx^2 + bx + a\right)\left(b^2 - 4\,ac\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

maple [B] time = 0.01, size = 237, normalized size = 2.19

$$-\frac{bcx}{\left(c\,x^2 + bx + a\right)\left(4ac - b^2\right)a} - \frac{6bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}}\,a} + \frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}}\,a^2} - \frac{b^2}{\left(c\,x^2 + bx + a\right)\left(4ac - b^2\right)a} - \frac{2c\ln}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^5,x)
```

```
[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)*c
-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/a^
2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
```

```
*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.10, size = 620, normalized size = 5.74

$$\frac{\ln(x)}{a^2}+\frac{\frac{2\,a\,c-b^2}{a\,\left(4\,a\,c-b^2\right)}-\frac{b\,c\,x}{a\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}+\frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-{\left(4\,a\,c\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] Timed out
```

3.429 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} \, dx$

Optimal. Leaf size=148

$$\frac{b \log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+}{ax\left(b^2-4ac\right)\left(a+\right.}$$

```
[Out] -2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^
2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2}{ax\left(b^2-4ac\right)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x\right)}{a^2\left(a+bx+cx^2\right)}\right)}{a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4\right.}{a+bx+cx^2}}{a^3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3} + \frac{\left(b^4-\right.}{}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx+cx^2\right)}{a^3} - \frac{\left(\right.}{}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2c}{\sqrt{b^2-4}}\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.27, size = 131, normalized size = 0.89

$$-\frac{\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{a\left(-3abc-2ac^2x+b^3+b^2cx\right)}{\left(b^2-4ac\right)(a+x(b+cx))} - b\log(a+x(b+cx)) + \frac{a}{x} + 2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

fricas [B] time = 1.21, size = 975, normalized size = 6.59

$$\left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\left(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3\right)x^2 + \left(\left(b^4c - 6\,ab^2c^2 + 6\,a^2c^3\right)x^3 + \left(b^5 - 6\,ab^3c + 6\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

giac [A] time = 0.42, size = 171, normalized size = 1.16

$$\frac{2\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cx^2 - 6\,ac^2x^2 + 2\,b^3x - 7\,abcx + ab^2 - 4\,a^2c}{\left(a^2b^2 - 4\,a^3c\right)\left(cx^3 + bx^2 + ax\right)} + \frac{b\log\left(cx^2 + bx\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

maple [B] time = 0.01, size = 328, normalized size = 2.22

$$-\frac{2c^2x}{\left(c\,x^2 + bx + a\right)\left(4ac - b^2\right)a} - \frac{12c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}}a} + \frac{b^2cx}{\left(c\,x^2 + bx + a\right)\left(4ac - b^2\right)a^2} + \frac{12b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}}a^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^6,x)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/(c*x^2+b*x
+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c+1/a^2/(c*x^2+b*x+
a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b-1/a^3/(4*a*c-b^2)*
ln(c*x^2+b*x+a)*b^3-12/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*c^2+12/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c-
2/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.13, size = 775, normalized size = 5.24

$$\ln\left(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

```
[Out] Timed out
```

3.430 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} dx$

Optimal. Leaf size=202

$$-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{a^3x(b^2-4ac)}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+\cdots}{a^4(b^2\cdots}$$

```
[Out] 1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/x
+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*
c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a
*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Rubi [A] time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{b(3b^2-11ac\cdots}{a^3x(b^2-4ac\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{\left(b^2-4ac\right)\left(-3b^2+2ac\right)}{a^3x} + \frac{b\left(3b^4-17ab^2c\right.}{}\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{b\left(3b^4-20ab^2c\right.}{a}$$

Mathematica [A] time = 0.34, size = 175, normalized size = 0.87

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^4+b^3cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)} - \frac{a^2}{x^2} + 2\log(x)\left(3b^2-2ac\right) + \left(2ac-3b^2\right)\log(a+x}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

fricas [B] time = 1.45, size = 1226, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

giac [A] time = 0.33, size = 229, normalized size = 1.13

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{2}-4\,a^{5}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{\left(3\,b^{2}-2\,ac\right)\log\left(cx^{2}+bx+a\right)}{2\,a^{4}}+\frac{\left(3\,b^{2}-2\,ac\right)\log\left(|x|\right)}{a^{4}}-\frac{a^{3}b^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

maple [B] time = 0.02, size = 418, normalized size = 2.07

$$\frac{3b\,c^{2}x}{\left(c\,x^{2}+bx+a\right)\left(4ac-b^{2}\right)a^{2}}+\frac{30b\,c^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{2}}-\frac{b^{3}cx}{\left(c\,x^{2}+bx+a\right)\left(4ac-b^{2}\right)a^{3}}-\frac{20b^{3}c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^7,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*ln(x)*b^2+2/a^3*b/x+3/a^2/(c*x^2+b*x+a)*b*
c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x-2/a/(c*x^2+b*x+a)
/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)-7/a^3/(4*a*c-b^2)*c*l
n(c*x^2+b*x+a)*b^2+3/2/a^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/a^2/(4*a*c-b^
2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2-20/a^3/(4*a*c-b^2)^(3/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c+3/a^4/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.30, size = 914, normalized size = 4.52

$$\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+307\,a^3\,b^4\,c^2-492\,a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

```
[Out] Timed out
```

3.431 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=238

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(\ldots}{2\left(b^2-4ac\ldots\right.}$$

```
[Out] 3*(10*a^2*c^2-7*a*b^2*c+b^4)*x/c^3/(-4*a*c+b^2)^2-3/2*b*(-6*a*c+b^2)*x^2/c^
2/(-4*a*c+b^2)^2+1/2*x^5*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+x^3*(a*(-10
*a*c+b^2)+b*(-7*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-3*(-20*a^3*c^3+3
0*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4
*a*c+b^2)^(5/2)-3/2*b*ln(c*x^2+b*x+a)/c^4
```

Rubi [A] time = 0.29, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1340, 738, 818, 800, 634, 618, 206, 628}

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2\ldots}{2\left(b^2-4ac\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \int \frac{x^6}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^4(10a+2bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{x^2\left(6a\left(b^2-10ac\right)+6b\left(b^2-6\right.\right.}{a+bx+cx^2}}{2c\left(b^2-4ac\right)^2}$$

$$= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \left(-\frac{6\left(b^4-7ab^2c+10a^2c^2\right)}{c^2}\right.}{}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-1\right.\right.}{c\left(b^2-4\right.}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-1\right.\right.}{c\left(b^2-4\right.}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-1\right.\right.}{c\left(b^2-4\right.}$$

$$= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2} - \frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3\left(a\left(b^2-1\right.\right.}{c\left(b^2-4\right.}$$

Mathematica [A] time = 0.37, size = 260, normalized size = 1.09

$$\frac{\frac{a^3c^2(2cx-5b)+a^2b^2c(5b-9cx)-ab^4(b-6cx)+b^6(-x)}{\left(b^2-4ac\right)(a+x(b+cx))^2} + \frac{6c\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{-78a^3bc^3+36a^3c^4x+61a^2b^3c^2-102a^2b^2}{\left(b^2-4ac\right)^2(a+}}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

fricas [B] time = 0.98, size = 1926, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*
c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b
```

```
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 1
23*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c +
 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^
3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3
*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2
- 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3
*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 +
(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 +
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b
*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*
b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24
*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^
5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c +
 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4
+ 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^
3 + 88*a^4*b*c^4)*x^2 + 6*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*
c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c +
10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c
 + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^
4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a
^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*
x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*
x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x
^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
+ (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 -
 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^
5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*
a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x)]
```

giac [A] time = 0.38, size = 282, normalized size = 1.18

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{4}-8\,ab^{2}c^{5}+16\,a^{2}c^{6}\right)\sqrt{-b^{2}+4\,ac}}+\frac{x}{c^{3}}-\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,c^{4}}-\frac{5\,a^{2}b^{5}-36\,a^{3}b^{3}c+58\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) +
x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c + 58
*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*
b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2
*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^4)
```

maple [B] time = 0.02, size = 1040, normalized size = 4.37

$$\frac{18a^3x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{51a^2b^2x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{24a\,b^4x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3,x)
```

```
[Out] x/c^3+18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^3-51/c/(c*x^2+b*x
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^2*b^2+24/c^2/(c*x^2+b*x+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^3*a*b^4-3/c^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3*b^6-21/c/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3-41/2/
c^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2+17/c^3/(c*x^2+b*
x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a-5/2/c^4/(c*x^2+b*x+a)^2*b^7/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2+14/c/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x-71/c^2/(c*x^2+b*x+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2+38/c^3/(
c*x^2+b*x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4-5/c^4/(c*x^2+b*x+a)^2*a
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6-29/c^2/(c*x^2+b*x+a)^2*b*a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)+18/c^3/(c*x^2+b*x+a)^2*b^3*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)-5/
2/c^4/(c*x^2+b*x+a)^2*b^5*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*b+12/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(
c*x^2+b*x+a)*a*b^3-3/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-6
0/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^3+90/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2-30/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4+3/c^4/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.00, size = 705, normalized size = 2.96

$$\frac{x}{c^3}-\frac{\frac{3\,x^3\,\left(-6\,a^3\,c^3+17\,a^2\,b^2\,c^2-8\,a\,b^4\,c+b^6\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{x^2\,\left(42\,a^3\,b\,c^3+41\,a^2\,b^3\,c^2-34\,a\,b^5\,c+5\,b^7\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(58\,a^2\,b\,c^2-36\,a\,b^3\,c+5\,b^5\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(-14\,a^3\,c^3+71\,a^2\,b^2\,c\right.}{c\,\left(16\,a^2\,c^2-8\,a\,l\right.}}{a^2\,c^3+c^5\,x^4+x^2\,\left(b^2\,c^3+2\,a\,c^4\right)+2\,b\,c^4\,x^3+2\,a\,b\,c^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x)^3,x)
```

```
[Out] x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*a
^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*b^
5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2 - 3
6*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3*c^3
 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*c
```

```
^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) + (log(
a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 + 20*a*b^
8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (3*atan(((
(3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*c - b^2)^5
) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5/2) - 16*a*b^
2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*c^2 - 30*a*b^4
*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*(4*a*c - b^2)^(
5/2))
```

sympy [**B**] time = 4.20, size = 1714, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66
*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) - 3*sqr
t(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 9
0*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c
 - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3
*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2
 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 27*a**2*b**3*
c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*
c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c
 - 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5 + x**3*(36*a**
3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b**6*c) + x**2*(-42*a**3*b
*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c - 5*b**7) + x*(28*a**4*c**3 - 142*a
**3*b**2*c**2 + 76*a**2*b**4*c - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c
**5 + 2*a**2*b**4*c**4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6)
 + x**3*(64*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b**3*c**5 +
 4*a*b**5*c**4)) + x/c**3
```

3.432 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x} dx$

Optimal. Leaf size=190

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{x^4(2a+$$

```
[Out] -b*(-7*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+
b*x+a)^2+1/2*x^2*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^
2+b*x+a)+b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/c^3/(-4*a*c+b^2)^(5/2)+1/2*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] time = 0.28, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\right.}{2c\left(b^2-4ac\right)^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 -
4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))
/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^
2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[
a + b*x + c*x^2]/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^3(8a+bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{x\left(2a\left(b^2-16ac\right)+2b\left(b^2\right.\right.}{a+bx+cx^2}}{2c\left(b^2-4ac\right.}$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} -$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} +$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} +$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} +$$

Mathematica [A] time = 0.31, size = 221, normalized size = 1.16

$$\frac{-\frac{2bc\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{2a^3c^2+a^2bc(5cx-4b)+ab^3(b-5cx)+b^5x}{\left(b^2-4ac\right)(a+x(b+cx))^2} + \frac{32a^3c^3-39a^2b^2c^2+50a^2bc^3x+11ab^4c-30ab^3c^2x-b^6+4b^5cx}{\left(b^2-4ac\right)^2(a+x(b+cx))} -}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c^
2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 + b^
5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*(b
+ c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4)
```

fricas [B] time = 0.90, size = 1603, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^
3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c
 - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
```

```
+ b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 12
4*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b
^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b
^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^
3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 -
128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*
b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2
*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 3
1*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a^2*b^5
 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^
4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a
^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5
- 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 +
 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3
- 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*
b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

giac [A] time = 0.40, size = 245, normalized size = 1.29

$$-\frac{\left(b^5 - 10\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^3} + \frac{3\,a^2b^4 - 21\,a^3b^2c + 24\,a^4c^2 + 2\left(2\,b^5c - 15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 +
b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c - 15
*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*
a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 + b*x +
 a)^2*(b^2 - 4*a*c)^2*c^3)
```

maple [B] time = 0.02, size = 530, normalized size = 2.79

$$-\frac{30a^2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c} + \frac{10a\,b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c^2} - \frac{b^5\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x,x)
```

```
[Out] (1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*(
32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3/2*
a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)
^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2-4/c^2/(16*a^2*c^2-8*a
```

```
*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2+1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x
^2+b*x+a)*b^4-30/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a^2*b+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^3/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.20, size = 620, normalized size = 3.26

$$\frac{\frac{3\,a^2\,\left(8\,a^2\,c^2-7\,a\,b^2\,c+b^4\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,x^3\,\left(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,b\,x\,\left(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4\right)}{c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{\ln\left(c\,x^2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c^2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2*c))
/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 +
 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 + 160*a^
2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(1024*a^5*
c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 - 1280*a^
4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2*(4*a*c -
 b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(32*a^2*
c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a
*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 3.21, size = 1510, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-
(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
```

```
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4*a*c - b**2)
**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*sqrt(-(4*a*c
- b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(3
0*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(30
*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a*
*2*b**4 + x**3*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3
*c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44*
a**2*b**3*c + 6*a*b**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**4*c*
*3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2*b*c
**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c**4 +
 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
```

3.433 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} dx$

Optimal. Leaf size=111

$$\frac{12a^2 \tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}} - \frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)} + \frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

```
[Out] 1/2*(b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)^2-3*a*(b+2*a/x)/(-4*a*c+b^2)^2/(c+
a/x^2+b/x)+12*a^2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1352, 614, 618, 206}

$$\frac{12a^2 \tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}} - \frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)} + \frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/(
(b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = -\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{b+\frac{2a}{x}}{2(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{2(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{(b^2-4ac)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{(6a^2)\,\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x\right.}{(b^2-4ac)^2}$$

$$= \frac{b+\frac{2a}{x}}{2(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{(b^2-4ac)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{(12a^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{(b^2-4ac)^2}$$

$$= \frac{b+\frac{2a}{x}}{2(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{(b^2-4ac)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Mathematica [A] time = 0.17, size = 174, normalized size = 1.57

$$\frac{1}{2}\left(\frac{24a^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{a^2c(2cx-3b)+ab^2(b-4cx)+b^4x}{c^3(4ac-b^2)(a+x(b+cx))^2} + \frac{22a^2bc^2-20a^2c^3x-8ab^3c+16ab^2c^2x+b^5}{c^3(b^2-4ac)^2(a+x(b+cx))}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3*
x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a
^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

fricas [B] time = 0.97, size = 953, normalized size = 8.59

$$\left[-\frac{a^2b^5-14\,a^3b^3c+40\,a^4bc^2+2\left(b^6c-12\,ab^4c^2+42\,a^2b^2c^3-40\,a^3c^4\right)x^3+\left(b^7-12\,ab^5c+30\,a^2b^3c^2+8\,a^3bc\right.}{2\left(a^2b^6c^2-12\,a^3b^4c^3+48\,a^4b^2c^4-64\,a^5c^5+\left(b^6c^4-12\,ab^4c^5+48\,a^2b^2c^6-64\,a^3c^7\right)x^4+2\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 +
 (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^
6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^
4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*
c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3
*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 1
```

```
28*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b
*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4
*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^
2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b
^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(
b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10
*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*
c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

giac [A] time = 0.42, size = 202, normalized size = 1.82

$$\frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-2}{2\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\left(cx^2+bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

```
[Out] 12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3*x
^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*c*x
 + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 260, normalized size = 2.34

$$\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{\left(10ac-b^2\right)a^2b}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{\left(2a^2c^2+8a\,b^2c-b^4\right)b\,x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}-\frac{\left(6a^2c^2-10a\,b^2c+b^4\right.}{\left(16a^2c^2-8a\,b^2c+b^4\right.}}{\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^2,x)
```

```
[Out] (-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a^
2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a*b
^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.19, size = 343, normalized size = 3.09

$$\frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2}{2\,c^2}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^3),x)
```

```
[Out] (12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2
)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 + 10*
a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a
*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^5 + 8
*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2*c^2 -
 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^
2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

sympy [B] time = 1.41, size = 547, normalized size = 4.93

$$-6a^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-72a^3b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+6a^2b^6\cdots}{12a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] -6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4*c
*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6*a*
*2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**5*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**
5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/(4*a*c
 - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**3*(-2
0*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*a*b**3*c
 - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4*c**4 - 1
6*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b**2*c**5 +
 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c**3) + x**2
*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c**4 - 32*a**
2*b**3*c**3 + 4*a*b**5*c**2))
```

3.434 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx$

Optimal. Leaf size=107

$$\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -1/2*x^3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3/2*b*x*(b*x+2*a)/(-4*a*c+b
^2)^2/(c*x^2+b*x+a)+6*a*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 728, 722, 618, 206}

$$-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a + b*
x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = \int \frac{x^3}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{(3b)\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(3ab)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6ab)\,\text{Subst}\left(\int \frac{1}{b^2-4ac}\right.}{\left(b^2-4\right.}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.19, size = 126, normalized size = 1.18

$$-\frac{8a^3c+a^2\left(b^2+10bcx+16c^2x^2\right)+abx\left(2b^2+bcx+6c^2x^2\right)+b^4x^2}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2} - \frac{6ab\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 10
*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

fricas [B] time = 0.87, size = 872, normalized size = 8.15

$$\left[-\frac{a^2b^4+4a^3b^2c-32a^4c^2+6\left(ab^3c^2-4a^2bc^3\right)x^3+\left(b^6-3ab^4c+12a^2b^2c^2-64a^3c^3\right)x^2-6\left(abc^3x^4+2a\right.}{2\left(a^2b^6c-12a^3b^4c^2+48a^4b^2c^3-64a^5c^4+\left(b^6c^3-12ab^4c^4+48a^2b^2c^5-64a^3c^6\right)x^4+2\left(b^7c^2-12ab^5c^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 + 2
*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a
^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b
```

```
^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*
a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 +
32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^
3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b
^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)
*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3
*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c -
 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^
4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2
*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64
*a^4*b*c^4)*x)]
```

giac [A] time = 0.31, size = 163, normalized size = 1.52

$$-\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

```
[Out] -6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*a^
2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 223, normalized size = 2.08

$$-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(5ac+b^2\right)abx}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(8ac+b^2\right)a^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}}{\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^3,x)
```

```
[Out] (-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x-
1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.43, size = 271, normalized size = 2.53

$$-\frac{\frac{x^2\left(16a^2c^2+ab^2c+b^4\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{a^2\left(b^2+8ac\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{3abcx^3}{16a^2c^2-8ab^2c+b^4}+\frac{abx\left(b^2+5ac\right)}{c\left(16a^2c^2-8ab^2c+b^4\right)}}{x^2\left(b^2+2ac\right)+a^2+c^2x^4+2abx+2bcx^3}-\frac{6ab\,\mathrm{atan}\left(\left(\frac{3ab^2}{\left(4ac-b^2\right)^{5/2}}+\frac{6ab}{\left(4ac-b^2\right)\cdots}\right.\right.}{\left(4ac-b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3)/
(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) -
 (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^2)^(5
/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 1.21, size = 513, normalized size = 4.79

$$3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{\cdots}}{6abc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5*c
*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**
2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c**3*s
qrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) +
 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b**2
)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x**3 + x
**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3))/(32*a*
*4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 - 16*a*b**
2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4*b**5*c**2
) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3*b*c**3 - 3
2*a**2*b**3*c**2 + 4*a*b**5*c))
```

3.435 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} dx$

Optimal. Leaf size=115

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+(3*a*b+(2*a*c+b^2)*x)/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)-2*(2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1354, 738, 638, 618, 206}

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a
*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \int \frac{x^2}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{2a-2bx}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(b^2+2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2\left(b^2+2ac\right)\right)\text{Subst}\left(\int \right.}{\left(b^2-\right.}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(b^2+2ac\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b^2}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 1.14

$$\frac{1}{2}\left(\frac{\left(2ac+b^2\right)(b+2cx)}{c\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))^2} + \frac{4\left(2ac+b^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 + 2
*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*c)
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

fricas [B] time = 0.88, size = 887, normalized size = 7.71

$$\left[\frac{6\,a^2b^3-24\,a^3bc+2\left(b^4c-2\,ab^2c^2-8\,a^2c^3\right)x^3+3\left(b^5-2\,ab^3c-8\,a^2bc^2\right)x^2+2\left(\left(b^2c^2+2\,ac^3\right)x^4+a^2b^2+2\right.}{2\left(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+\left(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5\right)x^4+2\left(b^7c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2
+ 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 +
 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^4 -
22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
```

```
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x
^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)
*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*
b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.44, size = 154, normalized size = 1.34

$$\frac{2\left(b^{2}+2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{2\,b^{2}cx^{3}+4\,ac^{2}x^{3}+3\,b^{3}x^{2}+6\,abcx^{2}+10\,ab^{2}x-4\,a^{2}cx+6\,a^{2}b}{2\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\left(cx^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

```
[Out] 2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*x^
2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c + 16*
a^2*c^2)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 262, normalized size = 2.28

$$\frac{4ac\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{2b^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{\frac{\left(2ac+b^{2}\right)c\,x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3\left(2ac+\cdots\right.}{2\left(16a^{2}c^{2}-\cdots\right.}}{\left(c\,x^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^4,x)
```

```
[Out] (c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c+2/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.50, size = 313, normalized size = 2.72

$$\frac{\frac{3\,a^2\,b}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{a\,x\,\left(2\,a\,c-5\,b^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(b^2+2\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}+\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^2+2\,a\,c\right)\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{\left(4\,a\,c-b^2\right)^{5/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x}{(4\,\dots}\right.}{b^2+2\,\dots}\right.}{\left(4\,a\,c\,\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c
 + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^2)*(
b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 1.28, size = 570, normalized size = 4.96

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-12\dots}{4ac^2+2\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/(
4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c*
*2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (64*a**
3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b**2*c) +
x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**2 - 16*a**
3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2)
 + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 -
 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.436 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx$

Optimal. Leaf size=103

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)+6*b*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 638, 614, 618, 206}

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b
^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx=\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx$$

$$=\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{(3b)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$=\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{(3bc)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$=\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{(6bc)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x}\right.}{\left(b^2-4ac\right.}$$

$$=\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 102, normalized size = 0.99

$$\frac{\frac{\left(b^2-4ac\right)(2a+bx)}{(a+x(b+cx))^2}-\frac{12bc\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b(b+2cx)}{a+x(b+cx)}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a + x
*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c])/(2*(b^2 - 4*a*c)^2)
```

fricas [B] time = 0.89, size = 788, normalized size = 7.65

$$\left[-\frac{ab^4+4a^2b^2c-32a^3c^2+6\left(b^3c^2-4abc^3\right)x^3+9\left(b^4c-4ab^2c^2\right)x^2-6\left(bc^3x^4+2b^2c^2x^3+2a\right.}{2\left(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^4+2\left(b^7c-12ab^5c^2+4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(
b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2
*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5
+ a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2*c - 32*a
```

```
^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 12*(b*
c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.41, size = 135, normalized size = 1.31

$$-\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

```
[Out] -6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*b*
c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

maple [A] time = 0.00, size = 130, normalized size = 1.26

$$-\frac{3bcx}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)}-\frac{6bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}-\frac{3b^2}{2\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)}+\frac{-bx-2a}{2\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^5,x)
```

```
[Out] 1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/(4*a*c-b^2)^2*b/(c*x^2+b*x+a)*
c*x-3/2/(4*a*c-b^2)^2*b^2/(c*x^2+b*x+a)-6/(4*a*c-b^2)^(5/2)*b*c*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.43, size = 253, normalized size = 2.46

$$-\frac{\frac{8\,c\,a^2+a\,b^2}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^2\,c\,x^2}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\,\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,b\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,(16}{3\,b\,c}\right.}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2
) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a*c -
 b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 1.09, size = 481, normalized size = 4.67

$$3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^5c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^7c\sqrt{}}{6bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c**2
*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*
c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c**4*
sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**
2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2 - 6*b*
c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2
*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*
b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*
b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.437 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{-b-2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[Out] 1/2*(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(c
*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1354, 614, 618, 206}

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^
2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3c)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(12c^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-\dots}\right.}{\left(b^2-4a\dots\right.}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 97, normalized size = 0.94

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{(b+2cx)\left(-2c\left(5a+3cx^2\right)+b^2-6bcx\right)}{(a+x(b+cx))^2}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2)
 + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(
b^2 - 4*a*c)^2)
```

fricas [B] time = 0.98, size = 785, normalized size = 7.62

$$\left[-\frac{b^5-14\,ab^3c+40\,a^2bc^2-12\left(b^2c^3-4\,ac^4\right)x^3-18\left(b^3c^2-4\,abc^3\right)x^2-12\left(c^4x^4+2\,bc^3x^3+2\,a\dots\right.}{2\left(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+\left(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5\right)x^4+2\left(b^7c-12\,ab^5c^2+4\dots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b
^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2
 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*
c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x
^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
```

```
4*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.34, size = 136, normalized size = 1.32

$$\frac{12\,c^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*a*
c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2
)
```

maple [A] time = 0.00, size = 129, normalized size = 1.25

$$\frac{6c^2x}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)}+\frac{12c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}+\frac{3bc}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)}+\frac{2cx+b}{2\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^6,x)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+6*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)
*x+3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b+12*c^2/(4*a*c-b^2)^(5/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.42, size = 285, normalized size = 2.77

$$\frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}+\frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{\left(4\,a\,c-b^2\right)^{5/2}}+\frac{6\,c^2\left(16\,a^2\,b\,c^2-8\,a\,b^3\cdots\right)}{\left(4\,a\,c-b^2\right)^{5/2}\left(16\,a^2\,c^2-8\cdots\right)}\right)\cdots}{6\,c^2\cdots}\right)}{\left(4\,a\,c-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*
```

```
c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) + a^2
 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c - b^2)
^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2)))/(4*a
*c - b^2)^(5/2)
```

sympy [B] time = 1.11, size = 474, normalized size = 4.60

$$-6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-72ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+6b^6c^2\right.}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6*b*
c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**5
*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5)
 + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c -
 b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*
c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a*
*2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**
2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c +
2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.438 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx$

Optimal. Leaf size=185

$$-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*b^4-15*a*b^2*c+
16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a^2
*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)
^(5/2)+ln(x)/a^3-1/2*ln(c*x^2+b*x+a)/a^3
```

Rubi [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15
*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a +
b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^
2]/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \frac{2\left(b^2-4ac\right)^2 \cdots}{x\left(a+\cdots\right.}}{2a^2\left(b \cdots\right.}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{2\left(-b^2+4a \cdots\right.}{ax}\cdots\right.}{\cdots}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} + \frac{\int \cdots}{\cdots}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} - \frac{\int \cdots}{\cdots}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} - \frac{\mathrm{lo}\cdots}{\cdots}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b\left(b^4-10a \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.35, size = 178, normalized size = 0.96

$$\frac{\frac{a^2\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))^2} - \frac{2b\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{a\left(16a^2c^2-15ab^2c-14abc^2x+2b^4+2b^3cx\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} - \log(a+x(b+cx)) + 2\log \cdots}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b^
4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2*(
a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)
])/(2*a^3)
```

fricas [B] time = 1.79, size = 1985, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
```

```
a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b
^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^
2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*
(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)
) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^
6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 -
128*a^7*c^4)*x^2 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^
3)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*
b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2
+ 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + 2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*
c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2
 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*
x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^7 - 10*a^2*b^5*c
+ 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 +
 (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^2 + 2*(a^4*b^
7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*x)]
```

giac [A] time = 0.32, size = 239, normalized size = 1.29

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 +
b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2
 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2
- 4*a*c)^2*a^3)
```

maple [B] time = 0.02, size = 781, normalized size = 4.22

$$-\frac{7b\,c^3x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{b^3c^2x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{29b^2c^2x^2}{2\left(c\,x^2+bx+a\right)^2\left(16a^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^7,x)
```

```
[Out] ln(x)/a^3-7/a/(c*x^2+b*x+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/a^2/(c
*x^2+b*x+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+8/(c*x^2+b*x+a)^2*c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-29/2/a/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^2*b^2+2/a^2/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^
4-1/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-6/a/(c*x^2+b*x+a)^2*
b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c+1/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*
a*b^2*c+b^4)*x+12*a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2-21/2/(c*
x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/2/a/(c*x^2+b*x+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*b^4-8/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)+
4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^2-1/2/a^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2+10/a^2/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/a^
3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.46, size = 1089, normalized size = 5.89

$$\frac{\ln(x)}{a^3}+\frac{\frac{3\left(8a^2c^2-7a\,b^2c+b^4\right)}{2a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{x^2\left(16a^2c^3-29a\,b^2c^2+4b^4c\right)}{2a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\,x\left(a^2c^2+6a\,b^2c-b^4\right)}{a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\,c^2x^3\left(7a\,c-b^2\right)}{a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{x^2\left(b^2+2a\,c\right)+a^2+c^2x^4+2a\,b\,x+2b\,c\,x^3}-\frac{\ln\left(1536a^6c^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

```
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (log(1
536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) + 39*a
^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160*a^4*b^
4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c^3*x - 25
60*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^9*c*x + 2
016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*
```

```
(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c - b^2)^5)^(1/
2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20*a*b^8*c + 30
*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))
/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a^6*c^5 + 2*a*b^
5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2
) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-
(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^
7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c -
 b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c -
b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(b^10 - 1024*a^5
*c^5 + b^5*(-(4*a*c - b^2)^5)^(1/2) + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1
280*a^4*b^2*c^4 - 20*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a
*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

```
[Out] Timed out
```

3.439 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} dx$

Optimal. Leaf size=239

$$\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3x\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-20a^3c^3\right.}{}$$

```
[Out] -3*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/2*(b*c*x-2*a*c+b^2)/a/(
-4*a*c+b^2)/x/(c*x^2+b*x+a)^2+1/2*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*
c+b^2)*x)/a^2/(-4*a*c+b^2)^2/x/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c^2-
10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(5/2
)-3*b*ln(x)/a^4+3/2*b*ln(c*x^2+b*x+a)/a^4
```

Rubi [A] time = 0.28, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b\right.}{a^3x\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \frac{6(b^2-5ac}{}}{2}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{6(b^2-5}{}\right.}{}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2x\,(a+}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2x\,(a+}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2x\,(a+}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2x\,(a+}$$

Mathematica [A] time = 0.44, size = 221, normalized size = 0.92

$$\frac{\frac{a^2\left(-3abc-2ac^2x+b^3+b^2cx\right)}{\left(4ac-b^2\right)(a+x(b+cx))^2} - \frac{a\left(46a^2bc^2+28a^2c^3x-29ab^3c-26ab^2c^2x+4b^5+4b^4cx\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{6\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + 3b\log}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 26*
a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b^6
- 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a
^4)
```

fricas [B] time = 2.17, size = 2280, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*
```

```
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 3
0*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*
a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 -
20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*
x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*
a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4
*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*
c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*
c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 +
 (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*
x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 +
2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*
a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 -
 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^
5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*
c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x), -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3
+ 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a
*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8
 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a
*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b
^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^
3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 -
20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3
)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b
^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12
*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c
 + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2
 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 6
4*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
```

giac [A] time = 0.47, size = 309, normalized size = 1.29

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,a^{4}}-\frac{3\,b\log\left(|x|\right)}{a^{4}}-\frac{2\,a^{3}b^{4}-16\,a^{4}b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) +
3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*
```

```
a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3
*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*
c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b
*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

maple [B] time = 0.02, size = 954, normalized size = 3.99

$$-\frac{14c^4x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{13b^2c^3x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{2b^4c^2x^3}{\left(c\,x^2+bx+a\right)^2\left(16a^2c^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^8,x)
```

```
[Out] -1/a^3/x-3*b*ln(x)/a^4-14/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+13/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2-2/a^3/(c*
x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-37/a/(c*x^2+b*x+a)^2*b*
c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+55/2/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2-4/a^3/(c*x^2+b*x+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^2-18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3-7/a/(c*x^2+b*x+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+12/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^4*c-2/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*b^6-29/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2+18/a/(c*x^2+b*x+a)
^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c-5/2/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*b^5+24/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)*b-12/
a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^3+3/2/a^4/(16*a^2*c^2-8*
a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-60/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3+90/a^2/(16*a^2*c^2-8*a*b^2*
c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2-30/a^3
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^4*c+3/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.55, size = 1255, normalized size = 5.25

$$-\frac{\frac{1}{a}+\frac{x^2\left(50\,a^3\,c^3+7\,a^2\,b^2\,c^2-18\,a\,b^4\,c+3\,b^6\right)}{a^3\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x\left(122\,a^2\,b\,c^2-68\,a\,b^3\,c+9\,b^5\right)}{2\,a^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,x^3\left(46\,a^2\,b\,c^3-29\,a\,b^3\,c^2+4\,b^5\,c\right)}{2\,a^3\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,c^2\,x^4\left(10\,a^2\,c^2-7\,a\,b^2\,c+b^4\right)}{a^3\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^3\left(b^2+2\,a\,c\right)+a^2\,x+c^2\,x^5+2\,a\,b\,x^2+2\,b\,c\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2
*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*a^2*
```

```
c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c + b^2)
+ a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*log(2*a*
b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^
6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^
2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b
^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x + 39*a^3
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) -
 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)
^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*
b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-(4*a*c - b^2)^5
)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*log(2*a*b^11 + 2*b
^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^6*b*c^5 +
320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^2 - 1170*a
^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/2) + 17*a^
2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b^6*c^3*x +
 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x - 39*a^3*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) + 58*a^3*b*
c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)^5)^(1/2))
*(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*b^7*c^2 -
640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 + 20*a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
20*a*b^9*c - 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] Timed out
```

3.440 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \int \frac{x^4}{2+13x+15x^2}\,dx$$
$$= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right)dx$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

fricas [A] time = 0.76, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

giac [A] time = 0.26, size = 32, normalized size = 0.80

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(|5\,x+1|\right)-\frac{16}{567}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(5x+1\right)}{4375}-\frac{16\ln\left(3x+2\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15+2/x^2+13/x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

maxima [A] time = 0.42, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.65

$$\frac{139\,x}{3375}-\frac{16\,\ln\left(x+\frac{2}{3}\right)}{567}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{13\,x^{2}}{450}+\frac{x^{3}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13/x + 2/x^2 + 15),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^{3}}{45}-\frac{13x^{2}}{450}+\frac{139x}{3375}+\frac{\log\left(x+\frac{1}{5}\right)}{4375}-\frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

3.441 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1354, 701, 632, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \int \frac{x^3}{2+13x+15x^2}\,dx$$
$$= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(15 + 2/x^2 + 13/x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

fricas [A] time = 0.88, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

giac [A] time = 0.28, size = 27, normalized size = 0.82

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln\left(5x+1\right)}{875}+\frac{8\ln\left(3x+2\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15+2/x^2+13/x),x)
[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)
```

maxima [A] time = 0.42, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

mupad [B] time = 1.31, size = 21, normalized size = 0.64

$$\frac{8\ \ln\left(x+\frac{2}{3}\right)}{189}-\frac{13\,x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13/x + 2/x^2 + 15),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x**2+13/x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

3.442 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal. Leaf size=26

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1340, 703, 632, 31}

$$\frac{x}{15} - \frac{4}{63}\log(3x+2) + \frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\int \frac{x^2}{2+13x+15x^2}\,dx$$
$$=\frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx$$
$$=\frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx$$
$$=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

fricas [A] time = 0.83, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

giac [A] time = 0.28, size = 22, normalized size = 0.85

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15}+\frac{\ln\left(5x+1\right)}{175}-\frac{4\ln\left(3x+2\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x),x)
[Out] 1/15*x-4/63*ln(3*x+2)+1/175*ln(5*x+1)
```

maxima [A] time = 0.42, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

mupad [B] time = 0.08, size = 16, normalized size = 0.62

$$\frac{x}{15}-\frac{4\,\ln\left(x+\frac{2}{3}\right)}{63}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13/x + 2/x^2 + 15),x)
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x),x)
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

3.443 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx &= \int \frac{x}{2+13x+15x^2}\,dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

fricas [A] time = 0.89, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

giac [A] time = 0.28, size = 19, normalized size = 0.90

$$-\frac{1}{35}\log\left(|5x+1|\right)+\frac{2}{21}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
```

```
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{\ln\left(5x+1\right)}{35}+\frac{2\ln\left(3x+2\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x,x)
```

```
[Out] 2/21*ln(3*x+2)-1/35*ln(5*x+1)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

mupad [B] time = 0.07, size = 13, normalized size = 0.62

$$\frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x,x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

3.444 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

```
[Out] 1/7*ln(5+1/x)-1/7*ln(3+2/x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 616, 31}

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{15+13x+2x^2}\,dx,x,\frac{1}{x}\right)\\
&= -\left(\frac{2}{7}\text{Subst}\left(\int \frac{1}{3+2x}\,dx,x,\frac{1}{x}\right)\right)+\frac{2}{7}\text{Subst}\left(\int \frac{1}{10+2x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

fricas [A] time = 1.01, size = 17, normalized size = 0.74

$$\frac{1}{7}\log\left(5x+1\right)-\frac{1}{7}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

giac [A] time = 0.22, size = 19, normalized size = 0.83

$$\frac{1}{7}\log\left(|5x+1|\right)-\frac{1}{7}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln\left(5x+1\right)}{7}-\frac{\ln\left(3x+2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^2,x)
```

```
[Out] -1/7*ln(3*x+2)+1/7*ln(5*x+1)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.74

$$\frac{1}{7}\log\left(5x+1\right)-\frac{1}{7}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

mupad [B] time = 1.37, size = 8, normalized size = 0.35

$$-\frac{2\,\mathrm{atanh}\left(\frac{30x}{7}+\frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(13/x + 2/x^2 + 15)),x)
```

```
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

sympy [A] time = 0.11, size = 15, normalized size = 0.65

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**2,x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

3.445 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3} \, dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx$$
$$= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx$$
$$= \frac{\log(x)}{2} + \frac{45}{14}\int \frac{1}{10+15x}\,dx - \frac{75}{7}\int \frac{1}{3+15x}\,dx$$
$$= \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2} + \frac{3}{14}\log(3x+2) - \frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

fricas [A] time = 0.87, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5\,x+1\right) + \frac{3}{14}\log\left(3\,x+2\right) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

giac [A] time = 0.25, size = 24, normalized size = 0.89

$$-\frac{5}{7}\log\left(|5\,x+1|\right) + \frac{3}{14}\log\left(|3\,x+2|\right) + \frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2} - \frac{5\ln\left(5x+1\right)}{7} + \frac{3\ln\left(3x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^3,x)
[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)
```

maxima [A] time = 0.42, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5\,x+1\right) + \frac{3}{14}\log\left(3\,x+2\right) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

mupad [B] time = 1.39, size = 17, normalized size = 0.63

$$\frac{3\ \ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ \ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**3,x)
```

```
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

3.446 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

fricas [A] time = 0.95, size = 30, normalized size = 0.88

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log(x)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

giac [A] time = 0.31, size = 29, normalized size = 0.85

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13\ln(x)}{4}+\frac{25\ln\left(5x+1\right)}{7}-\frac{9\ln\left(3x+2\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^4,x)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)
```

maxima [A] time = 0.43, size = 26, normalized size = 0.76

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.65

$$\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln(x)}{4}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(13/x + 2/x^2 + 15)),x)
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4} + \frac{25\log\left(x+\frac{1}{5}\right)}{7} - \frac{9\log\left(x+\frac{2}{3}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**4,x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

3.447 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\
&= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

fricas [A] time = 0.79, size = 39, normalized size = 0.95

$$-\frac{1000\,x^2\log(5\,x+1)-27\,x^2\log(3\,x+2)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

giac [A] time = 0.30, size = 34, normalized size = 0.83

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(|5\,x+1|)+\frac{27}{56}\log(|3\,x+2|)+\frac{139}{8}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139\ln(x)}{8}-\frac{125\ln(5x+1)}{7}+\frac{27\ln(3x+2)}{56}+\frac{13}{4x}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^5,x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(5*x+1)
```

maxima [A] time = 0.42, size = 31, normalized size = 0.76

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(5\,x+1)+\frac{27}{56}\log(3\,x+2)+\frac{139}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

mupad [B] time = 1.31, size = 26, normalized size = 0.63

$$\frac{27\,\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\,\ln(x)}{8}+\frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

sympy [**A**] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8} - \frac{125\log\left(x+\frac{1}{5}\right)}{7} + \frac{27\log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**5,x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

3.448 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx\\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

fricas [A] time = 1.03, size = 44, normalized size = 0.92

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

giac [A] time = 0.29, size = 39, normalized size = 0.81

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(|5\,x+1|\right)-\frac{81}{112}\log\left(|3\,x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417\ln(x)}{16}+\frac{625\ln\left(5x+1\right)}{7}-\frac{81\ln\left(3x+2\right)}{112}-\frac{139}{8x}+\frac{13}{8x^2}-\frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^6,x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(5*x+1)
```

maxima [A] time = 0.65, size = 36, normalized size = 0.75

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(5\,x+1\right)-\frac{81}{112}\log\left(3\,x+2\right)-\frac{1417}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.67

$$\frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln(x)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417\log(x)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**6,x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

3.449 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=204

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}-\frac{5\left(\frac{2c\left(12ac+b^2\right)}{x}+b\left(44a\right.\right.}{64}$$

```
[Out] -5/24*(a+c/x^2+b/x)^(3/2)*(7*b+6*c/x)+(a+c/x^2+b/x)^(5/2)*x+5/2*a^(3/2)*b*a
rctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))+5/128*(-48*a^2*c^2-24*a*b
^2*c+b^4)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))/c^(3/2)-5/64*(
b*(44*a*c+b^2)+2*c*(12*a*c+b^2)/x)*(a+c/x^2+b/x)^(1/2)/c
```

Rubi [A] time = 0.23, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}+\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)-\frac{5\left(\frac{2c\left(12ac+b^2\right)}{x}+b\left(44a\right.\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*
(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/
2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])
/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sq
rt[a + c/x^2 + b/x])])/(128*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
```

```
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{5/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x - \frac{5}{2}\text{Subst}\left(\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5\,\text{Subst}\left(\int \frac{\left(-8abc-c\left(b^2+12ac\right)x\right)\sqrt{a}}{x}\right.}{16c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c} + \left(a+\right.$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c} + \left(a+\right.$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c} + \left(a+\right.$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c} + \left(a+\right.$$

Mathematica [A] time = 0.52, size = 213, normalized size = 1.04

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(960a^{3/2}bc^{3/2}x^4\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+15x^4\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)-\right.}{384c^{3/2}x^3\sqrt{x(ax+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*Sqrt[c + x*(b + a*x)]*(48*c^3 + 15*b^3
*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 + 278*a*b*x - 96*a^2*x^2))
 + 960*a^(3/2)*b*c^(3/2)*x^4*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b +
 a*x)])] + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*ArcTanh[(2*c + b*x)/(2*Sq
rt[c]*Sqrt[c + x*(b + a*x)])]))/(384*c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])
```

fricas [A] time = 1.71, size = 959, normalized size = 4.70

$$\left[\frac{960\,a^{\frac{3}{2}}bc^2x^3\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac-4\left(2\,ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)-15\left(b^4-24\,ab^2c-48\,a^2c^2\right)\sqrt{c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2
*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c -
 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x
^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4
- 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108
*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqrt(-a)*
a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)
/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt(
(a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 -
(15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2
+ b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8
*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2
)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(192
*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*
b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/384*(9
60*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*
sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2
)/(a*c*x^2 + b*c*x + c^2)) - 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (1
5*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c^2*x^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 701, normalized size = 3.44

$$\left(\frac{a\,x^{2}+bx+c}{x^{2}}\right)^{\frac{5}{2}}\left(-720a^{\frac{7}{2}}c^{\frac{9}{2}}x^{4}\ln\left(\frac{bx+2c+2\sqrt{a\,x^{2}+bx+c}\;\sqrt{c}}{x}\right)+960a^{3}b\,c^{4}x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx+c}\;\sqrt{a}}{2\sqrt{a}}\right)-360a^{\frac{5}{2}}b^{2}c^{\frac{7}{2}}x^{4}\ln\left(\frac{bx\cdot}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(5/2),x)
```

```
[Out] 1/384*((a*x^2+b*x+c)/x^2)^(5/2)*x*(720*a^(7/2)*(a*x^2+b*x+c)^(1/2)*x^4*c^4+
6*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^3*b^3-6*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x^4*b^4
-720*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(9/2)*a^(7/2)*x^4-6*a^
(5/2)*(a*x^2+b*x+c)^(5/2)*x^5*b^3+144*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^4*c^2-1
44*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^2*c^2+240*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^4*
c^3-360*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(7/2)*a^(5/2)*x^4*b
^2+660*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^4*b^2*c^3+960*ln(1/2*(2*(a*x^2+b*x+c)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*x^4*b*c^4+148*a^(5/2)*(a*x^2+b*x+c)^(5/
2)*x^4*b^2*c+280*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^5*b*c^2-10*a^(5/2)*(a*x^2+b*
x+c)^(3/2)*x^5*b^3*c+15*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(5/
2)*a^(3/2)*x^4*b^4+260*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^4*b^2*c^2+600*a^(7/2)*
(a*x^2+b*x+c)^(1/2)*x^5*b*c^3-30*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^5*b^3*c^2+15
2*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^5*b*c-152*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^3*b
*c+4*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^2*b^2*c-10*a^(3/2)*(a*x^2+b*x+c)^(3/2)*x
^4*b^4*c+16*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x*b*c^2-30*a^(3/2)*(a*x^2+b*x+c)^(1
```

```
/2)*x^4*b^4*c^2-96*(a*x^2+b*x+c)^(7/2)*c^3*a^(3/2))/(a*x^2+b*x+c)^(5/2)/c^4
/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a + b/x + c/x^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(5/2),x)
[Out] int((a + b/x + c/x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(5/2),x)
[Out] Integral((a + b/x + c/x**2)**(5/2), x)
```

3.450 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=145

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.$$

```
[Out] (a+c/x^2+b/x)^(3/2)*x+3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/
2))*a^(1/2)-3/8*(4*a*c+b^2)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/
2))/c^(1/2)-3/4*(3*b+2*c/x)*(a+c/x^2+b/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x +
(3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3
*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8
*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
```

```
|| LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{3}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3\,\text{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}} dx, x, \frac{1}{x}\right.}{8c}$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + (3ab)\,\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{}{\sqrt{a}}\right.$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right.$$

Mathematica [A] time = 0.25, size = 163, normalized size = 1.12

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(-3x^2\left(4ac+b^2\right)\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)+12\sqrt{a}b\sqrt{c}x^2\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)-2\sqrt{c}(x(5b-4ax)+\cdots}{8\sqrt{c}x\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*(2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b
+ a*x)] + 12*Sqrt[a]*b*Sqrt[c]*x^2*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c +
x*(b + a*x)])] - 3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c
+ x*(b + a*x)])]))/(8*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

fricas [A] time = 1.39, size = 709, normalized size = 4.89

$$\left[\frac{12\sqrt{a}bcx\log\left(-8a^2x^2-8abx-b^2-4ac-4\left(2ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+3\left(b^2+4ac\right)\sqrt{c}x\log\left(-\frac{8bcx+\left(b^2+4ac\cdots\right.}{\cdots}\right.\cdots}{16cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2
 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*lo
g(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a
*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b*x)*s
qrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)
*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2
)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*a^2*x^2 -
 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x
^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt(
(a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2 - 5*b*c*x -
2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b*c*x*arctan(1
/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x +
a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt
((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c*x^2 - 5*b*c*x -
 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 334, normalized size = 2.30

$$-\frac{\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{3}{2}}\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}x^2\ln\left(\frac{bx+2c+2\sqrt{ax^2+bx+c}\,\sqrt{c}}{x}\right)-12a^2bc^2x^2\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+3a^{\frac{3}{2}}b^2c^{\frac{3}{2}}x^2\ln\left(\frac{bx+2c+2\cdots}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(3/2),x)
```

```
[Out] -1/8*((a*x^2+b*x+c)/x^2)^(3/2)*x*(12*a^(5/2)*c^(5/2)*ln((b*x+2*c+2*(a*x^2+b
*x+c)^(1/2)*c^(1/2))/x)*x^2-2*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^3*b-4*a^(5/2)*(
a*x^2+b*x+c)^(3/2)*x^2*c-6*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^3*b*c+3*a^(3/2)*c^
(3/2)*ln((b*x+2*c+2*(a*x^2+b*x+c)^(1/2)*c^(1/2))/x)*x^2*b^2-12*a^(5/2)*(a*x
^2+b*x+c)^(1/2)*x^2*c^2+2*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x*b-2*a^(3/2)*(a*x^2+
b*x+c)^(3/2)*x^2*b^2+4*(a*x^2+b*x+c)^(5/2)*c*a^(3/2)-6*a^(3/2)*(a*x^2+b*x+c
)^(1/2)*x^2*b^2*c-12*a^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(
1/2))*x^2*b*c^2)/(a*x^2+b*x+c)^(3/2)/c^2/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(3/2),x)
```

```
[Out] int((a + b/x + c/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^{2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(3/2), x)
```

3.451 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=105

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

```
[Out] 1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(1/2)-arctanh(1/
2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)+x*(a+c/x^2+b/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1342, 732, 843, 621, 206, 724}

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2
+ b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c
/x^2 + b/x])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right) - c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\right.\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x + b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - (2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx\right.\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}} - \sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.22

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(b\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\left(\sqrt{x(ax+b)+c}-\sqrt{c}\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)\right)\right)}{2\sqrt{a}\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(b*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b
+ a*x)])] + 2*Sqrt[a]*(Sqrt[c + x*(b + a*x)] - Sqrt[c]*ArcTanh[(2*c + b*x)/
(2*Sqrt[c]*Sqrt[c + x*(b + a*x)])])))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

fricas [A] time = 1.31, size = 590, normalized size = 5.62

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{a}\,b\log\left(-8\,a^2x^2 - 8\,abx - b^2 - 4\,ac - 4\left(2\,ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right) + 2\,a\sqrt{c}\,\log\left(-\frac{8\,bcx+(b}{}\right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) +
2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*s
qrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x +
 c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x
 + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*
c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2
))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(1/2*(b*x
^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
 sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2)
- sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2
)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-
c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 121, normalized size = 1.15

$$\frac{\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,\left(-2\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{bx+2c+2\sqrt{a\,x^{2}+bx+c}\,\sqrt{c}}{x}\right)+b\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}\right)x}{2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((b*x+2*c+2*(a*x^2+b*x+c)^(1/
2)*c^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(
1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2), x)
```

mupad [B] time = 0.13, size = 100, normalized size = 0.95

$$x\sqrt{\frac{1}{x^{2}}}\,\sqrt{a\,x^{2}+b\,x+c}-\sqrt{c}\,x\,\ln\left(\frac{2\,c+2\,\sqrt{c}\,\sqrt{a\,x^{2}+b\,x+c}\,+b\,x}{x}\right)\sqrt{\frac{1}{x^{2}}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,x^{2}+b\,x+c}+a\,x}{\sqrt{a}}\right)\sqrt{\frac{1}{x^{2}}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(1/2),x)
```

```
[Out] x*(1/x^2)^(1/2)*(c + b*x + a*x^2)^(1/2) - c^(1/2)*x*log((2*c + 2*c^(1/2)*(c
 + b*x + a*x^2)^(1/2) + b*x)/x)*(1/x^2)^(1/2) + (b*x*log((b/2 + a^(1/2)*(c
+ b*x + a*x^2)^(1/2) + a*x)/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2), x)
```

3.452 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(3/2)+x*(a+c/x^
2+b/x)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1342, 730, 724, 206}

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/(2*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a}$$
$$= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.33

$$\frac{2\sqrt{a}\,(x(ax+b)+c) - b\sqrt{x(ax+b)+c}\,\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)}{2a^{3/2}x\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/
(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

fricas [A] time = 1.15, size = 171, normalized size = 2.55

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{a}\,b\log\left(-8\,a^2x^2 - 8\,abx - b^2 - 4\,ac + 4\left(2\,ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a
^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*x^2
 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.00, size = 88, normalized size = 1.31

$$\frac{\sqrt{a\,x^{2}+bx+c}\,\left(-ab\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a\,x^{2}+bx+c}\;a^{\frac{3}{2}}\right)}{2\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\;a^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(1/2),x)
[Out] 1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*a*x+b+2*
(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a + b/x + c/x^2), x)
```

mupad [B] time = 1.45, size = 53, normalized size = 0.79

$$\frac{x\,\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}{a}-\frac{b\,\mathrm{atanh}\left(\frac{a+\frac{b}{2\,x}}{\sqrt{a}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(1/2),x)
[Out] (x*(a + b/x + c/x^2)^(1/2))/a - (b*atanh((a + b/(2*x))/(a^(1/2)*(a + b/x +
c/x^2)^(1/2))))/(2*a^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(1/2),x)
[Out] Integral(1/sqrt(a + b/x + c/x**2), x)
```

3.453 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}+\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

```
[Out] -3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(5/2)-2*(b^2-2*
a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(1/2)+(-8*a*c+3*b^2)*x*(a+c/x^2+b
/x)^(1/2)/a^2/(-4*a*c+b^2)
```

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1342, 740, 806, 724, 206}

$$\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2
*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(
2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x,\right.}{2a^2}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+}{\sqrt{a+}}\right.}{a^2}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.17, size = 138, normalized size = 1.04

$$-\frac{3b\left(b^2-4ac\right)\sqrt{x(ax+b)+c}\,\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\left(-b^2\left(ax^2+3c\right)+10abcx+4ac\left(ax^2+2c\right)-3b^3x\right)}{2a^{5/2}x\left(b^2-4ac\right)\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] -1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c + a
*x^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/(2*Sqr
t[a]*Sqrt[c + x*(b + a*x)])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x)/x
^2])
```

fricas [A] time = 1.31, size = 465, normalized size = 3.50

$$\left[\frac{3\left(b^3c-4\,abc^2+\left(ab^3-4\,a^2bc\right)x^2+\left(b^4-4\,ab^2c\right)x\right)\sqrt{a}\,\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\left(2\,ax^2+bx\right)\sqrt{a}\right.}{4\left(a^3b^2c-4\,a^4c^2+\left(a^4b^2-4\,a^5c\right)x^2+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)
*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)
*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*
a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/
2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sq
rt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2
*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*
x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c -
4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 197, normalized size = 1.48

$$\frac{\left(a\,x^2+bx+c\right)\left(8a^{\frac{7}{2}}c\,x^2-2a^{\frac{5}{2}}b^2x^2+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^3x-12\sqrt{a\,x^2+bx+c}\;a^2bc\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx+c}\;\sqrt{a}}{2\sqrt{a}}\right)+3\right.}{2\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}\left(4ac-b^2\right)a^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(3/2),x)
```

```
[Out] 1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*x^2*c-2*a^(5/2)*x^2*b^2+20*a^(5/2)*x*b
*c-6*a^(3/2)*x*b^3+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*a*x+b+2*(a*x
^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(2*a
*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)/((a
*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(3/2),x)
```

```
[Out] int(1/(a + b/x + c/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

3.454 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}}}{3a^3\left(b^2-4ac\right)^2}$$

```
[Out] -2/3*(b^2-2*a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(3/2)-5/2*b*arctanh(1
/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(7/2)-2/3*(5*b^4-32*a*b^2*c+32*
a^2*c^2+b*c*(-28*a*c+5*b^2)/x)*x/a^2/(-4*a*c+b^2)^2/(a+c/x^2+b/x)^(1/2)+1/3
*(128*a^2*c^2-100*a*b^2*c+15*b^4)*x*(a+c/x^2+b/x)^(1/2)/a^3/(-4*a*c+b^2)^2
```

Rubi [A] time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1342, 740, 822, 806, 724, 206}

$$-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2
- 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 +
 b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))
/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a +
b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+16ac\right)-3bcx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{4\,\text{Subst}}{}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-\right.}{3a}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-\right.}{3a}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-\right.}{3a}$$

Mathematica [A] time = 0.38, size = 256, normalized size = 1.16

$$\frac{2\sqrt{a}\left(3b^4\left(a^2x^4-30acx^2+5c^2\right)-4ab^2c\left(6a^2x^4-12acx^2+25c^2\right)+8a^2bc^2x\left(32ax^2+39c\right)+16a^2c^2\left(3a^2x^4+\right.\right.}{6a^{7/2}x\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] (2*Sqrt[a]*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2) - 2*a*b^3*c*x*(105
*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c^2 - 30*a*c*x^2 + a^2
*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) - 4*a*b^2*c*(25*c^2 - 1
2*a*c*x^2 + 6*a^2*x^4)) - 15*b*(b^2 - 4*a*c)^2*(c + x*(b + a*x))^(3/2)*ArcT
anh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(6*a^(7/2)*(b^2 - 4*a*c
)^2*x*(c + x*(b + a*x))*Sqrt[a + (c + b*x)/x^2])
```

fricas [B] time = 1.45, size = 1081, normalized size = 4.91

$$\left[\frac{15\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4+\left(a^2b^5-8\,a^3b^3c+16\,a^4bc^2\right)x^4+2\left(ab^6-8\,a^2b^4c+16\,a^3b^2c^2\right)x^3+\left(b^7-6\,ab^5c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6
*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*
sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*
sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^
5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b
^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 5
2*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((
a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4
 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)
*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*
c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b
^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*
x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*
a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35
*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^
3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^
6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^
5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 376, normalized size = 1.71

$$-\frac{\left(a x^2+bx+c\right)\left(-96a^{\frac{13}{2}}c^2x^4+48a^{\frac{11}{2}}b^2c\,x^4-6a^{\frac{9}{2}}b^4x^4-512a^{\frac{11}{2}}b\,c^2x^3+296a^{\frac{9}{2}}b^3c\,x^3-40a^{\frac{7}{2}}b^5x^3-384a^{\frac{11}{2}}c^3x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(5/2),x)
```

```
[Out] -1/6*(a*x^2+b*x+c)*(-96*a^(13/2)*x^4*c^2+48*a^(11/2)*x^4*b^2*c-512*a^(11/2)
*x^3*b*c^2-6*a^(9/2)*x^4*b^4-384*a^(11/2)*x^2*c^3+296*a^(9/2)*x^3*b^3*c-96*
a^(9/2)*x^2*b^2*c^2-40*a^(7/2)*x^3*b^5-624*a^(9/2)*x*b*c^3+180*a^(7/2)*x^2*
b^4*c-256*a^(9/2)*c^4+420*a^(7/2)*x*b^3*c^2-30*a^(5/2)*x^2*b^6+200*a^(7/2)*
b^2*c^3-60*a^(5/2)*x*b^5*c+240*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2
))/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^4*b*c^2-120*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+
c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^3*b^3*c+15*ln(1/2*(2*a*x+b
+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^2*b^5-30*a^(
5/2)*b^4*c^2)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(5/2),x)
```

```
[Out] int(1/(a + b/x + c/x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-5/2), x)
```

3.455 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal. Leaf size=73

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

```
[Out] a*x*(a^2+b^2/x^2+2*a*b/x)^(1/2)/(a+b/x)-b*ln(1/x)*(a^2+b^2/x^2+2*a*b/x)^(1/
2)/(a+b/x)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1342, 646, 43}

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (
2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\ \text{Subst}\left(\int \frac{ab+b^2x}{x^2}\,dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}}$$
$$= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\ \text{Subst}\left(\int \left(\frac{ab}{x^2}+\frac{b^2}{x}\right)dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}}$$
$$= \frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}} + \frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\log(x)}{a+\frac{b}{x}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}\,(ax+b\log(x))}{ax+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

fricas [A] time = 0.86, size = 8, normalized size = 0.11

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
[Out] a*x + b*log(x)
```

giac [A] time = 0.34, size = 29, normalized size = 0.40

$$ax\text{sgn}\left(ax^2+bx\right)+b\log\left(|x|\right)\text{sgn}\left(ax^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

maple [A] time = 0.01, size = 40, normalized size = 0.55

$$\frac{\sqrt{\frac{a^2x^2+2axb+b^2}{x^2}}\,(ax+b\ln(x))\,x}{ax+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^2+2*a*b/x)^(1/2),x)
[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

maxima [A] time = 0.78, size = 8, normalized size = 0.11

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] a*x + b*log(x)
```

mupad [B] time = 0.11, size = 134, normalized size = 1.84

$$x\sqrt{\frac{1}{x^2}}\sqrt{a^2x^2+2abx+b^2}-x\ln\left(\frac{2\sqrt{b^2}\sqrt{a^2x^2+2abx+b^2}+2b^2+2abx}{x}\right)\sqrt{b^2}\sqrt{\frac{1}{x^2}}+\frac{abx\ln\left(\frac{ab+\sqrt{a^2}\sqrt{a^2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)
```

```
[Out] x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^2
 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/2)
 + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x)/(
a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

3.456 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1340, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
```

```
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx &= \int \frac{x^4}{a+bx^2+cx^4}\,dx\\
&= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 202, normalized size = 1.13

$$
-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.91, size = 1059, normalized size = 5.92

$$
-\frac{\sqrt{\frac{1}{2}}\,c\sqrt{-\frac{b^3-3abc+\left(b^2c^3-4ac^4\right)\sqrt{\frac{b^4-2ab^2c+a^2c^2}{b^2c^6-4ac^7}}}{b^2c^3-4ac^4}}\log\left(-2\left(ab^2-a^2c\right)x+\sqrt{\frac{1}{2}}\left(b^4-5ab^2c+4a^2c^2-\left(b^3c^3-4abc^4\right)\sqrt{\frac{b^4-\cdots}{b^2\cdots}}\right.\right.}{\cdots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
```

```
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

giac [B] time = 1.66, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
```

```
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.01, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\, a \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^4+b/x^2),x)
```

```
[Out] 1/c*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/2/c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab^2c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^3c-2\,b^4c+16\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,a^2c^2+8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,abc^2+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 2.08, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^4 + b/x^2),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
```

```
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 2.12, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

3.457 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\ldots\right.}{\ldots}$$

```
[Out] x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(
-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3
)
```

Rubi [A] time = 1.17, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2 \cdots}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \cdots$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4a \cdots}}\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-4ac \cdots}\right)} \cdots$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4a \cdots}}\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-4ac \cdots}\right)} \cdots$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-\cdots}{\sqrt{b^2-\cdots}}\right)\cdots}{\cdots}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

fricas [B] time = 2.60, size = 5260, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6
```

- 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) - 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b

```
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 -
 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*
a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1
/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*
c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*
c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3
+ 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^
4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^
(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b
^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log
(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*
a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*
b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*
a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*
c^5))^(1/3)) + 6*x)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + \frac{b}{x^3} + \frac{a}{x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^3 + a/x^6), x)
```

maple [C] time = 0.01, size = 59, normalized size = 0.09

$$\frac{x}{c}+\frac{\left(-\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{3}b-a\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)+x\right)}{3c\left(2\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{5}c+\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^6+b/x^3),x)
```

```
[Out] 1/c*x+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z
^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\int\frac{bx^{3}+a}{cx^{6}+bx^{3}+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 4.54, size = 2280, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^6 + b/x^3),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
```

```
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

sympy [A] time = 6.97, size = 196, normalized size = 0.31

$$\mathrm{RootSum}\left(t^6\left(46656a^3c^7 - 34992a^2b^2c^6 + 8748ab^4c^5 - 729b^6c^4\right) + t^3\left(864a^3bc^3 - 864a^2b^3c^2 + 270ab^5c - 27b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**6+b/x**3),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
 x/c
```

3.458 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

```
[Out] x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(
2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*
c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))
*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.67, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1340, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx &= \int \frac{x^8}{a+bx^4+cx^8}\,dx \\
&= \frac{x}{c}-\frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} \\
&= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} \\
&= \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b+\frac{b^2}{\sqrt{b^2}}\right.}{2\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.19

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

fricas [B] time = 2.06, size = 5082, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(2*sqrt(1/2)*((b^10*c^
5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128
*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^11 - 1
3*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c
^5)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*
c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^
6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^2*b^4 - 3*a^3
*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^
3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b
*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^4*b^
4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/
4*(2*sqrt(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8
 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*
a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c
^4 - 32*a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4
*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
```

```
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^
2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^
4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c
^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))/(a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3
*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*
a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*
c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^
5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*x)/c
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.96Unable to convert to real 1/4 Error: Bad Argument Value

maple [C] time = 0.00, size = 59, normalized size = 0.16

$$\frac{x}{c}+\frac{\left(-\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{4}b-a\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)+x\right)}{4c\left(2\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{7}c+\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^8+b/x^4),x)

[Out] 1/c*x+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 3.78, size = 10382, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + a/x^8 + b/x^4),x)

[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^

6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4

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*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 +
 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b
^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c +
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((16*(a^3*b^6
 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*
b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3
 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4
)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4
*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*
b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
```

```
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 1
6*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 2
56*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/
4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((16*(a^3
*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)
/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*
a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6
*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^
(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 -
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4
*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*
c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
```

```
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c +
13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6
 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + x/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**8+b/x**4),x)
```

```
[Out] Timed out
```

3.459 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

Optimal. Leaf size=106

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

```
[Out] -2*arctanh(1/2*(2*a+b*x^(1/2))/a^(1/2)/(a+c*x+b*x^(1/2))^(1/2))*a^(1/2)+b*a
rctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+c*x+b*x^(1/2))^(1/2))/c^(1/2)+2*(a+c*
x+b*x^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 734, 843, 621, 206, 724}

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{a+b\sqrt{x}+cx} - \text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{a+b\sqrt{x}+cx} + (2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right) + b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\cdots}\right.\\
&= 2\sqrt{a+b\sqrt{x}+cx} - (4a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{x}}{\sqrt{a+b\sqrt{x}+cx}}\right) + (2b)\,\text{Subst}\left(\int \frac{}{4c\cdots}\right.\\
&= 2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.00

$$2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [A] time = 0.01, size = 84, normalized size = 0.79

$$-2\sqrt{a}\,\ln\left(\frac{b\sqrt{x}+2a+2\sqrt{cx+b\sqrt{x}+a}\,\sqrt{a}}{\sqrt{x}}\right)+\frac{b\ln\left(\frac{c\sqrt{x}+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx+b\sqrt{x}+a}\right)}{\sqrt{c}}+2\sqrt{cx+b\sqrt{x}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x+b*x^(1/2))^(1/2)/x,x)
```

```
[Out] 2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))^
(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1/2
))/x^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+c\,x+b\,\sqrt{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x + b*x^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + c*x + b*x^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

3.460 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

Optimal. Leaf size=40

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

```
[Out] -1/160*b*(b+2*c*x^(1/2))^5/c^4+1/192*(b+2*c*x^(1/2))^6/c^4
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 190, 43}

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx &= \frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4 dx}{c^2}\\
&= \frac{2\,\text{Subst}\left(\int x\left(\frac{b}{2}+cx\right)^4 dx, x, \sqrt{x}\right)}{c^2}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{b\left(\frac{b}{2}+cx\right)^4}{2c}+\frac{\left(\frac{b}{2}+cx\right)^5}{c}\right) dx, x, \sqrt{x}\right)}{c^2}\\
&= -\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.72

$$-\frac{\left(b-10c\sqrt{x}\right)\left(b+2c\sqrt{x}\right)^5}{960c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -1/960*((b - 10*c*Sqrt[x])*(b + 2*c*Sqrt[x])^5)/c^4
```

fricas [A] time = 1.20, size = 53, normalized size = 1.32

$$\frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\left(12\,bc^3x^2+5\,b^3cx\right)\sqrt{x}}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*c
*x)*sqrt(x))/c^2
```

giac [A] time = 0.33, size = 49, normalized size = 1.22

$$\frac{80\,c^4x^3+192\,bc^3x^{\frac{5}{2}}+180\,b^2c^2x^2+80\,b^3cx^{\frac{3}{2}}+15\,b^4x}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
+ 15*b^4*x)/c^2
```

maple [A] time = 0.00, size = 52, normalized size = 1.30

$$\frac{b^2x^2}{2}+\frac{\left(\frac{8c^2x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}\right)b}{2c}+\frac{\left(cx+\frac{b^2}{4c}\right)^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*b^2/c+c*x+b*x^(1/2))^2,x)
```

```
[Out] 1/2*b^2*x^2+1/2*b/c*(8/5*c^2*x^(5/2)+2/3*x^(3/2)*b^2)+1/3*(1/4*b^2/c+c*x)^3
/c
```

maxima [A] time = 0.87, size = 54, normalized size = 1.35

$$\frac{1}{3}\,c^2x^3+\frac{4}{5}\,bcx^{\frac{5}{2}}+\frac{1}{2}\,b^2x^2+\frac{b^4x}{16\,c^2}+\frac{\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)b^2}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x^
2 + 4*b*x^(3/2))*b^2/c
```

mupad [B] time = 0.04, size = 44, normalized size = 1.10

$$\frac{3\,b^2\,x^2}{4}+\frac{c^2\,x^3}{3}+\frac{b^4\,x}{16\,c^2}+\frac{b^3\,x^{3/2}}{3\,c}+\frac{4\,b\,c\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)
```

```
[Out] (3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^(3/2))/(3*c) + (4*b
*c*x^(5/2))/5
```

sympy [A] time = 0.33, size = 51, normalized size = 1.28

$$\frac{b^4x}{16c^2} + \frac{b^3x^{\frac{3}{2}}}{3c} + \frac{3b^2x^2}{4} + \frac{4bcx^{\frac{5}{2}}}{5} + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b**2+c*x+b*x**(1/2))**2,x)
```

```
[Out] b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5 +
 c**2*x**3/3
```

3.461 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

```
[Out] -2*a*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b^2/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)+2*(a
^2+b^2*x+2*a*b*x^(1/2))^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1341, 640, 608, 31}

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*
Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{\left(2a\left(a+b\sqrt{x}\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.67

$$\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sqr
t[x])^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.40, size = 45, normalized size = 0.60

$$-\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\,\text{sgn}(a)\text{sgn}(b)+|a|\right|\right)}{b^2} + \frac{2\,\sqrt{b^2x}}{b^2\text{sgn}(a)\text{sgn}(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*sqrt(b^2*x)/
(b^2*sgn(a)*sgn(b))
```

maple [A] time = 0.01, size = 50, normalized size = 0.67

$$\frac{2\sqrt{b^2x+2ab\sqrt{x}+a^2}\,\left(-a\ln\left(b\sqrt{x}+a\right)+b\sqrt{x}\right)}{\left(b\sqrt{x}+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x)
```

[Out] 2*(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)*(b*x^(1/2)-a*ln(a+b*x^(1/2)))/(a+b*x^(1/2
))/b^2

maxima [A] time = 0.96, size = 23, normalized size = 0.31

$$-\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")

[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)

[Out] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x), x)

3.462 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\right.}{8b^3}$$

```
[Out] 3/8*a^2*(a+b*x^(1/3))^7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-2/3*a*(a+
b*x^(1/3))^8*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/10*(a+b*x^(1/3))^9
*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3) -
 (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(3*b^3) +
(3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{7/2}dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^7dx, x, \sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^7}{b^2}-\frac{2a(ab+b^2x)^8}{b^3}+\frac{(ab+b^2x)^9}{b^4}\right)d\right.}{b^7\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3} - \frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+}}{3b^3}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(a^2-8ab\sqrt[3]{x}+36b^2x^{2/3}\right)}{120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] ((a + b*x^(1/3))^7*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 8*a*b*x^(1/3) + 36*b^2*x^
(2/3)))/(120*b^3)
```

fricas [A] time = 1.28, size = 84, normalized size = 0.61

$$\frac{7}{3}ab^6x^3+\frac{35}{2}a^4b^3x^2+a^7x+\frac{63}{40}\left(5a^2b^5x^2+8a^5b^2x\right)x^{\frac{2}{3}}+\frac{3}{20}\left(2b^7x^3+100a^3b^4x^2+35a^6bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^2
*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

giac [A] time = 0.51, size = 140, normalized size = 1.02

$$\frac{3}{10}b^7x^{\frac{10}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{7}{3}ab^6x^3\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{63}{8}a^2b^5x^{\frac{8}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+15a^3b^4x^{\frac{7}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{35}{2}a^4b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] 3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) + 6
3/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3) +
 a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x^(1
/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.02, size = 109, normalized size = 0.80

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2}\left(36b^7x^{\frac{10}{3}}+280ab^6x^3+945a^2b^5x^{\frac{8}{3}}+1800a^3b^4x^{\frac{7}{3}}+2100a^4b^3x^2+1512a^5b^2x^{\frac{5}{3}}+630a^6\right.}{120bx^{\frac{1}{3}}+120a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

```
[Out] 1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*b^7*x^(10/3)+945*a^2*b^5*x^
(8/3)+1800*a^3*b^4*x^(7/3)+1512*a^5*b^2*x^(5/3)+630*a^6*b*x^(4/3)+280*a*b^6
*x^3+2100*a^4*b^3*x^2+120*a^7*x)/(a+b*x^(1/3))
```

maxima [A] time = 0.91, size = 114, normalized size = 0.83

$$\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^2x^{\frac{1}{3}}}{8\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^3}{8\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}x^{\frac{1}{3}}}{10\,b^2}-\frac{11\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^2*x^(1/3)/b^2 + 3/8*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^3/b^3 + 3/10*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^(9/2)*x^(1/3)/b^2 - 11/30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(9
/2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(7/2), x)
```

3.463 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,\left(a+\right.}{2b^3}$$

```
[Out] 1/2*a^2*(a+b*x^(1/3))^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/7*a*(a+
b*x^(1/3))^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/8*(a+b*x^(1/3))^7*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{1341, 646, 43\}$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,\left(a+\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (
6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3
*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2} dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5 dx, x, \sqrt[3]{x}\right)}{b^5\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^5}{b^2}-\frac{2a(ab+b^2x)^6}{b^3}+\frac{(ab+b^2x)^7}{b^4}\right) dx, x\right.}{b^5\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/}}}{7b^3}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(a^2-6ab\sqrt[3]{x}+21b^2x^{2/3}\right)}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] ((a + b*x^(1/3))^5*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 6*a*b*x^(1/3) + 21*b^2*x^
(2/3)))/(56*b^3)
```

fricas [A] time = 0.76, size = 61, normalized size = 0.45

$$5\,a^2b^3x^2 + a^5x + \frac{3}{8}\left(b^5x^2 + 16\,a^3b^2x\right)x^{\frac{2}{3}} + \frac{15}{28}\left(4\,ab^4x^2 + 7\,a^4bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b
^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

giac [A] time = 0.53, size = 102, normalized size = 0.74

$$\frac{3}{8}\,b^5x^{\frac{8}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}\,ab^4x^{\frac{7}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+5\,a^2b^3x^2\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+6\,a^3b^2x^{\frac{5}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}\,a^4bx^{\frac{4}{3}}\text{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
+ 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a) +
 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.00, size = 87, normalized size = 0.64

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(21b^5x^{\frac{8}{3}}+120a\,b^4x^{\frac{7}{3}}+280a^2b^3x^2+336a^3b^2x^{\frac{5}{3}}+210a^4b\,x^{\frac{4}{3}}+56a^5x\right)}{56b\,x^{\frac{1}{3}}+56a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(5/2),x)
```

```
[Out] 1/56*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(21*b^5*x^(8/3)+120*a*b^4*x^(7/3
)+336*a^3*b^2*x^(5/3)+210*a^4*b*x^(4/3)+280*a^2*b^3*x^2+56*a^5*x)/(b*x^(1/3
)+a)
```

maxima [A] time = 0.89, size = 114, normalized size = 0.83

$$\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8\,b^2}-\frac{27\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)}{56\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/
2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/2), x)
```

3.464 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right.}{4b^3}$$

```
[Out] 3/4*a^2*(a+b*x^(1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/5*a*(a+
b*x^(1/3))^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+1/2*(a+b*x^(1/3))^5*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1341, 645}

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right.}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) -
 (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(5*b^3) +
((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^3}{b^2}-\frac{2a\left(ab+b^2x\right)^4}{b^3}+\frac{\left(ab+b^2x\right)^5}{b^4}\right)dx,x\right.}{b^3\left(a+b\sqrt[3]{x}\right)}$$

$$=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x\right.}{5b^3}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.47

$$\frac{x\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(20a^3+45a^2b\sqrt[3]{x}+36ab^2x^{2/3}+10b^3x\right)}{20\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(20*a^3 + 45*a^2*b*x^(1/3) + 36*a*b^2*x^(2/3) +
10*b^3*x))/(20*(a + b*x^(1/3)))
```

fricas [A] time = 1.39, size = 32, normalized size = 0.23

$$\frac{1}{2}b^3x^2 + \frac{9}{5}ab^2x^{\frac{5}{3}} + \frac{9}{4}a^2bx^{\frac{4}{3}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

giac [A] time = 0.37, size = 64, normalized size = 0.47

$$\frac{1}{2}b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{5}ab^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{4}a^2bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + a^3x\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/4
*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.00, size = 65, normalized size = 0.47

$$\frac{\sqrt{b^2x^{\frac{2}{3}} + 2ab\,x^{\frac{1}{3}} + a^2}\,\left(10b^3x^2 + 36a\,b^2x^{\frac{5}{3}} + 45a^2b\,x^{\frac{4}{3}} + 20a^3x\right)}{20b\,x^{\frac{1}{3}} + 20a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(3/2),x)
```

```
[Out] 1/20*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(36*a*b^2*x^(5/3)+45*a^2*b*x^(4/
3)+10*b^3*x^2+20*a^3*x)/(b*x^(1/3)+a)
```

maxima [A] time = 0.91, size = 114, normalized size = 0.83

$$\frac{3\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}}a^2x^{\frac{1}{3}}}{4\,b^2} + \frac{3\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}}a^3}{4\,b^3} + \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}}x^{\frac{1}{3}}}{2\,b^2} - \frac{7\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2
)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(3/2), x)
```

3.465 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

```
[Out] a*x*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))+3/4*b*x^(4/3)*(a^2+
2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a
^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = 3\,\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,\sqrt[3]{x}\right)$$
$$=\frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)dx,x,\sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}$$
$$=\frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int \left(abx^2+b^2x^3\right)dx,x,\sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}$$
$$=\frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.49

$$\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

fricas [A] time = 1.16, size = 10, normalized size = 0.11

$$\frac{3}{4}\,bx^{\frac{4}{3}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
[Out] 3/4*b*x^(4/3) + a*x
```

giac [A] time = 0.39, size = 26, normalized size = 0.30

$$\frac{3}{4}\,bx^{\frac{4}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right)+ax\text{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
[Out] 3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.00, size = 43, normalized size = 0.49

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(3b\,x^{\frac{4}{3}}+4ax\right)}{4b\,x^{\frac{1}{3}}+4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2),x)
[Out] 1/4*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(3*b*x^(4/3)+4*a*x)/(b*x^(1/3)+a)
```

maxima [A] time = 0.88, size = 114, normalized size = 1.30

$$\frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}\,a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}\,a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}x^{\frac{1}{3}}}{4\,b^2}-\frac{5\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a/b
^3
```

mupad [B] time = 1.56, size = 71, normalized size = 0.81

$$\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^3-4\,a^2\,b\,x^{1/3}-5\,a\,b^2\,x^{2/3}+3\,b\,x^{1/3}\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^3 - 4*a^2*b*x^(1/3) - 5*a*b^2
*x^(2/3) + 3*b*x^(1/3)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))))/(4*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

3.466 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal. Leaf size=147

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3/2*(a
+b*x^(1/3))*x^(2/3)/b/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*a^2*(a+b*x^(1
/3))*ln(a+b*x^(1/3))/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = 3\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\mathrm{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\mathrm{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\mathrm{lo}}{b^3\sqrt{a^2+2ab\sqrt[3]{}}}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(2a^2\log\left(a+b\sqrt[3]{x}\right)+b\sqrt[3]{x}\left(b\sqrt[3]{x}-2a\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)]
))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

fricas [A] time = 1.39, size = 33, normalized size = 0.22

$$\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

giac [A] time = 0.49, size = 61, normalized size = 0.41

$$\frac{3\left(bx^{\frac{2}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2}+\frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
[Out] 3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 + 3
*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

maple [A] time = 0.02, size = 103, normalized size = 0.70

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\left(4a^2\ln\left(b\,x^{\frac{1}{3}}+a\right)-2a^2\ln\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)+2a^2\ln\left(b^3x+a^3\right)+3b^2x^{\frac{2}{3}}-6ab\,x^{\frac{1}{3}}\right)}{2\left(b\,x^{\frac{1}{3}}+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2),x)
```

```
[Out] 1/2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(3*b^2*x^(2/3)-6*a*b*x^(1/3)+2*a^
2*ln(b^3*x+a^3)-2*a^2*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)+4*a^2*ln(b*x^(1/3)+a)
)/(b*x^(1/3)+a)/b^3
```

maxima [A] time = 0.88, size = 36, normalized size = 0.24

$$\frac{3\,a^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{3\,ax^{\frac{1}{3}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

3.467 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] 6*a/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2*a^2/b^3/(a+b*x^(1/3))/(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*(a+b*x^(1/3))*ln(a+b*x^(1/3))/b^3/(a^2
+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b
*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log
[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\right.}{b^3\sqrt{a^2}}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.55

$$\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*(
a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

fricas [A] time = 1.32, size = 113, normalized size = 0.87

$$\frac{3\left(6\,a^3b^3x+3\,a^6+2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(bx^{\frac{1}{3}}+a\right)+\left(4\,ab^5x+a^4b^2\right)x^{\frac{2}{3}}-\left(5\,a^2b^4x+2\,a^5b\right)x^{\frac{1}{3}}\right)}{2\left(b^9x^2+2\,a^3b^6x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*
x^2 + 2*a^3*b^6*x + a^6*b^3)
```

giac [A] time = 0.53, size = 64, normalized size = 0.49

$$\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)} + \frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2 b^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*a
^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

maple [A] time = 0.01, size = 92, normalized size = 0.71

$$\frac{3\sqrt{b^{2}x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^{2}}\,\left(2b^{2}x^{\frac{2}{3}}\ln\left(b\,x^{\frac{1}{3}}+a\right)+4ab\,x^{\frac{1}{3}}\ln\left(b\,x^{\frac{1}{3}}+a\right)+2a^{2}\ln\left(b\,x^{\frac{1}{3}}+a\right)+4ab\,x^{\frac{1}{3}}+3a^{2}\right)}{2\left(b\,x^{\frac{1}{3}}+a\right)^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(3/2),x)
```

```
[Out] 3/2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(2*x^(2/3)*ln(b*x^(1/3)+a)*b^2+4*
x^(1/3)*ln(b*x^(1/3)+a)*a*b+4*a*b*x^(1/3)+2*a^2*ln(b*x^(1/3)+a)+3*a^2)/(b*x
^(1/3)+a)^3/b^3
```

maxima [A] time = 0.62, size = 55, normalized size = 0.42

$$\frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^{3}}+\frac{6\,ax^{\frac{1}{3}}}{b^{4}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{2}}+\frac{9\,a^{2}}{2\,b^{5}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(b
^5*(x^(1/3) + a/b)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

3.468 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}}}$$

```
[Out] -3/4*a^2/b^3/(a+b*x^(1/3))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2*a/b^3/
(a+b*x^(1/3))^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2/b^3/(a+b*x^(1/3))
/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^7(a+bx)^5}-\frac{2a}{b^7(a+bx)^4}+\frac{1}{b^7(a+bx)^3}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.43

$$\frac{-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3))/(4*b^3*(a + b*x^(1/3))^3*Sqrt[(a + b
*x^(1/3))^2])
```

fricas [A] time = 1.10, size = 136, normalized size = 1.01

$$\frac{20\,ab^9x^3-60\,a^4b^6x^2-a^{10}-9\left(5\,a^2b^8x^2-4\,a^5b^5x\right)x^{\frac{2}{3}}-3\left(2\,b^{10}x^3-20\,a^3b^7x^2+5\,a^6b^4x\right)x^{\frac{1}{3}}}{4\left(b^{15}x^4+4\,a^3b^{12}x^3+6\,a^6b^9x^2+4\,a^9b^6x+a^{12}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x)
*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x^4
 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

giac [A] time = 0.55, size = 43, normalized size = 0.32

$$-\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(bx^{\frac{1}{3}}+a\right)^4b^3\text{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^(
1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.40

$$-\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(6b^2x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^2\right)}{4\left(b\,x^{\frac{1}{3}}+a\right)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(5/2),x)
```

```
[Out] -1/4*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2
)/(b*x^(1/3)+a)^5/b^3
```

maxima [A] time = 0.72, size = 53, normalized size = 0.39

$$-\frac{3}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{2\,a}{b^6\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3\,a^2}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] -3/2/(b^5*(x^(1/3) + a/b)^2) + 2*a/(b^6*(x^(1/3) + a/b)^3) - 3/4*a^2/(b^7*(
x^(1/3) + a/b)^4)
```

mupad [B] time = 2.80, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+6\,b^2\,x^{2/3}+4\,a\,b\,x^{1/3}\right)}{4\,b^3\,\left(a+b\,x^{1/3}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 6*b^2*x^(2/3) + 4*a*b*x^
(1/3)))/(4*b^3*(a + b*x^(1/3))^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

3.469 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2a}}$$

```
[Out] -1/2*a^2/b^3/(a+b*x^(1/3))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/5*a/b^
3/(a+b*x^(1/3))^4/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/4/b^3/(a+b*x^(1/3
))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (6
*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(
4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{7/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^7}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^9(a+bx)^7}-\frac{2a}{b^9(a+bx)^6}+\frac{1}{b^9(a+bx)^5}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.42

$$\frac{-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}}{20b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] (-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3))/(20*b^3*(a + b*x^(1/3))^5*Sqrt[(a +
 b*x^(1/3))^2])
```

fricas [A] time = 1.15, size = 209, normalized size = 1.53

$$-\frac{280\,a^2b^{12}x^4-1400\,a^5b^9x^3+735\,a^8b^6x^2-14\,a^{11}b^3x+a^{14}+3\left(5\,b^{14}x^4-210\,a^3b^{11}x^3+483\,a^6b^8x^2-112\,a^9b^5x\right)\cdots}{20\left(b^{21}x^6+6\,a^3b^{18}x^5+15\,a^6b^{15}x^4+20\,a^9b^{12}x^3+15\,a^{12}b^9x^2+6\,\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3*
x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b^5
*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35*a^
10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^9*b^
12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)
```

giac [A] time = 0.59, size = 43, normalized size = 0.31

$$-\frac{15\,b^2x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}+a^2}{20\left(bx^{\frac{1}{3}}+a\right)^6b^3\,\text{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] -1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^6*b^3*sgn(b*x
^(1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^2 x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(15b^2x^{\frac{2}{3}}+6ab\,x^{\frac{1}{3}}+a^2\right)}{20\left(b\,x^{\frac{1}{3}}+a\right)^7 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(7/2),x)
```

```
[Out] -1/20*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a
^2)/(b*x^(1/3)+a)^7/b^3
```

maxima [A] time = 0.54, size = 53, normalized size = 0.39

$$-\frac{3}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}+\frac{6\,a}{5\,b^8\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^5}-\frac{a^2}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] -3/4/(b^7*(x^(1/3) + a/b)^4) + 6/5*a/(b^8*(x^(1/3) + a/b)^5) - 1/2*a^2/(b^9
*(x^(1/3) + a/b)^6)
```

mupad [B] time = 3.23, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+15\,b^2\,x^{2/3}+6\,a\,b\,x^{1/3}\right)}{20\,b^3\,{\left(a+b\,x^{1/3}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 15*b^2*x^(2/3) + 6*a*b*x
^(1/3)))/(20*b^3*(a + b*x^(1/3))^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

3.470 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3/8*a^2/b^3/(a+b*x^(1/3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/7*a/b^
3/(a+b*x^(1/3))^6/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-1/2/b^3/(a+b*x^(1/3
))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) -
 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{9/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{11}(a+bx)^9}-\frac{2a}{b^{11}(a+bx)^8}+\frac{1}{b^{11}(a+bx)^7}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.42

$$\frac{-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}}{56b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3))/(56*b^3*(a + b*x^(1/3))^7*Sqrt[(a +
 b*x^(1/3))^2])
```

fricas [B] time = 1.17, size = 275, normalized size = 2.01

$$-\frac{28\,b^{18}x^6-2856\,a^3b^{15}x^5+18186\,a^6b^{12}x^4-20608\,a^9b^9x^3+4200\,a^{12}b^6x^2-48\,a^{15}b^3x+a^{18}-27\left(8\,ab^{17}x^5-2\right.}{56\left(b^{27}x^8+8\,a^3b^{24}x^7+28\,a^6b^{21}x^6+56\,a^9b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^9
*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*a^
4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3) +
 27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b^7*x
^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*x^6 +
 56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*x^2 +
8*a^21*b^6*x + a^24*b^3)
```

giac [A] time = 0.65, size = 43, normalized size = 0.31

$$-\frac{28\,b^2x^{\frac{2}{3}}+8\,abx^{\frac{1}{3}}+a^2}{56\left(bx^{\frac{1}{3}}+a\right)^8b^3\text{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")
```

```
[Out] -1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^8*b^3*sgn(b*x
^(1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^{2} x^{\frac{2}{3}}+2 a b \,x^{\frac{1}{3}}+a^{2}}\, \left(28 b^{2} x^{\frac{2}{3}}+8 a b \,x^{\frac{1}{3}}+a^{2}\right)}{56 \left(b \,x^{\frac{1}{3}}+a \right)^{9} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(9/2),x)
```

```
[Out] -1/56*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(28*b^2*x^(2/3)+8*a*b*x^(1/3)+a
^2)/(b*x^(1/3)+a)^9/b^3
```

maxima [A] time = 0.72, size = 53, normalized size = 0.39

$$-\frac{1}{2 \, b^{9} {\left(x^{\frac{1}{3}} + \frac{a}{b}\right)}^{6}} + \frac{6 \, a}{7 \, b^{10} {\left(x^{\frac{1}{3}} + \frac{a}{b}\right)}^{7}} - \frac{3 \, a^{2}}{8 \, b^{11} {\left(x^{\frac{1}{3}} + \frac{a}{b}\right)}^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^9*(x^(1/3) + a/b)^6) + 6/7*a/(b^10*(x^(1/3) + a/b)^7) - 3/8*a^2/(b^
11*(x^(1/3) + a/b)^8)
```

mupad [B] time = 3.65, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+28\,b^2\,x^{2/3}+8\,a\,b\,x^{1/3}\right)}{56\,b^3\,{\left(a+b\,x^{1/3}\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(9/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 28*b^2*x^(2/3) + 8*a*b*x
^(1/3)))/(56*b^3*(a + b*x^(1/3))^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-9/2), x)
```

3.471 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2}$$

```
[Out] -3/10*a^2/b^3/(a+b*x^(1/3))^9/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2/3*a/b
^3/(a+b*x^(1/3))^8/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/8/b^3/(a+b*x^(1/
3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
 + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
- 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^{11}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{13}(a+bx)^{11}}-\frac{2a}{b^{13}(a+bx)^{10}}+\frac{1}{b^{13}(a+bx)^9}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.42

$$\frac{-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}}{120b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3))/(120*b^3*(a + b*x^(1/3))^9*Sqrt[(a
 + b*x^(1/3))^2])
```

fricas [B] time = 1.22, size = 343, normalized size = 2.50

$$\frac{440\,ab^{21}x^7-25630\,a^4b^{18}x^6+186252\,a^7b^{15}x^5-326150\,a^{10}b^{12}x^4+154000\,a^{13}b^9x^3-16005\,a^{16}b^6x^2+110\,a^{19}b^3x}{120\left(b^{33}x^{10}+10\,a^3b^{30}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*a
^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x -
a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 10910*
a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22*x^7
 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*a^12*
b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 + 10*a^3
*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 + 252*a^
15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x^2 + 10*
a^27*b^6*x + a^30*b^3)
```

giac [A] time = 0.74, size = 43, normalized size = 0.31

$$-\frac{45\,b^2x^{\frac{2}{3}}+10\,abx^{\frac{1}{3}}+a^2}{120\left(bx^{\frac{1}{3}}+a\right)^{10}b^3\text{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")
```

```
[Out] -1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^10*b^3*sgn(
b*x^(1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(45b^2x^{\frac{2}{3}}+10ab\,x^{\frac{1}{3}}+a^2\right)}{120\left(b\,x^{\frac{1}{3}}+a\right)^{11}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(11/2),x)
```

```
[Out] -1/120*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(45*b^2*x^(2/3)+10*a*b*x^(1/3)
+a^2)/(b*x^(1/3)+a)^11/b^3
```

maxima [A] time = 0.54, size = 53, normalized size = 0.39

$$-\frac{3}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}+\frac{2\,a}{3\,b^{12}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3\,a^2}{10\,b^{13}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")
```

```
[Out] -3/8/(b^11*(x^(1/3) + a/b)^8) + 2/3*a/(b^12*(x^(1/3) + a/b)^9) - 3/10*a^2/(
b^13*(x^(1/3) + a/b)^10)
```

mupad [B] time = 4.34, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+45\,b^2\,x^{2/3}+10\,a\,b\,x^{1/3}\right)}{120\,b^3\,\left(a+b\,x^{1/3}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(11/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 45*b^2*x^(2/3) + 10*a*b*
x^(1/3)))/(120*b^3*(a + b*x^(1/3))^11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-11/2), x)
```

3.472 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

Optimal. Leaf size=77

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

```
[Out] (a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x*(d*x)^m*hypergeom([-2*p, 3+3*m],[4+3*m]
,-b*x^(1/3)/a)/(1+m)/((1+b*x^(1/3)/a)^(2*p))
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1356, 343, 341, 64}

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m
), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}(dx)^m\,dx$$

$$= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p} x^m$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\text{Subst}\left(\int x^{-1+3(1+}\right.$$

$$= \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m\,{}_2F_1\left(3(1+m),-2p;4+3\right.}{1+m}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.88

$$\frac{x(dx)^m\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}{}_2F_1\left(3(m+1),-2p;3(m+1)+1;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] (((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*p, 1 + 3*(
1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*(d*x)^m,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*(d*x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
```

```
[Out] Timed out
```

3.473 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal. Leaf size=468

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\right.}{b^9(2p+7)}$$

```
[Out] 3*a^9*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+2*p)-12*a^9*
(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+p)+84*a^9*(1+b*x
^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+2*p)-84*a^9*(1+b*x^(1/
3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(2+p)+210*a^9*(1+b*x^(1/3)/a)
^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(5+2*p)-84*a^9*(1+b*x^(1/3)/a)^6*(
a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+p)+84*a^9*(1+b*x^(1/3)/a)^7*(a^2+2*
a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(7+2*p)-12*a^9*(1+b*x^(1/3)/a)^8*(a^2+2*a*b*
x^(1/3)+b^2*x^(2/3))^p/b^9/(4+p)+3*a^9*(1+b*x^(1/3)/a)^9*(a^2+2*a*b*x^(1/3)
+b^2*x^(2/3))^p/b^9/(9+2*p)
```

Rubi [A] time = 0.22, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1356, 266, 43}

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\right.}{b^9(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 +
 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^
2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/
3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1
+ (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) -
 (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*
(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(b^9*(9 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
```

```
FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x^2\,dx$$

$$=\left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^8\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,\sqrt[3]{x}\right.$$

$$=\left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^8\left(1+\frac{bx}{a}\right)^{2p}}{b^8}-\frac{8a^8\left(1\right.}{}\right.\right.$$

$$=\frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)}-\frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x\right.}{b^9(1+p)}$$

Mathematica [A] time = 0.22, size = 207, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\frac{a^8}{2p+1}-\frac{4a^7\left(a+b\sqrt[3]{x}\right)}{p+1}+\frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{2p+3}-\frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{p+2}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{2p+5}-\frac{28a^3\left(a+b\sqrt[3]{x}\right)^5}{p+3}+\frac{28a^2\left(a+b\sqrt[3]{x}\right)^6}{2p+7}-\frac{4a\left(a+\right.}{p}\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/3
))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(1/
3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x^(1
/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8/(9
+ 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9
```

fricas [A] time = 1.48, size = 579, normalized size = 1.24

$$3\left(2520\,a^9+\left(16\,b^9p^8+288\,b^9p^7+2184\,b^9p^6+9072\,b^9p^5+22449\,b^9p^4+33642\,b^9p^3+29531\,b^9p^2+13698\,b^9p\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")
```

```
[Out] 3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 224
49*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3 +
28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 + 13
7*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a
^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^
8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 +
 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p^4 +
35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260*a^8*b*p
 + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b^7*p^4 +
812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4
+ 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^
```

```
9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 +
 128322*b^9*p + 22680*b^9)
```

giac [B] time = 0.63, size = 1564, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

```
[Out] 3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1/3
) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3) +
2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^5*x^3 +
 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 2800*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^
5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^4*x^(8/3) -
5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3) + 4760*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^4*x^(4/3)
- 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3*x^(7/3) + 6300*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 + 29531*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*b^9*p^2*x^3 - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)^p*a^4*b^5*p^3*x^(5/3) + 3267*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*
p^2*x^(8/3) + 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^3*x^(4/3
) - 3528*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^2*x^(7/3) - 3360*(
b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^6*b^3*p^3*x + 3836*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a^3*b^6*p^2*x^2 + 13698*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*b^9*p*x^3 - 4200*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^2*
x^(5/3) + 630*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p*x^(8/3) + 4620*
(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^2*x^(4/3) - 720*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p*x^(7/3) - 5040*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^p*a^6*b^3*p^2*x + 840*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
3*b^6*p*x^2 + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*x^3 + 5040*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p^2*x^(2/3) - 1008*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p*x^(5/3) + 1260*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*a^5*b^4*p*x^(4/3) - 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6*b^3*p*x + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p*x^(2/3)
 - 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^8*b*p*x^(1/3) + 2520*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^9)/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p
^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \left(b^2 x^{\frac{2}{3}} + 2ab\,x^{\frac{1}{3}} + a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x^2,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x^2,x)
```

maxima [A] time = 0.70, size = 362, normalized size = 0.77

$$\frac{3\left(\left(16\,p^{8}+288\,p^{7}+2184\,p^{6}+9072\,p^{5}+22449\,p^{4}+33642\,p^{3}+29531\,p^{2}+13698\,p+2520\right)b^{9}x^{3}+\left(16\,p^{8}+224\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")
```

```
[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531*
p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 + 6
769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6 +
350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 +
 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20
*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^
2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(2*p^2
 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3) + a)^(2
*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 269325*p^4 + 3
61840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

mupad [B] time = 3.52, size = 777, normalized size = 1.66

$$\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x^3\,\left(16\,p^8+288\,p^7+2184\,p^6+9072\,p^5+22449\,p^4+33642\,p^3+29531\,p^2+\right.}{32\,p^9+720\,p^8+6960\,p^7+37800\,p^6+126546\,p^5+269325\,p^4+361840\,p^3+293175}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642*
p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322*p
 + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
 + 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 + 361
840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
 + 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 361840*p^3
 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2268
0)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 1288*p^5 +
224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^
4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) + (84*a^
3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*(128322*p +
 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
 720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))/(b^6*(128322
*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p
^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 293175*p^2 + 361840*
p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2
2680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p + 293175*p^2 + 36184
0*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 +
 22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(b^5*(128322*p
+ 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
+ 720*p^8 + 32*p^9 + 22680)) - (504*a^4*p*x^(5/3)*(25*p + 35*p^2 + 20*p^3 +
 4*p^4 + 6))/(b^4*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546
*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.474 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal. Leaf size=315

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}\right.}{b^6(p+2)}$$

```
[Out] -3*a^6*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+2*p)+15/2*a
^6*(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+p)-30*a^6*(1+
b*x^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+2*p)+15*a^6*(1+b*x^
(1/3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(2+p)-15*a^6*(1+b*x^(1/3)/
a)^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(5+2*p)+3/2*a^6*(1+b*x^(1/3)/a)^
6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+p)
```

Rubi [A] time = 0.14, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1356, 266, 43}

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}\right.}{b^6(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1
+ 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(
a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1
/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p} x\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^5\left(1+\frac{bx}{a}\right)^{2p}dx, x, \sqrt[3]{\right.$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^5\left(1+\frac{bx}{a}\right)^{2p}}{b^5}+\frac{5a^5}{\right.\right.$$

$$= -\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+\right.}{2b^6(1+p)}$$

Mathematica [A] time = 0.17, size = 143, normalized size = 0.45

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(-\frac{2a^5}{2p+1}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{p+1}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{2p+3}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{p+2}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{2p+5}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{p+3}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*x
^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*x^
(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b*x^
(1/3))^2)^p)/(2*b^6)
```

fricas [A] time = 1.43, size = 297, normalized size = 0.94

$$-\frac{3\left(30\,a^6-\left(8\,b^6p^5+60\,b^6p^4+170\,b^6p^3+225\,b^6p^2+137\,b^6p+30\,b^6\right)x^2-20\left(2\,a^3b^3p^3+3\,a^3b^3p^2+a^3b^3p\right)x\right.}{2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")
```

```
[Out] -3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b^
6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*(3
0*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^3 +
 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12
*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 8
12*b^6*p^2 + 441*b^6*p + 90*b^6)
```

giac [B] time = 0.48, size = 745, normalized size = 2.37

$$3\left(8\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^6p^5x^2+8\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p ab^5p^5x^{\frac{5}{3}}+60\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^6p^4x^2+40\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
```

```
[Out] 3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^4*
x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3) + 17
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^
3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 50*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p*x^2 -
 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) + 12*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x^2 - 30*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 4
41*b^6*p + 90*b^6)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x \left(b^2 x^{\frac{2}{3}} + 2ab\, x^{\frac{1}{3}} + a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x,x)
```

maxima [A] time = 0.64, size = 198, normalized size = 0.63

$$\frac{3\left(\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)b^6x^2+2\left(4\,p^5+20\,p^4+35\,p^3+25\,p^2+6\,p\right)ab^5x^{\frac{5}{3}}-5\left(4\,p^4+12\right.}{2\left(8\,p^6+84\,p^5+350\,p^4+735\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")
```

```
[Out] 3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5 +
 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p^2
 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 + p)
*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p
^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

mupad [B] time = 2.17, size = 390, normalized size = 1.24

$$\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\left(\frac{3\,x^2\,\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)}{2\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}-\frac{}{b^6\,\left(8\,p^6+84\,p^5+350\,p^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 +
60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*
p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 +
8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350*p^4
 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(
```

```
2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (30*a^
3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5
 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*p^2 + 20*
p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6
+ 90)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.475 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal. Leaf size=142

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b\right.}{b^3(2p+1)}$$

```
[Out] 3*a^2*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+2*p)-3*a*(a+b*
x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+p)+3*(a+b*x^(1/3))^3*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(3+2*p)
```

Rubi [A] time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1341, 646, 43}

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b\right.}{b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p
)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1
+ p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3
 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx = 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^p dx, x, \sqrt[3]{x}\right)$$
$$= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{2p} dx, x\right.$$
$$= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{2p}}{b^2}-\frac{2}{\ }\right.\right.$$
$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)} - \frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x\right.}{b^3(1+p)}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.58

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(2p+1)\sqrt[3]{x}+b^2\left(2p^2+3p+1\right)x^{2/3}\right)}{b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^2
*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

fricas [A] time = 1.11, size = 110, normalized size = 0.77

$$-\frac{3\left(2\,a^2bpx^{\frac{1}{3}}-a^3-\left(2\,b^3p^2+3\,b^3p+b^3\right)x-\left(2\,ab^2p^2+ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

```
[Out] -3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
+ a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12*b
^3*p^2 + 11*b^3*p + 3*b^3)
```

giac [A] time = 0.48, size = 229, normalized size = 1.61

$$\frac{3\left(2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3p^2x+2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p ab^2p^2x^{\frac{2}{3}}+3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3px+\left(b^2x^{\frac{2}{3}}+\right.\right.}{4\,b^3p^3+12\,b^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

```
[Out] 3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2*(b
^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4*b^3*
p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p,x)
```

maxima [A] time = 0.80, size = 77, normalized size = 0.54

$$\frac{3\left(\left(2p^2+3p+1\right)b^3x+\left(2p^2+p\right)ab^2x^{\frac{2}{3}}-2a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2p}}{\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

```
[Out] 3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
+ a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

mupad [B] time = 1.54, size = 138, normalized size = 0.97

$$\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x\,\left(2\,p^2+3\,p+1\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{6\,a^2\,p\,x^{1/3}}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*p
^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^(1
/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(11*p
 + 12*p^2 + 4*p^3 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

3.476 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

Optimal. Leaf size=69

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

```
[Out] -3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([1, 1+2*p],[
2+2*p],1+b*x^(1/3)/a)/(1+2*p)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1356, 266, 65}

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometri
c2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x}\,dx$$
$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{1+2p}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p {}_2F_1\left(1,2p+1;2p+2;\frac{\sqrt[3]{x}b}{a}+1\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
[Out] (-3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 +
 2*p, 1 + (b*x^(1/3))/a])/(a*(1 + 2*p))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="fricas")
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="giac")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p/x,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="maxima")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x,x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
[Out] Integral(((a + b*x**(1/3))**2)**p/x, x)
```

3.477 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

Optimal. Leaf size=75

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}\,b}{a}+1\right)}{a^3(2p+1)}$$

```
[Out] 3*b^3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([4, 1+2*p
],[2+2*p],1+b*x^(1/3)/a)/a^3/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1356, 266, 65}

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}\,b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeome
tric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x^2}\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\, {}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{a^3(1+2p)}$$

Mathematica [**A**] time = 0.01, size = 61, normalized size = 0.81

$$\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\, {}_2F_1\left(4,2p+1;2p+2;\frac{\sqrt[3]{x}b}{a}+1\right)}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[4, 1 + 2*p,
2 + 2*p, 1 + (b*x^(1/3))/a])/(a^4*(1 + 2*p))
```

fricas [**F**] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

maple [**F**] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2 x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="maxima")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2,x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
[Out] Integral(((a + b*x**(1/3))**2)**p/x**2, x)
```

3.478 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right) d$

Optimal. Leaf size=146

$$\frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}}-\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax}-\frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right.}{a^3\sqrt[3]{.}}$$

```
[Out] -(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a/x+b*(1-p)*(a+b*x^(1/3))*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^2/x^(2/3)-b^2*(1-2*p)*(1-p)*(a+b*x^(1/3
))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x^(1/3)
```

Rubi [C] time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 3, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {1356, 266, 65}

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}+\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x
^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
)^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 +
2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(\frac{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p}{3a^3x} \right) dx = -\frac{\left(2b^3(1-2p)(1-p)p\right)\int \frac{\left(a^2+2a\right.}{}}{3a^3}$$

$$= \left(\left(1 + \frac{b\sqrt[3]{x}}{a}\right)^{-2p} \left(a^2 + 2ab\sqrt[3]{x} +\right.\right.$$

$$= \left(3\left(1 + \frac{b\sqrt[3]{x}}{a}\right)^{-2p} \left(a^2 + 2ab\sqrt[3]{x}\right.\right.$$

$$= \frac{2b^3(1-2p)(1-p)p\left(1 + \frac{b\sqrt[3]{x}}{a}\right)}{}$$

Mathematica [C] time = 0.08, size = 101, normalized size = 0.69

$$\frac{b^3\left(a + b\sqrt[3]{x}\right)\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p \left(2p\left(2p^2 - 3p + 1\right) {}_2F_1\left(1, 2p+1; 2(p+1); \frac{\sqrt[3]{x}b}{a} + 1\right) + 3\,{}_2F_1\left(4, 2p+1; 2(p+1); \frac{\sqrt[3]{x}b}{a}\right.\right.}{a^3(2ap + a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 -
 p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeom
etric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[4,
 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

fricas [A] time = 1.32, size = 82, normalized size = 0.56

$$-\frac{\left(a^2bpx^{\frac{1}{3}} + a^3 + \left(2\,b^3p^2 - 3\,b^3p + b^3\right)x + 2\left(ab^2p^2 - ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")
```

```
[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a*
b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p b^3\left(2\,p - 1\right)\left(p - 1\right)p}{3\,a^3x} + \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int -\frac{2\left(-2p+1\right)\left(-p+1\right)b^{3}p\left(b^{2}x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^{2}\right)^{p}}{3a^{3}x}+\frac{\left(b^{2}x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p-2/3*b^3*(1-2*p)*(1-p)*p*(b^2*x^
(2/3)+2*a*b*x^(1/3)+a^2)^p/a^3/x,x)
```

```
[Out] int(1/x^2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p-2/3*b^3*(1-2*p)*(1-p)*p*(b^2*x^
(2/3)+2*a*b*x^(1/3)+a^2)^p/a^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}b^{3}\left(2\,p-1\right)\left(p-1\right)p}{3\,a^{3}x}+\frac{\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

mupad [B] time = 1.65, size = 69, normalized size = 0.47

$$-\frac{\left(a^{2}+b^{2}\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{p}\,\left(\frac{b^{3}\,x\,\left(2\,p^{2}-3\,p+1\right)}{a^{3}}+\frac{b\,p\,x^{1/3}}{a}+\frac{2\,b^{2}\,p\,x^{2/3}\,\left(p-1\right)}{a^{2}}+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)*
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (b
*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3a^{3}\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{p}}{x^{2}}\right)dx+\int\frac{2b^{3}p\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{p}}{x}\,dx+\int\left(-\frac{6b^{3}p^{2}\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{p}}{x}\right)dx+\int\frac{4b^{3}p^{3}\left(a^{2}+2ab\sqrt[3]{x}+\right.}{x}}{3a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-p
)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

```
[Out] -(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + In
tegral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral
(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(4
*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)
```

3.479 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab}}$$

```
[Out] -12*a^2/b^4/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+2*a^3/b^4/(a+b*x^(1/4))/(
a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+4*(a+b*x^(1/4))*x^(1/4)/b^3/(a^2+2*a*b
*x^(1/4)+b^2*x^(1/2))^(1/2)-12*a*(a+b*x^(1/4))*ln(a+b*x^(1/4))/b^4/(a^2+2*a
*b*x^(1/4)+b^2*x^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log}{b^4\sqrt{a^2+2ab\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a +
 b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x
^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4
))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = 4\,\text{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[4]{x}\right)$$

$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$= -\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}} + \frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}} + \frac{}{b^3\sqrt{}}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.53

$$\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)+2b^3x^{3/4}\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a + b
*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4))^
2])
```

fricas [A] time = 9.06, size = 147, normalized size = 0.84

$$\frac{2\left(9\,a^5b^4x-5\,a^9-6\left(ab^8x^2-2\,a^5b^4x+a^9\right)\log\left(bx^{\frac{1}{4}}+a\right)-2\left(3\,a^2b^7x-a^6b^3\right)x^{\frac{3}{4}}+\left(7\,a^3b^6x-3\,a^7b^2\right)\sqrt{x}+2\right.}{b^{12}x^2-2\,a^4b^8x+a^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x)
+ 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x + a^
8*b^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 114, normalized size = 0.65

$$\frac{2\sqrt{b^2\sqrt{x}+2ab\,x^{\frac{1}{4}}+a^2}\,\left(-6a\,b^2\sqrt{x}\,\ln\left(b\,x^{\frac{1}{4}}+a\right)-12a^2b\,x^{\frac{1}{4}}\ln\left(b\,x^{\frac{1}{4}}+a\right)-6a^3\ln\left(b\,x^{\frac{1}{4}}+a\right)+2b^3x^{\frac{3}{4}}+4a\,b^2\right.}{\left(b\,x^{\frac{1}{4}}+a\right)^3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x)
```

```
[Out] 2*(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)*(2*x^(3/4)*b^3-6*x^(1/2)*ln(a+b*x^(
1/4))*a*b^2+4*x^(1/2)*a*b^2-12*x^(1/4)*ln(a+b*x^(1/4))*a^2*b-4*x^(1/4)*a^2*
b-6*ln(a+b*x^(1/4))*a^3-5*a^3)/(a+b*x^(1/4))^3/b^4
```

maxima [A] time = 0.58, size = 114, normalized size = 0.65

$$\frac{4\sqrt{x}}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}\,b^2}-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{b^4}+\frac{8\,a^2}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}\,b^4}-\frac{24\,a^2x^{\frac{1}{4}}}{b^5\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}-\frac{22\,a^3}{b^6\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4)
+ a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2*x
^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a^2+b^2\,\sqrt{x}+2\,a\,b\,x^{1/4}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[4]{x}+b^{2}\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

3.480 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2 \cdots}}$$

```
[Out] -60*a^2/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+3/2*a^5/b^6/(a+b*x^(1/6))
^3/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)-10*a^4/b^6/(a+b*x^(1/6))^2/(a^2+2*
a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+30*a^3/b^6/(a+b*x^(1/6))/(a^2+2*a*b*x^(1/6)+
b^2*x^(1/3))^(1/2)+6*(a+b*x^(1/6))*x^(1/6)/b^5/(a^2+2*a*b*x^(1/6)+b^2*x^(1/
3))^(1/2)-30*a*(a+b*x^(1/6))*ln(a+b*x^(1/6))/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(
1/3))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x} \cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a
 + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a
 + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a
 + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))
*x^(1/6))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1
/6))*Log[a + b*x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = 6\,\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[6]{x}\right)$$

$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}-\right.\right.}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$= -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{}{b^6\left(a\right.}$$

Mathematica [A] time = 0.10, size = 121, normalized size = 0.45

$$\frac{-77a^5-248a^4b\sqrt[6]{x}-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}+48ab^4x^{2/3}-60a\left(a+b\sqrt[6]{x}\right)^4\log\left(a+b\sqrt[6]{x}\right)+12b^5x^{5/6}}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] + 4
8*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/6)
])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.87, size = 237, normalized size = 0.88

$$\frac{3\,a^4|a|\log\left(\left|x^{\frac{1}{6}}|b|\text{sgn}(a)\text{sgn}(b)+|a|\right|\right)}{4\left(a^3b^5|a||b|\text{sgn}(a)\text{sgn}(b)-a^4b^6\right)}+\frac{3\left(24\,a^5b^2|b|\text{sgn}(a)\text{sgn}(b)-25\,a^4b^3|a|\right)\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{4\left(a^3b^8|a||b|\text{sgn}(a)\text{sgn}(b)-a^4b^9\right)}+\frac{6\,x^{\frac{1}{6}}}{b^4|b|\text{sgn}(a)\text{sgn}(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] 3/4*a^4*abs(a)*log(abs(x^(1/6)*abs(b)*sgn(a)*sgn(b) + abs(a)))/(a^3*b^5*abs
(a)*abs(b)*sgn(a)*sgn(b) - a^4*b^6) + 3/4*(24*a^5*b^2*abs(b)*sgn(a)*sgn(b)
- 25*a^4*b^3*abs(a))*log(abs(b*x^(1/6) + a))/(a^3*b^8*abs(a)*abs(b)*sgn(a)*
sgn(b) - a^4*b^9) + 6*x^(1/6)/(b^4*abs(b)*sgn(a)*sgn(b)) + 1/4*(70*a^5*abs(
b)*sgn(a)*sgn(b) - 70*a^4*b*abs(a) + 93*(a^3*b^2*abs(b)*sgn(a)*sgn(b) - a^2
*b^3*abs(a))*x^(1/3) + 159*(a^4*b*abs(b)*sgn(a)*sgn(b) - a^3*b^2*abs(a))*x^
(1/6))/((abs(a)*abs(b)*sgn(a)*sgn(b) - a*b)*(b*x^(1/6) + a)^3*b^6)
```

maple [A] time = 0.02, size = 174, normalized size = 0.65

$$\frac{\sqrt{b^2x^{\frac{1}{3}}+2ab\,x^{\frac{1}{6}}+a^2}\,\left(-60a\,b^4x^{\frac{2}{3}}\ln\left(b\,x^{\frac{1}{6}}+a\right)-240a^2b^3\sqrt{x}\,\ln\left(b\,x^{\frac{1}{6}}+a\right)-360a^3b^2x^{\frac{1}{3}}\ln\left(b\,x^{\frac{1}{6}}+a\right)-240a\right.}{2\left(b\,x^{\frac{1}{6}}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x)
```

```
[Out] 1/2*(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)*(12*x^(5/6)*b^5-60*x^(2/3)*ln(a+b
*x^(1/6))*a*b^4+48*x^(2/3)*a*b^4-240*x^(1/2)*ln(a+b*x^(1/6))*a^2*b^3-48*x^(
1/2)*a^2*b^3-360*x^(1/3)*ln(a+b*x^(1/6))*a^3*b^2-252*x^(1/3)*a^3*b^2-240*x^
(1/6)*ln(a+b*x^(1/6))*a^4*b-248*x^(1/6)*a^4*b-60*ln(a+b*x^(1/6))*a^5-77*a^5
)/(a+b*x^(1/6))^5/b^6
```

maxima [A] time = 0.88, size = 119, normalized size = 0.44

$$\frac{12\,b^5x^{\frac{5}{6}}+48\,ab^4x^{\frac{2}{3}}-48\,a^2b^3\sqrt{x}-252\,a^3b^2x^{\frac{1}{3}}-248\,a^4bx^{\frac{1}{6}}-77\,a^5}{2\left(b^{10}x^{\frac{2}{3}}+4\,ab^9\sqrt{x}+6\,a^2b^8x^{\frac{1}{3}}+4\,a^3b^7x^{\frac{1}{6}}+a^4b^6\right)}-\frac{30\,a\log\left(bx^{\frac{1}{6}}+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*x
^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*a^
2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2+b^2\,x^{1/3}+2\,a\,b\,x^{1/6}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

3.481 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

Optimal. Leaf size=179

$$\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

```
[Out] a^3*x*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))+3*a*b^2*ln(x)*(a^2+b^2/
x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))-2*b^3*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/
(a+b/x^(1/2))/x^(1/2)+6*a^2*b*x^(1/2)*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/
x^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqrt[x]) + (6
*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*
Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2
+ b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int\left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2}dx=2\,\text{Subst}\left(\int\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2}x\,dx,x,\sqrt{x}\right)$$
$$=\frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int\left(ab+\frac{b^2}{x}\right)^3x\,dx,x,\sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$
$$=\frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int\frac{\left(b^2+abx\right)^3}{x^2}dx,x,\sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$
$$=\frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int\left(3a^2b^4+\frac{b^6}{x^2}+\frac{3ab^5}{x}+a^3b^3x\right)dx,x,\sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$
$$=-\frac{2b^4\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(ab+\frac{b^2}{\sqrt{x}}\right)\sqrt{x}}+\frac{6a^2b^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt{x}}}+\frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\,x}{a+\frac{b}{\sqrt{x}}}+\frac{3ab^3}{}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.37

$$\frac{\sqrt{\frac{\left(a\sqrt{x}+b\right)^2}{x}}\left(a^3x^{3/2}+6a^2bx+3ab^2\sqrt{x}\log(x)-2b^3\right)}{a\sqrt{x}+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqrt
[x]*Log[x]))/(b + a*Sqrt[x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.39, size = 80, normalized size = 0.45

$$a^3x\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)+3\,ab^2\log\left(|x|\right)\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)+6\,a^2b\sqrt{x}\,\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)-\frac{2\,b^3\text{sgn}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
[Out] a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x)
)*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x + b*
sqrt(x))*sgn(x)/sqrt(x)
```

maple [A] time = 0.03, size = 68, normalized size = 0.38

$$\frac{\sqrt{\frac{a^{2} x^{\frac{3}{2}}+2 a b x+b^{2} \sqrt{x}}{x^{\frac{3}{2}}}}\, \left(a^{3} x^{\frac{3}{2}}+3 a\, b^{2} \sqrt{x}\, \ln \left(x \right)+6 a^{2} b x -2 b^{3}\right)}{a \sqrt{x}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x)
```

```
[Out] ((a^2*x^(3/2)+b^2*x^(1/2)+2*a*x*b)/x^(3/2))^(1/2)*(3*a*b^2*ln(x)*x^(1/2)+6*
a^2*b*x+x^(3/2)*a^3-2*b^3)/(a*x^(1/2)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3} x + 3 \, a b^{2} \int \frac{1}{x}\,{d x} + 6 \, a^{2} b \sqrt{x} - \frac{2 \, b^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+\frac{b^2}{x}+\frac{2\,a\,b}{\sqrt{x}}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)
```

```
[Out] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2 a b}{\sqrt{x}} + \frac{b^{2}}{x}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

3.482 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{}}{2\left(a\right.}$$

```
[Out] -3/4*b^7*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(4/3)-7*a*b^
6*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x-63/2*a^2*b^5*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-105*a^3*b^4*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+63*a^5*b^2*x^(1/3)*(a^2+
b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+21/2*a^6*b*x^(2/3)*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^7*x*(a^2+b^2/x^(2/3)+2*a*b/x^(
1/3))^(1/2)/(a+b/x^(1/3))+35*a^4*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)/(a+b/x^(1/3))
```

Rubi [A] time = 0.19, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1/3))*x^(4/
3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*
x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/
3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a
+ b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)
/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^2\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^7}{x^5}\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(21a^5b^9+\frac{b^{14}}{x^5}+\frac{7ab^{13}}{x^4}+\frac{21a^2b^{12}}{x^3}+\frac{35a^3b^{11}}{x^2}+\frac{35a^4b^{10}}{x}+\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{4/3}} - \frac{7ab^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x} - \frac{63a^2b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{105a}{}$$

Mathematica [A] time = 0.07, size = 125, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(4a^7x^{7/3}+42a^6bx^2+252a^5b^2x^{5/3}+140a^4b^3x^{4/3}\log(x)-420a^3b^4x-126a^2b^5x^{2/3}-28ab^6\sqrt[3]{x}-3b^7\right)}{4x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x
^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3)
 + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.54, size = 173, normalized size = 0.44

$$a^7x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+35\,a^4b^3\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+\frac{21}{2}\,a^6bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+63\,a^5b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2/3)
)*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x^(1/
3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^(4/3)
```

maple [A] time = 0.03, size = 115, normalized size = 0.29

$$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}\left(4a^7x^{\frac{10}{3}}+140a^4b^3x^{\frac{7}{3}}\ln(x)+42a^6b\,x^3+252a^5b^2x^{\frac{8}{3}}-420a^3b^4x^2-126a^2b^5x^{\frac{5}{3}}-28a\,b^6x^{\frac{4}{3}}-\right.}{4\left(a\,x^{\frac{1}{3}}+b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x)
```

```
[Out] 1/4*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*(42*a^6*b*x^3+140*a^4*b
^3*ln(x)*x^(7/3)+252*a^5*b^2*x^(8/3)+4*a^7*x^(10/3)-28*a*b^6*x^(4/3)-420*a^
3*b^4*x^2-126*a^2*b^5*x^(5/3)-3*b^7*x)/(b+a*x^(1/3))^7
```

maxima [A] time = 0.60, size = 79, normalized size = 0.20

$$35\,a^4b^3\log(x)+\frac{4\,a^7x^{\frac{7}{3}}+42\,a^6bx^2+252\,a^5b^2x^{\frac{5}{3}}-420\,a^3b^4x-126\,a^2b^5x^{\frac{2}{3}}-28\,ab^6x^{\frac{1}{3}}-3\,b^7}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3)
 - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)
```

3.483 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$-\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+$$

```
[Out] -3/2*b^5*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-15*a*b
^4*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+30*a^3*b^2*x
^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+15/2*a^4*b*x^(2/
3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^5*x*(a^2+b^2/x^(2/
3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+10*a^2*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/
3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))
*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/
(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/
3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)
/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Lo
g[x^(1/3)])/(a + b/x^(1/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^2\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^5}{x^3}\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(10a^3b^7+\frac{b^{10}}{x^3}+\frac{5ab^9}{x^2}+\frac{10a^2b^8}{x}+5a^4b^6x+a^5b^5x^2\right) dx\right.}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{15ab^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\sqrt[3]{x}} + \frac{30a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} +$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.34

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^5x^{5/3}+15a^4bx^{4/3}+60a^3b^2x+20a^2b^3x^{2/3}\log(x)-30ab^4\sqrt[3]{x}-3b^5\right)}{2x\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/
3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^2/
x^(2/3)]*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.51, size = 128, normalized size = 0.44

$$a^5x\,\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+10\,a^2b^3\log\left(|x|\right)\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+\frac{15}{2}\,a^4bx^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+30\,a^3b^2x^{\frac{1}{3}}\text{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x^(2
/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

maple [A] time = 0.01, size = 91, normalized size = 0.31

$$\frac{\left(\frac{a^{2} x^{\frac{2}{3}}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} \left(2 a^{5} x^{\frac{5}{3}}+20 a^{2} b^{3} x^{\frac{2}{3}} \ln \left(x \right)+15 a^{4} b \,x^{\frac{4}{3}}+60 a^{3} b^{2} x -30 a \,b^{4} x^{\frac{1}{3}}-3 b^{5}\right) x}{2 \left(a \,x^{\frac{1}{3}}+b \right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(15*a^4*b*x^(4/3)+60*
a^3*b^2*x+20*a^2*b^3*ln(x)*x^(2/3)+2*a^5*x^(5/3)-30*a*b^4*x^(1/3)-3*b^5)/(a
*x^(1/3)+b)^5
```

maxima [A] time = 0.64, size = 57, normalized size = 0.20

$$10 \, a^{2} b^{3} \log\left(x\right) + \frac{2 \, a^{5} x^{\frac{5}{3}} + 15 \, a^{4} b x^{\frac{4}{3}} + 60 \, a^{3} b^{2} x - 30 \, a b^{4} x^{\frac{1}{3}} - 3 \, b^{5}}{2 \, x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2 a b}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

3.484 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

Optimal. Leaf size=189

$$\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] 9*a*b^2*x^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+9/2*a^2
*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^3*x*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+b^3*ln(x)*(a^2+b^2/x^(2/3)+2*
a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3))
 + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^
(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
 + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^
(1/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x^2\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^3}{x}\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(3ab^5+\frac{b^6}{x}+3a^2b^4x+a^3b^3x^2\right) dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{9ab^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{9a^2b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.41

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2a^3x^{4/3}+9a^2bx+18ab^2x^{2/3}+2b^3\sqrt[3]{x}\log(x)\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(18*a*b^2*x^(2/3) + 9*a^2*b*x + 2*a^3*x^(4
/3) + 2*b^3*x^(1/3)*Log[x]))/(2*(b + a*x^(1/3)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 79, normalized size = 0.42

$$a^3x\,\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+b^3\log\left(|x|\right)\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+\frac{9}{2}\,a^2bx^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+9\,ab^2x^{\frac{1}{3}}\text{sgn}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sg
n(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sgn(
a*x + b*x^(2/3))*sgn(x)
```

maple [A] time = 0.00, size = 69, normalized size = 0.37

$$\frac{\left(\frac{a^{2} x^{\frac{2}{3}}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} \left(2 a^{3} x+2 b^{3} \ln \left(x \right)+9 a^{2} b\, x^{\frac{2}{3}}+18 a\, b^{2} x^{\frac{1}{3}}\right) x}{2 \left(a\, x^{\frac{1}{3}}+b \right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(9*a^2*b*x^(2/3)+18*a
*b^2*x^(1/3)+2*b^3*ln(x)+2*a^3*x)/(a*x^(1/3)+b)^3
```

maxima [A] time = 0.66, size = 30, normalized size = 0.16

$$a^{3} x + b^{3} \log\left(x\right) + \frac{9}{2} \, a^{2} b x^{\frac{2}{3}} + 9 \, a b^{2} x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2 a b}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

3.485 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] 3/2*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a*x*(a^2+
b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 1355, 14}

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3)))
 + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx = 3\,\text{Subst}\left(\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\left(ab+\frac{b^2}{x}\right)x^2\,dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\left(b^2x+abx^2\right)dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$= \frac{3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.56

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2ax^{4/3}+3bx\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(3*b*x + 2*a*x^(4/3)))/(2*(b + a*x^(1/3)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.40, size = 34, normalized size = 0.39

$$ax\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+\frac{3}{2}\,bx^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x)
```

maple [A] time = 0.00, size = 50, normalized size = 0.57

$$\frac{\sqrt{\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}}\left(2ax+3b\,x^{\frac{2}{3}}\right)x^{\frac{1}{3}}}{2a\,x^{\frac{1}{3}}+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x^(1/3)*(3*x^(2/3)*b+2*
a*x)/(a*x^(1/3)+b)
```

maxima [A] time = 0.73, size = 10, normalized size = 0.11

$$ax + \frac{3}{2}\,bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

mupad [B] time = 1.43, size = 39, normalized size = 0.44

$$\frac{x\left(a+\frac{3\,b}{2\,x^{1/3}}\right)\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}{a+\frac{b}{x^{1/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

```
[Out] (x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a
+ b/x^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

3.486 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal. Leaf size=190

$$-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[Out] 3*b^2*(a+b/x^(1/3))*x^(1/3)/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3/2*b
*(a+b/x^(1/3))*x^(2/3)/a^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a+b/x^(1/
3))*x/a/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3*b^3*(a+b/x^(1/3))*ln(b+a*x^
(1/3))/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

[p - 1/2]

Rubi steps

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{x}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^3}{b^2+abx}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{b}{a^3}-\frac{x}{a^2}+\frac{x^2}{ab}-\frac{b^2}{a^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\text{l}}{a^4\sqrt{a^2+}}$$

Mathematica [A] time = 0.04, size = 86, normalized size = 0.45

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^3x-3a^2bx^{2/3}-6b^3\log\left(a\sqrt[3]{x}+b\right)+6ab^2\sqrt[3]{x}\right)}{2a^4\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b
 + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.59, size = 77, normalized size = 0.41

$$-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)} + \frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

```
[Out] -3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a
^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

maple [A] time = 0.01, size = 78, normalized size = 0.41

$$-\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(-2a^{3}x+6b^{3}\ln\left(a\,x^{\frac{1}{3}}+b\right)+3a^{2}b\,x^{\frac{2}{3}}-6a\,b^{2}x^{\frac{1}{3}}\right)}{2\sqrt{\frac{a^{2}x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\;a^{4}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] -1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)*(a*x^(1/3)+b)*
(3*a^2*b*x^(2/3)-6*a*b^2*x^(1/3)+6*b^3*ln(a*x^(1/3)+b)-2*a^3*x)/a^4
```

maxima [A] time = 0.74, size = 44, normalized size = 0.23

$$-\frac{3\,b^{3}\log\left(ax^{\frac{1}{3}}+b\right)}{a^{4}}+\frac{2\,a^{2}x-3\,abx^{\frac{2}{3}}+6\,b^{2}x^{\frac{1}{3}}}{2\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3
))/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{a^{2}+\frac{b^{2}}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a^{2}+\frac{2ab}{\sqrt[3]{x}}+\frac{b^{2}}{x^{\frac{2}{3}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

3.487 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{18b^2\sqrt[3]{x}\,(}{a^5\sqrt{a^2+}}$$

```
[Out] 3/2*b^5*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)-15*b^4*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))
^(1/2)+18*b^2*(a+b/x^(1/3))*x^(1/3)/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/
2)-9/2*b*(a+b/x^(1/3))*x^(2/3)/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a
+b/x^(1/3))*x/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-30*b^3*(a+b/x^(1/3)
)*ln(b+a*x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sq
rt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2
*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*
Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b
+ a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(b^2+abx\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{6}{a^5b}-\frac{3x}{a^4b^2}+\frac{x^2}{a^3b^3}-\frac{b^2}{a^5(b+ax)^3}+\frac{5b}{a^5(b+ax)^2}-\frac{10}{a^5(b+ax)}\right)dx, x, \sqrt[3]{x}\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{15\left(ab^4+\frac{b^5}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{18\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\cdots}}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 0.42

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^5x^{5/3}-5a^4bx^{4/3}+20a^3b^2x+63a^2b^3x^{2/3}+6ab^4\sqrt[3]{x}-60b^3\left(a\sqrt[3]{x}+b\right)^2\log\left(a\sqrt[3]{x}+b\right)-27b^5\right)}{2a^6x\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b
^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b + a
*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.66, size = 121, normalized size = 0.40

$$-\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}-\frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}+\frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] -30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x^(2/3) + b*x^(1/3))) - 3/2*(10*
a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x^(2/3) + b*x^(1/3))) +
 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x^(2/3) +
b*x^(1/3)))
```

maple [A] time = 0.01, size = 141, normalized size = 0.47

$$\frac{\left(2a^5x^{\frac{5}{3}}-60a^2b^3x^{\frac{2}{3}}\ln\left(a\,x^{\frac{1}{3}}+b\right)-5a^4b\,x^{\frac{4}{3}}-120a\,b^4x^{\frac{1}{3}}\ln\left(a\,x^{\frac{1}{3}}+b\right)+20a^3b^2x-60b^5\ln\left(a\,x^{\frac{1}{3}}+b\right)+63a^2b^3\right.}{2\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)/x*(2*a^5*x^(5/3)-5*a^4*
b*x^(4/3)-60*x^(2/3)*ln(a*x^(1/3)+b)*a^2*b^3+63*x^(2/3)*a^2*b^3-120*x^(1/3)
*ln(a*x^(1/3)+b)*a*b^4+6*a*b^4*x^(1/3)-60*ln(a*x^(1/3)+b)*b^5+20*a^3*b^2*x-
27*b^5)*(a*x^(1/3)+b)/a^6
```

maxima [A] time = 1.07, size = 97, normalized size = 0.32

$$\frac{2\,a^5x^{\frac{5}{3}}-5\,a^4bx^{\frac{4}{3}}+20\,a^3b^2x+63\,a^2b^3x^{\frac{2}{3}}+6\,ab^4x^{\frac{1}{3}}-27\,b^5}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)}-\frac{30\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*b^
3*log(a*x^(1/3) + b)/a^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+\frac{2ab}{\sqrt[3]{x}}+\frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

3.488 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal. Leaf size=410

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{}{a^8\sqrt{a}}$$

```
[Out] 3/4*b^7*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)-7*b^6*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3)
)^(1/2)+63/2*b^5*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x
^(1/3))^(1/2)-105*b^4*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*
b/x^(1/3))^(1/2)+45*b^2*(a+b/x^(1/3))*x^(1/3)/a^7/(a^2+b^2/x^(2/3)+2*a*b/x^
(1/3))^(1/2)-15/2*b*(a+b/x^(1/3))*x^(2/3)/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3
))^(1/2)+(a+b/x^(1/3))*x/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-105*b^3*
(a+b/x^(1/3))*ln(b+a*x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{}{a^8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(
1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (4
5*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(
1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
```

```
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^7}{\left(b^2+abx\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{15}{a^7b^3}-\frac{5x}{a^6b^4}+\frac{x^2}{a^5b^5}-\frac{b^2}{a^7(b+ax)^5}+\frac{7b}{a^7(b+ax)^4}-\frac{21}{a^7(b+ax)^3}+\frac{3}{a^7b(b}\right.\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^7+\frac{b^8}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7\left(ab^6+\frac{b^7}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3} + \frac{63\left(\right.}{2a^8\sqrt{a^2+\frac{b^2}{x^{2}}}}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 0.37

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(4a^7x^{7/3}-14a^6bx^2+84a^5b^2x^{5/3}+556a^4b^3x^{4/3}+544a^3b^4x-444a^2b^5x^{2/3}-856ab^6\sqrt[3]{x}-420b^3\left(a\sqrt[3]{x}\right.\right.}{4a^8x^{5/3}\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*
a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7
*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a*x^
(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.72, size = 141, normalized size = 0.34

$$-\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}-\frac{420\,a^3b^4x+1134\,a^2b^5x^{\frac{2}{3}}+1036\,ab^6x^{\frac{1}{3}}+319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4a^8\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}+\frac{2\,a^{10}x-15\,a^9bx^{\frac{2}{3}}+90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x^(2/3) + b*x^(1/3))) - 1/4*(42
0*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x^(1
/3) + b)^4*a^8*sgn(a*x^(2/3) + b*x^(1/3))) + 1/2*(2*a^10*x - 15*a^9*b*x^(2/
3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x^(2/3) + b*x^(1/3)))
```

maple [A] time = 0.01, size = 199, normalized size = 0.49

$$\frac{\left(4a^7x^{\frac{7}{3}}-420a^4b^3x^{\frac{4}{3}}\ln\left(a\,x^{\frac{1}{3}}+b\right)-14a^6b\,x^2-1680a^3b^4x\ln\left(a\,x^{\frac{1}{3}}+b\right)+84a^5b^2x^{\frac{5}{3}}-2520a^2b^5x^{\frac{2}{3}}\ln\left(a\,x^{\frac{1}{3}}+l\right.\right.}{4\left(\frac{a^2x}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/4/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)/x^(5/3)*(4*x^(7/3)*a^7+
84*a^5*b^2*x^(5/3)-420*x^(4/3)*ln(a*x^(1/3)+b)*a^4*b^3+556*x^(4/3)*a^4*b^3-
2520*x^(2/3)*ln(a*x^(1/3)+b)*a^2*b^5-444*x^(2/3)*a^2*b^5-1680*x^(1/3)*ln(a*
x^(1/3)+b)*a*b^6-1680*x*ln(a*x^(1/3)+b)*a^3*b^4-14*a^6*b*x^2-856*x^(1/3)*a*
b^6-420*ln(a*x^(1/3)+b)*b^7+544*x*a^3*b^4-319*b^7)*(a*x^(1/3)+b)/a^8
```

maxima [A] time = 0.85, size = 139, normalized size = 0.34

$$\frac{4\,a^7x^{\frac{7}{3}}-14\,a^6bx^2+84\,a^5b^2x^{\frac{5}{3}}+556\,a^4b^3x^{\frac{4}{3}}+544\,a^3b^4x-444\,a^2b^5x^{\frac{2}{3}}-856\,ab^6x^{\frac{1}{3}}-319\,b^7}{4\left(a^{12}x^{\frac{4}{3}}+4\,a^{11}bx+6\,a^{10}b^2x^{\frac{2}{3}}+4\,a^9b^3x^{\frac{1}{3}}+a^8b^4\right)}-\frac{105\,b^3\log\left(ax^{\frac{1}{3}}\right.}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/3
) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a^1
2*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^4)
- 105*b^3*log(a*x^(1/3) + b)/a^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

3.489 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$

Optimal. Leaf size=289

$$-\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

```
[Out] -4*b^5*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))/x^(1/4)+40*a^2*b
^3*x^(1/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20/3*a^4*b*x
^(3/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+a^5*x*(a^2+2*a*b
/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+5*a*b^4*ln(x)*(a^2+2*a*b/x^(1/4)+
b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20*a^3*b^2*x^(1/2)*(a^2+2*a*b/x^(1/4)+b^2/
x^(1/2))^(1/2)/(a+b/x^(1/4))
```

Rubi [A] time = 0.14, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4))*x^(1/4)
) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x
^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a
 + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4)
)/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(
a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^
(1/4)])/(a + b/x^(1/4))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = 4\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^3\,dx, x, \sqrt[4]{x}\right)$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^3\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^5}{x^2}\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(10a^2b^8+\frac{b^{10}}{x^2}+\frac{5ab^9}{x}+10a^3b^7x+5a^4b^6x^2+a^5b^5x^3\right)\right.}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= -\frac{4b^6\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)\sqrt[4]{x}} + \frac{40a^2b^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\,\sqrt[4]{x}}{ab+\frac{b^2}{\sqrt[4]{x}}} + \frac{20a^3b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\,\sqrt{}}{ab+\frac{b^2}{\sqrt[4]{x}}}$$

Mathematica [A] time = 0.05, size = 98, normalized size = 0.34

$$\frac{\sqrt{\frac{\left(a\sqrt[4]{x}+b\right)^2}{\sqrt{x}}}\left(3a^5x^{5/4}+20a^4bx+60a^3b^2x^{3/4}+120a^2b^3\sqrt{x}+15ab^4\sqrt[4]{x}\log(x)-12b^5\right)}{3\left(a\sqrt[4]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^
2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b +
a*x^(1/4)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.49, size = 126, normalized size = 0.44

$$a^5x\,\text{sgn}\left(ax+bx^{\frac{3}{4}}\right)\text{sgn}(x)+5\,ab^4\log\left(|x|\right)\text{sgn}\left(ax+bx^{\frac{3}{4}}\right)\text{sgn}(x)+\frac{20}{3}\,a^4bx^{\frac{3}{4}}\text{sgn}\left(ax+bx^{\frac{3}{4}}\right)\text{sgn}(x)+20\,a^3b^2\sqrt{x}\,\text{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4)
)*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sqrt
(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4))*s
gn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

maple [A] time = 0.04, size = 94, normalized size = 0.33

$$\frac{\sqrt{\frac{a^{2}x^{\frac{3}{4}}+2ab\sqrt{x}+b^{2}x^{\frac{1}{4}}}{x^{\frac{3}{4}}}}\,\left(3a^{5}x^{\frac{5}{4}}+15a\,b^{4}x^{\frac{1}{4}}\ln(x)+20a^{4}bx+60a^{3}b^{2}x^{\frac{3}{4}}+120a^{2}b^{3}\sqrt{x}-12b^{5}\right)}{3a\,x^{\frac{1}{4}}+3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x)
```

```
[Out] 1/3*((a^2*x^(3/4)+b^2*x^(1/4)+2*a*b*x^(1/2))/x^(3/4))^(1/2)*(20*x*a^4*b+15*
ln(x)*x^(1/4)*a*b^4+120*a^2*b^3*x^(1/2)+60*x^(3/4)*a^3*b^2+3*x^(5/4)*a^5-12
*b^5)/(a*x^(1/4)+b)
```

maxima [A] time = 0.99, size = 57, normalized size = 0.20

$$5\,ab^{4}\log(x)+\frac{3\,a^{5}x^{\frac{5}{4}}+20\,a^{4}bx+60\,a^{3}b^{2}x^{\frac{3}{4}}+120\,a^{2}b^{3}\sqrt{x}-12\,b^{5}}{3\,x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 120
*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2\,a\,b}{x^{1/4}}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

3.490 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}\,dx$

Optimal. Leaf size=291

$$\frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}$$

```
[Out] 25*a*b^4*x^(1/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25*a^2
*b^3*x^(2/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+50/3*a^3*b
^2*x^(3/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25/4*a^4*b*x
^(4/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+a^5*x*(a^2+b^2/x
^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+b^5*ln(x)*(a^2+b^2/x^(2/5)+2*a*b/
x^(1/5))^(1/2)/(a+b/x^(1/5))
```

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)
) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x
^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3
*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(
4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*
x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x
^(1/5)])/(a + b/x^(1/5))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = 5\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^4\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^4\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x}\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(5ab^9+\frac{b^{10}}{x}+10a^2b^8x+10a^3b^7x^2+5a^4b^6x^3+a\right.\right.}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{25ab^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\,\sqrt[5]{x}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{25a^2b^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\,x^{2/5}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{50a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}}}{3\left(ab+\frac{b^2}{\sqrt[5]{x}}\right.}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.35

$$\frac{\sqrt{\frac{\left(a\sqrt[5]{x}+b\right)^2}{x^{2/5}}}\left(12a^5x^{6/5}+75a^4bx+200a^3b^2x^{4/5}+300a^2b^3x^{3/5}+300ab^4x^{2/5}+12b^5\sqrt[5]{x}\log(x)\right)}{12\left(a\sqrt[5]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*(300*a*b^4*x^(2/5) + 300*a^2*b^3*x^(3/5) +
 200*a^3*b^2*x^(4/5) + 75*a^4*b*x + 12*a^5*x^(6/5) + 12*b^5*x^(1/5)*Log[x])
)/(12*(b + a*x^(1/5)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.48, size = 125, normalized size = 0.43

$$a^5x\,\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+b^5\log(|x|)\,\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+\frac{50}{3}a^3b^2x^{\frac{3}{5}}\text{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*sg
n(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3/5
)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*sgn
(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

maple [A] time = 0.03, size = 91, normalized size = 0.31

$$\frac{\left(\frac{a^2 x^{\frac{2}{5}}+2 a b\, x^{\frac{1}{5}}+b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} \left(12 a^5 x+12 b^5 \ln(x)+75 a^4 b\, x^{\frac{4}{5}}+200 a^3 b^2 x^{\frac{3}{5}}+300 a^2 b^3 x^{\frac{2}{5}}+300 a\, b^4 x^{\frac{1}{5}}\right) x}{12 \left(a\, x^{\frac{1}{5}}+b\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x)
```

```
[Out] 1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(75*a^4*b*x^(4/5)+20
0*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+12*b^5*ln(x)+300*a*b^4*x^(1/5)+12*a^5
*x)/(x^(1/5)*a+b)^5
```

maxima [A] time = 0.94, size = 52, normalized size = 0.18

$$a^5 x+b^5 \log(x)+\frac{25}{4}\, a^4 b x^{\frac{4}{5}}+\frac{50}{3}\, a^3 b^2 x^{\frac{3}{5}}+25\, a^2 b^3 x^{\frac{2}{5}}+25\, a b^4 x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
```

```
[Out] a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^3
*x^(2/5) + 25*a*b^4*x^(1/5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{2/5}}+\frac{2\,a\,b}{x^{1/5}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2 a b}{\sqrt[5]{x}} + \frac{b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)
```

3.491 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{}{4b^5\left(a+b\sqrt[5]{x}\right.}$$

```
[Out] 20*a/b^5/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-5/4*a^4/b^5/(a+b*x^(1/5))^3/
(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)+20/3*a^3/b^5/(a+b*x^(1/5))^2/(a^2+2*a
*b*x^(1/5)+b^2*x^(2/5))^(1/2)-15*a^2/b^5/(a+b*x^(1/5))/(a^2+2*a*b*x^(1/5)+b
^2*x^(2/5))^(1/2)+5*(a+b*x^(1/5))*ln(a+b*x^(1/5))/b^5/(a^2+2*a*b*x^(1/5)+b^
2*x^(2/5))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\cdot}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a +
b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a
+ b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a
+ b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*
Log[a + b*x^(1/5)])/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = 5\,\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[5]{x}\right)}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right.\right.}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$= \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{}{3b^5\left(a\right.}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.44

$$\frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\sqrt[5]{x}\right)}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60*
(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a + b
*x^(1/5))^2])
```

fricas [A] time = 0.83, size = 302, normalized size = 1.36

$$\frac{5\left(300\,a^5b^{15}x^3+100\,a^{15}b^5x+25\,a^{20}+12\left(b^{20}x^4+4\,a^5b^{15}x^3+6\,a^{10}b^{10}x^2+4\,a^{15}b^5x+a^{20}\right)\log\left(bx^{\frac{1}{5}}+a\right)+\left(48\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

```
[Out] 5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b^
15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*a*
b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84*a^
2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (136*
a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5) - (20
7*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5))/(b^
25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5)
```

giac [A] time = 0.59, size = 84, normalized size = 0.38

$$\frac{5\log\left(\left|bx^{\frac{1}{5}}+a\right|\right)}{b^5\text{sgn}\left(bx^{\frac{1}{5}}+a\right)}+\frac{5\left(48\,ab^2x^{\frac{3}{5}}+108\,a^2bx^{\frac{2}{5}}+88\,a^3x^{\frac{1}{5}}+\frac{25\,a^4}{b}\right)}{12\left(bx^{\frac{1}{5}}+a\right)^4b^4\text{sgn}\left(bx^{\frac{1}{5}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
```

```
[Out] 5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5)
 + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*sg
n(b*x^(1/5) + a))
```

maple [A] time = 0.01, size = 152, normalized size = 0.68

$$\frac{5\sqrt{b^{2}x^{\frac{2}{5}}+2ab\,x^{\frac{1}{5}}+a^{2}}\,\left(12b^{4}x^{\frac{4}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+48a\,b^{3}x^{\frac{3}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+72a^{2}b^{2}x^{\frac{2}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+48a^{3}b\,x^{\frac{1}{5}}\ln\right.}{12\left(b\,x^{\frac{1}{5}}+a\right)^{5}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x)
```

```
[Out] 5/12*(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)*(12*x^(4/5)*ln(a+b*x^(1/5))*b^4+
48*x^(3/5)*ln(a+b*x^(1/5))*a*b^3+48*x^(3/5)*a*b^3+72*x^(2/5)*ln(a+b*x^(1/5)
)*a^2*b^2+108*x^(2/5)*a^2*b^2+48*x^(1/5)*ln(a+b*x^(1/5))*a^3*b+88*x^(1/5)*a
^3*b+12*ln(a+b*x^(1/5))*a^4+25*a^4)/(a+b*x^(1/5))^5/b^5
```

maxima [A] time = 1.02, size = 99, normalized size = 0.45

$$\frac{5\left(48\,ab^{3}x^{\frac{3}{5}}+108\,a^{2}b^{2}x^{\frac{2}{5}}+88\,a^{3}bx^{\frac{1}{5}}+25\,a^{4}\right)}{12\left(b^{9}x^{\frac{4}{5}}+4\,ab^{8}x^{\frac{3}{5}}+6\,a^{2}b^{7}x^{\frac{2}{5}}+4\,a^{3}b^{6}x^{\frac{1}{5}}+a^{4}b^{5}\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/(
b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a^4
*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+b^2\,x^{2/5}+2\,a\,b\,x^{1/5}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[5]{x}+b^{2}x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

3.492 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$-\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}$$

```
[Out] -6*b^7*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))/x^(1/6)+126*a^2*
b^5*x^(1/6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+105*a^3*b^4
*x^(1/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+63/2*a^5*b^2*x
^(2/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+42/5*a^6*b*x^(5/
6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+a^7*x*(a^2+b^2/x^(1/
3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+7*a*b^6*ln(x)*(a^2+b^2/x^(1/3)+2*a*b/
x^(1/6))^(1/2)/(a+b/x^(1/6))+70*a^4*b^3*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/
2)*x^(1/2)/(a+b/x^(1/6))
```

Rubi [A] time = 0.18, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6))*x^(1/6)
) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/
x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/
(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqr
t[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6
)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b
)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*
a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*
b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int\left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}dx = 6\,\text{Subst}\left(\int\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2}x^5\,dx,x,\sqrt[6]{x}\right)$$

$$=\frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int\left(ab+\frac{b^2}{x}\right)^7x^5\,dx,x,\sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$=\frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int\frac{(b^2+abx)^7}{x^2}\,dx,x,\sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$=\frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int\left(21a^2b^{12}+\frac{b^{14}}{x^2}+\frac{7ab^{13}}{x}+35a^3b^{11}x+35a^4b^{10}x^2+\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$=-\frac{6b^8\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)\sqrt[6]{x}}+\frac{126a^2b^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}}+\frac{105a^3b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+}}{ab+\frac{b^2}{\sqrt[6]{x}}}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[6]{x}+b\right)^2}{\sqrt[3]{x}}}\left(10a^7x^{7/6}+84a^6bx+315a^5b^2x^{5/6}+700a^4b^3x^{2/3}+1050a^3b^4\sqrt{x}+1260a^2b^5\sqrt[3]{x}+70ab^6\sqrt[6]{x}\log(x)\right.}{10\left(a\sqrt[6]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3
*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*
a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.46, size = 172, normalized size = 0.44

$$a^7x\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+7\,ab^6\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+\frac{42}{5}\,a^6bx^{\frac{5}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+\frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6)
)*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*x^
(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/6))
*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5*x^(
1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x^(1/6
)
```

maple [A] time = 0.03, size = 116, normalized size = 0.30

$$\frac{\sqrt{\frac{a^2\sqrt{x}+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{1}{6}}}{\sqrt{x}}}\left(10a^7x^{\frac{7}{6}}+70a\,b^6x^{\frac{1}{6}}\ln(x)+84a^6bx+315a^5b^2x^{\frac{5}{6}}+700a^4b^3x^{\frac{2}{3}}+1050a^3b^4\sqrt{x}+1260a^2b^5x^{\frac{1}{3}}\right.}{10a\,x^{\frac{1}{6}}+10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

```
[Out] 1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+b^2*x^(1/6))/x^(1/2))^(1/2)*(84*a^6*b*x+31
5*a^5*b^2*x^(5/6)+70*a*b^6*ln(x)*x^(1/6)+1050*a^3*b^4*x^(1/2)+1260*a^2*b^5*
x^(1/3)+700*a^4*b^3*x^(2/3)+10*a^7*x^(7/6)-60*b^7)/(a*x^(1/6)+b)
```

maxima [A] time = 0.95, size = 79, normalized size = 0.20

$$7\,ab^6\log(x)+\frac{10\,a^7x^{\frac{7}{6}}+84\,a^6bx+315\,a^5b^2x^{\frac{5}{6}}+700\,a^4b^3x^{\frac{2}{3}}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
```

```
[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)
/x^(1/6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+\frac{b^2}{x^{1/3}}+\frac{2\,a\,b}{x^{1/6}}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

```
[Out] Timed out
```

3.493 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=46

$$\frac{b^2\log(b+cx^n)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[Out] -b*x^n/c^2/n+1/2*x^(2*n)/c/n+b^2*ln(b+c*x^n)/c^3/n
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 43}

$$\frac{b^2\log(b+cx^n)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+3n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{b+cx}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log(b+cx^n)}{c^3n}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.83

$$\frac{2b^2\log(b+cx^n)+cx^n(cx^n-2b)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

fricas [A] time = 1.06, size = 38, normalized size = 0.83

$$\frac{c^{2}x^{2\,n} - 2\,bcx^{n} + 2\,b^{2}\log\left(cx^{n} + b\right)}{2\,c^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{cx^{2\,n} + bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.03, size = 62, normalized size = 1.35

$$\left(-\frac{b\,{\mathrm e}^{2n\ln(x)}}{c^{2}n} + \frac{{\mathrm e}^{3n\ln(x)}}{2cn}\right){\mathrm e}^{-n\ln(x)} + \frac{b^{2}\ln\left(c\,{\mathrm e}^{n\ln(x)} + b\right)}{c^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x)
[Out] (1/2/c/n*exp(n*ln(x))^3-b/c^2/n*exp(n*ln(x))^2)/exp(n*ln(x))+b^2/c^3/n*ln(c
*exp(n*ln(x))+b)
```

maxima [A] time = 0.93, size = 45, normalized size = 0.98

$$\frac{b^{2}\log\left(\frac{cx^{n}+b}{c}\right)}{c^{3}n} + \frac{cx^{2\,n} - 2\,bx^{n}}{2\,c^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4\,n-1}}{b\,x^{n} + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A] time = 19.83, size = 42, normalized size = 0.91

$$\frac{b^2\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{c^2n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] b**2*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c**2*n) - b*
x**n/(c**2*n) + x**(2*n)/(2*c*n)
```

3.494 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

```
[Out] x^n/c/n-b*ln(b+c*x^n)/c^2/n
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 43}

$$\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+2n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{b+cx}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= \frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{\frac{x^n}{c}-\frac{b\log(b+cx^n)}{c^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
[Out] (x^n/c - (b*Log[b + c*x^n])/c^2)/n
```

fricas [A] time = 0.86, size = 24, normalized size = 0.86

$$\frac{cx^n - b\log(cx^n + b)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.02, size = 33, normalized size = 1.18

$$-\frac{b\ln\left(c\,\mathrm{e}^{n\ln(x)} + b\right)}{c^2 n} + \frac{\mathrm{e}^{n\ln(x)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x)
[Out] 1/c/n*exp(n*ln(x))-b/c^2/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.87, size = 32, normalized size = 1.14

$$\frac{x^n}{cn} - \frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{3n-1}}{b\,x^n + c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A] time = 13.89, size = 26, normalized size = 0.93

$$-\frac{b\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{cn}+\frac{x^n}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -b*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c*n) + x**n/(c
*n)
```

3.495 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{cn}$$

[Out] ln(b+c*x^n)/c/n

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 260}

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[b + c*x^n]/(c*n)

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+n}}{b+cx^n}\,dx\\ &= \frac{\log(b+cx^n)}{cn}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[b + c*x^n]/(c*n)

fricas [A] time = 0.87, size = 15, normalized size = 1.00

$$\frac{\log(cx^n+b)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

```
[Out] log(c*x^n + b)/(c*n)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{cx^{2n} + bx^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A]   time = 0.02, size = 18, normalized size = 1.20

$$\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)} + b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x)
[Out] 1/c/n*ln(c*exp(n*ln(x))+b)
```

maxima [A]   time = 0.89, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{cx^{n}+b}{c}\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^n + b)/c)/(c*n)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^{2n-1}}{b\,x^{n} + c\,x^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A]   time = 11.28, size = 37, normalized size = 2.47

$$\begin{cases} \frac{\log(x)}{b} & \text{for}\ c = 0 \wedge n = 0 \\ \frac{\log(x)}{b+c} & \text{for}\ n = 0 \\ \frac{x^{n}}{bn} & \text{for}\ c = 0 \\ -\frac{\log(x)}{c} + \frac{\log\left(\frac{bx^{n}}{c} + x^{2n}\right)}{cn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (log(x)/(b + c), Eq(n, 0)), (x**
n/(b*n), Eq(c, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n))/(c*n), True))
```

3.496 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}$$

[Out] ln(x)/b-ln(b+c*x^n)/b/n

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[x]/b - Log[b + c*x^n]/(b*n)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx &= \int \frac{1}{x\left(b+cx^n\right)}\,dx\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{bn} - \frac{c\operatorname{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^n\right)}{bn}\\ &= \frac{\log(x)}{b} - \frac{\log\left(b+cx^n\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log\left(b+cx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
[Out] (n*Log[x] - Log[b + c*x^n])/(b*n)
```

fricas [A] time = 0.93, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log\left(cx^n+b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (n*log(x) - log(c*x^n + b))/(b*n)
```

giac [A] time = 0.25, size = 25, normalized size = 1.09

$$\frac{\log\left(|x|\right)}{b} - \frac{\log\left(|cx^n+b|\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

maple [A] time = 0.02, size = 26, normalized size = 1.13

$$\frac{\ln(x)}{b} - \frac{\ln\left(c\,e^{n\ln(x)}+b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(b*x^n+c*x^(2*n)),x)
[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.90, size = 27, normalized size = 1.17

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/b - log((c*x^n + b)/c)/(b*n)
```

mupad [B] time = 1.37, size = 20, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x^n}{b}+1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] -(2*atanh((2*c*x^n)/b + 1))/(b*n)
```

sympy [A] time = 16.49, size = 66, normalized size = 2.87

$$\begin{cases} \tilde{\infty}\log(x) & \text{for}\ b=0 \wedge c=0 \wedge n=0 \\ -\frac{x^{-n}}{cn} & \text{for}\ b=0 \\ \frac{\log(x)}{b+c} & \text{for}\ n=0 \\ \frac{\frac{n^2\log(x)}{n^2-n}-\frac{n\log(x)}{n^2-n}}{b} & \text{for}\ c=0 \\ \frac{2\log(x)}{b}-\frac{\log\left(\frac{bx^n}{c}+x^{2n}\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x**(-n)/(c*n), Eq
(b, 0)), (log(x)/(b + c), Eq(n, 0)), ((n**2*log(x)/(n**2 - n) - n*log(x)/(n
**2 - n))/b, Eq(c, 0)), (2*log(x)/b - log(b*x**n/c + x**(2*n))/(b*n), True)
)
```

3.497 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=57

$$-\frac{c^2\log(b+cx^n)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

```
[Out] -1/2/b/n/(x^(2*n))+c/b^2/n/(x^n)+c^2*ln(x)/b^3-c^2*ln(b+c*x^n)/b^3/n
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 44}

$$-\frac{c^2\log(b+cx^n)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^n]
)/(b^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-2n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.86

$$\frac{-2c^2\log(b+cx^n)+bx^{-2n}(2cx^n-b)+2c^2n\log(x)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] ((b*(-b + 2*c*x^n))/x^(2*n) + 2*c^2*n*Log[x] - 2*c^2*Log[b + c*x^n])/(2*b^3
*n)
```

fricas [A] time = 0.62, size = 59, normalized size = 1.04

$$\frac{2\,c^2 n x^{2\,n} \log(x) - 2\,c^2 x^{2\,n} \log\left(c x^n + b\right) + 2\,b c x^n - b^2}{2\,b^3 n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b^
2)/(b^3*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{c x^{2\,n} + b x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.02, size = 69, normalized size = 1.21

$$\left(\frac{c^2 e^{2n\ln(x)}\ln(x)}{b^3} + \frac{c\,e^{n\ln(x)}}{b^2 n} - \frac{1}{2bn}\right)e^{-2n\ln(x)} - \frac{c^2 \ln\left(c\,e^{n\ln(x)} + b\right)}{b^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c/b^2/n*exp(n*ln(x))-1/2/b/n+c^2/b^3*ln(x)*exp(n*ln(x))^2)/exp(n*ln(x))^2-
c^2/b^3/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.91, size = 58, normalized size = 1.02

$$\frac{c^2 \log(x)}{b^3} - \frac{c^2 \log\left(\frac{c x^n + b}{c}\right)}{b^3 n} + \frac{2\,c x^n - b}{2\,b^2 n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n*
x^(2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{n+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.498 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=76

$$\frac{c^3\log(b+cx^n)}{b^4n}-\frac{c^3\log(x)}{b^4}-\frac{c^2x^{-n}}{b^3n}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

```
[Out] -1/3/b/n/(x^(3*n))+1/2*c/b^2/n/(x^(2*n))-c^2/b^3/n/(x^n)-c^3*ln(x)/b^4+c^3*
ln(b+c*x^n)/b^4/n
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 44}

$$-\frac{c^2x^{-n}}{b^3n}+\frac{c^3\log(b+cx^n)}{b^4n}-\frac{c^3\log(x)}{b^4}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])/b
^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-3n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4n}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 62, normalized size = 0.82

$$-\frac{bx^{-3n}\left(2b^2-3bcx^n+6c^2x^{2n}\right)-6c^3\log\left(b+cx^n\right)+6c^3n\log(x)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*n*Log[x] - 6*
c^3*Log[b + c*x^n])/(b^4*n)
```

fricas [A] time = 0.59, size = 72, normalized size = 0.95

$$-\frac{6\,c^3nx^{3\,n}\log(x)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2*
n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.03, size = 88, normalized size = 1.16

$$\left(-\frac{c^3\mathrm{e}^{3n\ln(x)}\ln(x)}{b^4}+\frac{c\,\mathrm{e}^{n\ln(x)}}{2b^2n}-\frac{c^2\mathrm{e}^{2n\ln(x)}}{b^3n}-\frac{1}{3bn}\right)\mathrm{e}^{-3n\ln(x)}+\frac{c^3\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (-1/3/b/n+1/2/b^2*c/n*exp(n*ln(x))-c^2/b^3/n*exp(n*ln(x))^2-c^3/b^4*ln(x)*e
xp(n*ln(x))^3)/exp(n*ln(x))^3+c^3/b^4/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.91, size = 71, normalized size = 0.93

$$-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*b
*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{2\,n+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [A] time = 52.91, size = 73, normalized size = 0.96

$$-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}+\frac{c^4\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{b^4n}-\frac{c^3\log(x^n)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-3*n)/(3*b*n) + c*x**(-2*n)/(2*b**2*n) - c**2*x**(-n)/(b**3*n) + c**4*
Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(b**4*n) - c**3*lo
g(x**n)/(b**4*n)
```

3.499 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=93

$$-\frac{c^4\log\left(b+cx^n\right)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{c^3x^{-n}}{b^4n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

```
[Out] -1/4/b/n/(x^(4*n))+1/3*c/b^2/n/(x^(3*n))-1/2*c^2/b^3/n/(x^(2*n))+c^3/b^4/n/
(x^n)+c^4*ln(x)/b^5-c^4*ln(b+c*x^n)/b^5/n
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 44}

$$-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}-\frac{c^4\log\left(b+cx^n\right)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b^4
*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-4n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x^5(b+cx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^5}-\frac{c}{b^2x^4}+\frac{c^2}{b^3x^3}-\frac{c^3}{b^4x^2}+\frac{c^4}{b^5x}-\frac{c^5}{b^5(b+cx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(b+cx^n\right)}{b^5n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.81

$$-\frac{bx^{-4n}\left(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n}\right)+12c^4\log\left(b+cx^n\right)-12c^4n\log(x)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n)
 - 12*c^4*n*Log[x] + 12*c^4*Log[b + c*x^n])/(b^5*n)
```

fricas [A] time = 0.81, size = 85, normalized size = 0.91

$$\frac{12\,c^4nx^{4\,n}\log(x)-12\,c^4x^{4\,n}\log\left(cx^n+b\right)+12\,bc^3x^{3\,n}-6\,b^2c^2x^{2\,n}+4\,b^3cx^n-3\,b^4}{12\,b^5nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x^
(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.03, size = 105, normalized size = 1.13

$$\left(\frac{c^4e^{4n\ln(x)}\ln(x)}{b^5}+\frac{c\,e^{n\ln(x)}}{3b^2n}-\frac{c^2e^{2n\ln(x)}}{2b^3n}+\frac{c^3e^{3n\ln(x)}}{b^4n}-\frac{1}{4bn}\right)e^{-4n\ln(x)}-\frac{c^4\ln\left(c\,e^{n\ln(x)}+b\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c^3/b^4/n*exp(n*ln(x))^3-1/4/b/n+1/3/b^2*c/n*exp(n*ln(x))-1/2*c^2/b^3/n*ex
p(n*ln(x))^2+c^4/b^5*ln(x)*exp(n*ln(x))^4)/exp(n*ln(x))^4-c^4/b^5/n*ln(c*ex
p(n*ln(x))+b)
```

maxima [A] time = 0.92, size = 84, normalized size = 0.90

$$\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5n}+\frac{12\,c^3x^{3\,n}-6\,bc^2x^{2\,n}+4\,b^2cx^n-3\,b^3}{12\,b^4nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6*
b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [A] time = 105.73, size = 88, normalized size = 0.95

$$-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}-\frac{c^5\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log(b+cx^n)}{c} & \text{otherwise}\end{cases}\right)}{b^5n}+\frac{c^4\log(x^n)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-4*n)/(4*b*n) + c*x**(-3*n)/(3*b**2*n) - c**2*x**(-2*n)/(2*b**3*n) + c
**3*x**(-n)/(b**4*n) - c**5*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/
c, True))/(b**5*n) + c**4*log(x**n)/(b**5*n)
```

3.500 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=236

$$\frac{c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}+\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{b^{7/4}n}$$

```
[Out] -4/3/b/n/(x^(3/4*n))+1/2*c^(3/4)*ln(-b^(1/4)*c^(1/4)*x^(1/4*n)*2^(1/2)+b^(1
/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-1/2*c^(3/4)*ln(b^(1/4)*c^(1/4)*x^(
1/4*n)*2^(1/2)+b^(1/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-c^(3/4)*arctan(
-1+c^(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n-c^(3/4)*arctan(1+c^
(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n
```

Rubi [A] time = 0.21, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1584, 362, 345, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}+\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{b^{7/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/
4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^
(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{3n}{4}}}{b+cx^n}\,dx$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{c\int \frac{x^{\frac{1}{4}(-4+n)}}{b+cx^n}\,dx}{b}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{bn}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n}\right.}{b^{3/2}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x,\right.}{b^{3/2}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n}\right.}{\sqrt{2}b^{7/4}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} + \frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{\sqrt{}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.14

$$-\frac{4x^{-3n/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

fricas [A] time = 0.57, size = 272, normalized size = 1.15

$$-\frac{12\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5cn^3xx^{\frac{1}{4}n-1}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{3}{4}}-b^5n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^3}{b^7n^4}}+c^2x^2x^{\frac{1}{2}n-2}}{x^2}}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{3}{4}}}{c^3}\right)+3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right. \cdots}{3\,bnx^3x^{\frac{3}{4}n-3} \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/3*(12*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*arctan(-(b^5*c*n^3*x*
x^(1/4*n - 1)*(-c^3/(b^7*n^4))^(3/4) - b^5*n^3*x*sqrt((b^4*n^2*sqrt(-c^3/(b
^7*n^4)) + c^2*x^2*x^(1/2*n - 2))/x^2)*(-c^3/(b^7*n^4))^(3/4))/c^3) + 3*b*n
*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7*n^4))^(1/4)
 + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*l
og(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 4)/(b*n*x^3*x^(
3/4*n - 3))
```

giac [A] time = 0.35, size = 203, normalized size = 0.86

$$-\frac{\frac{6\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\left(x^n\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\left(x^n\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}n}+\sqrt{2}\left(x^n\right)^{\frac{1}{4}}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{c}}\right)}{b^2} - \cdots}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] -1/6*(6*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*(
x^n)^(1/4))/(b/c)^(1/4))/b^2 + 6*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 3*sqrt(2)*(b*c^3)^
(1/4)*log(x^(1/2*n) + sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqrt(b/c))/b^2 - 3*
sqrt(2)*(b*c^3)^(1/4)*log(x^(1/2*n) - sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqr
t(b/c))/b^2 + 8/(b*x^(3/4*n)))/n
```

maple [C] time = 0.12, size = 54, normalized size = 0.23

$$\mathrm{RootOf}\left(b^7n^4\_Z^4+c^3\right)\ln\left(-\frac{\mathrm{RootOf}\left(b^7n^4\_Z^4+c^3\right)b^2n}{c}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{3n}{4}}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x)
```

[Out] -4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*n
^4+c^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int \frac{x^{\frac{1}{4}\,n}}{bcxx^{n}+b^{2}x}\,dx-\frac{4}{3\,bnx^{\frac{3}{4}\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] -c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{b\,x^{n}+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)

[Out] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n}x^{\frac{n}{4}-1}}{b+cx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)

[Out] Integral(x**(-n)*x**(n/4 - 1)/(b + c*x**n), x)

3.501 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

```
[Out] -3/2/b/n/(x^(2/3*n))-c^(2/3)*ln(b^(1/3)+c^(1/3)*x^(1/3*n))/b^(5/3)/n+1/2*c^
(2/3)*ln(b^(2/3)-b^(1/3)*c^(1/3)*x^(1/3*n)+c^(2/3)*x^(2/3*n))/b^(5/3)/n+c^(
2/3)*arctan(1/3*(b^(1/3)-2*c^(1/3)*x^(1/3*n))/b^(1/3)*3^(1/2))*3^(1/2)/b^(5
/3)/n
```

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1584, 362, 345, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(1/3)*x^(n/
3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1/3) + c^(1/3)*x^(n/3
)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*
x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{2n}{3}}}{b+cx^n}\,dx$$

$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c\int \frac{x^{\frac{1}{3}(-3+n)}}{b+cx^n}\,dx}{b}$$

$$= -\frac{3x^{-2n/3}}{2bn} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{b+cx^3}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{bn}$$

$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{c}\,x}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n} - \frac{c\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}\,x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx, x, x^{1+\frac{1}{3}(-3}\right.}{b^{5/3}n}$$

$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2c^{2/3}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{2b^{5/3}n} - \frac{(3c)}{}$$

$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n} - \frac{\left(3c^{2/3}\right)\text{Subst}}{}$$

$$= -\frac{3x^{-2n/3}}{2bn} + \frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x^{n/}\right.}{2b^{5/3}n}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.21

$$-\frac{3x^{-2n/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{cx^n}{b}\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

fricas [A] time = 0.74, size = 212, normalized size = 1.32

$$\frac{2\sqrt{3}x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)+2x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right.}{2bnx^2x^{\frac{2}{3}n-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x
*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c^2
/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3*n
- 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*(-c^
2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))
```

giac [A] time = 0.42, size = 136, normalized size = 0.85

$$\frac{\frac{2c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right|\right)}{b^2}-\frac{2\sqrt{3}\left(-bc^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2}-\frac{\left(-bc^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2}-\frac{3}{b\left(x^n\right)^{\frac{2}{3}}}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*(-b/c)^(1/3)*log(abs(x^(1/3*n) - (-b/c)^(1/3)))/b^2 - 2*sqrt(3)*(-
b*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-b/c)^(1/3))/(-b/c)^(1/3))/
b^2 - (-b*c^2)^(1/3)*log(x^(1/3*n)*(-b/c)^(1/3) + (x^n)^(2/3) + (-b/c)^(2/3
))/b^2 - 3/(b*(x^n)^(2/3)))/n
```

maple [C] time = 0.07, size = 54, normalized size = 0.34

$$\mathrm{RootOf}\left(b^5n^3\_Z^3+c^2\right)\ln\left(-\frac{\mathrm{RootOf}\left(b^5n^3\_Z^3+c^2\right)b^2n}{c}+x^{\frac{n}{3}}\right)-\frac{3x^{-\frac{2n}{3}}}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-_R*b^2/c*n),_R=RootOf(_Z^3*b^5*n
^3+c^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int\frac{x^{\frac{1}{3}n}}{bcxx^n+b^2x}\,dx-\frac{3}{2bnx^{\frac{2}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{3}-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.502 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

```
[Out] -2/b/n/(x^(1/2*n))+2*arctan(b^(1/2)/(x^(1/2*n))/c^(1/2))*c^(1/2)/b^(3/2)/n
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 345, 193, 321, 205}

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(3/2)*n
)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx \\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx, x, x^{-n/2}\right)}{n} \\
&= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx, x, x^{-n/2}\right)}{n} \\
&= -\frac{2x^{-n/2}}{bn} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx, x, x^{-n/2}\right)}{bn} \\
&= -\frac{2x^{-n/2}}{bn} + \frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.64

$$
-\frac{2x^{-n/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{cx^n}{b}\right)}{bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

fricas [A] time = 0.84, size = 151, normalized size = 3.02

$$
\left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}n-1}}, \frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}n-1}}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sqr
t(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1/2
*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^(1/
2*n - 1))]
```

giac [A] time = 0.38, size = 38, normalized size = 0.76

$$
-\frac{2\left(\frac{c\arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}+\frac{1}{b\sqrt{x^n}}\right)}{n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] -2*(c*arctan(c*sqrt(x^n)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(x^n)))/n
```

maple [A] time = 0.07, size = 79, normalized size = 1.58

$$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^{2}n}-\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)-1/b^2*
(-b*c)^(1/2)/n*ln(x^(1/2*n)+(-b*c)^(1/2)/c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int\frac{x^{\frac{1}{2}n}}{bcxx^{n}+b^{2}x}\,dx-\frac{2}{bnx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^{\frac{n}{2}-1}}{b\,x^{n}+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A] time = 13.00, size = 36, normalized size = 0.72

$$-\frac{2\operatorname{atan}\left(\frac{x^{\frac{n}{2}}}{\sqrt{\frac{b}{c}}}\right)}{bn\sqrt{\frac{b}{c}}}-\frac{2x^{-\frac{n}{2}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*atan(x**(n/2)/sqrt(b/c))/(b*n*sqrt(b/c)) - 2*x**(-n/2)/(b*n)
```

3.503 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

```
[Out] -2/3/b/n/(x^(3/2*n))+2*c/b^2/n/(x^(1/2*n))-2*c^(3/2)*arctan(b^(1/2)/(x^(1/2
*n))/c^(1/2))/b^(5/2)/n
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1584, 362, 345, 193, 321, 205}

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/
(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{3n}{2}}}{b+cx^n}\,dx\\
&= -\frac{2x^{-3n/2}}{3bn}-\frac{c\int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx}{b}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx,x,x^{-n/2}\right)}{bn}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx,x,x^{-n/2}\right)}{bn}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx,x,x^{-n/2}\right)}{b^2n}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.50

$$-\frac{2x^{-3n/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

fricas [A] time = 1.05, size = 161, normalized size = 2.37

$$\left[-\frac{2\,bx^3x^{-\frac{3}{2}\,n-3}-6\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^2x^{-n-2}-2\,bxx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-c}{bx^2x^{-n-2}+c}\right)}{3\,b^2n},-\frac{2\left(bx^3x^{-\frac{3}{2}\,n-3}-3\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{\frac{c}{b}}\text{ a}\right.}{3\,b^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log((
b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2) +
 c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*sqrt
(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.10, size = 97, normalized size = 1.43

$$-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{2c\,x^{-\frac{n}{2}}}{b^{2}n}-\frac{\sqrt{-bc}\;c\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^{3}n}+\frac{\sqrt{-bc}\;c\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/
2*n)+(-b*c)^(1/2)/c)-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\int\frac{x^{\frac{1}{2}n}}{b^{2}cxx^{n}+b^{3}x}\,dx+\frac{2\left(3\,cx^{n}-b\right)}{3\,b^{2}nx^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2*
n*x^(3/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{\frac{n}{2}+1}\left(b\,x^{n}+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [A] time = 24.92, size = 58, normalized size = 0.85

$$-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{2cx^{-\frac{n}{2}}}{b^{2}n}-\frac{2c^{2}\operatorname{atan}\left(\frac{x^{-\frac{n}{2}}}{\sqrt{\frac{c}{b}}}\right)}{b^{3}n\sqrt{\frac{c}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*x**(-3*n/2)/(3*b*n) + 2*c*x**(-n/2)/(b**2*n) - 2*c**2*atan(x**(-n/2)/sqr
t(c/b))/(b**3*n*sqrt(c/b))
```

3.504 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=176

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}\,x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-$$

```
[Out] -3/4/b/n/(x^(4/3*n))+3*c/b^2/n/(x^(1/3*n))-c^(4/3)*ln(c^(1/3)+b^(1/3)/(x^(1
/3*n)))/b^(7/3)/n+1/2*c^(4/3)*ln(c^(2/3)+b^(2/3)/(x^(2/3*n))-b^(1/3)*c^(1/3
)/(x^(1/3*n)))/b^(7/3)/n+c^(4/3)*arctan(1/3*(1-2*b^(1/3)/c^(1/3)/(x^(1/3*n)
))*3^(1/2))*3^(1/2)/b^(7/3)/n
```

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1584, 362, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}\,x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)*ArcTan[(c
^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7/3)*n) - (c^(4/3)*Lo
g[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (c^(4/3)*Log[c^(2/3) + b^(2/3)/
x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(n/3)])/(2*b^(7/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[(((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{4n}{3}}}{b+cx^n}\,dx$$
$$= -\frac{3x^{-4n/3}}{4bn} - \frac{c\int \frac{x^{-1-\frac{n}{3}}}{b+cx^n}\,dx}{b}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^3}}\,dx, x, x^{-n/3}\right)}{bn}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\,\text{Subst}\left(\int \frac{x^3}{c+bx^3}\,dx, x, x^{-n/3}\right)}{bn}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{c+bx^3}\,dx, x, x^{-n/3}\right)}{b^2n}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{b}x}\,dx, x, x^{-n/3}\right)}{b^2n} - \frac{c^{4/3}\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\right.}{b^2n}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2b^{2/3}x}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\,dx, x, x^{-n}\right.}{2b^{7/3}n}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n}$$
$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} + \frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/\right.}{}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.19

$$-\frac{3x^{-4n/3}\,{}_2F_1\left(-\frac{4}{3},1;-\frac{1}{3};-\frac{cx^n}{b}\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

fricas [A] time = 0.83, size = 171, normalized size = 0.97

$$-\frac{3\,bx^4x^{-\frac{4}{3}n-4} - 12\,cxx^{-\frac{1}{3}n-1} - 4\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right) - 4c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}n-1}-\left(-\frac{c}{b}\right)^{\frac{1}{3}}}{x}\right) +}{4\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^(
1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c)
- 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)^(1
/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))
/x^2))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [C] time = 0.08, size = 73, normalized size = 0.41

$$\mathrm{RootOf}\left(b^7n^3\_Z^3+c^4\right)\ln\left(\frac{\mathrm{RootOf}\left(b^7n^3\_Z^3+c^4\right)^2b^5n^2}{c^3}+x^{\frac{n}{3}}\right)-\frac{3x^{-\frac{4n}{3}}}{4bn}+\frac{3c\,x^{-\frac{n}{3}}}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^3
*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\int \frac{x^{\frac{2}{3}n}}{b^2cxx^n+b^3x}\,dx+\frac{3\,(4\,cx^n-b)}{4\,b^2nx^{\frac{4}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2*
n*x^(4/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{3}+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.505 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=252

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\right.}{b^{9/4}n}$$

```
[Out] -4/5/b/n/(x^(5/4*n))+4*c/b^2/n/(x^(1/4*n))+1/2*c^(5/4)*ln(-b^(1/4)*c^(1/4)*
2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(9/4)/n*2^(1/2)-1/2*c^(5
/4)*ln(b^(1/4)*c^(1/4)*2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(
9/4)/n*2^(1/2)+c^(5/4)*arctan(1-b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1/2
)/b^(9/4)/n-c^(5/4)*arctan(1+b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1/2)/b
^(9/4)/n
```

Rubi [A] time = 0.22, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1584, 362, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\right.}{b^{9/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)*ArcTan[1
- (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (Sqrt[2]*c^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) + (c^(5/4)*Log[Sqr
t[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/
4)*n) - (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/
x^(n/4)])/(Sqrt[2]*b^(9/4)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{5n}{4}}}{b+cx^n}\,dx \\
&= -\frac{4x^{-5n/4}}{5bn}-\frac{c\int \frac{x^{-1-\frac{n}{4}}}{b+cx^n}\,dx}{b} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{(4c)\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^4}}\,dx,x,x^{-n/4}\right)}{bn} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{(4c)\,\text{Subst}\left(\int \frac{x^4}{c+bx^4}\,dx,x,x^{-n/4}\right)}{bn} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}-\frac{\left(4c^2\right)\text{Subst}\left(\int \frac{1}{c+bx^4}\,dx,x,x^{-n/4}\right)}{b^2n} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}-\frac{\left(2c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{b}\,x^2}{c+bx^4}\,dx,x,x^{-n/4}\right)}{b^2n}-\frac{\left(2c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{b}\,x^2}{c+bx^4}\,d\cdots\right.}{b^2n} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{c^{5/4}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,x^{-n/4}\right)}{\sqrt{2}\,b^{9/4}n}+\frac{c^{5/4}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}-2\cdots}{-\frac{\sqrt{c}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt[4]{b}}}\right.}{\sqrt{2}\,b^{9/4}n\cdots} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}\,x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}\right)}{\sqrt{2}\,b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}\,x^{-n/\cdots}\right.}{\sqrt{2}\,b^{\cdots}} \\
&= -\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}+\frac{c^{5/\cdots}}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.13

$$
-\frac{4x^{-5n/4}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^n}{b}\right)}{5bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

fricas [A] time = 0.68, size = 259, normalized size = 1.03

$$
-\frac{4\,bx^5x^{-\frac{5}{4}n-5}+20\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^7cn^3xx^{-\frac{1}{4}n-1}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}-b^7n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^5}{b^9n^4}}+c^2x^2x^{-\frac{1}{2}n-2}}{x^2}}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}}{c^5}\right)+5\,b^2n\left(-\cdots\right.}{5\,b^2n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) + 20*b^2*n*(-c^5/(b^9*n^4))^(1/4)*arctan(-(b^7
*c*n^3*x*x^(-1/4*n - 1)*(-c^5/(b^9*n^4))^(3/4) - b^7*n^3*x*sqrt((b^4*n^2*sq
```

```
rt(-c^5/(b^9*n^4)) + c^2*x^2*x^(-1/2*n - 2))/x^2)*(-c^5/(b^9*n^4))^(3/4))/c
^5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*
x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log(-(b^2*n*(-c^5/(b^
9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [C] time = 0.09, size = 73, normalized size = 0.29

$$\mathrm{RootOf}\left(b^{9}n^{4}\_Z^{4}+c^{5}\right)\ln\left(\frac{\mathrm{RootOf}\left(b^{9}n^{4}\_Z^{4}+c^{5}\right)^{3}b^{7}n^{3}}{c^{4}}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{5n}{4}}}{5bn}+\frac{4c\,x^{-\frac{n}{4}}}{b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^4
*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\int \frac{x^{\frac{3}{4}n}}{b^{2}cxx^{n}+b^{3}x}\,dx+\frac{4\left(5\,cx^{n}-b\right)}{5\,b^{2}nx^{\frac{5}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2*
n*x^(5/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1}\,\left(b\,x^{n}+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.506 $\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=37

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn(p+1)}$$

```
[Out] (b*x^n+c*x^(2*n))^(1+p)/c/n/(1+p)/(x^(n*(1+p)))
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2014}

$$\frac{x^{-n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(-1+p)} \left(bx^n + cx^{2n}\right)^p \, dx = \frac{x^{-n(1+p)} \left(bx^n + cx^{2n}\right)^{1+p}}{cn(1+p)}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.03

$$\frac{x^{-np} \left(b + cx^n\right) \left(x^n \left(b + cx^n\right)\right)^p}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

fricas [A] time = 0.68, size = 59, normalized size = 1.59

$$\frac{\left(cxx^{-np+n-1}x^n + bxx^{-np+n-1}\right)\left(cx^{2n} + bx^n\right)^p}{\left(cnp + cn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((c
*n*p + c*n)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n}\right)^{p} x^{-n(p-1)-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^{-(p-1)n-1} \left(b\,x^{n} + c\,x^{2n}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

maxima [A] time = 1.11, size = 43, normalized size = 1.16

$$\frac{(cx^{n} + b)e^{\left(-np\log(x) + p\log(cx^{n} + b) + p\log(x^{n})\right)}}{cn\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^{n} + c\,x^{2\,n}\right)^{p}}{x^{n\,(p-1)+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

3.507 $\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=38

$$-\frac{x^{-2n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{bn(p+1)}$$

```
[Out] -(b*x^n+c*x^(2*n))^(1+p)/b/n/(1+p)/(x^(2*n*(1+p)))
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2014}

$$-\frac{x^{-2n(p+1)} \left(bx^n + cx^{2n}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx = -\frac{x^{-2n(1+p)} \left(bx^n + cx^{2n}\right)^{1+p}}{bn(1+p)}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.13

$$-\frac{x^{-n(2p+1)} (b + cx^n) (x^n (b + cx^n))^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

fricas [A] time = 0.79, size = 59, normalized size = 1.55

$$-\frac{\left(cxx^{-2\,np-n-1}x^n + bxx^{-2\,np-n-1}\right)\left(cx^{2\,n} + bx^n\right)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)^
p/(b*n*p + b*n)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n}\right)^{p} x^{-n(2p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

maple **[F]** time = 0.07, size = 0, normalized size = 0.00

$$\int x^{-(2p+1)n-1} \left(b\,x^{n} + c\,x^{2n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(2*p+1))*(b*x^n+c*x^(2*n))^p,x)
[Out] int(x^(-1-n*(2*p+1))*(b*x^n+c*x^(2*n))^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n}\right)^{p} x^{-n(2p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^{n} + c\,x^{2\,n}\right)^{p}}{x^{n\,(2\,p+1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1),x)
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

3.508 $\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^{5/2} dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^7 \sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)} - \frac{a\,(a+bx^n)^6 \sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

```
[Out] -1/6*a*(a+b*x^n)^6*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/7*
(a+b*x^n)^7*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1355, 266, 43}

$$\frac{(a+bx^n)^7 \sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)} - \frac{a\,(a+bx^n)^6 \sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b^3
*x^n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^5\,dx}{b^4\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\mathrm{Subst}\left(\int x\left(ab+b^2x\right)^5\,dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\mathrm{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right)dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}$$
$$=-\frac{a\left(a+bx^n\right)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}+\frac{\left(a+bx^n\right)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.36

$$-\frac{\left(a-6bx^n\right)\left(a+bx^n\right)^5\sqrt{\left(a+bx^n\right)^2}}{42b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -1/42*((a - 6*b*x^n)*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])/(b^2*n)
```

fricas [A] time = 0.73, size = 74, normalized size = 0.66

$$\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

maple [A] time = 0.05, size = 208, normalized size = 1.86

$$\frac{\sqrt{\left(b\,x^n+a\right)^2}\;a^5x^{2n}}{2\left(b\,x^n+a\right)n}+\frac{5\sqrt{\left(b\,x^n+a\right)^2}\;a^4b\,x^{3n}}{3\left(b\,x^n+a\right)n}+\frac{5\sqrt{\left(b\,x^n+a\right)^2}\;a^3b^2x^{4n}}{2\left(b\,x^n+a\right)n}+\frac{2\sqrt{\left(b\,x^n+a\right)^2}\;a^2b^3x^{5n}}{\left(b\,x^n+a\right)n}+\frac{5\sqrt{\left(b\,x^n+a\right)^2}\;a\,b^4}{6\left(b\,x^n+a\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] 1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*a*b^4/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+5/
2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)*a^4*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^2
```

maxima [A] time = 0.92, size = 74, normalized size = 0.66

$$\frac{6\,b^5x^{7\,n} + 35\,ab^4x^{6\,n} + 84\,a^2b^3x^{5\,n} + 105\,a^3b^2x^{4\,n} + 70\,a^4bx^{3\,n} + 21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

3.509 $\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}$$

```
[Out] -1/4*a*(a+b*x^n)^4*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/5*
(a+b*x^n)^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(5*n*(a*b^2 + b^3
*x^n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^3dx}{b^2\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^3dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right)dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$=-\frac{a\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}+\frac{\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.36

$$-\frac{\left(a-4bx^n\right)\left(a+bx^n\right)^3\sqrt{\left(a+bx^n\right)^2}}{20b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] -1/20*((a - 4*b*x^n)*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])/(b^2*n)
```

fricas [A] time = 0.76, size = 48, normalized size = 0.43

$$\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

maple [A] time = 0.03, size = 135, normalized size = 1.21

$$\frac{\sqrt{\left(b\,x^n+a\right)^2}\;a^3x^{2n}}{2\left(b\,x^n+a\right)n}+\frac{\sqrt{\left(b\,x^n+a\right)^2}\;a^2b\,x^{3n}}{\left(b\,x^n+a\right)n}+\frac{3\sqrt{\left(b\,x^n+a\right)^2}\;a\,b^2x^{4n}}{4\left(b\,x^n+a\right)n}+\frac{\sqrt{\left(b\,x^n+a\right)^2}\;b^3x^{5n}}{5\left(b\,x^n+a\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/5*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3/n*(x^n)^5+3/4*((b*x^n+a)^2)^(1/2)/(b*
x^n+a)*a*b^2/n*(x^n)^4+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/n*(x^n)^3+1/2*((
b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3/n*(x^n)^2
```

maxima [A] time = 0.92, size = 48, normalized size = 0.43

$$\frac{4\,b^3x^{5\,n} + 15\,ab^2x^{4\,n} + 20\,a^2bx^{3\,n} + 10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

3.510 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=99

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}$$

```
[Out] 1/2*a*x^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a+b*x^n)+1/3*b^2*x^(3*n)
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)
```

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1355, 14}

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^
(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(abx^{-1+2n}+b^2x^{-1+3n}\right)dx}{ab+b^2x^n}\\ &= \frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.44

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\,(3a+2bx^n)}{6n(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

fricas [A] time = 0.72, size = 22, normalized size = 0.22

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

maple [A] time = 0.02, size = 64, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n+a)^2}\;a\,x^{2n}}{2\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;b\,x^{3n}}{3\,(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/n*(x^n)^3+1/2*((b*x^n+a)^2)^(1/2)/(b*x^
n+a)*a/n*(x^n)^2
```

maxima [A] time = 0.89, size = 22, normalized size = 0.22

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

3.511 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=90

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x^n*(a+b*x^n)/b/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-a*(a+b*x^n)*ln(a+b*x^n)
/b^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n
)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{x^{-1+2n}}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{\left(ab+b^2x^n\right)\mathrm{Subst}\left(\int \frac{x}{ab+b^2x}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{\left(ab+b^2x^n\right)\mathrm{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a}{b^2(a+bx)}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.51

$$\frac{(a+bx^n)\left(\frac{x^n}{b}-\frac{a\log(a+bx^n)}{b^2}\right)}{n\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((a + b*x^n)*(x^n/b - (a*Log[a + b*x^n])/b^2))/(n*Sqrt[(a + b*x^n)^2])
```

fricas [A] time = 0.75, size = 24, normalized size = 0.27

$$\frac{bx^n-a\log(bx^n+a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{b^2x^{2n}+2abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

maple [A] time = 0.03, size = 71, normalized size = 0.79

$$-\frac{\sqrt{(bx^n+a)^2}\,a\ln\left(x^n+\frac{a}{b}\right)}{(bx^n+a)\,b^2n}+\frac{\sqrt{(bx^n+a)^2}\,x^n}{(bx^n+a)\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/b/n*x^n-((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/b^2/n
*ln(x^n+a/b)
```

maxima [A] time = 0.89, size = 32, normalized size = 0.36

$$\frac{x^n}{bn}-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{\sqrt{a^2+b^2x^{2n}+2abx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

3.512 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

[Out] 1/2*x^(2*n)/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1355, 264}

$$\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.73

$$\frac{x^{2n}\left(a+bx^n\right)}{2an\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^(2*n)*(a + b*x^n))/(2*a*n*((a + b*x^n)^2)^(3/2))
```

fricas [A] time = 0.68, size = 41, normalized size = 0.85

$$-\frac{2\,bx^{n}+a}{2\,\left(b^{4}nx^{2\,n}+2\,ab^{3}nx^{n}+a^{2}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

maple [A] time = 0.03, size = 37, normalized size = 0.77

$$-\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\,\left(2b\,x^{n}+a\right)}{2\left(b\,x^{n}+a\right)^{3}b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^3*(2*b*x^n+a)/b^2/n
```

maxima [A] time = 0.92, size = 41, normalized size = 0.85

$$-\frac{2\,bx^{n}+a}{2\,\left(b^{4}nx^{2\,n}+2\,ab^{3}nx^{n}+a^{2}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\left(\left(a+bx^{n}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)
```

3.513 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/4*a/b^2/n/(a+b*x^n)^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/3/b^2/n/(a+b*x^
n)^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*
(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^5}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\operatorname{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^5}\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\operatorname{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.45

$$-\frac{a+4bx^n}{12b^2n\left(a+bx^n\right)^3\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -1/12*(a + 4*b*x^n)/(b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])
```

fricas [A] time = 0.66, size = 69, normalized size = 0.78

$$-\frac{4\,bx^n+a}{12\left(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

maple [A] time = 0.03, size = 37, normalized size = 0.42

$$-\frac{\sqrt{\left(b\,x^n+a\right)^2}\;\left(4b\,x^n+a\right)}{12\left(b\,x^n+a\right)^5b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] -1/12*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^5*(4*b*x^n+a)/b^2/n
```

maxima [A] time = 0.95, size = 69, normalized size = 0.78

$$-\frac{4\,bx^{n}+a}{12\,\left(b^{6}nx^{4\,n}+4\,ab^{5}nx^{3\,n}+6\,a^{2}b^{4}nx^{2\,n}+4\,a^{3}b^{3}nx^{n}+a^{4}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

3.514 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{6b^2n\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/6*a/b^2/n/(a+b*x^n)^5/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/5/b^2/n/(a+b*x^
n)^4/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{a}{6b^2n\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*
(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx = \frac{\left(b^6\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^7}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^7}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^8(a+bx)^7}+\frac{1}{b^8(a+bx)^6}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [**A**] time = 0.03, size = 40, normalized size = 0.45

$$-\frac{a+6bx^n}{30b^2n\left(a+bx^n\right)^5\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] -1/30*(a + 6*b*x^n)/(b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])
```

fricas [**A**] time = 0.64, size = 97, normalized size = 1.10

$$-\frac{6\,bx^n+a}{30\left(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

maple [**A**] time = 0.03, size = 37, normalized size = 0.42

$$-\frac{\sqrt{\left(b\,x^n+a\right)^2}\,\left(6b\,x^n+a\right)}{30\left(b\,x^n+a\right)^7b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)
```

```
[Out] -1/30*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^7*(6*b*x^n+a)/b^2/n
```

maxima [A] time = 0.97, size = 97, normalized size = 1.10

$$-\frac{6\,bx^{n}+a}{30\,\left(b^{8}nx^{6\,n}+6\,ab^{7}nx^{5\,n}+15\,a^{2}b^{6}nx^{4\,n}+20\,a^{3}b^{5}nx^{3\,n}+15\,a^{4}b^{4}nx^{2\,n}+6\,a^{5}b^{3}nx^{n}+a^{6}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

```
[Out] Timed out
```

3.515 $\int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}\, dx$

Optimal. Leaf size=108

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

```
[Out] a*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+b^2*x^(1+
n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1355, 14, 20, 30}

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n
)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m +
n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (dx)^m\left(ab+b^2x^n\right)dx}{ab+b^2x^n}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(ab(dx)^m+b^2x^n(dx)^m\right)dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{\left(b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^m\,dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{\left(b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m}}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.51

$$\frac{x(dx)^m\sqrt{(a+bx^n)^2}\,(a(m+n+1)+b(m+1)x^n)}{(m+1)(m+n+1)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(1
 + m + n)*(a + b*x^n))
```

fricas [A] time = 0.86, size = 57, normalized size = 0.53

$$\frac{(bm+b)xx^ne^{(m\log(d)+m\log(x))}+(am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

giac [A] time = 0.35, size = 173, normalized size = 1.60

$$\frac{bmxx^ne^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)+amxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)+bmxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] (b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*n*
x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

maple [C] time = 0.04, size = 132, normalized size = 1.22

$$\frac{\sqrt{(b\,x^n+a)^2}\,\left(bm\,x^n+am+an+b\,x^n+a\right)x\,\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(ix)\mathrm{csgn}(idx)+i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(idx)^2+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(idx)^2-i\pi\mathrm{csgn}(idx)^3+2\ln(d)}{2}}}{(b\,x^n+a)(m+1)(m+n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(m+1)/(1+m+n)*exp
(1/2*m*(-I*Pi*csgn(I*d*x)^3+I*Pi*csgn(I*d*x)^2*csgn(I*d)+I*Pi*csgn(I*d*x)^2
*csgn(I*x)-I*Pi*csgn(I*d*x)*csgn(I*d)*csgn(I*x)+2*ln(d)+2*ln(x)))
```

maxima [A] time = 0.94, size = 47, normalized size = 0.44

$$\frac{ad^m(m+n+1)xx^m + bd^m(m+1)xe^{\left(m\log(x)+n\log(x)\right)}}{m^2+m(n+2)+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
m*(n + 2) + n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

3.516 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\,(a+bx^n)}$$

```
[Out] 1/3*a*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(3+n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx^2+b^2x^{2+n}\right)dx}{ab+b^2x^n}\\ &= \frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\,(a+bx^n)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^3\sqrt{(a+bx^n)^2}\,\left(a(n+3)+3bx^n\right)}{3(n+3)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

fricas [A] time = 0.83, size = 28, normalized size = 0.30

$$\frac{3\,bx^3x^n + (an + 3\,a)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

giac [A] time = 0.26, size = 53, normalized size = 0.57

$$\frac{3\,bx^3x^n\mathrm{sgn}\left(bx^n + a\right) + anx^3\mathrm{sgn}\left(bx^n + a\right) + 3\,ax^3\mathrm{sgn}\left(bx^n + a\right)}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] 1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x^
n + a))/(n + 3)
```

maple [A] time = 0.02, size = 61, normalized size = 0.66

$$\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\;b\,x^{3}x^{n}}{\left(b\,x^{n}+a\right)\left(n+3\right)}+\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\;a\,x^{3}}{3b\,x^{n}+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x^3+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(3+
n)*x^3*x^n
```

maxima [A] time = 0.95, size = 25, normalized size = 0.27

$$\frac{3\,bx^3x^n + a(n+3)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{\left(a + bx^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)
```

3.517 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

```
[Out] 1/2*a*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(2+n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx+b^2x^{1+n}\right)dx}{ab+b^2x^n}\\
&= \frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^2\sqrt{\left(a+bx^n\right)^2}\left(a(n+2)+2bx^n\right)}{2(n+2)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

fricas [A] time = 0.60, size = 28, normalized size = 0.30

$$\frac{2\,bx^2x^n + (an + 2\,a)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

giac [A] time = 0.29, size = 53, normalized size = 0.57

$$\frac{2\,bx^2x^n\mathrm{sgn}\,(bx^n + a) + anx^2\mathrm{sgn}\,(bx^n + a) + 2\,ax^2\mathrm{sgn}\,(bx^n + a)}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x^
n + a))/(n + 2)
```

maple [A] time = 0.02, size = 61, normalized size = 0.66

$$\frac{\sqrt{(b\,x^n + a)^2}\;b\,x^2x^n}{(b\,x^n + a)\,(n+2)} + \frac{\sqrt{(b\,x^n + a)^2}\;a\,x^2}{2b\,x^n + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(2+
n)*x^2*x^n
```

maxima [A] time = 0.88, size = 25, normalized size = 0.27

$$\frac{2\,bx^2x^n + a(n+2)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{(a + bx^n)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a + b*x**n)**2), x)
```

3.518 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] a*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(1+n)*(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1343}

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(2ab+2b^2x^n\right)dx}{2ab+2b^2x^n}\\ &= \frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.44

$$\frac{x\sqrt{(a+bx^n)^2}\,(an+a+bx^n)}{(n+1)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

fricas [A] time = 0.72, size = 20, normalized size = 0.23

$$\frac{bxx^n+(an+a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

giac [A] time = 0.22, size = 25, normalized size = 0.28

$$\left(ax+\frac{bx^{n+1}}{n+1}\right)\operatorname{sgn}\left(bx^{n}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] (a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

maple [A] time = 0.02, size = 56, normalized size = 0.64

$$\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\;bx\,x^{n}}{\left(b\,x^{n}+a\right)\left(n+1\right)}+\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\;ax}{b\,x^{n}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(1+n)*x*x
^n
```

maxima [A] time = 1.04, size = 19, normalized size = 0.22

$$\frac{a(n+1)x+bxx^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a^{2}+b^{2}\,x^{2\,n}+2\,a\,b\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2}+2abx^{n}+b^{2}x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

3.519 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal. Leaf size=85

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[Out] b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a*ln(x)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x}\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x}+b^2x^{-1+n}\right)dx}{ab+b^2x^n}\\ &= \frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.44

$$\frac{\sqrt{(a+bx^n)^2}\left(an\log(x)+bx^n\right)}{n\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]

[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*n*Log[x]))/(n*(a + b*x^n))

fricas [A] time = 0.89, size = 15, normalized size = 0.18

$$\frac{an\log(x) + bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")

[Out] (a*n*log(x) + b*x^n)/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)

maple [A] time = 0.02, size = 54, normalized size = 0.64

$$\frac{\sqrt{(b\,x^n + a)^2}\;a\ln(x)}{b\,x^n + a} + \frac{\sqrt{(b\,x^n + a)^2}\;b\,x^n}{(b\,x^n + a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*ln(x)+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/n*x^n

maxima [A] time = 0.80, size = 13, normalized size = 0.15

$$a\log(x) + \frac{bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")

[Out] a*log(x) + b*x^n/n

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)

[Out] Integral(sqrt((a + b*x**n)**2)/x, x)

3.520 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

```
[Out] -a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-b^2*x^(-1+n)*(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^2}\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^2}+b^2x^{-2+n}\right)dx}{ab+b^2x^n}\\
&= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.45

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(-an+a+bx^n\right)}{(n-1)x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

fricas [A] time = 0.57, size = 23, normalized size = 0.24

$$-\frac{an - bx^n - a}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2 x^{2n} + 2\,abx^n + a^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

maple [A] time = 0.02, size = 61, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n + a)^2}\;b\,x^n}{(b\,x^n + a)\,(n-1)\,x} - \frac{\sqrt{(b\,x^n + a)^2}\;a}{(b\,x^n + a)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x)
```

```
[Out] -((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+n)*b/x
*x^n
```

maxima [A] time = 0.67, size = 22, normalized size = 0.23

$$-\frac{a(n-1) - bx^n}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

3.521 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

```
[Out] -1/2*a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-b^2*x^(-2+n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b^2*x^(-2 +
 n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^3}\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^3}+b^2x^{-3+n}\right)dx}{ab+b^2x^n}\\ &= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^n\right)^2}\,\left(2bx^n-a(n-2)\right)}{2(n-2)x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n)
)
```

fricas [A] time = 0.88, size = 23, normalized size = 0.24

$$-\frac{an - 2\,bx^n - 2\,a}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

maple [A] time = 0.02, size = 61, normalized size = 0.64

$$\frac{\sqrt{(b\,x^n+a)^2}\;b\,x^n}{(b\,x^n+a)\,(n-2)\,x^2} - \frac{\sqrt{(b\,x^n+a)^2}\;a}{2\,(b\,x^n+a)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-2+
n)*b/x^2*x^n
```

maxima [A] time = 0.91, size = 22, normalized size = 0.23

$$-\frac{a(n-2) - 2\,bx^n}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

3.522 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=238

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2}}{(m+2n+1)\left(ab+b^2x^n\right)}$$

```
[Out] a^3*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+3*a^2*b
^2*x^(1+n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)+
3*a*b^3*x^(1+2*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+2*n)/(a*b+
b^2*x^n)+b^4*x^(1+3*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+3*n)/
(a*b+b^2*x^n)
```

Rubi [A] time = 0.10, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1355, 270, 20, 30}

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2}}{(m+3n+1)\left(ab+b^2x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x
^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((
1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 + 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])/((1 + m + 2*n)*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*
x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(ab+b^2x^n\right)^3 dx}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(a^3b^3(dx)^m+3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)^m+b^6x\right.}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(3a^2b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^n}{ab+b^2x^n}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(3a^2b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)}{ab+b^2x^n}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)} +$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.38

$$\frac{x(dx)^m\left((a+bx^n)^2\right)^{3/2}\left(\frac{a^3}{m+1}+\frac{3a^2bx^n}{m+n+1}+\frac{3ab^2x^{2n}}{m+2n+1}+\frac{b^3x^{3n}}{m+3n+1}\right)}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) +
 (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x^n
)^3
```

fricas [A] time = 0.81, size = 390, normalized size = 1.64

$$\left(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\left(b^3m+b^3\right)n^2+3\left(b^3m^2+2\,b^3m+b^3\right)n\right)xx^{3\,n}e^{\left(m\log(d)+m\log(x)\right)}+3\left(ab^2m^3+3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^
2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^
2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*
m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*
m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*
m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)
*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m +
1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

giac [B] time = 0.95, size = 2719, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x
^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(m*
```

```
log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3
*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2
*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(2*n)*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2
*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^n*e^(m
*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*
m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3*x*e^(m*log(d)
+ m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^
n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*m*n^2*x*e^(m*log(d) + m*log
(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 9*a*b^2*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x
*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n^3*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n +
 a) + 6*b^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^
2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^(2*n)*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(2*n)*e^(m*log(d) +
m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 6*b^3*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 9*a*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*
x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)
)*sgn(b*x^n + a) + 3*b^3*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 30*a^2*b*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x
*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^n*e^(m*log(d) +
 m*log(x))*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(
b*x^n + a) + 9*a*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b
^3*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*e^(m*log(d) + m*log(x))*sgn(b
*x^n + a) + 9*a*b^2*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^
2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a^3*m*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 30*a^2*b*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 24*a*b^2*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*n^2*x*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 18*a^2*b*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9
*a*b^2*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(3*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
9*a*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(2
*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*n)*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 9*a^2*b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*
a*b^2*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 12*a*b^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 3*b^3*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x
^n + a) + 9*a*b^2*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n*x*e^(m*log(d) + m*log(x))*sg
n(b*x^n + a) + 15*a^2*b*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b
```

```
^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*x^(2*n
)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*x^n*e^(m*log(d) + m*lo
g(x))*sgn(b*x^n + a) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + b^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*x*e^(m*log(d) + m
*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 3*a*b^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a))/(m^4 + 6*m^3*n + 11*m^2*n^2 + 6*m*n^3 + 4*m^3 + 18*
m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n + 11*n^2 + 4*m + 6*n + 1)
```

maple [C] time = 0.06, size = 532, normalized size = 2.24

$$\frac{\sqrt{(b x^n + a)^2}\, \left(3a^2 b\, m^3 x^n + 15a^2 b\, m^2 n\, x^n + 18a^2 bm\, n^2 x^n + 3a\, b^2 m^3 x^{2n} + 12a\, b^2 m^2 n\, x^{2n} + 9a\, b^2 m\, n^2 x^{2n} + b^3 m^3 x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x*(b^3*(x^n)^3+a^3*m^3+3*a^3*m^2+11*a^3*n^2+6
*a^3*n+6*a^3*n^3+a^3+15*a^2*b*n*x^n+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3
+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n*(x^n)^3+3*a^2*b*m^3*x^n+9*a*b^2*m^2*(x^n)^2+
9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*a*b^2*(x^n)^2+12*a
*b^2*(x^n)^2*n+9*m*a^2*b*x^n+12*a*b^2*m^2*n*(x^n)^2+b^3*m^3*(x^n)^3+3*b^3*m
^2*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x^n)^3*n+3*a^2*b*x^n+3*
(x^n)^2*a*b^2+6*a^3*m^2*n+11*a^3*m*n^2+12*a^3*m*n+9*a*b^2*m*n^2*(x^n)^2+15*
a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+3*m*a^3+24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*
n*x^n)/(m+1)/(m+n+1)/(1+m+2*n)/(1+m+3*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)
*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi
*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 0.99, size = 276, normalized size = 1.16

$$\frac{\left(m^3 + 3\,m^2(2\,n+1) + 6\,n^3 + \left(11\,n^2 + 12\,n + 3\right)m + 11\,n^2 + 6\,n + 1\right)a^3 d^m x x^m + \left(m^3 + 3\,m^2(n+1) + \left(2\,n^2 + 6\,n\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)
*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n +
 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^2 +
 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m
^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m*x*e^
(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6
*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left((a+bx^n)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)
```

3.523 $\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=212

$$\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2a}}{3(a+}$$

```
[Out] 1/3*a^3*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+1/3*b^4*x^(3+3*n)*(
a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(3+n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)+3*a*b^3*x^(3+2*n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(3+2*n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.06, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2a}}{3(a+}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^4*x^(3*(
1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)*(a*b + b^2*x^n)) +
(3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b
^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 +
2*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)^3 dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x^2+3ab^5x^{2(1+n)}+3a^2b^4x^{2+n}+b^6x^{2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2}}{(3+n)\left(\right.}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 123, normalized size = 0.58

$$\frac{x^3\sqrt{(a+bx^n)^2}\left(a^3\left(2n^3+11n^2+18n+9\right)+9a^2b\left(2n^2+5n+3\right)x^n+9ab^2\left(n^2+4n+3\right)x^{2n}+b^3\left(2n^2+9n\right.\right.}{3(n+1)(n+3)(2n+3)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5*
n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*x^
(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

fricas [A] time = 0.74, size = 144, normalized size = 0.68

$$\frac{\left(2\,b^3n^2+9\,b^3n+9\,b^3\right)x^3x^{3\,n}+9\left(ab^2n^2+4\,ab^2n+3\,ab^2\right)x^3x^{2\,n}+9\left(2\,a^2bn^2+5\,a^2bn+3\,a^2b\right)x^3x^n+\left(2\,a^3n^3\right.}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n +
 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (2*
a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

giac [A] time = 0.43, size = 292, normalized size = 1.38

$$\frac{2\,b^3n^2x^3x^{3\,n}\mathrm{sgn}\left(bx^n+a\right)+9\,ab^2n^2x^3x^{2\,n}\mathrm{sgn}\left(bx^n+a\right)+18\,a^2bn^2x^3x^n\mathrm{sgn}\left(bx^n+a\right)+2\,a^3n^3x^3\mathrm{sgn}\left(bx^n+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n + a
) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^n +
 a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a) + 9
*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a) + 27*
a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^3*sgn(
b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

maple [A] time = 0.02, size = 146, normalized size = 0.69

$$\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^3x^n}{(b\,x^n+a)\,(n+3)}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^3x^{2n}}{(b\,x^n+a)\,(2n+3)}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^3x^{3n}}{3\,(b\,x^n+a)\,(n+1)}+\frac{\sqrt{(b\,x^n+a)^2}\;a^3x^3}{3b\,x^n+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x^3*a^3+1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)
*b^3*x^3/(n+1)*(x^n)^3+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/(3+2*n)*x^3*(x
^n)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/(n+3)*x^3*x^n
```

maxima [A] time = 0.93, size = 108, normalized size = 0.51

$$\frac{\left(2\,n^2+9\,n+9\right)b^3x^3x^{3\,n}+9\left(n^2+4\,n+3\right)ab^2x^3x^{2\,n}+9\left(2\,n^2+5\,n+3\right)a^2bx^3x^n+\left(2\,n^3+11\,n^2+18\,n+9\right)a^3}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*n
) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x^3
)/(2*n^3 + 11*n^2 + 18*n + 9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a^2 + b^2 x^{2n} + 2 a b x^n\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left((a + bx^n)^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**n)**2)**(3/2), x)
```

3.524 $\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=211

$$\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2 \cdots}}{2(\cdots}$$

```
[Out] 1/2*a^3*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+3/2*a*b^3*x^(2+2*n)
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(2+n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)+b^4*x^(2+3*n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(2+3*n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.06, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2 \cdots}}{2(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^
(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)
) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b
 + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 +
3*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x+3a^2b^4x^{1+n}+3ab^5x^{1+2n}+b^6x^{1+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{2+n \cdots}}{(2+\cdots}\end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.59

$$\frac{x^2\sqrt{(a+bx^n)^2}\,\left(a^3\left(3n^3+11n^2+12n+4\right)+6a^2b\left(3n^2+5n+2\right)x^n+3ab^2\left(3n^2+8n+4\right)x^{2n}+2b^3\left(n^2+3n+\right.\right.}{2(n+1)(n+2)(3n+2)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5*
n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2)*
x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

fricas [A] time = 0.79, size = 145, normalized size = 0.69

$$\frac{2\left(b^3n^2+3\,b^3n+2\,b^3\right)x^2x^{3\,n}+3\left(3\,ab^2n^2+8\,ab^2n+4\,ab^2\right)x^2x^{2\,n}+6\left(3\,a^2bn^2+5\,a^2bn+2\,a^2b\right)x^2x^n+\left(3\,a^3n^3+\right.}{2\left(3\,n^3+11\,n^2+12\,n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n
 + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (
3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

giac [A] time = 0.37, size = 292, normalized size = 1.38

$$\frac{2\,b^3n^2x^2x^{3\,n}\mathrm{sgn}\left(bx^n+a\right)+9\,ab^2n^2x^2x^{2\,n}\mathrm{sgn}\left(bx^n+a\right)+18\,a^2bn^2x^2x^n\mathrm{sgn}\left(bx^n+a\right)+3\,a^3n^3x^2\mathrm{sgn}\left(bx^n+a\right)+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n + a
) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^n +
 a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a) + 4
*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a) + 12*
a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^2*sgn(
b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

maple [A] time = 0.02, size = 145, normalized size = 0.69

$$\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^2x^n}{(b\,x^n+a)\,(n+2)}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^2x^{2n}}{2\,(b\,x^n+a)\,(n+1)}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^2x^{3n}}{(b\,x^n+a)\,(3n+2)}+\frac{\sqrt{(b\,x^n+a)^2}\;a^3x^2}{2b\,x^n+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x^2*a^3+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3
/(2+3*n)*x^2*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2*x^2/(n+1)*(x^n
)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/(n+2)*x^2*x^n
```

maxima [A] time = 0.94, size = 109, normalized size = 0.52

$$\frac{2\left(n^2+3\,n+2\right)b^3x^2x^{3\,n}+3\left(3\,n^2+8\,n+4\right)ab^2x^2x^{2\,n}+6\left(3\,n^2+5\,n+2\right)a^2bx^2x^n+\left(3\,n^3+11\,n^2+12\,n+4\right)a^3x}{2\left(3\,n^3+11\,n^2+12\,n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(2
*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3*x
^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**n)**2)**(3/2), x)
```

3.525 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=206

$$\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2\right.}{}$$

```
[Out] a^3*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/(a+b*x^n)^3+3*a^2*b^4*x^(1+n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+n)/(a*b+b^2*x^n)^3+3*a*b^5*x^(1+2*n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+2*n)/(a*b+b^2*x^n)^3+b^6*x^(1+3*n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(3/2)/(1+3*n)/(a*b+b^2*x^n)^3
```

Rubi [A] time = 0.05, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1343, 244}

$$\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^
(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3)
+ (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a
*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/
((1 + 3*n)*(a*b + b^2*x^n)^3)
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(2ab+2b^2x^n\right)^3\,dx}{\left(2ab+2b^2x^n\right)^3}\\ &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^n+24ab^5x^{2n}+8b^6x^{3n}\right)dx}{\left(2ab+2b^2x^n\right)^3}\\ &= \frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(a+bx^n)^3}+\frac{3a^2b^4x^{1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(1+n)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{1+2n}\left(\right.}{(1+2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 122, normalized size = 0.59

$$\frac{x\sqrt{(a+bx^n)^2}\left(a^3\left(6n^3+11n^2+6n+1\right)+3a^2b\left(6n^2+5n+1\right)x^n+3ab^2\left(3n^2+4n+1\right)x^{2n}+b^3\left(2n^2+3n+1\right)\right.}{(n+1)(2n+1)(3n+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n +
 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x^(
3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

fricas [A] time = 0.76, size = 130, normalized size = 0.63

$$\frac{\left(2 b^{3} n^{2} + 3 b^{3} n + b^{3}\right) x x^{3 n} + 3 \left(3 a b^{2} n^{2} + 4 a b^{2} n + a b^{2}\right) x x^{2 n} + 3 \left(6 a^{2} b n^{2} + 5 a^{2} b n + a^{2} b\right) x x^{n} + \left(6 a^{3} n^{3} + 11 a\right.}{6 n^{3} + 11 n^{2} + 6 n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2
)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a
^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [A] time = 0.49, size = 263, normalized size = 1.28

$$\frac{6 a^{3} n^{3} x \operatorname{sgn}\left(b x^{n} + a\right) + 2 b^{3} n^{2} x x^{3 n} \operatorname{sgn}\left(b x^{n} + a\right) + 9 a b^{2} n^{2} x x^{2 n} \operatorname{sgn}\left(b x^{n} + a\right) + 18 a^{2} b n^{2} x x^{n} \operatorname{sgn}\left(b x^{n} + a\right) + 11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*
n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3*n
^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*
n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x^n +
 a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a) + 3*a
^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n + 1
)
```

maple [A] time = 0.02, size = 138, normalized size = 0.67

$$\frac{3 \sqrt{\left(b x^{n} + a\right)^{2}}\, a^{2} b x x^{n}}{\left(b x^{n} + a\right) \left(n + 1\right)} + \frac{3 \sqrt{\left(b x^{n} + a\right)^{2}}\, a b^{2} x x^{2 n}}{\left(b x^{n} + a\right) \left(2 n + 1\right)} + \frac{\sqrt{\left(b x^{n} + a\right)^{2}}\, b^{3} x x^{3 n}}{\left(b x^{n} + a\right) \left(3 n + 1\right)} + \frac{\sqrt{\left(b x^{n} + a\right)^{2}}\, a^{3} x}{b x^{n} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3*x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3/(1+3*
n)*x*(x^n)^3+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/(1+2*n)*x*(x^n)^2+3*((b*
x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/(n+1)*x*x^n
```

maxima [A] time = 0.95, size = 101, normalized size = 0.49

$$\frac{\left(2 n^{2} + 3 n + 1\right) b^{3} x x^{3 n} + 3 \left(3 n^{2} + 4 n + 1\right) a b^{2} x x^{2 n} + 3 \left(6 n^{2} + 5 n + 1\right) a^{2} b x x^{n} + \left(6 n^{3} + 11 n^{2} + 6 n + 1\right) a^{3} x}{6 n^{3} + 11 n^{2} + 6 n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3*
(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3 +
11*n^2 + 6*n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

3.526 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=196

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2a}}{a+bx}$$

```
[Out] 3*a^2*b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3/2*a*b^3*x
^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/3*b^4*x^(3*n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a^3*ln(x)*(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)/(a+b*x^n)
```

Rubi [A] time = 0.05, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1355, 266, 43}

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2a}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (
3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n))
 + (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) +
 (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x}\,dx}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx, x, x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\ \text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx, x, x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$= \frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)} + \frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)} + \frac{b^4x^{3n}\sqrt{a^2+2a}}{3n\left(ab+\right.}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.35

$$\frac{\left((a+bx^n)^2\right)^{3/2}\left(a^3n\log(x)+3a^2bx^n+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}\right)}{n\,(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (((a + b*x^n)^2)^(3/2)*(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/2 + (b^3*x^(3*n))/3
 + a^3*n*Log[x]))/(n*(a + b*x^n)^3)
```

fricas [A] time = 0.94, size = 44, normalized size = 0.22

$$\frac{6\,a^3n\log(x)+2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

maple [A] time = 0.02, size = 127, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n+a)^2}\;a^3\ln(x)}{b\,x^n+a}+\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^n}{(b\,x^n+a)\,n}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^{2n}}{2\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^{3n}}{3\,(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3*ln(x)+1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b
^3/n*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/n*(x^n)^2+3*((b*x^n+a)
^2)^(1/2)/(b*x^n+a)*a^2*b/n*x^n
```

maxima [A] time = 0.90, size = 43, normalized size = 0.22

$$a^3 \log(x) + \frac{2\, b^3 x^{3\,n} + 9\, a b^2 x^{2\,n} + 18\, a^2 b x^n}{6\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\, x^{2\,n} + 2\, a\, b\, x^n\right)^{3/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
[Out] Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

3.527 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=212

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+b\ldots}$$

```
[Out] -a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-3*a^2*b^2*x^(-1+n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)-3*a*b^3*x^(-1+2*n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(1-2*n)/(a*b+b^2*x^n)-b^4*x^(-1+3*n)*(a^2+2*a*b*
x^n+b^2*x^(2*n))^(1/2)/(1-3*n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.07, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x
^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) -
(3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b
+ b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 -
3*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x^{-2+n}+3ab^5x^{2(-1+n)}+b^6x^{-2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{-1+2n}\sqrt{a^2\ldots}}{(1-2n)\left(\ldots\right.}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.58

$$\frac{\sqrt{(a+bx^n)^2}\left(a^3\left(-6n^3+11n^2-6n+1\right)+3a^2b\left(6n^2-5n+1\right)x^n+3ab^2\left(3n^2-4n+1\right)x^{2n}+b^3\left(2n^2-3n+\right.\right.}{(n-1)(2n-1)(3n-1)x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n + 6
*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(3*
n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

fricas [A] time = 0.75, size = 131, normalized size = 0.62

$$-\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-\left(2\,b^3n^2-3\,b^3n+b^3\right)x^{3\,n}-3\left(3\,ab^2n^2-4\,ab^2n+ab^2\right)x^{2\,n}-3\left(6\,a^2bn^2-5\,a^2\right.}{\left(6\,n^3-11\,n^2+6\,n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(3
*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^2*
b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

maple [A] time = 0.02, size = 147, normalized size = 0.69

$$\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^n}{(b\,x^n+a)\,(n-1)\,x}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^{2n}}{(b\,x^n+a)\,(2n-1)\,x}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^{3n}}{(b\,x^n+a)\,(3n-1)\,x}-\frac{\sqrt{(b\,x^n+a)^2}\;a^3}{(b\,x^n+a)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] -((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3/x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+3*n)
*b^3/x*(x^n)^3+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+2*n)*a*b^2/x*(x^n)^2+3*(
(b*x^n+a)^2)^(1/2)/(b*x^n+a)/(n-1)*a^2*b/x*x^n
```

maxima [A] time = 0.94, size = 101, normalized size = 0.48

$$\frac{\left(2\,n^2-3\,n+1\right)b^3x^{3\,n}+3\left(3\,n^2-4\,n+1\right)ab^2x^{2\,n}+3\left(6\,n^2-5\,n+1\right)a^2bx^n-\left(6\,n^3-11\,n^2+6\,n-1\right)a^3}{\left(6\,n^3-11\,n^2+6\,n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*n
^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^2
+ 6*n - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{n}\right)^{2}\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**2, x)
```

3.528 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=218

$$-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+}}{2x}$$

```
[Out] -1/2*a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-3/2*a*b^3*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(x^(2-2*n))/(a*b+b^2*x^n)-3*a^2*b^2*x^(-2+n
)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)-b^4*x^(-2+3*n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-3*n)/(a*b+b^2*x^n)
```

Rubi [A] time = 0.07, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (3*a*b^3*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1 - n))*(a*b + b^2*x^n
)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a
*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2
 - 3*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3} \, dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \frac{\left(ab+b^2x^n\right)^3}{x^3} \, dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(\frac{a^3b^3}{x^3}+3a^2b^4x^{-3+n}+b^6x^{3(-1+n)}+3ab^5x^{-3+2n}\right) dx}{b^2\left(ab+b^2x^n\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{-2+n}}{(2-}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.57

$$\frac{\sqrt{(a+bx^n)^2}\left(a^3\left(-3n^3+11n^2-12n+4\right)+6a^2b\left(3n^2-5n+2\right)x^n+3ab^2\left(3n^2-8n+4\right)x^{2n}+2b^3\left(n^2-3n+2\right.\right.}{2(n-2)(n-1)(3n-2)x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^(3
*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

fricas [A] time = 0.56, size = 134, normalized size = 0.61

$$-\frac{3\,a^3n^3-11\,a^3n^2+12\,a^3n-4\,a^3-2\left(b^3n^2-3\,b^3n+2\,b^3\right)x^{3\,n}-3\left(3\,ab^2n^2-8\,ab^2n+4\,ab^2\right)x^{2\,n}-6\left(3\,a^2bn^2-5\right.}{2\left(3\,n^3-11\,n^2+12\,n-4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*
b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b*n
^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

maple [A] time = 0.03, size = 145, normalized size = 0.67

$$\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^n}{(b\,x^n+a)\,(n-2)\,x^2}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^{2n}}{2\,(b\,x^n+a)\,(n-1)\,x^2}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^{3n}}{(b\,x^n+a)\,(3n-2)\,x^2}-\frac{\sqrt{(b\,x^n+a)^2}\;a^3}{2\,(b\,x^n+a)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3/x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-
2+3*n)*b^3/x^2*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(n-1)*a*b^2/x^2*(x
^n)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(n-2)*a^2*b/x^2*x^n
```

maxima [A] time = 1.17, size = 101, normalized size = 0.46

$$\frac{2\left(n^2-3\,n+2\right)b^3x^{3\,n}+3\left(3\,n^2-8\,n+4\right)ab^2x^{2\,n}+6\left(3\,n^2-5\,n+2\right)a^2bx^n-\left(3\,n^3-11\,n^2+12\,n-4\right)a^3}{2\left(3\,n^3-11\,n^2+12\,n-4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6*
(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 - 11
*n^2 + 12*n - 4)*x^2)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + b^2 x^{2n} + 2 a b x^n\right)^{3/2}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

3.529 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{m+1}\left(a+bx^n\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/d/(1+m
)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1355, 364}

$$\frac{(dx)^{m+1}\left(a+bx^n\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{(dx)^m}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(dx)^{1+m}\left(a+bx^n\right)\,_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.82

$$\frac{x(dx)^m\left(a+bx^n\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a(m+1)\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b*
x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{2ab\,x^n + b^2x^{2n} + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

3.530 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/3*x^3*(a+b*x^n)*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{x^3(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{x^2}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{x^3(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.83

$$\frac{x^3(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a*Sqr
t[(a + b*x^n)^2])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{2ab\,x^n + b^2x^{2n} + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

```
[Out] int(x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a + b*x**n)**2), x)
```

3.531 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2 (a+bx^n)\ {}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^2*(a+b*x^n)*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 364}

$$\frac{x^2 (a+bx^n)\ {}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{x}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^2 (a+bx^n)\ {}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.83

$$\frac{x^2 (a+bx^n)\ {}_2F_1\left(1,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a*Sqr
t[(a + b*x^n)^2])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{2ab\,x^n + b^2x^{2n} + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
[Out] int(x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\left(a + bx^n\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x/sqrt((a + b*x**n)**2), x)
```

3.532 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=55

$$\frac{x\left(a+bx^n\right) {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x*(a+b*x^n)*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b^2*x^(2*
n))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1343, 245}

$$\frac{x\left(a+bx^n\right) {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(2ab+2b^2x^n\right)\int \frac{1}{2ab+2b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x\left(a+bx^n\right) {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.80

$$\frac{x\left(a+bx^n\right) {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[(a + b*x^n)^2])

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b^{2} x^{2 \, n} + 2 \, a b x^{n} + a^{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^{2} x^{2 \, n} + 2 \, a b x^{n} + a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2ab\,x^{n} + b^{2} x^{2n} + a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)

[Out] int(1/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^{2} x^{2 \, n} + 2 \, a b x^{n} + a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)

[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2} + 2abx^{n} + b^{2}x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)

[Out] Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)

3.533 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=85

$$\frac{\log(x)(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (a+b*x^n)*ln(x)/a/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a+b*x^n)*ln(a+b*x^n)/a
/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)
*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\left(ab+b^2x^n\right)\int \frac{1}{x\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(ab+b^2x^n\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(ab+b^2x^n\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{abn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{\left(b\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.49

$$\frac{(a+bx^n)\left(n\log(x)-\log\left(a+bx^n\right)\right)}{an\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] ((a + b*x^n)*(n*Log[x] - Log[a + b*x^n]))/(a*n*Sqrt[(a + b*x^n)^2])
```

fricas [A] time = 0.74, size = 22, normalized size = 0.26

$$\frac{n\log(x)-\log\left(bx^n+a\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

maple [A] time = 0.02, size = 66, normalized size = 0.78

$$\frac{\sqrt{(b\,x^n+a)^2}\;\ln(x)}{(b\,x^n+a)\,a} - \frac{\sqrt{(b\,x^n+a)^2}\;\ln\left(x^n+\frac{a}{b}\right)}{(b\,x^n+a)\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a*ln(x)-((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a/n*ln(
x^n+a/b)
```

maxima [A] time = 1.07, size = 27, normalized size = 0.32

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

3.534 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(a+b*x^n)*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/x/(a^2+2*a*b*x^n+b^2*
x^(2*n))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{1}{x^2\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.78

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(a*
x*Sqrt[(a + b*x^n)^2]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}{b^2x^2x^{2\,n}+2\,abx^2x^n+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x
^n + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2ab\,x^n+b^2x^{2n}+a^2}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

```
[Out] int(1/x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\left(a+bx^n\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

3.535 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -1/2*(a+b*x^n)*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/x^2/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a*
x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{1}{x^3\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.79

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a*x^2
*Sqrt[(a + b*x^n)^2])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{b^2x^3x^{2n} + 2\,abx^3x^n + a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x
^n + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2ab\,x^n + b^2x^{2n} + a^2}\;x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

```
[Out] int(1/x^3/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{(a + bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

3.536 $\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{m+1}\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/d/(1
+m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1355, 364}

$$\frac{(dx)^{m+1}\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(dx)^{1+m}\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.80

$$\frac{x(dx)^m\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3(m+1)\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*
x^n)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^n)^2])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n}+2\,abx^n+a^2}\,(dx)^m}{b^4x^{4n}+4\,a^2b^2x^{2n}+4\,a^3bx^n+a^4+2\left(2\,ab^3x^n+a^2b^2\right)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m/(b^4*x^(4*n) + 4*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(2ab\,x^n+b^2x^{2n}+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(m^2-m(3\,n-2)+2\,n^2-3\,n+1\right)d^m\int\frac{x^m}{2\left(a^2bn^2x^n+a^3n^2\right)}\,dx-\frac{ad^m(m-3\,n+1)xx^m+bd^m(m-2\,n+1)xe^{(m\log}}{2\left(a^2b^2n^2x^{2n}+2\,a^3bn^2x^n+a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*b*n^2*x^n
+ a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m*(m - 2*n + 1)*x*e^(
m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**n)**2)**(3/2), x)
```

3.537 $\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/3*x^3*(a+b*x^n)*hypergeom([3, 3/n],[(3+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$\frac{x^3\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x^2}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3\left(a+bx^n\right)\,{}_2F_1\left(3,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.86

$$\frac{x^3\left(a+bx^n\right)^3\,{}_2F_1\left(3,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a^3\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)^3*Hypergeometric2F1[3, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a^3
*((a + b*x^n)^2)^(3/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}\,x^2}{b^4x^{4n} + 4\,a^2b^2x^{2n} + 4\,a^3bx^n + a^4 + 2\left(2\,ab^3x^n + a^2b^2\right)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2/(b^4*x^(4*n) + 4*a^2*b^2*x
^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(2ab\,x^n + b^2x^{2n} + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] int(x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,n^2 - 9\,n + 9\right)\int \frac{x^2}{2\left(a^2bn^2x^n + a^3n^2\right)}\,dx + \frac{b(2\,n-3)x^3x^n + 3\,a(n-1)x^3}{2\left(a^2b^2n^2x^{2n} + 2\,a^3bn^2x^n + a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*
(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n
 + a^4*n^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

3.538 $\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\left(a+bx^n\right){}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^2*(a+b*x^n)*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 364}

$$\frac{x^2\left(a+bx^n\right){}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^2\left(a+bx^n\right){}_2F_1\left(3,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.86

$$\frac{x^2\left(a+bx^n\right)^3{}_2F_1\left(3,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)^3*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^3
*((a + b*x^n)^2)^(3/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x}{b^4x^{4\,n}+4\,a^2b^2x^{2\,n}+4\,a^3bx^n+a^4+2\,\left(2\,ab^3x^n+a^2b^2\right)x^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x/(b^4*x^(4*n) + 4*a^2*b^2*x^(
2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2ab\,x^n+b^2x^{2n}+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] int(x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(n^2-3\,n+2\right)\int \frac{x}{a^2bn^2x^n+a^3n^2}\,dx+\frac{2\,b(n-1)x^2x^n+a(3\,n-2)x^2}{2\left(a^2b^2n^2x^{2\,n}+2\,a^3bn^2x^n+a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n - 1
)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**n)**2)**(3/2), x)
```

3.539 $\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^n)^3\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

```
[Out] x*(a+b*x^n)^3*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n+b^2*x
^(2*n))^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1343, 245}

$$\frac{x(a+bx^n)^3\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(2ab+2b^2x^n\right)^3\int \frac{1}{\left(2ab+2b^2x^n\right)^3}\,dx}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\\ &= \frac{x(a+bx^n)^3\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.81

$$\frac{x(a+bx^n)^3\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*((a + b*x^n)^2)^(3/2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}{b^4x^{4n}+4\,a^2b^2x^{2n}+4\,a^3bx^n+a^4+2\left(2\,ab^3x^n+a^2b^2\right)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^(4*n) + 4*a^2*b^2*x^(2*
n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2ab\,x^n+b^2x^{2n}+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] int(1/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,n^2-3\,n+1\right)\int \frac{1}{2\left(a^2bn^2x^n+a^3n^2\right)}\,dx+\frac{b(2\,n-1)xx^n+a(3\,n-1)x}{2\left(a^2b^2n^2x^{2n}+2\,a^3bn^2x^n+a^4n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*n
 - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

3.540 $\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\,(a+bx^n)\,\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\,(a+bx^n)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\,(a+ }{a^3n\sqrt{a^2+2abx^n+}}$$

```
[Out] 1/a^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+1/2/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)+(a+b*x^n)*ln(x)/a^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a
+b*x^n)*ln(a+b*x^n)/a^3/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1355, 266, 44}

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\,(a+bx^n)\,\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\,(a+bx^n)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\,(a+ }{a^3n\sqrt{a^2+2abx^n+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a
^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{(a+bx}{a^3\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.48

$$\frac{\left(a+bx^n\right)^3\left(-\frac{\log\left(a+bx^n\right)}{a^3}+\frac{n\log(x)}{a^3}+\frac{1}{a^2\left(a+bx^n\right)}+\frac{1}{2a\left(a+bx^n\right)^2}\right)}{n\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] ((a + b*x^n)^3*(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + (n*Log[x])/a^
3 - Log[a + b*x^n]/a^3))/(n*((a + b*x^n)^2)^(3/2))
```

fricas [A] time = 1.04, size = 106, normalized size = 0.67

$$\frac{2\,b^2nx^{2\,n}\log(x)+2\,a^2n\log(x)+3\,a^2+2\left(2\,abn\log(x)+ab\right)x^n-2\left(b^2x^{2\,n}+2\,abx^n+a^2\right)\log\left(bx^n+a\right)}{2\left(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

maple [A] time = 0.02, size = 104, normalized size = 0.65

$$\frac{\sqrt{\left(b\,x^n+a\right)^2}\ \ln(x)}{\left(b\,x^n+a\right)a^3}+\frac{\sqrt{\left(b\,x^n+a\right)^2}\ \left(2b\,x^n+3a\right)}{2\left(b\,x^n+a\right)^3a^2n}-\frac{\sqrt{\left(b\,x^n+a\right)^2}\ \ln\left(x^n+\frac{a}{b}\right)}{\left(b\,x^n+a\right)a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a^3*ln(x)+1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^3
*(2*b*x^n+3*a)/a^2/n-((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a^3/n*ln(x^n+a/b)
```

maxima [A] time = 1.16, size = 70, normalized size = 0.44

$$\frac{2\,bx^n + 3\,a}{2\left(a^2b^2nx^{2\,n} + 2\,a^3bnx^n + a^4n\right)} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{bx^n + a}{b}\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a^
3 - log((b*x^n + a)/b)/(a^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a + bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

3.541 $\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(a+b*x^n)*hypergeom([3, -1/n],[(-1+n)/n],-b*x^n/a)/a^3/x/(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x^2\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.82

$$-\frac{(a+bx^n)^3\,{}_2F_1\left(3,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3x\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)^3*Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(
a^3*x*((a + b*x^n)^2)^(3/2)))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{b^4x^2x^{4n} + 4\,a^2b^2x^2x^{2n} + 4\,a^3bx^2x^n + a^4x^2 + 2\left(2\,ab^3x^2x^n + a^2b^2x^2\right)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^2*x^(4*n) + 4*a^2*b^2*x
^2*x^(2*n) + 4*a^3*b*x^2*x^n + a^4*x^2 + 2*(2*a*b^3*x^2*x^n + a^2*b^2*x^2)*
x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2ab\,x^n + b^2x^{2n} + a^2\right)^{\frac{3}{2}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] int(1/x^2/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,n^2 + 3\,n + 1\right)\int \frac{1}{2\left(a^2bn^2x^2x^n + a^3n^2x^2\right)}\,dx + \frac{b(2\,n + 1)x^n + a(3\,n + 1)}{2\left(a^2b^2n^2xx^{2n} + 2\,a^3bn^2xx^n + a^4n^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/2
*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x^n
 + a^4*n^2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**2*((a + b*x**n)**2)**(3/2)), x)
```

3.542 $\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -1/2*(a+b*x^n)*hypergeom([3, -2/n],[(-2+n)/n],-b*x^n/a)/a^3/x^2/(a^2+2*a*b*
x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 364}

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^
3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x^3\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.82

$$-\frac{(a+bx^n)^3\,{}_2F_1\left(3,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -1/2*((a + b*x^n)^3*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a^3
*x^2*((a + b*x^n)^2)^(3/2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}{b^4x^3x^{4n}+4\,a^2b^2x^3x^{2n}+4\,a^3bx^3x^n+a^4x^3+2\left(2\,ab^3x^3x^n+a^2b^2x^3\right)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^3*x^(4*n) + 4*a^2*b^2*x
^3*x^(2*n) + 4*a^3*b*x^3*x^n + a^4*x^3 + 2*(2*a*b^3*x^3*x^n + a^2*b^2*x^3)*
x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(2ab\,x^n+b^2x^{2n}+a^2\right)^{\frac{3}{2}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] int(1/x^3/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(n^2+3\,n+2\right)\int\frac{1}{a^2bn^2x^3x^n+a^3n^2x^3}\,dx+\frac{2\,b(n+1)x^n+a(3\,n+2)}{2\left(a^2b^2n^2x^2x^{2n}+2\,a^3bn^2x^2x^n+a^4n^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2*
b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x^n
 + a^4*n^2*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^3\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**3*((a + b*x**n)**2)**(3/2)), x)
```

3.543 $\int \left(a^2+b^2 x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

```
[Out] x*(a+b*x^(1/(-1-2*p)))*(a^2+2*a*b*x^(1/(-1-2*p))+b^2/(x^(2/(1+2*p))))^p/a
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1343, 191}

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 +
 2*p)))^p)/a
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\left(\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{-2p}\right)\int \left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)$$

$$=\frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.12

$$\frac{x^{\frac{2p}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)\left(x^{-\frac{2}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)^2\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

[Out] (x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/x^(2/(1 + 2*p)))^p)/a

fricas [A] time = 0.70, size = 79, normalized size = 1.52

$$\frac{\left(axx^{\left(\frac{1}{2p+1}\right)} + bx\right)\left(\frac{a^2x^{\frac{2}{2p+1}}+2\,abx^{\left(\frac{1}{2p+1}\right)}+b^2}{x^{\frac{2}{2p+1}}}\right)^p}{ax^{\left(\frac{1}{2p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="fricas")

[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1)) + b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2\,ab}{x^{\left(\frac{1}{2p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="giac")

[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(2ab\,x^{-\frac{1}{2p+1}} + b^2x^{-\frac{2}{2p+1}} + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/(x^(2/(2*p+1)))+2*a*b/(x^(1/(2*p+1))))^p,x)

[Out] int((a^2+b^2/(x^(2/(2*p+1)))+2*a*b/(x^(1/(2*p+1))))^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2\,ab}{x^{\left(\frac{1}{2p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="maxima")

[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2\,a\,b}{x^{\frac{1}{2p+1}}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p,x)
[Out] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
[Out] Timed out
```

3.544 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx$

Optimal. Leaf size=43

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

```
[Out] x*(a+b*x^n)/a/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n))
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1343, 191}

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\,dx &= \left(\left(2ab+2b^2x^n\right)^{\frac{1+n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{-\frac{1+n}{n}}\,dx\\ &= \frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.74

$$\frac{x\left(a+bx^n\right)\left(\left(a+bx^n\right)^2\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

fricas [A] time = 0.95, size = 45, normalized size = 1.05

$$\frac{bxx^n+ax}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{n+1}{2\,n}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="frica
s")
```

```
[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

maple [A] time = 0.04, size = 51, normalized size = 1.19

$$\left(\frac{bx\,\mathrm{e}^{n\ln(x)}}{a} + x\right)\mathrm{e}^{-\frac{(n+1)\ln\left(2ab\,\mathrm{e}^{n\ln(x)} + b^2\mathrm{e}^{2n\ln(x)} + a^2\right)}{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(n+1)/n)),x)
```

```
[Out] (x+1/a*b*x*exp(n*ln(x)))/exp(1/2*(n+1)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(
n*ln(x))^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{\frac{\frac{n}{2}+\frac{1}{2}}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{-\frac{\frac{n}{2}+\frac{1}{2}}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-(n/2 + 1/2)/n), x)
```

3.545 $\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx$

Optimal. Leaf size=130

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

```
[Out] 2*(1+p)*x*(a+b/(x^(1/2/(1+p))))*(a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))
))^p/a/(1+2*p)-x*(a+b/(x^(1/2/(1+p))))^2*(a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1
/2/(1+p))))^p/a^2/(1+2*p)
```

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1343, 192, 191}

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/
x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 +
 b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p)/(a^2*(1 + 2*p))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{-2p}\right)\int \left(2ab+2b^2x^{-\frac{1}{2(1+p}}\right.$$

$$= \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)} - \frac{\left(\left(a^2+b^2x^{-\frac{1}{1+p}}+\right.\right.}{}$$

$$= \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)} - \frac{x\left(a+bx^{-\frac{1}{2(1+p)}}\right)^2}{}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.62

$$\frac{x^{\frac{p}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)\left(x^{-\frac{1}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)^2\right)^p\left(a(2p+1)x^{\frac{1}{2p+2}}-b\right)}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (x^(p/(1 + p))*(b + a*x^(2 + 2*p)^(-1))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 +
p)^(-1))^p*(-b + a*(1 + 2*p)*x^(2 + 2*p)^(-1)))/(a^2*(1 + 2*p))
```

fricas [A] time = 0.88, size = 103, normalized size = 0.79

$$\frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^2x+\left(2\,a^2p+a^2\right)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^2x^{\left(\frac{1}{p+1}\right)}+b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p}{\left(2\,a^2p+a^2\right)x^{\left(\frac{1}{p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fri
cas")
```

```
[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*a
*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p +
a^2)*x^(1/(p + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="gia
c")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(2ab\,x^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1/2/(p+1))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1/2/(p+1))))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}} + \frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="max
ima")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b^2}{x^{\frac{1}{p+1}}} + a^2 + \frac{2\,a\,b}{x^{\frac{1}{2\,(p+1)}}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p,x)
```

```
[Out] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

```
[Out] Timed out
```

3.546 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$

Optimal. Leaf size=102

$$\frac{nx\left(a+bx^n\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

```
[Out] x*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a/(1+n)+n*x*(a+b*x^n)^2*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a^2/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 192, 191}

$$\frac{nx\left(a+bx^n\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n
)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/
(a^2*(1 + n))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx=\left(\left(2ab+2b^2x^n\right)^{\frac{1+2n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{-\frac{1+2n}{n}}\,dx$$

$$=\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{\left(n\left(2ab+2b^2x^n\right)^{\frac{1+2n}{n}}\left(a^2+2abx^n\right.\right.}{2a}$$

$$=\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{nx\left(a+bx^n\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)}{a^2(1+n)}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.58

$$\frac{x\left((a+bx^n)^2\right)^{-\frac{1}{2}/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

fricas [A] time = 0.95, size = 82, normalized size = 0.80

$$\frac{b^2nxx^{2n}+(2abn+ab)xx^n+\left(a^2n+a^2\right)x}{\left(a^2n+a^2\right)\left(b^2x^{2n}+2abx^n+a^2\right)^{\frac{2n+1}{2n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fri
cas")
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*
(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^2x^{2n}+2abx^n+a^2\right)^{\frac{2n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\left(2ab\,x^n+b^2x^{2n}+a^2\right)^{-\frac{2n+1}{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(2*n+1)/n)),x)
```

```
[Out] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(2*n+1)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^2x^{2n}+2abx^n+a^2\right)^{\frac{2n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{\frac{n+\frac{1}{2}}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n),x)
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
[Out] Timed out
```

3.547 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=117

$$\frac{(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2a^2dn(p+1)(2p+1)}-\frac{(a+bx^n)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(2p+1)}$$

```
[Out] -(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/a/d/n/(1+2*p)/((d*x)^(2*n*(1+p)))+
1/2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1+p)/a^2/d/n/(1+p)/(1+2*p)/((d*x)^(2*n*(1+
p)))
```

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1356, 273, 264}

$$\frac{\left(\frac{bx^n}{a}+1\right)^2(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(2p^2+3p+1\right)}-\frac{\left(\frac{bx^n}{a}+1\right)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2*p)*(d*x)^
(2*n*(1 + p)))) + ((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*
d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p dx = \left(\left(1 + \frac{bx^n}{a}\right)^{-2p} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p\right) \int (dx)^{-1-2n(1+p)} \left(1 + \frac{bx^n}{a}\right)^{2p}$$

$$= -\frac{(dx)^{-2n(1+p)} \left(1 + \frac{bx^n}{a}\right) \left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{dn(1 + 2p)} + \frac{\left((-2n(1 + p) + n(1\right.}{}$$

$$= -\frac{(dx)^{-2n(1+p)} \left(1 + \frac{bx^n}{a}\right) \left(a^2 + 2abx^n + b^2x^{2n}\right)^p}{dn(1 + 2p)} + \frac{(dx)^{-2n(1+p)} \left(1 + \frac{bx^n}{a}\right.}{2dn\left(1\right.}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.64

$$-\frac{x(dx)^{-2n(p+1)-1} \left((a + bx^n)^2\right)^p \left(\frac{bx^n}{a} + 1\right)^{-2p} {}_2F_1\left(-2p, -2(p + 1); 1 - 2(p + 1); -\frac{bx^n}{a}\right)}{2n(p + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p,
-2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p))
```

fricas [A] time = 0.82, size = 165, normalized size = 1.41

$$-\frac{\left(2\, abpxx^n e^{\left(-\left(2\, np+2\, n+1\right) \log(d) - \left(2\, np+2\, n+1\right) \log(x)\right)} - b^2 xx^{2\, n} e^{\left(-\left(2\, np+2\, n+1\right) \log(d) - \left(2\, np+2\, n+1\right) \log(x)\right)} + \left(2\, a^2 p + a^2\right) xe^{\left(-\left(2\, n\right.\right.}}{2\left(2\, a^2 np^2 + 3\, a^2 np + a^2 n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x))
+ (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2 x^{2\, n} + 2\, abx^n + a^2\right)^p (dx)^{-2\, n\left(p+1\right)-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^{-2\left(p+1\right)n-1} \left(2ab\, x^n + b^2 x^{2n} + a^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(-1-2*n*(p+1))*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)
```

```
[Out] int((d*x)^(-1-2*n*(p+1))*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{2n} + 2\,abx^n + a^2\right)^p (dx)^{-2n(p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
maxima")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^p}{(d\,x)^{2\,n\,(p+1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.548 $\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

```
[Out] -a^2*(1+b*x^n/a)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+2*p)+1/2*a^2*(1+b*x
^n/a)^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1356, 266, 43}

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 +
 p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int x^{-1+2n}\left(1+\frac{bx^n}{a}\right)^{2p}\,dx$$
$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^n\right)}{n}$$
$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right.\right.}{n}$$
$$=-\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^2\right.}{2b^2n(1+p)}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.52

$$\frac{(a+bx^n)\left((a+bx^n)^2\right)^p\left(b(2p+1)x^n-a\right)}{2b^2n(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
+ 2*p))
```

fricas [A] time = 0.94, size = 78, normalized size = 0.76

$$\frac{\left(2\,abpx^n-a^2+\left(2\,b^2p+b^2\right)x^{2\,n}\right)\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p}{2\left(2\,b^2np^2+3\,b^2np+b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
+ a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

maple [C] time = 0.07, size = 148, normalized size = 1.44

$$-\frac{\left(-2abp\,x^n-2b^2p\,x^{2n}-b^2x^{2n}+a^2\right)\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}\left(i\left(b\,x^n+a\right)\right)^2\mathrm{csgn}\left(i\left(b\,x^n+a\right)^2\right)+2i\pi\,\mathrm{csgn}\left(i\left(b\,x^n+a\right)\right)\mathrm{csgn}\left(i\left(b\,x^n+a\right)^2\right)^2-i\pi\,\mathrm{csgn}\left(i\left(b\,x^n+a\right)^2\right)^3+4\ln\left(b\,x^n\right.\right.}{2}}}{2\left(2p+1\right)\left(p+1\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)
```

```
[Out] -1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(2*p+1)/(p+1)/n/b^2*exp
(1/2*p*(-I*Pi*csgn(I*(b*x^n+a)^2)^3+2*I*Pi*csgn(I*(b*x^n+a)^2)^2*csgn(I*(b*
x^n+a))-I*Pi*csgn(I*(b*x^n+a)^2)*csgn(I*(b*x^n+a))^2+4*ln(b*x^n+a)))
```

maxima [A] time = 1.23, size = 59, normalized size = 0.57

$$\frac{\left(b^2\left(2\,p+1\right)x^{2\,n}+2\,abpx^n-a^2\right)(bx^n+a)^{2\,p}}{2\left(2\,p^2+3\,p+1\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2 +
 3*p + 1)*b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.549 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=111

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[Out] -b*x^n/c^2/n+1/2*x^(2*n)/c/n+1/2*(-a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/c^3/n+b*(
-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c^3/n/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 701, 634, 618, 206, 628}

$$\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x
^n + c*x^(2*n)])/(2*c^3*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^n\right)}{n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\text{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx,x,x^n\right)}{c^2n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^3n}+\frac{\left(b^2-ac\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,d\right.}{2c^3n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{c^3n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.84

$$\frac{\left(b^2-ac\right)\log\left(a+x^n\left(b+cx^n\right)\right)+\frac{2b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+cx^n\left(cx^n-2b\right)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 -
 4*a*c]])/Sqrt[b^2 - 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c^3*
n)
```

fricas [A] time = 0.98, size = 353, normalized size = 3.18

$$\left[-\frac{\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)x^n-\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)-\left(b^2c^2-4\,ac^3\right)x^{2\,n}+2\left(b^3c-4\,abc^2\right)x^n-}{2\left(b^2c^3-4\,ac^4\right)n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4 -
5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n),
```

```
1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*
x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2*n) -
2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b
*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.16, size = 973, normalized size = 8.77

$$\frac{4a^2c^2n^2\ln(x)}{4a\,c^4n^2 - b^2c^3n^2} - \frac{5a\,b^2c\,n^2\ln(x)}{4a\,c^4n^2 - b^2c^3n^2} + \frac{b^4n^2\ln(x)}{4a\,c^4n^2 - b^2c^3n^2} - \frac{2a^2\ln\left(x^n - \frac{-3a\,b^2c + b^4 + \sqrt{-36a^3b^2c^3 + 33a^2b^4c^2 - 10a\,b^6c + b^8}}{2\left(3ac - b^2\right)bc}\right)}{\left(4ac - b^2\right)cn} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-b
^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2*c
+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(3*
a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a
*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*
a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a
*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*
c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c/(4*a*c-b^
2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2
*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3
+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/c^3/(4*a*c-
b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*
c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b
^8)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2 - ac\right)\log(x)}{c^3} + \frac{cx^{2n} - 2\,bx^n}{2\,c^2n} + \int -\frac{ab^2 - a^2c + \left(b^3 - 2\,abc\right)x^n}{c^4xx^{2n} + bc^3xx^n + ac^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a*
b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4\,n-1}}{a + b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.550 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=87

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

```
[Out] x^n/c/n-1/2*b*ln(a+b*x^n+c*x^(2*n))/c^2/n-(-2*a*c+b^2)*arctanh((b+2*c*x^n)/
(-4*a*c+b^2)^(1/2))/c^2/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*S
qrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\
&= \frac{x^n}{cn}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^n\right)}{cn}\\
&= \frac{x^n}{cn}-\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^2n}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^2n}\\
&= \frac{x^n}{cn}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{c^2n}\\
&= \frac{x^n}{cn}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 80, normalized size = 0.92

$$\frac{-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}-\frac{b\log(a+x^n(b+cx^n))}{2c}+x^n}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^n - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + x^n*(b + c*x^n)])/(2*c))/(c*n)
```

fricas [A] time = 0.85, size = 285, normalized size = 3.28

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)-2\left(b^2c-4\,ac^2\right)x^n+\left(b^3-4\,abc\right)\log\left(cx^{2\,n}\right.}{2\left(b^2c^2-4\,ac^3\right)n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2
*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n
+ a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a
))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 2*(b
^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2
 - 4*a*c^3)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.13, size = 664, normalized size = 7.63

$$\frac{4abc\,n^2\ln(x)}{4a\,c^3n^2-b^2c^2n^2}-\frac{b^3n^2\ln(x)}{4a\,c^3n^2-b^2c^2n^2}-\frac{2ab\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16a^3c^3+20b^2a^2c^2-8b^4ac+b^6}}{2(2ac-b^2)c}\right)}{(4ac-b^2)\,cn}-\frac{2ab\ln\left(x^n+\frac{2abc-b^3+\sqrt{-1}}{}\right)}{(4ac-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^3
*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2/c^
2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^
4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b
*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-1
6*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*
(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*a*b+1/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^
2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/c^2/(4*a*c-b^2)/n*ln(x^n
+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*
c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\log(x)}{c^2}+\frac{x^n}{cn}-\int-\frac{ab+(b^2-ac)x^n}{c^3xx^{2n}+bc^2xx^n+ac^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2*
n) + b*c^2*x*x^n + a*c^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3n-1}}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.551 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

```
[Out] 1/2*ln(a+b*x^n+c*x^(2*n))/c/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c/n
/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*n) + Log[
a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2cn} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2cn}$$
$$= \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{cn}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,n} + \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Mathematica [A] time = 0.07, size = 62, normalized size = 0.91

$$\frac{\frac{2b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \log\left(a+x^n\left(b+cx^n\right)\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((2*b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + Log[a +
 x^n*(b + c*x^n)])/(2*c*n)
```

fricas [A] time = 0.93, size = 231, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\left(b^2c-4\,ac^2\right)n}, \frac{2\sqrt{-b^2+4\,ac}\,b\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(
b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^2
- 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)
*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.10, size = 402, normalized size = 5.91

$$-\frac{4ac\,n^2\ln(x)}{4a\,c^2n^2-b^2c\,n^2}+\frac{b^2n^2\ln(x)}{4a\,c^2n^2-b^2c\,n^2}+\frac{2a\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{\left(4ac-b^2\right)n}+\frac{2a\ln\left(x^n+\frac{b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{\left(4ac-b^2\right)n}-\frac{b^2\ln\left(x^n-\frac{-b^2\ldots}{\ldots}\right)}{2\left(4ac-\ldots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2)
*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c
)*a-1/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1
/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*
c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*a
-1/2/(4*a*c-b^2)/c/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/(
4*a*c-b^2)/c/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(x)}{c} - \int \frac{bx^n + a}{c^2 x x^{2n} + bcx x^n + acx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{a + b\, x^n + c\, x^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.552 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=39

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

```
[Out] -2*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1352, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{n}\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

fricas [B] time = 0.62, size = 159, normalized size = 4.08

$$\left[\frac{\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}\,c\right)x^n-\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)}{\sqrt{b^2-4\,ac}\,n}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)}{\left(b^2-4\,ac\right)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt(-
b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

giac [A] time = 0.41, size = 39, normalized size = 1.00

$$\frac{2\,\arctan\left(\frac{2\,cx^n+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

maple [B] time = 0.06, size = 113, normalized size = 2.90

$$-\frac{\ln\left(x^n+\frac{-4ac+b^2+\sqrt{-4ac+b^2}\,b}{2\sqrt{-4ac+b^2}\,c}\right)}{\sqrt{-4ac+b^2}\,n}+\frac{\ln\left(x^n+\frac{4ac-b^2+\sqrt{-4ac+b^2}\,b}{2\sqrt{-4ac+b^2}\,c}\right)}{\sqrt{-4ac+b^2}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*((-4*a*c+b^2)^(1/2)*b-4*a*c+b^2)/c/(-4*a
*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*((-4*a*c+b^2)^(1/2)*b+4*a*
c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [B] time = 1.47, size = 39, normalized size = 1.00

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)}{n\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] (2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.553 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=98

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

```
[Out] -1/a/n/(x^n)-b*ln(x)/a^2+1/2*b*ln(a+b*x^n+c*x^(2*n))/a^2/n-(-2*a*c+b^2)*arc
tanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^2/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/
(2*a^2*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx,x,x^n\right)}{n}$$

$$= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx,x,x^n\right)}{an}$$

$$= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)dx,x,x^n\right)}{an}$$

$$= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^n\right)}{a^2n}$$

$$= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2a^2n}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,\right.}{2a^2n}$$

$$= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c\right.}{a^2n}$$

$$= -\frac{x^{-n}}{an}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,n}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}$$

Mathematica [A] time = 0.67, size = 135, normalized size = 1.38

$$-\frac{-\frac{4c^2\log\left(x^{-n}\left(b-\sqrt{b^2-4ac}\right)+2c\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^2}+\frac{4c^2\log\left(x^{-n}\left(\sqrt{b^2-4ac}+b\right)+2c\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^2}+\frac{x^{-n}}{a}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((1/(a*x^n) - (4*c^2*Log[2*c + (b - Sqrt[b^2 - 4*a*c])/x^n])/(Sqrt[b^2 - 4
*a*c]*(b - Sqrt[b^2 - 4*a*c])^2) + (4*c^2*Log[2*c + (b + Sqrt[b^2 - 4*a*c])
/x^n])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^2))/n)
```

fricas [A] time = 0.90, size = 333, normalized size = 3.40

$$\left[-\frac{2\left(b^{3}-4\,abc\right)nx^{n}\log(x)+\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,x^{n}\log\left(\frac{2\,c^{2}x^{2\,n}+b^{2}-2\,ac+2\left(bc+\sqrt{b^{2}-4\,ac}\,c\right)x^{n}+\sqrt{b^{2}-4\,ac}\,b}{cx^{2\,n}+bx^{n}+a}\right)+2\,ab^{2}-8\,a^{2}\cdots}{2\left(a^{2}b^{2}-4\,a^{3}c\right)nx^{n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^n
*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a
*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2*(2*
(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*arcta
n(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + 2*a
*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2
- 4*a^3*c)*n*x^n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{cx^{2\,n}+bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.15, size = 658, normalized size = 6.71

$$-\frac{4abc\,n^{2}\ln(x)}{4a^{3}c\,n^{2}-a^{2}b^{2}n^{2}}+\frac{b^{3}n^{2}\ln(x)}{4a^{3}c\,n^{2}-a^{2}b^{2}n^{2}}+\frac{2bc\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{\left(4ac-b^{2}\right)an}+\frac{2bc\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16\cdots}}{\cdots}\right)}{\left(4ac-b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^2
*b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3
*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*b*c-1/2/a^2/(4*a*c
-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)
^(1/2))/(2*a*c-b^2)/c)*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(
-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*(-16*a^3*c^
3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c
-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*b*c-1
/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*
a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*
a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{anx^{n}}-\int \frac{cx^{n}+b}{acxx^{2\,n}+abxx^{n}+a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{n+1}\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.554 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=126

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

```
[Out] -1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(a+b
*x^n+c*x^(2*n))/a^3/n+b*(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2)
)/a^3/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)
/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3 -
 ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-2n}}{2an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx,x,x^n\right)}{an}\\
&= -\frac{x^{-2n}}{2an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)dx,x,x^n\right)}{an}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{(b^2-ac)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx,x,x^n\right)}{a^3n}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2a^3n}-\frac{(b^2-\ldots}{\ldots}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Su}\ldots}{\ldots}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+\ldots}{2a^3n\ldots}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 112, normalized size = 0.89

$$\frac{-a^2x^{-2n}-(b^2-ac)\log(a+x^n(b+cx^n))+\frac{2b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+2n\log(x)(b^2-ac)+2abx^{-n}}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-(a^2/x^(2*n)) + (2*a*b)/x^n + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sq
rt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*n*Log[x] - (b^2 - a*c)*
Log[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

fricas [A] time = 0.76, size = 429, normalized size = 3.40

$$\left[-\frac{a^2b^2 - 4\,a^3c - 2\left(b^4 - 5\,ab^2c + 4\,a^2c^2\right)nx^{2\,n}\log(x) + \left(b^3 - 3\,abc\right)\sqrt{b^2 - 4\,ac}\,x^{2\,n}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc-\sqrt{b^2-4\,ac}\,c\right.}{cx^{2\,n}+bx^n+a}\right.}{2\left(a^3b^2 - 4\,a^4c\right)nx^{2\,n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x)
 + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2*a
*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) +
b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x^n +
 a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2*(a^2
*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n +
 sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*
n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^
4*c)*n*x^(2*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{cx^{2\,n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.17, size = 958, normalized size = 7.60

$$-\frac{4a^2c^2n^2\ln(x)}{4a^4c\,n^2 - a^3b^2n^2}+\frac{5a\,b^2c\,n^2\ln(x)}{4a^4c\,n^2 - a^3b^2n^2}-\frac{b^4n^2\ln(x)}{4a^4c\,n^2 - a^3b^2n^2}+\frac{2c^2\ln\left(x^n - \frac{-3a\,b^2c+b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10a\,b^6c+b^8}}{2\left(3ac-b^2\right)bc}\right)}{\left(4ac - b^2\right)an}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^2
+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2)*
n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+
33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/(3*a*c-b^2)/b/c)*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^
2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/
b/c)*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+3
3*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*(-36*a^3*b^2*c^3+33*a
^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*
c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^
2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^
8)^(1/2))/(3*a*c-b^2)/b/c)*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c
+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c
)*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bx^{n}-a}{2\,a^{2}nx^{2\,n}}+\int\frac{bcx^{n}+b^{2}-ac}{a^{2}cxx^{2\,n}+a^{2}bxx^{n}+a^{3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c*
x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{2\,n+1}\,\left(a+b\,x^{n}+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.555 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}} dx$

Optimal. Leaf size=164

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c\cdots}{\sqrt{b^2-4\cdots}}\right)}{a^4n\sqrt{b^2-4ac}}$$

```
[Out] -1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))+(a*c-b^2)/a^3/n/(x^n)-b*(-2*a*c+b^
2)*ln(x)/a^4+1/2*b*(-2*a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/a^4/n-(2*a^2*c^2-4*a*
b^2*c+b^4)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^4/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^4
- 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqr
t[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a +
 b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
```

```
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a^3\left(a+bx+cx^2\right)}\right) dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{\left(b^2-ac\right)x^{-n}}{a^3n} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{\text{Subst}\left(\int \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right.}{a+bx+cx^2}\right.}{a^4n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{\left(b^2-ac\right)x^{-n}}{a^3n} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{\left(b\left(b^2-2ac\right)\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+c}\right.}{2a^4n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{\left(b^2-ac\right)x^{-n}}{a^3n} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^2\right.}{2a^4n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{\left(b^2-ac\right)x^{-n}}{a^3n} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,n} - \frac{b\left(b^2-2ac\right.}{a^4}$$

Mathematica [A] time = 0.44, size = 143, normalized size = 0.87

$$\frac{-2a^3x^{-3n} - \frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + 3a^2bx^{-2n} + 6ax^{-n}\left(ac-b^2\right) + 3b\left(b^2-2ac\right)\log\left(a+x^n\left(b+cx^n\right)\right) -}{6a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*a^3)/x^(3*n) + (3*a^2*b)/x^(2*n) + (6*a*(-b^2 + a*c))/x^n - (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 -
 4*a*c] - 6*b*(b^2 - 2*a*c)*n*Log[x] + 3*b*(b^2 - 2*a*c)*Log[a + x^n*(b + c
*x^n)])/(6*a^4*n)
```

fricas [A] time = 0.96, size = 522, normalized size = 3.18

$$\left[-\frac{2\,a^{3}b^{2}-8\,a^{4}c+6\left(b^{5}-6\,ab^{3}c+8\,a^{2}bc^{2}\right)nx^{3\,n}\log(x)-3\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)\sqrt{b^{2}-4\,ac}\,x^{3\,n}\log\left(\frac{2\,c^{2}x^{2\,n}+b^{2}-2\,a\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*lo
g(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c^2
*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a
*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(3*n)
*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2*n) -
 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(2*a^3*
b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) + 6*(b^4
 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqrt(-b^2 +
 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x
^(3*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.20, size = 1300, normalized size = 7.93

$$\frac{8a^{2}b\,c^{2}n^{2}\ln(x)}{4a^{5}c\,n^{2}-a^{4}b^{2}n^{2}}-\frac{6a\,b^{3}c\,n^{2}\ln(x)}{4a^{5}c\,n^{2}-a^{4}b^{2}n^{2}}+\frac{b^{5}n^{2}\ln(x)}{4a^{5}c\,n^{2}-a^{4}b^{2}n^{2}}-\frac{4b\,c^{2}\ln\left(x^{n}-\frac{-2a^{2}b\,c^{2}+4a\,b^{3}c-b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52\cdots}}{2\left(2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)c}\right.}{\left(4ac-b^{2}\right)a^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2/a^2*b/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4*
a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*
ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n
*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b
^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*
b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^
5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2
*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^
2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-1
2*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)
/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4)
)*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10
)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5
*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c
```

```
/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*
c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2
-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-
b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-9
6*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c
+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-1
6*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/
2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3
+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,abx^n-2\,a^2-6\left(b^2-ac\right)x^{2\,n}}{6\,a^3nx^{3\,n}}+\int-\frac{b^3-2\,abc+\left(b^2c-ac^2\right)x^n}{a^3cxx^{2\,n}+a^3bxx^n+a^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrate
(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n + a^
4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{3\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.556 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=353

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}}{n\sqrt{b^2-4a}}$$

```
[Out] 2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))/n/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+2*2^(3/4)*c^(3/4)
*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/n/(-b-(-4
*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)-2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*
c^(1/4)*x^(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(
-4*a*c+b^2)^(1/2))^(3/4)-2*2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(3/4)
```

Rubi **[A]** time = 0.63, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1381, 1347, 212, 208, 205}

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}}{n\sqrt{b^2-4a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)
*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2
 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2
^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{4\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, x^{n/4}\right)}{n}$$

$$= \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\,n} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}\,x^2}\,d\ldots\right.}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\ldots}$$

$$= \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\,2^{3/4}c^{3/4}\tan\ldots}{\sqrt{b^2-4ac}\left(\ldots\right.}$$

Mathematica [A] time = 0.90, size = 340, normalized size = 0.96

$$\frac{2\,2^{3/4}c^{3/4}\left(-\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}-\frac{\tanh^{-1}\left(\ldots\right)}{\sqrt{b^2-4ac}\left(\ldots\right.}\ldots\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/
4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((-b - Sqrt[b^2 - 4*a*c
])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])
/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))))/n
```

fricas [B] time = 1.79, size = 4426, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 6
4*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4
)))*arctan(1/16*sqrt(2)*(2*sqrt(2)*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b
^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*n^7*x*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)*n^8)) - (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4
*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
)*n^4)) + sqrt(2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c
^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x
^(1/2*n - 2) - sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*
b^3*c^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^8 - 8*a*b^6*c + 21*a^2
*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)*n^4)))/x^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
)*n^4)))*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))
)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(1/8*(2*((a^3*b^10*c - 15*a^
4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^
6)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8
*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*
a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48
*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2)*n^4)) + ((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^
2*c^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^7 - 9*a*b^5*c + 24*a^2*
b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)*n^4)))*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2
*n - 2) + sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c
^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^8 - 8*a*b^6*c + 21*a^2*b^4*
c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*n^4)))/x^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))
)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/(
```

```
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/
4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c
)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^
(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b
*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*
x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt
(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*
b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x
*x^(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.75, size = 280, normalized size = 0.79

$$\mathrm{RootOf}\left(\left(256a^7c^4n^8 - 256a^6b^2c^3n^8 + 96a^5b^4c^2n^8 - 16a^4b^6c\,n^8 + a^3b^8n^8\right)\_Z^8 + \left(-48a^3b\,c^3n^4 + 40a^2b^3c^2n^4 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^3
*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*c^
2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8-25
6*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8+(-4
8*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a + b\,x^n + c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.557 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=610

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\right.}{}$$

```
[Out] 2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b-(-4*a*c+b^2)^(1/2))^(1/3))/
n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/2*c^(2/3)*ln(2^(2/3)*c^
(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(
-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^
2)^(1/2)-2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1/3*n)/(b-(-4*a
*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a
*c+b^2)^(1/2)-2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b+(-4*a*c+b^2)^
(1/2))^(1/3))/n/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/2*c^(2/3)
*ln(2^(2/3)*c^(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b+(-4*a*c+b^2)^(1/
2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(-4*a*c+b^2)^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)+2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1
/3*n)/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(-4*a*c+b^2)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 1.15, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1381, 1347, 200, 31, 634, 617, 204, 628}

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3
))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) +
 2^(1/3)*c^(1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x
^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+bx^3+cx^6}\,dx, x, x^{n/3}\right)}{n}$$

$$= \frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{\left(2^{2/3}c\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3} n} + \frac{\left(2^{2/3}c\right)\mathrm{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}} - \cdots}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-\cdots}}}{\sqrt[3]{2}}\cdots}\cdots\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4a\cdots}\right.}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3} n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\cdots\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3} n}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3} n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\cdots\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3} n}$$

$$= -\frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3} n} + \frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3} n} + \frac{2^{2/3}c^{2/3}\log\left(\cdots\right.}{\sqrt{b^2-\cdots}}$$

Mathematica [A] time = 0.76, size = 526, normalized size = 0.86

$$\frac{c^{2/3}\left(-\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)+\left(b-\sqrt{b^2\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b - Sq
rt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt
[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - (b +
Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3
)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)] + (b
 - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(
1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]))
/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n)
```

fricas [B] time = 1.43, size = 4699, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/
((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b
^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5
*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 -
16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3
)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 -
4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c
^3)*n^6)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2
 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 1
0*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c
+ 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
- (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*((
(a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3
))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*
b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 -
 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^8 - 10*a*b^6*c + 36
*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(((a^2*b^2 - 4*a^3*c)*n^3*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)*(((a^2*b^
2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*
c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3
) + 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 +
 4*a^2*c^4)) - 2*sqrt(3)*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*
c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)
*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 +
128*a^6*c^5)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 2
0*a^2*b^3*c^3 - 16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n
^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1
/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n
^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)
*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3
)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt(
(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 6
4*a^7*c^3)*n^6)) + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)
*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2
)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*
b^2 - 4*a^3*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*
b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8
 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*
b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2
/3))/x^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a
```

```
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 -
 4*a^3*c)*n^3))^(2/3) - 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4
*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) + (1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sq
rt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2
- 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c -
2*a*c^2)*x*x^(1/3*n - 1) + (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c
^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^
6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(((a^2*b^2
 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c
 + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)
)/x) + (1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((
a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) -
(1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6
)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) - 1/2*(1/2)^(1/3)*(((
a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^
5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3)
)^(1/3)*log(8*(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1
/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4
*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c
^4)*n*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)
/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c +
72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c
 + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
 - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(
((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^
3))^(2/3))/x^2) - 1/2*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(8*(2*(b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^
2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c - 8
*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-((a^2*b^2 -
4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) +
(1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 1
28*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4
*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^
2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.46, size = 260, normalized size = 0.43

$$\mathrm{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)\ln\left(\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n^
4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*c^
2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64*a^
5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2*b*c
^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.558 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 2*arctan(x^(1/2*n)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^
(1/2)/n/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctan(x^(1/2*n)*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/n/(-4*a*c+b^2
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1381, 1093, 205}

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^{n/2}\right)}{n}$$
$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}\,n}$$
$$= \frac{2\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,n} - \frac{2\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}\,n}$$

Mathematica [A] time = 0.26, size = 145, normalized size = 0.86

$$\frac{2\sqrt{2}\,\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*
a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[
b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]*n)
```

fricas [B] time = 0.71, size = 801, normalized size = 4.74

$$\frac{1}{2}\sqrt{2}\sqrt{-\frac{\left(ab^2-4\,a^2c\right)n^2\sqrt{\frac{1}{\left(a^2b^2-4\,a^3c\right)n^4}}+b}{\left(ab^2-4\,a^2c\right)n^2}}\log\left(\frac{4\,cxx^{\frac{1}{2}\,n-1}+\sqrt{2}\left(\left(ab^3-4\,a^2bc\right)n^3\sqrt{\frac{1}{\left(a^2b^2-4\,a^3c\right)n^4}}-\left(b^2-4\,ac\right)n\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
+ b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 -
4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt(-(
(a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^
2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2
- 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - s
qrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 -
4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) +
b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2*c)*n^2*s
qrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x
^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)
*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*
a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*sqrt(((a*b^2 -
 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2
))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1
/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x)
```

giac [B] time = 1.85, size = 1035, normalized size = 6.12

$$\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c-2b^4c+16\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)
*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b
^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2
*c^3)*abs(c)) + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b
^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b - sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*
c^2 - 4*a^2*c^3)*abs(c)))/n
```

maple [C] time = 0.19, size = 114, normalized size = 0.67

$$\mathrm{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)\ln\left(\left(\left(4a^2b\,n^3-\frac{a\,b^3n^3}{c}\right)\mathrm{RootOf}\left(\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R),
_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b^3
*n^2)*_Z^2+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.559 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

```
[Out] -2/a/n/(x^(1/2*n))+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*2^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1381, 1340, 1122, 1166, 205}

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx &= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+\frac{c}{x^4}+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{n}\\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{x^4}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{n}\\
&= -\frac{2x^{-n/2}}{an}+\frac{2\,\mathrm{Subst}\left(\int \frac{c+bx^2}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{an}\\
&= -\frac{2x^{-n/2}}{an}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^2}\,dx, x, x^{-n/2}\right)}{an}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,n}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 127, normalized size = 0.62

$$\frac{4cx^{-n/2}\left(\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/2))
```

fricas [B] time = 0.74, size = 1229, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)
```

```
)*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)
*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4
*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*
a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((a^3*b^3 -
 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))
 - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^
3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*(
(a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*
c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*
b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) -
sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a
^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3
 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - 4*x*x^(-1/2*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.35, size = 268, normalized size = 1.31

$$\mathrm{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2c\,n^4+a^3b^4n^4\right)\_Z^4+c^3+\left(12a^2b\,c^2n^2-7a\,b^3c\,n^2+b^5n^2\right)\_Z^2\right)\ln\left(\left(-\frac{8a^5c^2n^3}{a\,c^3-b^2c^2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(a
*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^3-
b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_
R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b*c^
2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{anx^{\frac{1}{2}n}}-\int \frac{cx^{\frac{3}{2}n}+bx^{\frac{1}{2}n}}{acxx^{2n}+abxx^n+a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{2}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(-n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.560 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=699

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\ldots$$

```
[Out] -3/a/n/(x^(1/3*n))+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2)
)^(1/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^
2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b-(-4*a
*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/2*a
rctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^
(1/2))*3^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*
a*c+b^2)^(1/2))^(2/3)+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1
/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*
c+b^2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b+(
-4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*c+b^2)^(1/2))^(2/3)-
1/2*arctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b+(-4*a*c+b^2)^(1/2))^(1/3
))*3^(1/2))*3^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(
b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 1.49, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1381, 1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1
 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(
2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (Sqrt[3]*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a
*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/
3)*n) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^
(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c]
)^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqr
t[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
```

```
ntegerQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+\frac{c}{x^6}+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3\,\mathrm{Subst}\left(\int \frac{x^6}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{3\,\mathrm{Subst}\left(\int \frac{c+bx^3}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(3\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^3}\,dx, x, x^{-n/3}\right)}{2an} + \frac{\left(3\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right.\cdots}{\cdots}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\cdots\right.}{\sqrt[3]{2}\,a^{4/3}\cdots}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\cdots\right.}{\sqrt[3]{2}\,a^{4/3}\cdots}$$

$$= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{a}\cdots}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n\cdots}$$

Mathematica [C] time = 0.13, size = 127, normalized size = 0.18

$$\frac{6cx^{-n/3}\left(\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/3))
```

fricas [B] time = 5.40, size = 6279, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(3)*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*
c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^4*b^12*c - 17*a^5*b^1
0*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^4*c^5 - 480*a^9*b^2*c
^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9*c^3 - 252*a^3*b^7*c^
4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*n^2*x)*x^(-1/3*n - 1)
*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c
^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - sqrt(2)*(1/2)
^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3
+ 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b
*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c
^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c
^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*
b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10*c - 12*a*b^8*c^2 + 52*a^2
*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1
)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3
*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*
c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) - (1/2)^(2/3)*
((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*
c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*
b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
- 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^
4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*
a^5*c)*n^3))^(2/3) + 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16
*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3
*b^2*c^7 + 4*a^4*c^8)) - 4*sqrt(3)*(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n
^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*
c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^
4*b^12*c - 17*a^5*b^10*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^
4*c^5 - 480*a^9*b^2*c^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
```

b^2*c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9
*c^3 - 252*a^3*b^7*c^4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*
n^2*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^
10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))
^(2/3) - sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*
c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*
a^3*b^3*c^3 + 24*a^4*b*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*
b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) + (1/2)^(1/3)*((a^
4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*
n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^10*c -
12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c
^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3
))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7
*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200
*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(-((a^4
*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(-((a^4*b^2 - 4*
a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*
c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3
+ 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*
c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5
+ 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8)) + 2*(1/2)^(1/3)*a*n*(((a^4*
b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)
) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2
*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c +
16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6))
- (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(((a^4*b^2 - 4*a^5*c)*
n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((
a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b
*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + 2*(1/2)^(1/3)*a*n*(-((a^4*b^2 -
4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^
3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 +
2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^
6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/(
(a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) - (1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)
*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*
b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b^8*c^2 - 8*a*b^6*c^3 + 20*
a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*
((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c
^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10
*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a

```
^5*c^6)*n*x)*x^(-1/3*n - 1)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c
 + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*
n^3))^(1/3) - (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*
a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 -
200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a
^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n
^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2) - (1/2)^(1/3)*
a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^
11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b
^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(
-2/3*n - 2) + (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 8
0*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)*n^6)) + (b^10*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c
^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c
)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a
*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^
9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n
^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a
*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*
c^5 + 32*a^6*c^6)*n^2)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3)
)^(2/3))/x^2) - 6*x*x^(-1/3*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.75, size = 534, normalized size = 0.76

$$\mathrm{RootOf}\left(\left(64a^7c^3n^6 - 48a^6b^2c^2n^6 + 12a^5b^4c\,n^6 - a^4b^6n^6\right)\_Z^6 + c^4 + \left(-32a^3b\,c^3n^3 + 32a^2b^3c^2n^3 - 10a\,b^5c\,n^3 + b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^2*
c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a
^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*
a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*
n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*b^4*c*n^
```

```
6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5*c*n^3+b^
7*n^3)*_Z^3+c^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{anx^{\frac{1}{3}n}} - \int \frac{cx^{\frac{5}{3}n} + bx^{\frac{2}{3}n}}{acxx^{2n} + abxx^{n} + a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{3}+1}\,\left(a + b\,x^{n} + c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.561 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=414

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -4/a/n/(x^(1/4*n))-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+b
^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a*c
+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a*
c+b^2)^(1/2))^(3/4)-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.79, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1381, 1340, 1367, 1422, 212, 208, 205}

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4
))])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(
1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^8}+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{n}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^8}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{n}$$

$$= -\frac{4x^{-n/4}}{an} + \frac{4\,\text{Subst}\left(\int \frac{c+bx^4}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{an}$$

$$= -\frac{4x^{-n/4}}{an} + \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^4}\,dx, x, x^{-n/4}\right)}{an} + \frac{\left(2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\cdots}{\cdots}$$

$$= -\frac{4x^{-n/4}}{an} - \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}\,x^2}\,dx, x, x^{-n/4}\right)}{a\sqrt{-b+\sqrt{b^2-4ac}}\,n} - \frac{\left(2\left(b-\cdots\right.\right.}{\cdots}$$

$$= -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4} n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\cdots}{\sqrt[4]{-b+\sqrt{b\cdots}}}\right.}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4} n\cdots}$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.31

$$\frac{8cx^{-n/4}\left(\frac{{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/4))
```

fricas [B] time = 2.70, size = 5712, normalized size = 13.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^
3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/16
*sqrt(2)*(2*sqrt(2)*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10*c^3 - 590
*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^2*c^7 - 12
8*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8))
+ (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 + 605*a^4*b^
7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*x^(-1/4*n
- 1)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)) - sqrt(2)*((a^5*b^10 - 16*a^6*b^8*c + 9
8*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^11 - 13*a*b^9*c
+ 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)
*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)
*x^2*x^(-1/2*n - 2) + sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 -
220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2
 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*n^2)*sqr
t(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*
b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(sqrt
(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7
 - 6*a^3*b^2*c^8 + a^4*c^9)) - 4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
```

```
^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*n^4)))*arctan(1/8*(2*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10
*c^3 - 590*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^
2*c^7 - 128*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)*n^8)) - (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 +
605*a^4*b^7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*
x^(-1/4*n - 1)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(-((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2)*n^4)) - ((a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^
4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)*n^8)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3
*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)*sqrt(sqrt(2)*sqrt(-((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)*n^4)))*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^
3*b^2*c^5 + a^4*c^6)*x^2*x^(-1/2*n - 2) - sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c
 + 85*a^7*b^7*c^2 - 220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^12 - 12*a*b^1
0*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 +
 8*a^6*c^6)*n^2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*
c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(-((a^5*b^4 - 8*
a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4*
c^9)) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b
^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*
b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3
)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sq
rt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*
a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b
^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1)
 - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 -
4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c
```

```
+ 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a
*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^
2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2
 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c
^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7
*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b
^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*
log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5
 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(s
qrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - 8*x*x^(-1/4*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 1.22, size = 630, normalized size = 1.52

$$\mathrm{RootOf}\left(\left(256a^9c^4n^8 - 256a^8b^2c^3n^8 + 96a^7b^4c^2n^8 - 16a^6b^6c\,n^8 + a^5b^8n^8\right)\_Z^8 + c^5 + \left(80a^4b\,c^4n^4 - 120a^3b^3c^3n^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c^6
-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*
b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b
^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b
*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a
*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^7*
a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c^3*n^8+96
*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a
^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4}{anx^{\frac{1}{4}n}} - \int \frac{cx^{\frac{7}{4}n} + bx^{\frac{3}{4}n}}{acxx^{2n} + abxx^n + a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.562 $\int \frac{x^2}{a+bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=140

$$-\frac{2cx^3 \, {}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3 \, {}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] -2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b
^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1383, 364}

$$-\frac{2cx^3 \, {}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3 \, {}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeometric2
F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c
+ b*Sqrt[b^2 - 4*a*c]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a+bx^n+cx^{2n}} \, dx &= \frac{(2c)\int \frac{x^2}{b-\sqrt{b^2-4ac}+2cx^n} \, dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{x^2}{b+\sqrt{b^2-4ac}+2cx^n} \, dx}{\sqrt{b^2-4ac}}\\ &= -\frac{2cx^3 \, {}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3 \, {}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)} \end{aligned}$$

Mathematica [A] time = 0.65, size = 265, normalized size = 1.89

$$-\frac{2}{3}cx^3\left(\frac{1-\left(\frac{x^n}{x^n-\frac{\sqrt{b^2-4ac}-b}{2c}}\right)^{-3/n}{}_2F_1\left(-\frac{3}{n},-\frac{3}{n};\frac{n-3}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{1-8^{-1/n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-3/n}{}_2F_1\left(-\frac{3}{n},-\frac{3}{n};\right.}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*((1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b - Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c
])/c + x^n))^(3/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometr
ic2F1[-3/n, -3/n, (-3 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)]/(8^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/n)))
/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/3
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\,x^n+c\,x^{2n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a + b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^2/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n)), x)
```

3.563 $\int \frac{x}{a+bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=136

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4
*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1383, 364}

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric2F1[1,
2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^n+cx^{2n}}\,dx &= \frac{(2c)\int \frac{x}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{x}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.58, size = 263, normalized size = 1.93

$$-cx^2\left(\frac{1-\left(\frac{x^n}{x^n-\frac{\sqrt{b^2-4ac}-b}{2c}}\right)^{-2/n}\,{}_2F_1\left(-\frac{2}{n},-\frac{2}{n};\frac{n-2}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{1-4^{-1/n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-2/n}\,{}_2F_1\left(-\frac{2}{n},-\frac{2}{n};\frac{n-}{n}\right.}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(c*x^2*((1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b - Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])
/c + x^n))^(2/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometric
2F1[-2/n, -2/n, (-2 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/(4^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)))/(
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(x/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{b\,x^n + c\,x^{2n} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] int(x/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{a + b\,x^n + c\,x^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x/(a + b*x**n + c*x**(2*n)), x)
```

3.564 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=124

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a
*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = $\{1347, 245\}$

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Hypergeometric2F1[1
, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx^n+cx^{2n}}\,dx &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [B] time = 0.53, size = 261, normalized size = 2.10

$$-2cx\left(\frac{1-\left(\frac{x^n}{x^n-\frac{\sqrt{b^2-4ac}-b}{2c}}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{n-1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{1-2^{-1/n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{n-1}{n};\right.}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] -2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*
a*c])/c + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeo
metric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)]/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
))^n^(-1)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{cx^{2n} + bx^{n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^{2n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,x^{n} + c\,x^{2n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] int(1/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^{2n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a + b\,x^{n} + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(1/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/(a + b*x**n + c*x**(2*n)), x)
```

3.565 $\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/2*ln(a+b*x^n+c*x^(2*n))/a/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1
/2))/a/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*n) + Log[
x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^n\right)}{an}\\
&= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2an}\\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^n+cx^{2n}\right)}{2an} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{an}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.00

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}} + \frac{\log(a+x^n(b+cx^n))}{n} - 2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)
 - 2*Log[x] + Log[a + x^n*(b + c*x^n)]/n)/a
```

fricas [A] time = 0.63, size = 259, normalized size = 3.50

$$\left[\frac{2\left(b^2-4ac\right)n\log(x)+\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc+\sqrt{b^2-4ac}\,c\right)x^n+\sqrt{b^2-4ac}\,b}{cx^{2n}+bx^n+a}\right)-\left(b^2-4ac\right)\log\left(cx^{2n}+bx^n-\right.}{2\left(ab^2-4a^2c\right)n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b
^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^
(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4
*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-
```

```
(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 -
 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

maple [B] time = 0.10, size = 397, normalized size = 5.36

$$\frac{4ac\,n^2\ln(x)}{4a^2c\,n^2 - a\,b^2n^2} - \frac{b^2n^2\ln(x)}{4a^2c\,n^2 - a\,b^2n^2} + \frac{b^2\ln\left(x^n - \frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{2\left(4ac-b^2\right)an} + \frac{b^2\ln\left(x^n + \frac{b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{2\left(4ac-b^2\right)an} - \frac{2c\ln\left(x^n - \frac{-b^2+}{}\right.}{\left(4ac-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+c*x^(2*n)+a),x)
[Out] 4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)
*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/
(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*
c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/
2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*
a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^
2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

mupad [B] time = 1.61, size = 224, normalized size = 3.03

$$\frac{\ln\left(-\frac{1}{c\,x} - \frac{(2\,a\,n+b\,n\,x^n)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,c\,x\left(a\,b^2\,n-4\,a^2\,c\,n\right)}\right)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)} - \frac{\ln\left(\frac{(2\,a\,n+b\,n\,x^n)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,c\,x\left(a\,b^2\,n-4\,a^2\,c\,n\right)} - \frac{1}{c\,x}\right)\left(\right.}{2\,\left(a\,b^2\,n-4\,a^2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))),x)
[Out] (log(- 1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/
(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*(a
*b^2*n - 4*a^2*c*n)) - (log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) - 4*a
*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1/2)
- 4*a*c + b^2))/(2*(a*b^2*n - 4*a^2*c*n)) + (log(x)*(n - 1))/(a*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.566 $\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=142

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] 2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))/x/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1383, 364}

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeometric2F1
[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a
*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx &= \frac{(2c)\int \frac{1}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{1}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 240, normalized size = 1.69

$$\frac{c2^{\frac{1}{n}+1}\left(\frac{\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{\frac{1}{n}}{}_2F_1\left(1+\frac{1}{n},1+\frac{1}{n};2+\frac{1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}-b-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{\frac{1}{n}+1}{}_2F_1\left(1+\frac{1}{n},1+\frac{1}{n};2+\frac{1}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{c}\right)}{(n+1)x\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^(1 + n^(-1))*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)*Hype
rgeometric2F1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b - Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((
c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1 + n^(-1))*Hypergeometric2F1[1
+ n^(-1), 1 + n^(-1), 2 + n^(-1), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(1 + n)*x)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{cx^2x^{2n}+bx^2x^n+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^{2n}+bx^n+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n+c\,x^{2n}+a)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] int(1/x^2/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^{2n}+bx^n+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.567 $\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=140

$$\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x^2/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))+c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))/x^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1383, 364}

$$\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hypergeometric2F1[1, -
2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*S
qrt[b^2 - 4*a*c])*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx&=\frac{(2c)\int \frac{1}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{1}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\&=\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2}+\frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}\end{aligned}$$

Mathematica [A] time = 0.42, size = 258, normalized size = 1.84

$$\frac{c2^{\frac{n+2}{n}}\left(\frac{\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{2/n}{}_2F_1\left(\frac{n+2}{n},\frac{n+2}{n};2+\frac{2}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}-b-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{\frac{n+2}{n}}{}_2F_1\left(\frac{n+2}{n},\frac{n+2}{n};2+\frac{2}{n};\frac{b+\sqrt{b^2-4ac}}{2cx^n+b+\sqrt{b^2-4ac}}\right)}{c}\right)}{(n+2)x^2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^((2 + n)/n)*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)*Hyperg
eometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((c*x^n)/
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((2 + n)/n)*Hypergeometric2F1[(2 + n)/n,
 (2 + n)/n, 2 + 2/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(2 + n)*x^2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{cx^3x^{2n}+bx^3x^n+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n+c\,x^{2n}+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] int(1/x^3/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.568 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,-1/2,-1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx&=\frac{\sqrt{a+bx^n+cx^{2n}}\int x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\&=\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.75, size = 365, normalized size = 2.47

$$\frac{x^4\left(2bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+4}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+an(n+4)\sqrt{\frac{-\sqrt{b^2-4ac}+}{b-\sqrt{b^2-}}}\right.}{4(n+4)^2\sqrt{a+x^n\left(b+c\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*(4*(4 + n)*(a + x^n*(b + c*x^n)) + a*n*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*n*x^n*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4 + n)/n, 1/2
, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])]))/(4*(4 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + c\,x^{2n} + a}\; x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(x^3*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.569 $\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,-1/2,-1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx &= \frac{\sqrt{a+bx^n+cx^{2n}}\int x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.72, size = 366, normalized size = 2.47

$$\frac{x^3\left(3bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+3}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+2an(n+3)\sqrt{\frac{-\sqrt{b^2-}}{b-}}\right.}{6(n+3)^2\sqrt{a+x^n\left(b\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*(6*(3 + n)*(a + x^n*(b + c*x^n)) + 2*a*n*(3 + n)*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + n)/n, 1
/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])]))/(6*(3 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^{n}+c\,x^{2n}+a}\;x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(x^2*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.570 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,-1/2,-1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a+bx^n+cx^{2n}}\,dx &= \frac{\sqrt{a+bx^n+cx^{2n}}\int x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{x^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [B] time = 0.65, size = 364, normalized size = 2.46

$$\frac{x^2\left(bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+2}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+an(n+2)\sqrt{\frac{-\sqrt{b^2-4ac}+b\cdots}{b-\sqrt{b^2-4a\cdots}}}\cdots\right.}{2(n+2)^2\sqrt{a+x^n\left(b+c\cdots\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*(2*(2 + n)*(a + x^n*(b + c*x^n)) + a*n*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2 + n)/n, 1/2,
1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])]))/(2*(2 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + c\,x^{2n} + a}\;x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(x*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.571 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] x*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^n+cx^{2n}}\,dx &= \frac{\sqrt{a+bx^n+cx^{2n}}\int \sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 0.65, size = 351, normalized size = 2.53

$$\frac{x\left(bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+2(n+1)\left(an\sqrt{\frac{-\sqrt{b^2}}{b}}\right.\right.}{2(n+1)^2\sqrt{a+x^n\left(b+c\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1
 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1
 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + c\,x^{2n} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int((b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\,x^n + c\,x^{2\,n}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx^n + cx^{2n}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.572 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))*a^(1/2)/n+1/2*b
*arctanh(1/2*(b+2*c*x^n)/c^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/c^(1/2)+(a+b*
x^n+c*x^(2*n))^(1/2)/n
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*S
qrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[
a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{2n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{2n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \cdots\right)}{n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

Mathematica [A] time = 0.15, size = 110, normalized size = 0.92

$$\frac{\sqrt{a+x^n(b+cx^n)} - \sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+x^n(b+cx^n)}}\right)}{2\sqrt{c}}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] (Sqrt[a + x^n*(b + c*x^n)] - Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[
a + x^n*(b + c*x^n)])] + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + x^n*(
b + c*x^n)])])/(2*Sqrt[c]))/n
```

fricas [A] time = 1.12, size = 658, normalized size = 5.53

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^{2n} - 8bcx^n - b^2 - 4ac - 4\left(2c^{\frac{3}{2}}x^n + b\sqrt{c}\right)\sqrt{cx^{2n}+bx^n+a}\right) + 2\sqrt{a}\,c\log\left(-\frac{8abx^n+8a^2+\left(b^2+4ac\right)x^{2n}\cdots}{\cdots}\right)\cdots}{4cn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)
*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*x^
n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^
(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), -1/2
*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n + 8*a^2
 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b
*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(4*sqrt(-
a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)
/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n
- b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a))
+ 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arctan(1/2*(sqr
t(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*
x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*
x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt(c*x^(2*n) + b*
x^n + a)*c)/(c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

maple [A] time = 0.12, size = 125, normalized size = 1.05

$$-\frac{\sqrt{a}\,\ln\left(\left(b\,e^{n\ln(x)}+2a+2\sqrt{b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}+a}\,\sqrt{a}\right)e^{-n\ln(x)}\right)}{n}+\frac{b\ln\left(\frac{c\,e^{n\ln(x)}+\frac{b}{2}}{\sqrt{c}}+\sqrt{b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}+\right.}{2\sqrt{c}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(1/2)/x,x)
```

```
[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln(
x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/2)
*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)
)/exp(n*ln(x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n + c\,x^{2\,n}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x,x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

3.573 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -AppellF1(-1/n,-1/2,-1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}} \int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^n+cx^{2n}}\, F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.64, size = 365, normalized size = 2.45

$$\frac{bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n-1}{n};\frac{1}{2},\frac{1}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-2a(n-1)n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}}{2(n-1)^2x\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] (2*(-1 + n)*(a + x^n*(b + c*x^n)) - 2*a*(-1 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-1 + n)/n,
1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])/(2*(-1 + n)^2*x*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n}+bx^n+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n+c\,x^{2n}+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(1/2)/x^2,x)
```

```
[Out] int((b*x^n+c*x^(2*n)+a)^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n + c\,x^{2\,n}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

3.574 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -1/2*AppellF1(-2/n,-1/2,-1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.61, size = 365, normalized size = 2.42

$$\frac{bnx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n-2}{n};\frac{1}{2},\frac{1}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-a(n-2)n\sqrt{\frac{-\sqrt{b^2-4ac}+b+}{b-\sqrt{b^2-4ac}}}}{2(n-2)^2x^2\sqrt{a+x^n(b+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] (2*(-2 + n)*(a + x^n*(b + c*x^n)) - a*(-2 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-2 + n)/n, 1/2,
1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])/(2*(-2 + n)^2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n}+bx^n+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n+c\,x^{2n}+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(1/2)/x^3,x)
```

```
[Out] int((b*x^n+c*x^(2*n)+a)^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n + c\,x^{2\,n}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

3.575 $\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=149

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/4*a*x^4*AppellF1(4/n,-3/2,-3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}dx &= \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 1.49, size = 518, normalized size = 3.48

$$\frac{x^4\left(2(n+4)\left(32a^2c\left(n^2+3n+2\right)+a\left(3b^2n^2+2bc\left(23n^2+84n+64\right)x^n+8c^2\left(5n^2+18n+16\right)x^{2n}\right)+x^n(b+c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(2*(4 + n)*(32*a^2*c*(2 + 3*n + n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*
b*c*(32 + 36*n + 7*n^2)*x^n + 8*c^2*(8 + 6*n + n^2)*x^(2*n)) + a*(3*b^2*n^2
 + 2*b*c*(64 + 84*n + 23*n^2)*x^n + 8*c^2*(16 + 18*n + 5*n^2)*x^(2*n))) - 6
*a*n^2*(4 + n)*(b^2 - 2*a*c*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(8 + n) - 4*a
*c*(8 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(2 + n)*(4 + n)^2*(4 + 3*n)*Sqrt[a
+ x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(x^3*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.576 $\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=149

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/3*a*x^3*AppellF1(3/n,-3/2,-3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2} dx &= \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 1.46, size = 524, normalized size = 3.52

$$\frac{x^3\left(2(n+3)\left(4a^2c\left(8n^2+18n+9\right)+a\left(3b^2n^2+2bc\left(23n^2+63n+36\right)x^n+4c^2\left(10n^2+27n+18\right)x^{2n}\right)+x^n\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(2*(3 + n)*(4*a^2*c*(9 + 18*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 +
2*b*c*(18 + 27*n + 7*n^2)*x^n + 4*c^2*(9 + 9*n + 2*n^2)*x^(2*n)) + a*(3*b^2
*n^2 + 2*b*c*(36 + 63*n + 23*n^2)*x^n + 4*c^2*(18 + 27*n + 10*n^2)*x^(2*n))
) + 2*a*n^2*(3 + n)*(-3*b^2 + 4*a*c*(3 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(-12*a*c
*(2 + n) + b^2*(6 + n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(24*c*(1 + n)*(3 + n)^2*(3 + 2*
n)*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(b\,x^{n}+c\,x^{2n}+a\right)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(x^2*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.577 $\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 1/2*a*x^2*AppellF1(2/n,-3/2,-3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.49, size = 520, normalized size = 3.49

$$\frac{x^2\left(2(n+2)\left(16a^2c\left(2n^2+3n+1\right)+a\left(3b^2n^2+2bc\left(23n^2+42n+16\right)x^n+8c^2\left(5n^2+9n+4\right)x^{2n}\right)+x^n\left(b+cx\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(2*(2 + n)*(16*a^2*c*(1 + 3*n + 2*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 +
2*b*c*(8 + 18*n + 7*n^2)*x^n + 8*c^2*(2 + 3*n + n^2)*x^(2*n)) + a*(3*b^2*n^
2 + 2*b*c*(16 + 42*n + 23*n^2)*x^n + 8*c^2*(4 + 9*n + 5*n^2)*x^(2*n))) - 6*
a*n^2*(2 + n)*(b^2 - 4*a*c*(1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(4 + n) - 4*a*
c*(4 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(1 + n)*(2 + n)^2*(2 + 3*n)*Sqrt[a +
 x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(b\,x^{n}+c\,x^{2n}+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(x*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(x*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.578 $\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=140

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] a*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx &= \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{ax\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] time = 1.52, size = 513, normalized size = 3.66

$$x\left(2(n+1)\left(4a^2c\left(8n^2+6n+1\right)-3an^2\left(b^2-4ac(2n+1)\right)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 +
 n)*(4*a^2*c*(1 + 6*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c*(2 + 9*
n + 7*n^2)*x^n + 4*c^2*(1 + 3*n + 2*n^2)*x^(2*n)) + a*(3*b^2*n^2 + 2*b*c*(4
 + 21*n + 23*n^2)*x^n + 4*c^2*(2 + 9*n + 10*n^2)*x^(2*n)) - 3*a*n^2*(b^2 -
4*a*c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*Sqrt[a
 + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int((b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.579 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx$

Optimal. Leaf size=173

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn}$$

```
[Out] 1/3*(a+b*x^n+c*x^(2*n))^(3/2)/n-a^(3/2)*arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+
b*x^n+c*x^(2*n))^(1/2))/n-1/16*b*(-12*a*c+b^2)*arctanh(1/2*(b+2*c*x^n)/c^(1
/2)/(a+b*x^n+c*x^(2*n))^(1/2))/c^(3/2)/n+1/8*(b^2+8*a*c+2*b*c*x^n)*(a+b*x^n
+c*x^(2*n))^(1/2)/c/n
```

Rubi [A] time = 0.16, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^
n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt
[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*S
qrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(16*c^(3/2)*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} - \frac{\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{2n}$$

$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12a\right.}{x\sqrt{a+bx+cx^2}}\right.}{8cn}$$

$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.}{n}$$

$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,d\right.}{n}$$

$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} - \frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n}}\right.}{n}$$

Mathematica [A] time = 0.28, size = 158, normalized size = 0.91

$$\frac{-48a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right) - 3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+x^n(b+cx^n)}}\right) + 2\sqrt{c}\sqrt{a+x^n\left(b+cx^n\right)}\left(8c\left(4a\right.\right.}{48c^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(
2*n))) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(
b + c*x^n)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a
+ x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

fricas [A] time = 0.96, size = 827, normalized size = 4.78

$$\left[\frac{48\,a^{\frac{3}{2}}c^{2}\log\left(-\frac{8\,abx^{n}+8\,a^{2}+\left(b^{2}+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}\,bx^{n}+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^{n}+a}}{x^{2\,n}}\right)-3\left(b^{3}-12\,abc\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{2\,n}-8\,bcx^{n}-b\right.}{96\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(
sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3 -
 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3
/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b
*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(
24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)
*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 - 12*a*b
*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(96*sqrt(-a
)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n +
 a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*
x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^
(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)
*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c^2*arctan(1/2*(
sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a
*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n +
 b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2
*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n
 + a))/(c^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2\,n}+bx^{n}+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

maple [A] time = 0.05, size = 209, normalized size = 1.21

$$-\frac{a^{\frac{3}{2}}\ln\left(\left(b\,\mathrm{e}^{n\ln(x)}+2a+2\sqrt{b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}+a}\,\sqrt{a}\right)\mathrm{e}^{-n\ln(x)}\right)}{n}+\frac{3ab\ln\left(\frac{c\,\mathrm{e}^{n\ln(x)}+\frac{b}{2}}{\sqrt{c}}+\sqrt{b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}}\right.}{4\sqrt{c}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(3/2)/x,x)
```

```
[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*c*exp(n*ln(x))+32*a*c+3*b^2)*(a+b*exp(n*ln(
x))+c*exp(n*ln(x))^2)^(1/2)/c/n+3/4/c^(1/2)/n*a*b*ln((c*exp(n*ln(x))+1/2*b)
/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/16/c^(3/2)/n*b^3*ln((
c*exp(n*ln(x))+1/2*b)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/
n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))
^2)^(1/2))/exp(n*ln(x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n} + bx^{n} + a\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,x^{n} + c\,x^{2\,n}\right)^{3/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)
```

3.580 $\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -a*AppellF1(-1/n,-3/2,-3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.39, size = 526, normalized size = 3.51

$$\frac{2(n-1)\left(4a^2c\left(8n^2-6n+1\right)+a\left(3b^2n^2+2bc\left(23n^2-21n+4\right)x^n+4c^2\left(10n^2-9n+2\right)x^{2n}\right)+x^n\left(b+cx^n\right)\left(3b\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (2*(-1 + n)*(4*a^2*c*(1 - 6*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c
*(2 - 9*n + 7*n^2)*x^n + 4*c^2*(1 - 3*n + 2*n^2)*x^(2*n)) + a*(3*b^2*n^2 +
2*b*c*(4 - 21*n + 23*n^2)*x^n + 4*c^2*(2 - 9*n + 10*n^2)*x^(2*n))) - 6*a*(-
1 + n)*n^2*(b^2 + 4*a*c*(-1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(2 - 3*n) +
 b^2*(-2 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]])/(8*c*(-1 + n)^2*(-1 + 2*n)*(-1 +
3*n)*x*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(3/2)/x^2,x)
[Out] int((b*x^n+c*x^(2*n)+a)^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**2, x)
```

3.581 $\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -1/2*a*AppellF1(-2/n,-3/2,-3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x
^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^3}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 1.37, size = 520, normalized size = 3.42

$$\frac{2(n-2)\left(16a^2c\left(2n^2-3n+1\right)+a\left(3b^2n^2+2bc\left(23n^2-42n+16\right)x^n+8c^2\left(5n^2-9n+4\right)x^{2n}\right)+x^n\left(b+cx^n\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (2*(-2 + n)*(16*a^2*c*(1 - 3*n + 2*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*
c*(8 - 18*n + 7*n^2)*x^n + 8*c^2*(2 - 3*n + n^2)*x^(2*n)) + a*(3*b^2*n^2 +
2*b*c*(16 - 42*n + 23*n^2)*x^n + 8*c^2*(4 - 9*n + 5*n^2)*x^(2*n))) - 6*a*(b
^2 + 4*a*c*(-1 + n))*(-2 + n)*n^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(4 - 3*n) + b^2*
(-4 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]])/(16*c*(-2 + n)^2*(-1 + n)*(-2 + 3*n)*x^2*
Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(3/2)/x^3,x)
```

```
[Out] int((b*x^n+c*x^(2*n)+a)^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**3, x)
```

3.582 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,1/2,1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 1.18

$$\frac{x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{4\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2
, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(4*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{b\,x^n+c\,x^{2n}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(x^3/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^3/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.583 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,1/2,1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 175, normalized size = 1.18

$$\frac{x^3\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2
, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(3*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b\,x^n + c\,x^{2n} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(x^2/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a + b\,x^n + c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.584 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,1/2,1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 175, normalized size = 1.18

$$\frac{x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{2\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2
, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\,x^n+c\,x^{2n}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(x/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.585 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] x*AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*x^(
2*n)]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 166, normalized size = 1.19

$$\frac{x\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/
2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n+c\,x^{2n}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(1/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.586 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}\\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

fricas [A] time = 0.89, size = 148, normalized size = 3.15

$$\left[\frac{\log\left(-\frac{8\,abx^n+8\,a^2+\left(b^2+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}\,n}, \frac{\sqrt{-a}\,\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\,\sqrt{-a}\,a\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\left(acx^{2\,n}+abx^n+a^2\right)}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2
*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arcta
n(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2
*n) + a*b*x^n + a^2))/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2\,n} + bx^n + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n + c\,x^{2n} + a}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+c*x^(2*n)+a)^(1/2),x)
[Out] int(1/x/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2\,n} + bx^n + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)),x)
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.587 $\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -AppellF1(-1/n,1/2,1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^n +
 c*x^(2*n)]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\ &= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 173, normalized size = 1.16

$$-\frac{\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{n-1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{x\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1
/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x*Sqrt[a + x^n*(b + c*x^n)]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n+c\,x^{2n}+a}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.588 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -1/2*AppellF1(-2/n,1/2,1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\
&= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 175, normalized size = 1.16

$$-\frac{\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n-2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{2x^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -1/2*(Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1
/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n+c\,x^{2n}+a}\;x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.589 $\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,3/2,3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [B] time = 0.86, size = 398, normalized size = 2.64

$$\frac{x^4\left(32bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+4}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+4)\left(b^2(n-8)\right.\right.}{4an(n+4)\left(4a\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(-8*(4 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-8 + n) - 4*a*c*(-4 + n))*
(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n,
1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 32*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(4*a*(-b^2 + 4*a*c)*n*(4 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(x^3/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.590 $\int \frac{x^2}{\left(a+bx^n+cx^{2n}\right)^{3/2}} dx$

Optimal. Leaf size=151

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,3/2,3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^n+cx^{2n}\right)^{3/2}} dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}} dx}{a\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [B] time = 0.84, size = 398, normalized size = 2.64

$$\frac{x^3\left(18bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+3}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+3)\left(b^2(n-6)-4\right.\right.}{3an(n+3)\left(4ac-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(-6*(3 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-6 + n) - 4*a*c*(-3 + n))*
(3 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n,
1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 18*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*a*(-b^2 + 4*a*c)*n*(3 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(x^2/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.591 $\int \frac{x}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,3/2,3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 510}

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [B] time = 0.84, size = 398, normalized size = 2.64

$$\frac{x^2\left(8bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{n+2}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+2)\left(b^2(n-4)-\right.\right.}{2an(n+2)\left(4ac\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(-4*(2 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-4 + n) - 4*a*c*(-2 + n))*
(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n,
1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 8*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*n*(2 + n)
*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(x/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.592 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] x*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1348, 429}

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [B] time = 0.94, size = 384, normalized size = 2.70

$$\frac{x\left(2bcx^n\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-(n+1)\left(\left(b^2(n-2)-\right.\right.\right.}{an(n+1)\left(4ac-b^2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) + (b^2
*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sq
rt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(1/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(1/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

3.593 $\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/a^(3/2)/n+2*(b^
2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*
n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^n\right)}{n}\\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{2\,\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{a\left(b^2-4ac\right)n}\\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{an}\\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{an}\\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 94, normalized size = 0.96

$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a\left(b^2-4ac\right)\sqrt{a+x^n(b+cx^n)}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right)}{a^{3/2}}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*Sqrt[a + x^n*(b + c*x^n)]) -
ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)])]/a^(3/2))/n
```

fricas [B] time = 1.14, size = 449, normalized size = 4.58

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)\sqrt{a}\,x^{2\,n}+\left(b^3-4\,abc\right)\sqrt{a}\,x^n+\left(ab^2-4\,a^2c\right)\sqrt{a}\right)\log\left(-\frac{8\,abx^n+8\,a^2+\left(b^2+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+}}{x^{2\,n}}\right.}{2\left(\left(a^2b^2c-4\,a^3c^2\right)nx^{2\,n}+\left(a^2b^3-4\,a^3bc\right)nx^n+\left(a^3b^2-4\,a^4c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) -
4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*(a*
```

```
b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3
*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n), (((
b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a*b^2 -
 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*
n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b^2 - 2*a
^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^
2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n + c\,x^{2n} + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(1/x/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a + b\,x^n + c\,x^{2n}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.594 $\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -AppellF1(-1/n,3/2,3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^n
 + c*x^(2*n)]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\ &= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [B] time = 0.78, size = 395, normalized size = 2.60

$$\frac{(n-1)\left(b^2(n+2)-4ac(n+1)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{n-1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-2\left(\cdots\right.}{a(n-1)nx\left(4ac-b\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-1 + n)*(-4*a*c*(1 + n) + b^2*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*((-1 + n)*(b^2
- 2*a*c + b*c*x^n) + b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(a*(-b^2 + 4*a*c)*(-1 + n)
*n*x*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(1/x^2/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(a + b\, x^n + c\, x^{2\,n}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.595 $\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -1/2*AppellF1(-2/n,3/2,3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*x^2*Sqrt[a + b*x^n
 + c*x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\
&= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [B] time = 0.80, size = 399, normalized size = 2.59

$$\frac{(n-2)\left(b^2(n+4)-4ac(n+2)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n-2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)-}{2a(n-2)nx^2\left(4a\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-2 + n)*(-4*a*c*(2 + n) + b^2*(4 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*((-2 + n)*(b^2 - 2
*a*c + b*c*x^n) + 2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*(-2 + n)*n
*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(1/x^3/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(a + b\, x^n + c\, x^{2\,n}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.596 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx$

Optimal. Leaf size=182

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2}{m}$$

```
[Out] 3*a^2*b*x^(1+n)*(d*x)^m/(1+m+n)+3*a*(a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+b
*(6*a*c+b^2)*x^(1+3*n)*(d*x)^m/(1+m+3*n)+3*c*(a*c+b^2)*x^(1+4*n)*(d*x)^m/(1
+m+4*n)+3*b*c^2*x^(1+5*n)*(d*x)^m/(1+m+5*n)+c^3*x^(1+6*n)*(d*x)^m/(1+m+6*n)
+a^3*(d*x)^(1+m)/d/(1+m)
```

Rubi [A] time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1353, 20, 30}

$$\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*x
)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) +
(3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*n)*
(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*
x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 dx = \int \left(a^3(dx)^m + 3a^2bx^n(dx)^m + 3ab^2\left(1+\frac{ac}{b^2}\right)x^{2n}(dx)^m + b^3\left(1+\frac{6ac}{b^2}\right)x^{3n}(dx)^m +\right.$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int x^n(dx)^m\,dx + \left(3bc^2\right)\int x^{5n}(dx)^m\,dx + c^3\int x^{6n}(dx)^m\,dx +$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \left(3a^2bx^{-m}(dx)^m\right)\int x^{m+n}\,dx + \left(3bc^2x^{-m}(dx)^m\right)\int x^{m+5n}\,dx + \left(c^3x^{-}\right.$$
$$= \frac{3a^2bx^{1+n}(dx)^m}{1+m+n} + \frac{3a\left(b^2+ac\right)x^{1+2n}(dx)^m}{1+m+2n} + \frac{b\left(b^2+6ac\right)x^{1+3n}(dx)^m}{1+m+3n} + \frac{3c\left(b^2+\right.}{1+}$$

Mathematica [A] time = 0.35, size = 137, normalized size = 0.75

$$x(dx)^m\left(\frac{a^3}{m+1} + \frac{3a^2bx^n}{m+n+1} + \frac{3ax^{2n}\left(ac+b^2\right)}{m+2n+1} + \frac{bx^{3n}\left(6ac+b^2\right)}{m+3n+1} + \frac{3cx^{4n}\left(ac+b^2\right)}{m+4n+1} + \frac{3bc^2x^{5n}}{m+5n+1} + \frac{c^3x^{6n}}{m+6n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2*
n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 + a
*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6*n)
)/(1 + m + 6*n))
```

fricas [B] time = 0.85, size = 2303, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 + 1
5*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 + 3*
c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2 +
4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2 + 5*
c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b*c^2*
m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*c^2*m^
2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2*m^3 +
3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*c^2*m^3
+ 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m^4 + 10*b
*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*log(d) + m*l
og(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b^2*c + a*c^
2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*c + a*c^2 +
(b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a*c^2)*m^3 + 3
07*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m^2 + 3*(b^2*c
+ a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 107*((b^2*c + a*
c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c + a*c^2)*m^2 +
4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*c + a*c^2)*m^5 +
5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 10*(b^2*c
+ a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) +
((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(b^3 + 6*a*b*c + (b^3 +
6*a*b*c)*m)*n^5 + 15*(b^3 + 6*a*b*c)*m^4 + 508*(b^3 + 6*a*b*c + (b^3 + 6*a*
b*c)*m^2 + 2*(b^3 + 6*a*b*c)*m)*n^4 + 20*(b^3 + 6*a*b*c)*m^3 + 372*((b^3 +
6*a*b*c)*m^3 + b^3 + 6*a*b*c + 3*(b^3 + 6*a*b*c)*m^2 + 3*(b^3 + 6*a*b*c)*m)
*n^3 + b^3 + 6*a*b*c + 15*(b^3 + 6*a*b*c)*m^2 + 121*((b^3 + 6*a*b*c)*m^4 +
4*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 6*(b^3 + 6*a*b*c)*m^2 + 4*(b^3 + 6*
```

```
a*b*c)*m)*n^2 + 6*(b^3 + 6*a*b*c)*m + 18*((b^3 + 6*a*b*c)*m^5 + 5*(b^3 + 6*
a*b*c)*m^4 + 10*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 10*(b^3 + 6*a*b*c)*m^
2 + 5*(b^3 + 6*a*b*c)*m)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*((a*b^2 +
 a^2*c)*m^6 + 6*(a*b^2 + a^2*c)*m^5 + 360*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*
m)*n^5 + 15*(a*b^2 + a^2*c)*m^4 + 702*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m^2
+ 2*(a*b^2 + a^2*c)*m)*n^4 + 20*(a*b^2 + a^2*c)*m^3 + 461*((a*b^2 + a^2*c)*
m^3 + a*b^2 + a^2*c + 3*(a*b^2 + a^2*c)*m^2 + 3*(a*b^2 + a^2*c)*m)*n^3 + a*
b^2 + a^2*c + 15*(a*b^2 + a^2*c)*m^2 + 137*((a*b^2 + a^2*c)*m^4 + 4*(a*b^2
+ a^2*c)*m^3 + a*b^2 + a^2*c + 6*(a*b^2 + a^2*c)*m^2 + 4*(a*b^2 + a^2*c)*m)
*n^2 + 6*(a*b^2 + a^2*c)*m + 19*((a*b^2 + a^2*c)*m^5 + 5*(a*b^2 + a^2*c)*m^
4 + 10*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 10*(a*b^2 + a^2*c)*m^2 + 5*(a*
b^2 + a^2*c)*m)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^6 + 6*a^2
*b*m^5 + 15*a^2*b*m^4 + 20*a^2*b*m^3 + 720*(a^2*b*m + a^2*b)*n^5 + 15*a^2*b
*m^2 + 1044*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n^4 + 6*a^2*b*m + 580*(a^2*b*m^
3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n^3 + a^2*b + 155*(a^2*b*m^4 + 4*a^2*b
*m^3 + 6*a^2*b*m^2 + 4*a^2*b*m + a^2*b)*n^2 + 20*(a^2*b*m^5 + 5*a^2*b*m^4 +
 10*a^2*b*m^3 + 10*a^2*b*m^2 + 5*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*
log(x)) + (a^3*m^6 + 720*a^3*n^6 + 6*a^3*m^5 + 15*a^3*m^4 + 20*a^3*m^3 + 17
64*(a^3*m + a^3)*n^5 + 15*a^3*m^2 + 1624*(a^3*m^2 + 2*a^3*m + a^3)*n^4 + 6*
a^3*m + 735*(a^3*m^3 + 3*a^3*m^2 + 3*a^3*m + a^3)*n^3 + a^3 + 175*(a^3*m^4
+ 4*a^3*m^3 + 6*a^3*m^2 + 4*a^3*m + a^3)*n^2 + 21*(a^3*m^5 + 5*a^3*m^4 + 10
*a^3*m^3 + 10*a^3*m^2 + 5*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^7 +
 720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^5 + 21*m^5 + 1624*(m^3 +
3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 3
5*m^3 + 175*(m^5 + 5*m^4 + 10*m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^
6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.11, size = 3798, normalized size = 20.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^3,x)
[Out] x*(c^3*(x^n)^6+a^3*m^6+6*a^3*m^5+1764*a^3*n^5+15*a^3*m^4+1624*a^3*n^4+720*a
^3*n^6+b^3*(x^n)^3+20*a^3*m^3+15*a^3*m^2+175*a^3*n^2+21*a^3*n+735*a^3*n^3+a
^3+972*b*c^2*n^4*(x^n)^5+45*b*c^2*m^4*(x^n)^5+240*b^3*m*n^5*(x^n)^3+18*b^2*
c*m^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+150*c^3*m^3*n*(x^n)^6+510*c^3*m^2*n^2*(
x^n)^6+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+540*a*c^2*n^5*(x^n)^4+18*
b^3*m^5*n*(x^n)^3+508*b^3*m^2*n^4*(x^n)^3+15*c^3*m^5*n*(x^n)^6+85*c^3*m^4*n
^2*(x^n)^6+225*c^3*m^3*n^3*(x^n)^6+274*c^3*m^2*n^4*(x^n)^6+120*c^3*m*n^5*(x
^n)^6+3*b*c^2*m^6*(x^n)^5+75*c^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^n)^6+675*
c^3*m^2*n^3*(x^n)^6+548*c^3*m*n^4*(x^n)^6+3*a*c^2*m^6*(x^n)^4+3*b^2*c*m^6*(
x^n)^4+18*b*c^2*m^5*(x^n)^5+432*b*c^2*n^5*(x^n)^5+1116*b^3*m^2*n^3*(x^n)^3+
1016*b^3*m*n^4*(x^n)^3+45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2
*m^3*(x^n)^5+780*b*c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*a^
2*c*m^5*(x^n)^2+1080*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*
(x^n)^2+60*a*c^2*m^3*(x^n)^4+921*a*c^2*n^3*(x^n)^4+18*m*b*c^2*(x^n)^5+48*b*
c^2*(x^n)^5*n+45*a^2*b*m^4*x^n+3132*a^2*b*n^4*x^n+121*b^3*m^4*n^2*(x^n)^3+3
72*b^3*m^3*n^3*(x^n)^3+340*c^3*m*n^2*(x^n)^6+3*a^2*c*m^6*(x^n)^2+3*a*b^2*m^
```

6*(x^n)^2+45*a*c^2*m^4*(x^n)^4+1188*a*c^2*n^4*(x^n)^4+90*b^3*m^4*n*(x^n)^3+
484*b^3*m^3*n^2*(x^n)^3+45*b*c^2*m^2*(x^n)^5+285*b*c^2*n^2*(x^n)^5+18*a^2*b
*m^5*x^n+2160*a^2*b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+2106*a^2*c*n^4*(x^n)^2+45*
a*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(x^n)^2+45*a*c^2*m^2*(x^n)^4+321*a*c^2*n^2
*(x^n)^4+45*b^2*c*m^2*(x^n)^4+321*b^2*c*n^2*(x^n)^4+18*b^2*c*(x^n)^4*m+51*b
^2*c*(x^n)^4*n+1740*a^2*b*n^3*x^n+45*a^2*c*m^2*(x^n)^2+411*a^2*c*n^2*(x^n)^
2+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x^n)^3+1116*b^3*m*n^3*(x^n)^3+60*b
^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(x^n)^4+18*a^2*c*(x^n)^2*m+57*a^2*c*(x^n)^2*
n+6*a*b*c*(x^n)^3+60*a^2*c*m^3*(x^n)^2+1383*a^2*c*n^3*(x^n)^2+1383*a*b^2*n^
3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*a*c^2*(x^n)^4*n+60*a^2*b*n*x^n+180*b^3*m^2*
n*(x^n)^3+484*b^3*m*n^2*(x^n)^3+60*a*b^2*m^3*(x^n)^2+90*b^3*m*n*(x^n)^3+60*
a^2*b*m^3*x^n+45*a*b^2*m^2*(x^n)^2+411*a*b^2*n^2*(x^n)^2+45*a^2*b*m^2*x^n+4
65*a^2*b*n^2*x^n+18*m*a*b^2*(x^n)^2+57*a*b^2*(x^n)^2*n+18*a^2*b*m*x^n+1080*
a*b*c*m^3*n*(x^n)^3+570*a*b^2*m^2*n*(x^n)^2+2904*a*b*c*m*n^2*(x^n)^3+540*a*
b*c*m*n*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*c^3*n^2*(x^n)^6+1
5*b^3*m^4*(x^n)^3+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a^2*b*
m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m*n^4*x^
n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*(x^n)
^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m*n^3*(x^n
)^2+120*a*b*c*m^3*(x^n)^3+2232*a*b*c*n^3*(x^n)^3+6696*a*b*c*m^2*n^3*(x^n)^3
+150*c^3*m^2*n*(x^n)^6+105*a^3*m^4*n+700*a^3*m^3*n^2+2205*a^3*m^2*n^3+3248*
a^3*m*n^4+210*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*n^3+20*b^3*m^3*(x^n)^3+
15*b^3*m^2*(x^n)^3+121*b^3*n^2*(x^n)^3+6*m*b^3*(x^n)^3+18*b^3*(x^n)^3*n+3*a
^2*b*x^n+3*(x^n)^2*a*b^2+57*a*b^2*m^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1
383*a*b^2*m^3*n^3*(x^n)^2+2106*a*b^2*m^2*n^4*(x^n)^2+1080*a*b^2*m*n^5*(x^n)
^2+36*a*b*c*m^5*(x^n)^3+1440*a*b*c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^4+1926
*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+510*b^2*c*m^3*n*(x^n)^4+192
6*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^3*(x^n)^4+480*b*c^2*m^2*n*(x^n)^5+43
56*a*b*c*m^2*n^2*(x^n)^3+6696*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+
1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x^n+465*a^2*b*m^4*n^2*x^n+1740*a^2*
b*m^3*n^3*x^n+3132*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+285*a^2*c*m^4*n*(
x^n)^2+1644*a^2*c*m^3*n^2*(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*m*n
^4*(x^n)^2+285*a*b^2*m^4*n*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+4149*a*b^2*m^
2*n^3*(x^n)^2+4212*a*b^2*m*n^4*(x^n)^2+3*(x^n)^5*b*c^2+108*a*b*c*m^5*n*(x^n
)^3+90*a*b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)^3+510*a*c^2*m^2*n*(x^n)^4+128
4*a*c^2*m*n^2*(x^n)^4+510*b^2*c*m^2*n*(x^n)^4+210*a^3*m^2*n+700*a^3*m*n^2+1
05*a^3*m*n+1644*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+1860*a^2*b*m*n^2*x^
n+6*a^3*m+1440*a*b*c*m*n^5*(x^n)^3+2904*a*b*c*m^3*n^2*(x^n)^3+48*b*c^2*m^5*
n*(x^n)^5+285*b*c^2*m^4*n^2*(x^n)^5+780*b*c^2*m^3*n^3*(x^n)^5+972*b*c^2*m^2
n^4(x^n)^5+432*b*c^2*m*n^5*(x^n)^5+51*a*c^2*m^5*n*(x^n)^4+321*a*c^2*m^4*n
^2*(x^n)^4+21*a^3*m^5*n+175*a^3*m^4*n^2+735*a^3*m^3*n^3+1624*a^3*m^2*n^4+17
64*a^3*m*n^5+540*a*b*c*m^4*n*(x^n)^3+c^3*m^6*(x^n)^6+6*c^3*m^5*(x^n)^6+120*
c^3*n^5*(x^n)^6+15*c^3*m^4*(x^n)^6+274*c^3*n^4*(x^n)^6+b^3*m^6*(x^n)^3+20*c
^3*m^3*(x^n)^6+225*c^3*n^3*(x^n)^6+6*b^3*m^5*(x^n)^3+255*a*c^2*m*n*(x^n)^4+
255*b^2*c*m*n*(x^n)^4+600*a^2*b*m^3*n*x^n+2790*a^2*b*m^2*n^2*x^n+5220*a^2*b
*m*n^3*x^n+570*a^2*c*m^2*n*(x^n)^2+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x
^n)^3+726*a*b*c*n^2*(x^n)^3+285*a^2*c*m*n*(x^n)^2+36*a*b*c*(x^n)^3*m+108*a*
b*c*(x^n)^3*n+1284*a*c^2*m^3*n^2*(x^n)^4+2763*a*c^2*m^2*n^3*(x^n)^4+2376*a*
c^2*m*n^4*(x^n)^4+255*b^2*c*m^4*n*(x^n)^4+1284*b^2*c*m^3*n^2*(x^n)^4+2763*b
^2*c*m^2*n^3*(x^n)^4+2376*b^2*c*m*n^4*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+1710*
b*c^2*m^2*n^2*(x^n)^5+2340*b*c^2*m*n^3*(x^n)^5+57*a^2*c*m^5*n*(x^n)^2+411*a
^2*c*m^4*n^2*(x^n)^2+1383*a^2*c*m^3*n^3*(x^n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+
1080*a^2*c*m*n^5*(x^n)^2+372*b^3*n^3*(x^n)^3+3*(x^n)^2*a^2*c+3*(x^n)^4*a*c^
2+3*(x^n)^4*b^2*c+6096*a*b*c*m*n^4*(x^n)^3+508*b^3*n^4*(x^n)^3+6*m*c^3*(x^n
)^6+15*c^3*(x^n)^6*n+726*a*b*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+3
048*a*b*c*m^2*n^4*(x^n)^3+921*a*c^2*m^3*n^3*(x^n)^4+1188*a*c^2*m^2*n^4*(x^n
)^4+540*a*c^2*m*n^5*(x^n)^4+51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^
4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m^2*n^4*(x^n)^4+540*b^2*c*m*n^5*(x^n
)^4+240*b*c^2*m^4*n*(x^n)^5+1140*b*c^2*m^3*n^2*(x^n)^5+2340*b*c^2*m^2*n^3*(

```
x^n)^5+1944*b*c^2*m*n^4*(x^n)^5+6*a*b*c*m^6*(x^n)^3+255*a*c^2*m^4*n*(x^n)^4
+285*a*b^2*m*n*(x^n)^2+300*a^2*b*m*n*x^n)/(m+1)/(m+n+1)/(m+2*n+1)/(m+3*n+1)
/(1+m+4*n)/(1+m+5*n)/(1+m+6*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*
x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*
x)^3+2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 1.30, size = 273, normalized size = 1.50

$$\frac{c^3 d^m x e^{\left(m \log(x)+6\,n \log(x)\right)}}{m+6\,n+1}+\frac{3\,bc^2 d^m x e^{\left(m \log(x)+5\,n \log(x)\right)}}{m+5\,n+1}+\frac{3\,b^2 c d^m x e^{\left(m \log(x)+4\,n \log(x)\right)}}{m+4\,n+1}+\frac{3\,ac^2 d^m x e^{\left(m \log(x)+4\,n \log(x)\right)}}{m+4\,n+1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*b*c^2*d^m*x*e^(m*log(
x) + 5*n*log(x))/(m + 5*n + 1) + 3*b^2*c*d^m*x*e^(m*log(x) + 4*n*log(x))/(m
 + 4*n + 1) + 3*a*c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + b^3*d
^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*a*b*c*d^m*x*e^(m*log(x) +
3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*
n + 1) + 3*a^2*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*a^2*b*d^
m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

mupad [B] time = 2.16, size = 1734, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] (a^3*x*(d*x)^m)/(m + 1) + (c^3*x*x^(6*n)*(d*x)^m*(5*m + 15*n + 60*m*n + 255
*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 +
10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 +
 225*m^2*n^3 + 85*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2
*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*
n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 +
 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*
n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a*x*x^(2*n)*(d*x)^m*(a*c + b^2)*(
5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n + 702*m*
n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^3 + 702*n^
4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (b*x*x^(
3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^2*n + 7
44*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5
+ 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^3 + 121*m
^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 2
10*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3
 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6
 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 +
 175*m^4*n^2 + 1) + (3*c*x*x^(4*n)*(d*x)^m*(a*c + b^2)*(5*m + 17*n + 68*m*n
 + 321*m*n^2 + 102*m^2*n + 614*m*n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10
*m^2 + 10*m^3 + 5*m^4 + m^5 + 107*n^2 + 307*n^3 + 396*n^4 + 180*n^5 + 321*m
^2*n^2 + 307*m^2*n^3 + 107*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2
+ 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5
+ 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1
624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 +
1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a^2*b*x*x^n*(d*x)^m*(5*m
```

```
 + 20*n + 80*m*n + 465*m*n^2 + 120*m^2*n + 1160*m*n^3 + 80*m^3*n + 1044*m*n
^4 + 20*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 155*n^2 + 580*n^3 + 1044*n^
4 + 720*n^5 + 465*m^2*n^2 + 580*m^2*n^3 + 155*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*b*c^2
*x*x^(5*n)*(d*x)^m*(5*m + 16*n + 64*m*n + 285*m*n^2 + 96*m^2*n + 520*m*n^3
+ 64*m^3*n + 324*m*n^4 + 16*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 95*n^2
+ 260*n^3 + 324*n^4 + 144*n^5 + 285*m^2*n^2 + 260*m^2*n^3 + 95*m^3*n^2 + 1)
)/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n +
3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 +
 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2
*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^
2 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

3.597 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx$

Optimal. Leaf size=117

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

```
[Out] 2*a*b*x^(1+n)*(d*x)^m/(1+m+n)+(2*a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+2*b*c
*x^(1+3*n)*(d*x)^m/(1+m+3*n)+c^2*x^(1+4*n)*(d*x)^m/(1+m+4*n)+a^2*(d*x)^(1+m
)/d/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1353, 20, 30}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)
/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*
n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx &= \int \left(a^2(dx)^m+2abx^n(dx)^m+b^2\left(1+\frac{2ac}{b^2}\right)x^{2n}(dx)^m+2bcx^{3n}(dx)^m+c^2x^{4n}(dx\right.\\
&= \frac{a^2(dx)^{1+m}}{d(1+m)}+(2ab)\int x^n(dx)^m\,dx+(2bc)\int x^{3n}(dx)^m\,dx+c^2\int x^{4n}(dx)^m\,dx+\\
&= \frac{a^2(dx)^{1+m}}{d(1+m)}+\left(2abx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(2bcx^{-m}(dx)^m\right)\int x^{m+3n}\,dx+\left(c^2x^{-}\right.\\
&= \frac{2abx^{1+n}(dx)^m}{1+m+n}+\frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n}+\frac{2bcx^{1+3n}(dx)^m}{1+m+3n}+\frac{c^2x^{1+4n}(dx)^m}{1+m+4n}+
\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.74

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^{2n}\left(2ac+b^2\right)}{m+2n+1}+\frac{2abx^n}{m+n+1}+\frac{2bcx^{3n}}{m+3n+1}+\frac{c^2x^{4n}}{m+4n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))/
(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*n)
)
```

fricas [B] time = 0.94, size = 706, normalized size = 6.03

$$\frac{\left(c^2m^4+4\,c^2m^3+6\,c^2m^2+6\left(c^2m+c^2\right)n^3+4\,c^2m+11\left(c^2m^2+2\,c^2m+c^2\right)n^2+c^2+6\left(c^2m^3+3\,c^2m^2+3\,c^2m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^2
*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)*n
)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2 +
8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7*
(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c
)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 2*(b
^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*a*c)*m^3
 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b)
*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a*
b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*m^4 + 24*a^2
*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2
 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m + a^2)*n)*x
*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50*(m^2 + 2*m + 1
)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 + 10*(m^4 + 4*m^3
+ 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

giac [B] time = 0.80, size = 5454, normalized size = 46.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*
m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d) +
 m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m
^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)
```

) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n*e^(m*log(d) + m*lo
g(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^n*e^(m*log(
d) + m*log(x)) + c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*x^n
*e^(m*log(d) + m*log(x)) + 8*b^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a
*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^n*e^(m*log(d) + m
*log(x)) + 6*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 3
8*a*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^n*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*m*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^n*e^(m*l
og(d) + m*log(x)) + 6*c^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + a^2*m^4*x*e
^(m*log(d) + m*log(x)) + 2*a*b*m^4*x*e^(m*log(d) + m*log(x)) + b^2*m^4*x*e^
(m*log(d) + m*log(x)) + 2*a*c*m^4*x*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*e
^(m*log(d) + m*log(x)) + c^2*m^4*x*e^(m*log(d) + m*log(x)) + 10*a^2*m^3*n*x
*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*e^(m*log(d) + m*log(x)) + 8*b^2*m
^3*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 1
4*b*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*e^(m*log(d) + m*log(x
)) + 35*a^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*e^(m*log(d
) + m*log(x)) + 19*b^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x
*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 50*a^2*m*n^3*x*e^(m*log(d) + m*log(x)
) + 48*a*b*m*n^3*x*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*e^(m*log(d) + m
*log(x)) + 24*a*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*e^(m*log
(d) + m*log(x)) + 6*c^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 24*a^2*n^4*x*e^(m
*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*c^2*
m^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 42*b*c*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(3*n)
*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
 22*c^2*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^(3*n)*e^(m
*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m
^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m^3*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 8*b*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(2*n)
*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 48*a*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38
*b^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 22*c^
2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 24*a*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 8*a*b*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*x^n*e^(m*log(d)
 + m*log(x)) + 8*a*c*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x^n*e^
(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m^
2*n*x*x^n*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^n*e^(m*log(d) + m*log(
x)) + 48*a*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^n*e^(m*
log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 104*a*b*m
*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 38*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log
(x)) + 76*a*c*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^n*e^(m
*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^n*e^(m*log(d) + m*log(x))

+ 24*a*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*a^2*m^3*x*e^(m*log(d) + m*log(x)) + 8*a*b*m^3*x*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*e^(m*log(d) + m*log(x)) + 8*a*c*m^3*x*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*e^(m*log(d) + m*log(x)) + 30*a^2*m^2*n*x*e^(m*log(d) + m*log(x)) + 54*a*b*m^2*n*x*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*e^(m*log(d) + m*log(x)) + 70*a^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 104*a*b*m*n^2*x*e^(m*log(d) + m*log(x)) + 38*b^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 50*a^2*n^3*x*e^(m*log(d) + m*log(x)) + 48*a*b*n^3*x*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*e^(m*log(d) + m*log(x)) + 24*a*c*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*b^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 52*a*b*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*a^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*e^(m*log(d) + m*log(x)) + 6*b^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*e^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*e^(m*log(d) + m*log(x)) + 30*a^2*m*n*x*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*e^(m*log(d) + m*log(x)) + 35*a^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*n^2*x*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*b^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*x^n*e^(m*log(d) + m*log(x)) + 8*b^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^n*e^(m*log(d) + m*log

```
(x)) + 4*a^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*e^(m*log(d) + m*log(x)
) + 4*b^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*e^(m*log(d) + m*log(x)) +
 8*b*c*m*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*e^(m*log(d) + m*log(x)) + 10
*a^2*n*x*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*e^(m*log(d) + m*log(x)) + 8*b
^2*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*e^(m*log(d) + m*log(x)) + 14*b*
c*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*e^(m*log(d) + m*log(x)) + c^2*x*x
^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
c^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 2*a*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + c^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(
d) + m*log(x)) + b^2*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*c*x*x^n*e^(m*log(d
) + m*log(x)) + 2*b*c*x*x^n*e^(m*log(d) + m*log(x)) + c^2*x*x^n*e^(m*log(d)
 + m*log(x)) + a^2*x*e^(m*log(d) + m*log(x)) + 2*a*b*x*e^(m*log(d) + m*log(
x)) + b^2*x*e^(m*log(d) + m*log(x)) + 2*a*c*x*e^(m*log(d) + m*log(x)) + 2*b
*c*x*e^(m*log(d) + m*log(x)) + c^2*x*e^(m*log(d) + m*log(x)))/(m^5 + 10*m^4
*n + 35*m^3*n^2 + 50*m^2*n^3 + 24*m*n^4 + 5*m^4 + 40*m^3*n + 105*m^2*n^2 +
100*m*n^3 + 24*n^4 + 10*m^3 + 60*m^2*n + 105*m*n^2 + 50*n^3 + 10*m^2 + 40*m
*n + 35*n^2 + 5*m + 10*n + 1)
```

maple [C] time = 0.07, size = 1065, normalized size = 9.10

$$\frac{\left(2ab\,m^4x^n+18ab\,m^3n\,x^n+52ab\,m^2n^2x^n+48abm\,n^3x^n+2ac\,m^4x^{2n}+16ac\,m^3n\,x^{2n}+38ac\,m^2n^2x^{2n}+24acn\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^2,x)
```

```
[Out] x*(2*(x^n)^3*b*c+a^2+c^2*(x^n)^4+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^2+35*
a^2*n^2+24*a^2*n^4+4*a^2*m+10*a^2*n+b^2*(x^n)^2+6*c^2*m^3*n*(x^n)^4+11*c^2*
m^2*n^2*(x^n)^4+6*c^2*m*n^3*(x^n)^4+2*b*c*m^4*(x^n)^3+18*c^2*m^2*n*(x^n)^4+
22*c^2*m*n^2*(x^n)^4+52*a*b*n^2*x^n+8*a*c*(x^n)^2*m+12*a*b*m^2*x^n+12*a*c*m
^2*(x^n)^2+38*a*c*n^2*(x^n)^2+24*b^2*m*n*(x^n)^2+8*m*b*c*(x^n)^3+14*b*c*(x^
n)^3*n+18*c^2*m*n*(x^n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x^n)^2+24*a*c*n^3*(x^n)^
2+24*b^2*m^2*n*(x^n)^2+38*b^2*m*n^2*(x^n)^2+12*b*c*m^2*(x^n)^3+28*b*c*n^2*(
x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+16*a*c*(x^n)^2*n+8*a*b*x^n*m+18*a*b*x^n
*n+2*a*c*m^4*(x^n)^2+8*b^2*m^3*n*(x^n)^2+19*b^2*m^2*n^2*(x^n)^2+12*b^2*m*n^
3*(x^n)^2+8*b*c*m^3*(x^n)^3+16*b*c*n^3*(x^n)^3+2*(x^n)^2*a*c+c^2*m^4*(x^n)^
4+4*c^2*m^3*(x^n)^4+56*b*c*m*n^2*(x^n)^3+18*a*b*m^3*n*x^n+52*a*b*m^2*n^2*x^
n+48*a*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+76*a*c*m*n^2*(x^n)^2+42*b*c*m*n*(x^
n)^3+54*a*b*m^2*n*x^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(x^n)^2+54*a*b*m*n*x^n+6
*c^2*n^3*(x^n)^4+b^2*m^4*(x^n)^2+6*c^2*m^2*(x^n)^4+11*c^2*n^2*(x^n)^4+4*b^2
*m^3*(x^n)^2+12*b^2*n^3*(x^n)^2+4*m*c^2*(x^n)^4+6*c^2*(x^n)^4*n+6*b^2*m^2*(
x^n)^2+14*b*c*m^3*n*(x^n)^3+28*b*c*m^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n)^3+16*
a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+24*a*c*m*n^3*(x^n)^2+42*b*c*m^2*n*
(x^n)^3+19*b^2*n^2*(x^n)^2+4*b^2*(x^n)^2*m+8*b^2*(x^n)^2*n+30*a^2*m*n+2*a*b
*x^n+10*a^2*m^3*n+35*a^2*m^2*n^2+50*a^2*m*n^3+30*a^2*m^2*n+70*a^2*m*n^2)/(m
+1)/(m+n+1)/(m+2*n+1)/(m+3*n+1)/(1+m+4*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x
)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*P
i*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 1.27, size = 152, normalized size = 1.30

$$\frac{c^2d^mxe^{\left(m\log(x)+4\,n\log(x)\right)}}{m+4\,n+1}+\frac{2\,bcd^mxe^{\left(m\log(x)+3\,n\log(x)\right)}}{m+3\,n+1}+\frac{b^2d^mxe^{\left(m\log(x)+2\,n\log(x)\right)}}{m+2\,n+1}+\frac{2\,acd^mxe^{\left(m\log(x)+2\,n\log(x)\right)}}{m+2\,n+1}+\frac{2\,ab\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*b*c*d^m*x*e^(m*log(x)
 + 3*n*log(x))/(m + 3*n + 1) + b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n
 + 1) + 2*a*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*d^m*x*e
^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

mupad [B] time = 1.62, size = 543, normalized size = 4.64

$$\frac{a^2\,x\,{\left(d\,x\right)}^m}{m+1}+\frac{x\,x^{2\,n}\,{\left(d\,x\right)}^m\,\left(b^2+2\,a\,c\right)\,\left(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n^2\right.}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] (a^2*x*(d*x)^m)/(m + 1) + (x*x^(2*n)*(d*x)^m*(2*a*c + b^2)*(3*m + 8*n + 16*
m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*n
+ 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4
+ 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^(4*n)*(d*x)^m*(3*m
+ 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4
*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*
m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^n*(d*x)
^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3
 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6
*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*b*c*x*
x^(3*n)*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14
*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 +
10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

3.598 $\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx$

Optimal. Leaf size=58

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

```
[Out] b*x^(1+n)*(d*x)^m/(1+m+n)+c*x^(1+2*n)*(d*x)^m/(1+m+2*n)+a*(d*x)^(1+m)/d/(1+
m)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {14, 20, 30}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) +
 (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx &= \int \left(a(dx)^m+bx^n(dx)^m+cx^{2n}(dx)^m\right) dx\\
&= \frac{a(dx)^{1+m}}{d(1+m)}+b\int x^n(dx)^m\,dx+c\int x^{2n}(dx)^m\,dx\\
&= \frac{a(dx)^{1+m}}{d(1+m)}+\left(bx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(cx^{-m}(dx)^m\right)\int x^{m+2n}\,dx\\
&= \frac{bx^{1+n}(dx)^m}{1+m+n}+\frac{cx^{1+2n}(dx)^m}{1+m+2n}+\frac{a(dx)^{1+m}}{d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a}{m+1}+x^n\left(\frac{b}{m+n+1}+\frac{cx^n}{m+2n+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

fricas [B] time = 0.92, size = 142, normalized size = 2.45

$$\frac{\left(cm^2 + 2\,cm + (cm + c)n + c\right)xx^{2\,n}e^{\left(m\log(d)+m\log(x)\right)} + \left(bm^2 + 2\,bm + 2\,(bm + b)n + b\right)xx^{n}e^{\left(m\log(d)+m\log(x)\right)} + \left(am\right.}{m^3 + 2\,(m+1)n^2 + 3\,m^2 + 3\left(m^2 + 2\,m + 1\right)n + 3\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*m
^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 + 2*
a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(m +
 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

giac [B] time = 0.40, size = 557, normalized size = 9.60

$$\frac{cm^2xx^{2\,n}e^{\left(m\log(d)+m\log(x)\right)} + cmnxx^{2\,n}e^{\left(m\log(d)+m\log(x)\right)} + bm^2xx^{n}e^{\left(m\log(d)+m\log(x)\right)} + cm^2xx^{n}e^{\left(m\log(d)+m\log(x)\right)} + 2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(d)
+ m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m*log
(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log(x))
+ 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x)) + 2*a
*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m*log(x))
 + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) + m*log(x)
) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*log(x)) + 2
*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x)) + 3*a*n*x*
e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n*x*e^(m*log(
d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n*e^(m*log(d)
+ m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(d) + m*log(x))
 + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*
n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

maple [C] time = 0.05, size = 205, normalized size = 3.53

$$\frac{\left(b\,m^2x^n + 2bmn\,x^n + c\,m^2x^{2n} + cmn\,x^{2n} + a\,m^2 + 3amn + 2a\,n^2 + 2bm\,x^n + 2bn\,x^n + 2cm\,x^{2n} + cn\,x^{2n} + 2am +\right.}{(m+1)\,(m+n+1)\,(m+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+b*m^2*x^n+2*b*m*n*x^n+2*m*c*(x^n)^2+c*(x^n)^
2*n+a*m^2+3*a*m*n+2*a*n^2+2*b*m*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n+a
)/(m+1)/(m+n+1)/(m+2*n+1)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*
Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+
2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 1.12, size = 65, normalized size = 1.12

$$\frac{cd^m x e^{\left(m\log(x)+2\,n\log(x)\right)}}{m+2\,n+1}+\frac{bd^m x e^{\left(m\log(x)+n\log(x)\right)}}{m+n+1}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + b*d^m*x*e^(m*log(x) + n*l
og(x))/(m + n + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [B] time = 1.41, size = 83, normalized size = 1.43

$$(d\,x)^m\,\left(\frac{a\,x}{m+1}+\frac{b\,x\,x^n\,\left(m+2\,n+1\right)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{c\,x\,x^{2\,n}\,\left(m+n+1\right)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] (d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 +
 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 +
1))
```

sympy [A] time = 43.00, size = 1239, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**n
/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((log(x), Eq(n, 0)), (-x*
*(-2*n)*(0**(1/n))**(-2*n)/(2*n), Eq(d, 0**(1/n))), (-d**(-2*n)*x**(-2*n)/(
2*n), True))/d + b*Piecewise((log(x), Eq(n, 0)), (-x**n/(2*0**(1/n)*zoo**(1
/n)*n*x**(2*n)*(0**(1/n))**(2*n) - n*x**(2*n)*(0**(1/n))**(2*n)), Eq(d, 0**
(1/n))), (-d**(-2*n)*x**(-n)/n, True))/d + c*Piecewise((d**(-2*n)*log(x), A
bs(x) < 1), (-d**(-2*n)*log(1/x), 1/Abs(x) < 1), (-d**(-2*n)*meijerg(((), (
1, 1)), ((0, 0), ()), x) + d**(-2*n)*meijerg(((1, 1), ()), ((), (0, 0)), x)
, True))/d, Eq(m, -2*n - 1)), (a*Piecewise((log(x), Eq(n, 0)), (-x**(-n)*(0
**(1/n))**(-n)/n, Eq(d, 0**(1/n))), (-d**(-n)*x**(-n)/n, True))/d + b*Piece
wise((d**(-n)*log(x), Abs(x) < 1), (-d**(-n)*log(1/x), 1/Abs(x) < 1), (-d**
(-n)*meijerg(((), (1, 1)), ((0, 0), ()), x) + d**(-n)*meijerg(((1, 1), ()),
 ((), (0, 0)), x), True))/d + c*Piecewise((log(x), Eq(n, 0)), (-x**(2*n)/(0
**(1/n)*zoo**(1/n)*n*x**n*(0**(1/n))**n - 2*n*x**n*(0**(1/n))**n), Eq(d, 0*
*(1/n))), (d**(-n)*x**n/n, True))/d, Eq(m, -n - 1)), (a*d**m*m**2*x*x**m/(m
**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*
d**m*m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
 + 3*n + 1) + 2*a*d**m*m*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*
n + 3*m + 2*n**2 + 3*n + 1) + 2*a*d**m*n**2*x*x**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*d**m*n*x*x**m/(m**3 +
3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a*d**m*x*x
**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1)
+ b*d**m*m**2*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*
m + 2*n**2 + 3*n + 1) + 2*b*d**m*m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2
+ 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*d**m*m*x*x**m*x**n/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*d**
m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
 + 3*n + 1) + b*d**m*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m
```

```
*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*m**2*x*x**m*x**(2*n)/(m**3 + 3*m**2*n
 + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*m*n*x*x**m*
x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n
+ 1) + 2*c*d**m*m*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*n*x*x**m*x**(2*n)/(m**3 + 3*m**2*n +
 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*x*x**m*x**(2*
n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1),
True))
```

3.599 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=175

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m)*
hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)
/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1383, 364}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1
 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1383

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[(2*c)/q, Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx&=\frac{(2c)\int \frac{(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{(dx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\&=\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.85, size = 307, normalized size = 1.75

$$-\frac{x(dx)^m\left(\frac{2c\left(1-2^{-\frac{m+1}{n}}\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{-\frac{m+1}{n}}{}_2F_1\left(-\frac{m+1}{n},-\frac{m+1}{n};1-\frac{m+1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{2c\left(1-2^{-\frac{m+1}{n}}\left(\frac{cx^n}{\sqrt{b^2-4ac}+b+2cx^n}\right)^{-\frac{m+1}{n}}{}_2F_1\left(-\frac{m+1}{n},\right.\right.}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}\right.}\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((x*(d*x)^m*((2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (
1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^((1
 + m)/n)*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(b^2 -
4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -(
(1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/(2^((1 + m)/n)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1
+ m)/n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/(1 + m))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,x^n+c\,x^{2n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] int((d*x)^m/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n)), x)
```

3.600 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^2} dx$

Optimal. Leaf size=328

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4a\right.}$$

```
[Out] (d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))+c*(d
*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
))*(-b*(1+m-n)+(4*a*c*(1+m-2*n)-b^2*(1+m-n))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+
b^2)/d/(1+m)/n/(b-(-4*a*c+b^2)^(1/2))-c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],
[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(1+m-2*n)-b^2*(1+m-n)+b
*(1+m-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/n/(b+(-4*a*c+b^2)
^(1/2))
```

Rubi [A] time = 0.96, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1384, 1560, 364}

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c
*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c]
- b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*d*(1 + m)*n) - (c*(4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2
 - 4*a*c]*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1384

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p + 1)
 + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
```

```
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(dx)^m\left(-2ac(1+m-2n)+b^2(1+m-n)+bc(1+m-n)x^n\right)}{a+bx^n+cx^{2n}}\,dx}{a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \left(\frac{\left(bc(1+m-n)+\frac{c\left(b^2-4ac+b^2m-4acm-b^2n+8acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n} + \frac{\left(bc(1 \cdots\right.}{\cdots}\right.}{a\left(b^2-4ac\right) \cdots}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{\left(c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4 \cdots}\right.\right.}{a\left(b^2-4ac\right)^{3/2} \cdots}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4a \cdots}\right.}{a\left(b^2-4ac\right)^{3/2}\left(b \cdots\right.}$$

Mathematica [B] time = 6.36, size = 3515, normalized size = 10.72

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(d*x)^m*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*(a + b*x^n + c*x^(
2*n))) - (b*c*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n - (1 + m + n)/n)*(-(((x^n/
(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[
-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c
*(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]) + ((x^n/(-1/
2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1
 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1
/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]))/(a*(-b^2 + 4*a*
c)*(1 + m)) + (b*c*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n - (1 + m + n)/n)*(-((
(x^n/(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometri
c2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c
])/(c*(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]) + ((x^n
/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1
[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(
c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]))/(a*(-b^2 +
 4*a*c)*(1 + m)*n) + (b*c*m*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n - (1 + m + n
)/n)*(-(((x^n/(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hype
rgeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^
2 - 4*a*c])/(c*(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]
) + ((x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeo
metric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b + Sqrt[b^2 -
4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]))/(
a*(-b^2 + 4*a*c)*(1 + m)*n) + (b^2*x*(d*x)^m*((1 - (x^n/(-1/2*(-b - Sqrt[b^
2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b - Sqrt[b
^2 - 4*a*c])/c + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b
```

```
^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^
(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n
, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))
])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c)))
)/(a*(-b^2 + 4*a*c)*(1 + m)) - (4*c*x*(d*x)^m*((1 - (x^n/(-1/2*(-b - Sqrt[b
^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1
 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b - Sqrt[
b^2 - 4*a*c])/c + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[
b^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))
^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/
n, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n)
)])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))
))/((-b^2 + 4*a*c)*(1 + m)) - (b^2*x*(d*x)^m*((1 - (x^n/(-1/2*(-b - Sqrt[b^
2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b - Sqrt[b
^2 - 4*a*c])/c + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b
^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^
(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n
, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))
])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c)))
)/(a*(-b^2 + 4*a*c)*(1 + m)*n) + (2*c*x*(d*x)^m*((1 - (x^n/(-1/2*(-b - Sqrt
[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -(
(1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b - Sqr
t[b^2 - 4*a*c])/c + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqr
t[b^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n
))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m
)/n, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^
n))])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c
))))/((-b^2 + 4*a*c)*(1 + m)*n) - (b^2*m*x*(d*x)^m*((1 - (x^n/(-1/2*(-b - S
qrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n),
 -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b -
Sqrt[b^2 - 4*a*c])/c + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b -
Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c +
x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1
+ m)/n, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c +
 x^n))])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(
2*c))))/(a*(-b^2 + 4*a*c)*(1 + m)*n) + (2*c*m*x*(d*x)^m*((1 - (x^n/(-1/2*(-
b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m
)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(
-b - Sqrt[b^2 - 4*a*c])/c + x^n))])/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (
-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - (x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])
/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1
- (1 + m)/n, -1/2*(-b + Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c]
)/c + x^n))])/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c]
)^2/(2*c))))/((-b^2 + 4*a*c)*(1 + m)*n)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{c^2x^{4n} + b^2x^{2n} + 2\,abx^n + a^2 + 2\,(bcx^n + ac)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n
+ a*c)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(cx^{2n} + bx^n + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b\,x^n + c\,x^{2n} + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^2,x)
[Out] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcd^m x e^{(m\log(x)+n\log(x))} + (b^2 d^m - 2\,acd^m)xx^m}{a^2b^2n - 4\,a^3cn + (ab^2cn - 4\,a^2c^2n)x^{2n} + (ab^3n - 4\,a^2bcn)x^n} + \int -\frac{bcd^m(m-n+1)e^{(m\log(x)+n\log(x))} + (b^2d^m(}{a^2b^2n - 4\,a^3cn + (ab^2cn - 4\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
[Out] (b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)/(a^2*b^2*
n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)
*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*log(x) + n*log(x)) + (b^2*d^m*
(m - n + 1) - 2*a*c*d^m*(m - 2*n + 1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2
*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{(a + b\,x^n + c\,x^{2\,n})^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^2,x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
[Out] Timed out
```

3.601 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^3} dx$

Optimal. Leaf size=615

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)+6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b}\right.}{2a^2d(m+1)n^2\left(b^2\right.}$$

```
[Out] 1/2*(d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))^
2-1/2*(d*x)^(1+m)*(4*a^2*c^2*(1+m-4*n)-5*a*b^2*c*(1+m-3*n)+b^4*(1+m-2*n)-b*
c*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/d/n^2/(a+b*x^n+
c*x^(2*n))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))*(-b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2*c*(1+m^2+m*
(2-4*n)-4*n+3*n^2)-8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b*(2*a*c*(2+2*m-7*
n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/d/(1+m
)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m
+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*
a*b^2*c*(1+m^2+m*(2-4*n)-4*n+3*n^2)+8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b
*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(5/2)/d/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 10.72, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1384, 1558, 1560, 364}

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)+6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b(m-n+1)\sqrt{b}\right.}{2a^2d(m+1)n^2\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
 c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n) - 5*a*b^2*c*(1 + m
 - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n
))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt
[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) -
 4*n + 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)^(1 +
 m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n
^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(
1 + m - n) + b^4*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2
 + m*(2 - 4*n) - 4*n + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*
n^2))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m)*n^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1384

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p + 1)
 + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[p + 1, 0]
```

Rule 1558

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^3}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(dx)^m\left(-2ac(1+m-4n)+b^2(1+m-2n)+bc(1+m-3n)x^n\right)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx}{2a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-\cdots\right.}{2a^2\left(b^2\cdots\right.}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-\cdots\right.}{2a^2\left(b^2\cdots\right.}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-\cdots\right.}{2a^2\left(b^2\cdots\right.}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-\cdots\right.}{2a^2\left(b^2\cdots\right.}$$

Mathematica [B] time = 6.86, size = 12289, normalized size = 19.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{c^3x^{6n}+b^3x^{3n}+3\,ab^2x^{2n}+3\,a^2bx^n+a^3+3\left(bc^2x^n+ac^2\right)x^{4n}+3\left(b^2cx^{2n}+2\,abcx^n+a^2c\right)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
 + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a
^2*c)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\left(cx^{2n}+bx^n+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{(dx)^m}{\left(b\,x^n+c\,x^{2n}+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^3,x)
```

```
[Out] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(a^2b^2cd^m(5\,m-21\,n+5)-ab^4d^m(m-3\,n+1)-4\,a^3c^2d^m(m-6\,n+1)\right)xx^m+\left(2\,abc^3d^m(2\,m-7\,n+2)-b^3c^2d^{\prime}\right.}{2\left(a^4b^4n^2-8\,a^5b^2cn^2+16\,a^6c^2n^2+\left(a^2b^4c^2n^2-8\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*b^2*c*d^m*(5*m - 21*n + 5) - a*b^4*d^m*(m - 3*n + 1) - 4*a^3*c^2*
d^m*(m - 6*n + 1))*x*x^m + (2*a*b*c^3*d^m*(2*m - 7*n + 2) - b^3*c^2*d^m*(m
- 2*n + 1))*x*e^(m*log(x) + 3*n*log(x)) + (a*b^2*c^2*d^m*(9*m - 29*n + 9) -
 2*b^4*c*d^m*(m - 2*n + 1) - 4*a^2*c^3*d^m*(m - 4*n + 1))*x*e^(m*log(x) + 2
*n*log(x)) - (b^5*d^m*(m - 2*n + 1) - 4*a*b^3*c*d^m*(m - 3*n + 1) + 2*a^2*b
*c^2*d^m*n)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*
a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n
) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2
*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a
^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*(((m^2 - m*(3*n - 2)
 + 2*n^2 - 3*n + 1)*b^4*d^m - (5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*a
*b^2*c*d^m + 4*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*d^m)*x^m + (
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d^m - 2*(2*m^2 - m*(9*n - 4) +
```

```
7*n^2 - 9*n + 2)*a*b*c^2*d^m)*e^(m*log(x) + n*log(x)))/(a^3*b^4*n^2 - 8*a^4
*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c
^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{\left(a+b\,x^n+c\,x^{2\,n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
[Out] Timed out
```

3.602 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1((1+m)/n,-3/2,-3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/
(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/2
, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int (dx)^m \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1+m}{n};-\frac{3}{2},-\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 3.21, size = 618, normalized size = 3.84

$$\frac{x(dx)^m\left((m+1)\left(2(m+n+1)\left(4a^2c\left(m^2+m(6n+2)+8n^2+6n+1\right)+a\left(3b^2n^2+2bc\left(4m^2+m(21n+8)+\right.\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-6*a*n^2*(1 + m + n)*(b^2*(1 + m) - 4*a*c*(1 + m + 2*n))*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2
, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*(4*a^2*c*(1 + m^2 + 6*n + 8*n^2 + m*
(2 + 6*n)) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c*(2 + 2*m^2 + 9*n + 7*n^2 +
m*(4 + 9*n))*x^n + 4*c^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n))*x^(2*n)) + a
*(3*b^2*n^2 + 2*b*c*(4 + 4*m^2 + 21*n + 23*n^2 + m*(8 + 21*n))*x^n + 4*c^2*
(2 + 2*m^2 + 9*n + 10*n^2 + m*(4 + 9*n))*x^(2*n))) - 3*b*n^2*(b^2*(2 + 2*m
+ n) - 4*a*c*(2 + 2*m + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + m)
*(1 + m + n)^2*(1 + m + 2*n)*(1 + m + 3*n)*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n} + a\right)^{\frac{3}{2}} (dx)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{m} \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.603 $\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,-1/2,-1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/(1
+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
(1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx &= \frac{\sqrt{a+bx^n+cx^{2n}}\int (dx)^m\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1+m}{n};-\frac{1}{2},-\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [B] time = 0.69, size = 388, normalized size = 2.42

$$\frac{x(dx)^m\left(2an(m+n+1)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+(m+1\right.}{2(m+1)(m+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(2*a*n*(1 + m + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*
(a + x^n*(b + c*x^n)) + b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(2*(1 + m)*(1
+ m + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^{2n}+bx^n+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int\sqrt{b\,x^n+c\,x^{2n}+a}\;(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^{2n}+bx^n+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\,\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.604 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,1/2,1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/
2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(a+b*x^n+c*x^(2*n))^(1/
2)
```

Rubi [A] time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(
1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 183, normalized size = 1.14

$$\frac{x(dx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b\,x^n+c\,x^{2n}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.605 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=163

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,3/2,3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/
2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(a+b*x^n+c*x^(2*n))^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*d
*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{3}{2},\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [B] time = 1.46, size = 428, normalized size = 2.63

$$x(dx)^m\left((m+n+1)\left(b^2(2m-n+2)-4ac(m-n+1)\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};\cdot\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((-4*a*c*(1 + m - n) + b^2*(2 + 2*m - n))*(1 + m + n)*Sqrt[(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1
 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] - 2*(1 + m)*((1 + m + n)*(b^2 - 2*a*c + b*c*x^n) - b*c*(1 + m)*x^
n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)
/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*(1 + m)*n*(1 + m + n)*Sqrt
[a + x^n*(b + c*x^n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\,x^n+c\,x^{2n}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int((d*x)^m/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.606 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=158

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p dx = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (dx)^m \left(1+\right.$$

$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\right.}{d(1+m)}$$

Mathematica [A] time = 0.36, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+x^n\left(b+cx^n\right)\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(d*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^{2n} + bx^{n} + a\right)^{p} (dx)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n} + a\right)^{p} (dx)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^{m} \left(b\,x^{n} + c\,x^{2n} + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^p,x)
```

```
[Out] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n} + a\right)^{p} (dx)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{m} \left(a + b\,x^{n} + c\,x^{2\,n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.607 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal. Leaf size=46

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

```
[Out] 1/4*a*(e*x+d)^4/e+1/6*b*(e*x+d)^6/e+1/8*c*(e*x+d)^8/e
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1142, 14}

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right) dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) dx, x, d+ex\right)}{e}\\ &= \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}\end{aligned}$$

Mathematica [B] time = 0.04, size = 150, normalized size = 3.26

$$\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3
*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/
4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c
*e^7*x^8)/8
```

fricas [B] time = 0.78, size = 175, normalized size = 3.80

$$\frac{1}{8}x^8e^7c+x^7e^6dc+\frac{7}{2}x^6e^5d^2c+7x^5e^4d^3c+\frac{35}{4}x^4e^3d^4c+\frac{1}{6}x^6e^5b+7x^3e^2d^5c+x^5e^4db+\frac{7}{2}x^2ed^6c+\frac{5}{2}x^4e^3d^2b+xd^7c+\frac{10}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^7*c + x^7*e^6*d*c + 7/2*x^6*e^5*d^2*c + 7*x^5*e^4*d^3*c + 35/4*x^
4*e^3*d^4*c + 1/6*x^6*e^5*b + 7*x^3*e^2*d^5*c + x^5*e^4*d*b + 7/2*x^2*e*d^6
*c + 5/2*x^4*e^3*d^2*b + x*d^7*c + 10/3*x^3*e^2*d^3*b + 5/2*x^2*e*d^4*b + 1
/4*x^4*e^3*a + x*d^5*b + x^3*e^2*d*a + 3/2*x^2*e*d^2*a + x*d^3*a
```

giac [B] time = 0.41, size = 169, normalized size = 3.67

$$\frac{1}{2}\left(x^2e+2\,dx\right)cd^6+\frac{3}{4}\left(x^2e+2\,dx\right)^2cd^4e+\frac{1}{2}\left(x^2e+2\,dx\right)^3cd^2e^2+\frac{1}{2}\left(x^2e+2\,dx\right)bd^4+\frac{1}{8}\left(x^2e+2\,dx\right)^4ce^3+\frac{1}{2}\left(x^2e+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c*d^6 + 3/4*(x^2*e + 2*d*x)^2*c*d^4*e + 1/2*(x^2*e + 2*
d*x)^3*c*d^2*e^2 + 1/2*(x^2*e + 2*d*x)*b*d^4 + 1/8*(x^2*e + 2*d*x)^4*c*e^3
+ 1/2*(x^2*e + 2*d*x)^2*b*d^2*e + 1/6*(x^2*e + 2*d*x)^3*b*e^2 + 1/2*(x^2*e
+ 2*d*x)*a*d^2 + 1/4*(x^2*e + 2*d*x)^2*a*e
```

maple [B] time = 0.00, size = 298, normalized size = 6.48

$$\frac{c\,e^7x^8}{8}+cd\,e^6x^7+\frac{\left(15c\,d^2e^5+\left(6c\,d^2e^2+b\,e^2\right)e^3\right)x^6}{6}+\frac{\left(13c\,d^3e^4+3\left(6c\,d^2e^2+b\,e^2\right)d\,e^2+\left(4c\,d^3e+2deb\right)e^3\right)x^5}{5}+\left(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*c*x^8+d*e^6*c*x^7+1/6*(15*d^2*e^5*c+e^3*(6*c*d^2*e^2+b*e^2))*x^6+1/
5*(13*d^3*c*e^4+3*d*e^2*(6*c*d^2*e^2+b*e^2)+e^3*(4*c*d^3*e+2*b*d*e))*x^5+1/
4*(4*d^4*c*e^3+3*d^2*e*(6*c*d^2*e^2+b*e^2)+3*d*e^2*(4*c*d^3*e+2*b*d*e)+e^3*
(c*d^4+b*d^2+a))*x^4+1/3*(d^3*(6*c*d^2*e^2+b*e^2)+3*d^2*e*(4*c*d^3*e+2*b*d*
e)+3*d*e^2*(c*d^4+b*d^2+a))*x^3+1/2*(d^3*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4
+b*d^2+a))*x^2+d^3*(c*d^4+b*d^2+a)*x
```

maxima [B] time = 1.02, size = 142, normalized size = 3.09

$$\frac{1}{8}ce^7x^8+cde^6x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5x^6+\left(7\,cd^3+bd\right)e^4x^5+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)*
e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3
+ 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*d^5
 + a*d^3)*x
```

mupad [B] time = 0.08, size = 141, normalized size = 3.07

$$x\left(c\,d^7+b\,d^5+a\,d^3\right)+\frac{e^5\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,x^8}{8}+\frac{e^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}+\frac{d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] x*(a*d^3 + b*d^5 + c*d^7) + (e^5*x^6*(b + 21*c*d^2))/6 + (c*e^7*x^8)/8 + (e
^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*x^2*(3*a + 5*b*d^2 + 7*c*d^4))
/2 + (d*e^2*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*e^4*x^5*(b + 7*c*d^2) +
c*d*e^6*x^7
```

sympy [B] time = 0.11, size = 178, normalized size = 3.87

$$cde^6x^7+\frac{ce^7x^8}{8}+x^6\left(\frac{be^5}{6}+\frac{7cd^2e^5}{2}\right)+x^5\left(bde^4+7cd^3e^4\right)+x^4\left(\frac{ae^3}{4}+\frac{5bd^2e^3}{2}+\frac{35cd^4e^3}{4}\right)+x^3\left(ade^2+\frac{10bd^3e^2}{3}+7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(b
*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e**
3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d**2*
e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```

3.608 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$

Optimal. Leaf size=89

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

```
[Out] 1/4*a^2*(e*x+d)^4/e+1/3*a*b*(e*x+d)^6/e+1/8*(2*a*c+b^2)*(e*x+d)^8/e+1/5*b*c
*(e*x+d)^10/e+1/12*c^2*(e*x+d)^12/e
```

Rubi [A] time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1142, 1114, 631}

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x
)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule steps

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^2 dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int x\left(a+bx+cx^2\right)^2 dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right) dx, x, (d\right.}{2e}\\ &= \frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{\left(b^2+2ac\right)(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}\end{aligned}$$

Mathematica [B] time = 0.11, size = 401, normalized size = 4.51

$$\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4+72\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c
*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*
d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2
 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10
*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((
2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d
*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d
^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(2*b +
 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12
```

fricas [B] time = 0.67, size = 571, normalized size = 6.42

$$\frac{1}{12}x^{12}e^{11}c^2+x^{11}e^{10}dc^2+\frac{11}{2}x^{10}e^9d^2c^2+\frac{55}{3}x^9e^8d^3c^2+\frac{165}{4}x^8e^7d^4c^2+\frac{1}{5}x^{10}e^9cb+66x^7e^6d^5c^2+2x^9e^8dcb+77x^6e^5d^6c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^11*c^2 + x^11*e^10*d*c^2 + 11/2*x^10*e^9*d^2*c^2 + 55/3*x^9*e^8
*d^3*c^2 + 165/4*x^8*e^7*d^4*c^2 + 1/5*x^10*e^9*c*b + 66*x^7*e^6*d^5*c^2 +
2*x^9*e^8*d*c*b + 77*x^6*e^5*d^6*c^2 + 9*x^8*e^7*d^2*c*b + 66*x^5*e^4*d^7*c
^2 + 24*x^7*e^6*d^3*c*b + 165/4*x^4*e^3*d^8*c^2 + 42*x^6*e^5*d^4*c*b + 1/8*
x^8*e^7*b^2 + 1/4*x^8*e^7*c*a + 55/3*x^3*e^2*d^9*c^2 + 252/5*x^5*e^4*d^5*c*
b + x^7*e^6*d*b^2 + 2*x^7*e^6*d*c*a + 11/2*x^2*e*d^10*c^2 + 42*x^4*e^3*d^6*
c*b + 7/2*x^6*e^5*d^2*b^2 + 7*x^6*e^5*d^2*c*a + x*d^11*c^2 + 24*x^3*e^2*d^7
*c*b + 7*x^5*e^4*d^3*b^2 + 14*x^5*e^4*d^3*c*a + 9*x^2*e*d^8*c*b + 35/4*x^4*
e^3*d^4*b^2 + 35/2*x^4*e^3*d^4*c*a + 1/3*x^6*e^5*b*a + 2*x*d^9*c*b + 7*x^3*
e^2*d^5*b^2 + 14*x^3*e^2*d^5*c*a + 2*x^5*e^4*d*b*a + 7/2*x^2*e*d^6*b^2 + 7*
x^2*e*d^6*c*a + 5*x^4*e^3*d^2*b*a + x*d^7*b^2 + 2*x*d^7*c*a + 20/3*x^3*e^2*
d^3*b*a + 5*x^2*e*d^4*b*a + 1/4*x^4*e^3*a^2 + 2*x*d^5*b*a + x^3*e^2*d*a^2 +
 3/2*x^2*e*d^2*a^2 + x*d^3*a^2
```

giac [B] time = 0.41, size = 493, normalized size = 5.54

$$\frac{1}{2}\left(x^2e+2\,dx\right)c^2d^{10}+\frac{5}{4}\left(x^2e+2\,dx\right)^2c^2d^8e+\frac{5}{3}\left(x^2e+2\,dx\right)^3c^2d^6e^2+\left(x^2e+2\,dx\right)bcd^8+\frac{5}{4}\left(x^2e+2\,dx\right)^4c^2d^4e^3+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c^2*d^10 + 5/4*(x^2*e + 2*d*x)^2*c^2*d^8*e + 5/3*(x^2*e
 + 2*d*x)^3*c^2*d^6*e^2 + (x^2*e + 2*d*x)*b*c*d^8 + 5/4*(x^2*e + 2*d*x)^4*c
^2*d^4*e^3 + 2*(x^2*e + 2*d*x)^2*b*c*d^6*e + 1/2*(x^2*e + 2*d*x)^5*c^2*d^2*
e^4 + 2*(x^2*e + 2*d*x)^3*b*c*d^4*e^2 + 1/2*(x^2*e + 2*d*x)*b^2*d^6 + (x^2*
e + 2*d*x)*a*c*d^6 + 1/12*(x^2*e + 2*d*x)^6*c^2*e^5 + (x^2*e + 2*d*x)^4*b*c
*d^2*e^3 + 3/4*(x^2*e + 2*d*x)^2*b^2*d^4*e + 3/2*(x^2*e + 2*d*x)^2*a*c*d^4*
e + 1/5*(x^2*e + 2*d*x)^5*b*c*e^4 + 1/2*(x^2*e + 2*d*x)^3*b^2*d^2*e^2 + (x^
2*e + 2*d*x)^3*a*c*d^2*e^2 + (x^2*e + 2*d*x)*a*b*d^4 + 1/8*(x^2*e + 2*d*x)^
4*b^2*e^3 + 1/4*(x^2*e + 2*d*x)^4*a*c*e^3 + (x^2*e + 2*d*x)^2*a*b*d^2*e + 1
/3*(x^2*e + 2*d*x)^3*a*b*e^2 + 1/2*(x^2*e + 2*d*x)*a^2*d^2 + 1/4*(x^2*e + 2
*d*x)^2*a^2*e
```

maple [B] time = 0.00, size = 1314, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*c^2*x^12+d*e^10*c^2*x^11+1/10*(27*d^2*e^9*c^2+e^3*(2*(6*c*d^2*e^2
+b*e^2)*c*e^4+16*c^2*d^2*e^6))*x^10+1/9*(25*d^3*c^2*e^8+3*d*e^2*(2*(6*c*d^2
*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+e^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d
^2*e^2+b*e^2)*c*d*e^3))*x^9+1/8*(8*d^4*c^2*e^7+3*d^2*e*(2*(6*c*d^2*e^2+b*e^
2)*c*e^4+16*c^2*d^2*e^6)+3*d*e^2*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^
2+b*e^2)*c*d*e^3)+e^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^
3+(6*c*d^2*e^2+b*e^2)^2))*x^8+1/7*(d^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*
d^2*e^6)+3*d^2*e*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3
)+3*d*e^2*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e
^2+b*e^2)^2)+e^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*
e^2+b*e^2)))*x^7+1/6*(d^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2
)*c*d*e^3)+3*d^2*e*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(
6*c*d^2*e^2+b*e^2)^2)+3*d*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d
*e)*(6*c*d^2*e^2+b*e^2))+e^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^
3*e+2*b*d*e)^2))*x^6+1/5*(d^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e
)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*e*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*
d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*e^2*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e))*x
^5+1/4*(d^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b
*e^2))+3*d^2*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2
)+6*d*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a)^2)*x^4+1/
3*(d^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*
e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2
*d^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a)^2)*x^2+d^3
*(c*d^4+b*d^2+a)^2*x
```

maxima [B] time = 1.06, size = 403, normalized size = 4.53

$$\frac{1}{12}c^2e^{11}x^{12}+c^2de^{10}x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8x^9+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10 +
 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 +
 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 + 1/6
*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(
330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*
(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x
^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a
^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*
b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*
b*d^5 + a^2*d^3)*x
```

mupad [B] time = 1.48, size = 383, normalized size = 4.30

$$\frac{e^7\,x^8\,\left(b^2+72\,b\,c\,d^2+330\,c^2\,d^4+2\,a\,c\right)}{8}+\frac{e^5\,x^6\,\left(21\,b^2\,d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2\,d^6+42\,a\,c\,d^2\right)}{6}+\frac{e^3\,x^4\,\left(a^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^7*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*x^6*(2*a*b + 21
*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (e^3*x^4*(a^2 + 35*
b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 168*b*c*d^6))/4 + (c^2*e^
11*x^12)/12 + d^3*x*(a + b*d^2 + c*d^4)^2 + (c*e^9*x^10*(2*b + 55*c*d^2))/1
0 + c^2*d*e^10*x^11 + (d^2*e*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d
^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*
d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*x^7*(2*a*c + b^2 + 6
6*c^2*d^4 + 24*b*c*d^2) + (d*e^4*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 7
0*a*c*d^2 + 252*b*c*d^4))/5 + (c*d*e^8*x^9*(6*b + 55*c*d^2))/3
```

sympy [B] time = 0.19, size = 559, normalized size = 6.28

$$c^2de^{10}x^{11}+\frac{c^2e^{11}x^{12}}{12}+x^{10}\left(\frac{bce^9}{5}+\frac{11c^2d^2e^9}{2}\right)+x^9\left(2bcde^8+\frac{55c^2d^3e^8}{3}\right)+x^8\left(\frac{ace^7}{4}+\frac{b^2e^7}{8}+9bcd^2e^7+\frac{165c^2d^4e^7}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**2
*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 + b
**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**6 +
 b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7
*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5)
 + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*
e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35*a*c*
d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**8*e**3/4
) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*b**2*d**5
*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d**2*e/2 + 5
*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10
*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c
**2*d**11)
```

3.609 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$

Optimal. Leaf size=138

$$\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}$$

```
[Out] 1/4*a^3*(e*x+d)^4/e+1/2*a^2*b*(e*x+d)^6/e+3/8*a*(a*c+b^2)*(e*x+d)^8/e+1/10*
b*(6*a*c+b^2)*(e*x+d)^10/e+1/4*c*(a*c+b^2)*(e*x+d)^12/e+3/14*b*c^2*(e*x+d)^
14/e+1/16*c^3*(e*x+d)^16/e
```

Rubi [A] time = 0.37, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1142, 1114, 631}

$$\frac{a^2b(d+ex)^6}{2e}+\frac{a^3(d+ex)^4}{4e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d +
 e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d
+ e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e
)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx=\frac{\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^3 dx,x,d+ex\right)}{e}$$

$$=\frac{\text{Subst}\left(\int x\left(a+bx+cx^2\right)^3 dx,x,(d+ex)^2\right)}{2e}$$

$$=\frac{\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4+3c\left(\right.\right.\right.}{2e}$$

$$=\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)(d+ex)^8}{8e}+\frac{b\left(b^2+6a\right.}{1}$$

Mathematica [B] time = 0.28, size = 797, normalized size = 5.78

$$\frac{1}{16}c^3e^{15}x^{16}+c^3de^{14}x^{15}+\frac{3}{14}c^2\left(35cd^2+b\right)e^{13}x^{14}+c^2d\left(35cd^2+3b\right)e^{12}x^{13}+\frac{1}{4}c\left(455c^2d^4+78bcd^2+b^2+ac\right)e^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5
*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12
*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*
c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 +
84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10
 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 +
 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8
 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^
3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 10
01*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*
d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*
x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 33
0*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c
+ 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^
3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6
)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c
*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^
2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3
*e^15*x^16)/16
```

fricas [B] time = 0.77, size = 1335, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^15*c^3 + x^15*e^14*d*c^3 + 15/2*x^14*e^13*d^2*c^3 + 35*x^13*e^1
2*d^3*c^3 + 455/4*x^12*e^11*d^4*c^3 + 3/14*x^14*e^13*c^2*b + 273*x^11*e^10*
d^5*c^3 + 3*x^13*e^12*d*c^2*b + 1001/2*x^10*e^9*d^6*c^3 + 39/2*x^12*e^11*d^
2*c^2*b + 715*x^9*e^8*d^7*c^3 + 78*x^11*e^10*d^3*c^2*b + 6435/8*x^8*e^7*d^8
*c^3 + 429/2*x^10*e^9*d^4*c^2*b + 1/4*x^12*e^11*c*b^2 + 1/4*x^12*e^11*c^2*a
 + 715*x^7*e^6*d^9*c^3 + 429*x^9*e^8*d^5*c^2*b + 3*x^11*e^10*d*c*b^2 + 3*x^
11*e^10*d*c^2*a + 1001/2*x^6*e^5*d^10*c^3 + 1287/2*x^8*e^7*d^6*c^2*b + 33/2
*x^10*e^9*d^2*c*b^2 + 33/2*x^10*e^9*d^2*c^2*a + 273*x^5*e^4*d^11*c^3 + 5148
/7*x^7*e^6*d^7*c^2*b + 55*x^9*e^8*d^3*c*b^2 + 55*x^9*e^8*d^3*c^2*a + 455/4*
x^4*e^3*d^12*c^3 + 1287/2*x^6*e^5*d^8*c^2*b + 495/4*x^8*e^7*d^4*c*b^2 + 1/1
0*x^10*e^9*b^3 + 495/4*x^8*e^7*d^4*c^2*a + 3/5*x^10*e^9*c*b*a + 35*x^3*e^2*
d^13*c^3 + 429*x^5*e^4*d^9*c^2*b + 198*x^7*e^6*d^5*c*b^2 + x^9*e^8*d*b^3 +
198*x^7*e^6*d^5*c^2*a + 6*x^9*e^8*d*c*b*a + 15/2*x^2*e*d^14*c^3 + 429/2*x^4
*e^3*d^10*c^2*b + 231*x^6*e^5*d^6*c*b^2 + 9/2*x^8*e^7*d^2*b^3 + 231*x^6*e^5
*d^6*c^2*a + 27*x^8*e^7*d^2*c*b*a + x*d^15*c^3 + 78*x^3*e^2*d^11*c^2*b + 19
8*x^5*e^4*d^7*c*b^2 + 12*x^7*e^6*d^3*b^3 + 198*x^5*e^4*d^7*c^2*a + 72*x^7*e
^6*d^3*c*b*a + 39/2*x^2*e*d^12*c^2*b + 495/4*x^4*e^3*d^8*c*b^2 + 21*x^6*e^5
*d^4*b^3 + 495/4*x^4*e^3*d^8*c^2*a + 126*x^6*e^5*d^4*c*b*a + 3/8*x^8*e^7*b^
2*a + 3/8*x^8*e^7*c*a^2 + 3*x*d^13*c^2*b + 55*x^3*e^2*d^9*c*b^2 + 126/5*x^5
*e^4*d^5*b^3 + 55*x^3*e^2*d^9*c^2*a + 756/5*x^5*e^4*d^5*c*b*a + 3*x^7*e^6*d
*b^2*a + 3*x^7*e^6*d*c*a^2 + 33/2*x^2*e*d^10*c*b^2 + 21*x^4*e^3*d^6*b^3 + 3
3/2*x^2*e*d^10*c^2*a + 126*x^4*e^3*d^6*c*b*a + 21/2*x^6*e^5*d^2*b^2*a + 21/
2*x^6*e^5*d^2*c*a^2 + 3*x*d^11*c*b^2 + 12*x^3*e^2*d^7*b^3 + 3*x*d^11*c^2*a
+ 72*x^3*e^2*d^7*c*b*a + 21*x^5*e^4*d^3*b^2*a + 21*x^5*e^4*d^3*c*a^2 + 9/2*
```

```
x^2*e*d^8*b^3 + 27*x^2*e*d^8*c*b*a + 105/4*x^4*e^3*d^4*b^2*a + 105/4*x^4*e^
3*d^4*c*a^2 + 1/2*x^6*e^5*b*a^2 + x*d^9*b^3 + 6*x*d^9*c*b*a + 21*x^3*e^2*d^
5*b^2*a + 21*x^3*e^2*d^5*c*a^2 + 3*x^5*e^4*d*b*a^2 + 21/2*x^2*e*d^6*b^2*a +
 21/2*x^2*e*d^6*c*a^2 + 15/2*x^4*e^3*d^2*b*a^2 + 3*x*d^7*b^2*a + 3*x*d^7*c*
a^2 + 10*x^3*e^2*d^3*b*a^2 + 15/2*x^2*e*d^4*b*a^2 + 1/4*x^4*e^3*a^3 + 3*x*d
^5*b*a^2 + x^3*e^2*d*a^3 + 3/2*x^2*e*d^2*a^3 + x*d^3*a^3
```

giac [B] time = 0.62, size = 1109, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c^3*d^14 + 7/4*(x^2*e + 2*d*x)^2*c^3*d^12*e + 7/2*(x^2*
e + 2*d*x)^3*c^3*d^10*e^2 + 3/2*(x^2*e + 2*d*x)*b*c^2*d^12 + 35/8*(x^2*e +
2*d*x)^4*c^3*d^8*e^3 + 9/2*(x^2*e + 2*d*x)^2*b*c^2*d^10*e + 7/2*(x^2*e + 2*
d*x)^5*c^3*d^6*e^4 + 15/2*(x^2*e + 2*d*x)^3*b*c^2*d^8*e^2 + 3/2*(x^2*e + 2*
d*x)*b^2*c*d^10 + 3/2*(x^2*e + 2*d*x)*a*c^2*d^10 + 7/4*(x^2*e + 2*d*x)^6*c^
3*d^4*e^5 + 15/2*(x^2*e + 2*d*x)^4*b*c^2*d^6*e^3 + 15/4*(x^2*e + 2*d*x)^2*b
^2*c*d^8*e + 15/4*(x^2*e + 2*d*x)^2*a*c^2*d^8*e + 1/2*(x^2*e + 2*d*x)^7*c^3
*d^2*e^6 + 9/2*(x^2*e + 2*d*x)^5*b*c^2*d^4*e^4 + 5*(x^2*e + 2*d*x)^3*b^2*c*
d^6*e^2 + 5*(x^2*e + 2*d*x)^3*a*c^2*d^6*e^2 + 1/2*(x^2*e + 2*d*x)*b^3*d^8 +
 3*(x^2*e + 2*d*x)*a*b*c*d^8 + 1/16*(x^2*e + 2*d*x)^8*c^3*e^7 + 3/2*(x^2*e
+ 2*d*x)^6*b*c^2*d^2*e^5 + 15/4*(x^2*e + 2*d*x)^4*b^2*c*d^4*e^3 + 15/4*(x^2
*e + 2*d*x)^4*a*c^2*d^4*e^3 + (x^2*e + 2*d*x)^2*b^3*d^6*e + 6*(x^2*e + 2*d*
x)^2*a*b*c*d^6*e + 3/14*(x^2*e + 2*d*x)^7*b*c^2*e^6 + 3/2*(x^2*e + 2*d*x)^5
*b^2*c*d^2*e^4 + 3/2*(x^2*e + 2*d*x)^5*a*c^2*d^2*e^4 + (x^2*e + 2*d*x)^3*b^
3*d^4*e^2 + 6*(x^2*e + 2*d*x)^3*a*b*c*d^4*e^2 + 3/2*(x^2*e + 2*d*x)*a*b^2*d
^6 + 3/2*(x^2*e + 2*d*x)*a^2*c*d^6 + 1/4*(x^2*e + 2*d*x)^6*b^2*c*e^5 + 1/4*
(x^2*e + 2*d*x)^6*a*c^2*e^5 + 1/2*(x^2*e + 2*d*x)^4*b^3*d^2*e^3 + 3*(x^2*e
+ 2*d*x)^4*a*b*c*d^2*e^3 + 9/4*(x^2*e + 2*d*x)^2*a*b^2*d^4*e + 9/4*(x^2*e +
 2*d*x)^2*a^2*c*d^4*e + 1/10*(x^2*e + 2*d*x)^5*b^3*e^4 + 3/5*(x^2*e + 2*d*x
)^5*a*b*c*e^4 + 3/2*(x^2*e + 2*d*x)^3*a*b^2*d^2*e^2 + 3/2*(x^2*e + 2*d*x)^3
*a^2*c*d^2*e^2 + 3/2*(x^2*e + 2*d*x)*a^2*b*d^4 + 3/8*(x^2*e + 2*d*x)^4*a*b^
2*e^3 + 3/8*(x^2*e + 2*d*x)^4*a^2*c*e^3 + 3/2*(x^2*e + 2*d*x)^2*a^2*b*d^2*e
 + 1/2*(x^2*e + 2*d*x)^3*a^2*b*e^2 + 1/2*(x^2*e + 2*d*x)*a^3*d^2 + 1/4*(x^2
*e + 2*d*x)^2*a^3*e
```

maple [B] time = 0.00, size = 7550, normalized size = 54.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.13, size = 872, normalized size = 6.32

$$\frac{1}{16}c^3e^{15}x^{16}+c^3de^{14}x^{15}+\frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}x^{14}+\left(35\,c^3d^3+3\,bc^2d\right)e^{12}x^{13}+\frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
+ (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
```

```
e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x^8 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
 a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*x^
4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*x^3 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

mupad [B] time = 1.66, size = 777, normalized size = 5.63

$$\frac{3\,e^7\,x^8\,\left(a^2\,c+a\,b^2+72\,a\,b\,c\,d^2+330\,a\,c^2\,d^4+12\,b^3\,d^2+330\,b^2\,c\,d^4+1716\,b\,c^2\,d^6+2145\,c^3\,d^8\right)}{8}+\frac{e^5\,x^6\,\left(a^2\,b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 330
*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*x^6*(a^2*b + 42*b^3*d
^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + 462*b^2*
c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*x^10*(b^3 + 5005*c^3*d^6
+ 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 + (c^3*e^15
*x^16)/16 + d^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*x^4*(a^3 + 84*b^3*d^6 + 455*
c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 495*a*c^2*d^8 + 4
95*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 + (3*c^2*e^13*x^14*(b + 3
5*c*d^2))/14 + c^3*d*e^14*x^15 + d*e^2*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12
+ 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8
+ 78*b*c^2*d^10 + 72*a*b*c*d^6) + (c*e^11*x^12*(a*c + b^2 + 455*c^2*d^4 + 7
8*b*c*d^2))/4 + (d*e^6*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8
 + 1386*a*c^2*d^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (
3*d*e^4*x^5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 330*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d
*e^8*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 +
 6*a*b*c) + (3*d^2*e*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 +
 c^2*d*e^12*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*x^11*(a*c + b^2 + 91*c^2*d^4
 + 26*b*c*d^2)
```

sympy [B] time = 0.36, size = 1314, normalized size = 9.52

$$c^3de^{14}x^{15}+\frac{c^3e^{15}x^{16}}{16}+x^{14}\left(\frac{3bc^2e^{13}}{14}+\frac{15c^3d^2e^{13}}{2}\right)+x^{13}\left(3bc^2de^{12}+35c^3d^3e^{12}\right)+x^{12}\left(\frac{ac^2e^{11}}{4}+\frac{b^2ce^{11}}{4}+\frac{39bc^2d^2e}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/14 + 15*c*
*3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d**3*e**12) + x**12*(a
*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d**2*e**11/2 + 455*c**3*d**4*e**
11/4) + x**11*(3*a*c**2*d*e**10 + 3*b**2*c*d*e**10 + 78*b*c**2*d**3*e**10 +
 273*c**3*d**5*e**10) + x**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2 + b**
```

```
3*e**9/10 + 33*b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**3*d**6
*e**9/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8 + 55*b*
*2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8) + x**8*(3*a**2*
c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 + 495*a*c**2*d**4*e**7/4 +
9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**7/4 + 1287*b*c**2*d**6*e**7/2 + 643
5*c**3*d**8*e**7/8) + x**7*(3*a**2*c*d*e**6 + 3*a*b**2*d*e**6 + 72*a*b*c*d*
*3*e**6 + 198*a*c**2*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c*d**5*e**6 +
 5148*b*c**2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*e**5/2 + 21*a
**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e**5 + 231*a*c**
2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5 + 1287*b*c**2*d**8*e
**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**2*b*d*e**4 + 21*a**2*c*d**3*e*
*4 + 21*a*b**2*d**3*e**4 + 756*a*b*c*d**5*e**4/5 + 198*a*c**2*d**7*e**4 + 1
26*b**3*d**5*e**4/5 + 198*b**2*c*d**7*e**4 + 429*b*c**2*d**9*e**4 + 273*c**
3*d**11*e**4) + x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/2 + 105*a**2*c*d**4
*e**3/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e**3 + 495*a*c**2*d**8*e*
*3/4 + 21*b**3*d**6*e**3 + 495*b**2*c*d**8*e**3/4 + 429*b*c**2*d**10*e**3/2
 + 455*c**3*d**12*e**3/4) + x**3*(a**3*d*e**2 + 10*a**2*b*d**3*e**2 + 21*a*
*2*c*d**5*e**2 + 21*a*b**2*d**5*e**2 + 72*a*b*c*d**7*e**2 + 55*a*c**2*d**9*
e**2 + 12*b**3*d**7*e**2 + 55*b**2*c*d**9*e**2 + 78*b*c**2*d**11*e**2 + 35*
c**3*d**13*e**2) + x**2*(3*a**3*d**2*e/2 + 15*a**2*b*d**4*e/2 + 21*a**2*c*d
**6*e/2 + 21*a*b**2*d**6*e/2 + 27*a*b*c*d**8*e + 33*a*c**2*d**10*e/2 + 9*b*
*3*d**8*e/2 + 33*b**2*c*d**10*e/2 + 39*b*c**2*d**12*e/2 + 15*c**3*d**14*e/2
) + x*(a**3*d**3 + 3*a**2*b*d**5 + 3*a**2*c*d**7 + 3*a*b**2*d**7 + 6*a*b*c*
d**9 + 3*a*c**2*d**11 + b**3*d**9 + 3*b**2*c*d**11 + 3*b*c**2*d**13 + c**3*
d**15)
```

3.610 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal. Leaf size=55

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

```
[Out] 1/4*a*f^3*(e*x+d)^4/e+1/6*b*f^3*(e*x+d)^6/e+1/8*c*f^3*(e*x+d)^8/e
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1142, 14}

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)
/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx &= \frac{f^3 \operatorname{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right) dx, x, d+ex\right)}{e}\\
&= \frac{f^3 \operatorname{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) dx, x, d+ex\right)}{e}\\
&= \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 154, normalized size = 2.80

$$f^3\left(\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
(d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*
x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7
 + (c*e^7*x^8)/8)
```

fricas [B] time = 0.77, size = 229, normalized size = 4.16

$$\frac{1}{8}x^8f^3e^7c+x^7f^3e^6dc+\frac{7}{2}x^6f^3e^5d^2c+7x^5f^3e^4d^3c+\frac{35}{4}x^4f^3e^3d^4c+\frac{1}{6}x^6f^3e^5b+7x^3f^3e^2d^5c+x^5f^3e^4db+\frac{7}{2}x^2f^3ed^6c+\frac{5}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*f^3*e^7*c + x^7*f^3*e^6*d*c + 7/2*x^6*f^3*e^5*d^2*c + 7*x^5*f^3*e^4
*d^3*c + 35/4*x^4*f^3*e^3*d^4*c + 1/6*x^6*f^3*e^5*b + 7*x^3*f^3*e^2*d^5*c +
 x^5*f^3*e^4*d*b + 7/2*x^2*f^3*e*d^6*c + 5/2*x^4*f^3*e^3*d^2*b + x*f^3*d^7*
c + 10/3*x^3*f^3*e^2*d^3*b + 5/2*x^2*f^3*e*d^4*b + 1/4*x^4*f^3*e^3*a + x*f^
3*d^5*b + x^3*f^3*e^2*d*a + 3/2*x^2*f^3*e*d^2*a + x*f^3*d^3*a
```

giac [B] time = 0.30, size = 213, normalized size = 3.87

$$\frac{1}{2}\left(fx^2e+2\,dfx\right)cd^6f^2+\frac{1}{2}\left(fx^2e+2\,dfx\right)bd^4f^2+\frac{1}{2}\left(fx^2e+2\,dfx\right)ad^2f^2+\frac{18\left(fx^2e+2\,dfx\right)^2cd^4f^2e+12\left(fx^2e+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c*d^6*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b*d^4*f^2 + 1/2
*(f*x^2*e + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(f*x^2*e + 2*d*f*x)^2*c*d^4*f^2*e
 + 12*(f*x^2*e + 2*d*f*x)^3*c*d^2*f*e^2 + 12*(f*x^2*e + 2*d*f*x)^2*b*d^2*f^
2*e + 3*(f*x^2*e + 2*d*f*x)^4*c*e^3 + 4*(f*x^2*e + 2*d*f*x)^3*b*f*e^2 + 6*(
f*x^2*e + 2*d*f*x)^2*a*f^2*e)/f
```

maple [B] time = 0.00, size = 349, normalized size = 6.35

$$\frac{c\,e^7f^3x^8}{8}+cd\,e^6f^3x^7+\left(c\,d^4+b\,d^2+a\right)d^3f^3x+\frac{\left(15c\,d^2e^5f^3+\left(6c\,d^2e^2+b\,e^2\right)e^3f^3\right)x^6}{6}+\frac{\left(13c\,d^3e^4f^3+3\left(6c\,d^2e^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*f^3*c*x^8+d*f^3*e^6*c*x^7+1/6*(15*d^2*f^3*e^5*c+e^3*f^3*(6*c*d^2*e^
2+b*e^2))*x^6+1/5*(13*d^3*f^3*c*e^4+3*d*f^3*e^2*(6*c*d^2*e^2+b*e^2)+e^3*f^3
*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(4*d^4*f^3*c*e^3+3*d^2*f^3*e*(6*c*d^2*e^2+b*e
^2)+3*d*f^3*e^2*(4*c*d^3*e+2*b*d*e)+e^3*f^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*f
^3*(6*c*d^2*e^2+b*e^2)+3*d^2*f^3*e*(4*c*d^3*e+2*b*d*e)+3*d*f^3*e^2*(c*d^4+b
*d^2+a))*x^3+1/2*(d^3*f^3*(4*c*d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a))*
x^2+d^3*f^3*(c*d^4+b*d^2+a)*x
```

maxima [B] time = 1.08, size = 166, normalized size = 3.02

$$\frac{1}{8}ce^7f^3x^8+cde^6f^3x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5f^3x^6+\left(7\,cd^3+bd\right)e^4f^3x^5+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3f^3x^4+\frac{1}{3}\left(21\,cd^5+10\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*c
*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2
)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

mupad [B] time = 0.08, size = 164, normalized size = 2.98

$$\frac{e^5\,f^3\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,f^3\,x^8}{8}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)+\frac{e^3\,f^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,f^3\,x^2\,\left(7\,c\,d^4\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (e^5*f^3*x^6*(b + 21*c*d^2))/6 + (c*e^7*f^3*x^8)/8 + d^3*f^3*x*(a + b*d^2 +
 c*d^4) + (e^3*f^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*f^3*x^2*(3*a +
 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*f^3*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*
e^4*f^3*x^5*(b + 7*c*d^2) + c*d*e^6*f^3*x^7
```

sympy [B] time = 0.11, size = 240, normalized size = 4.36

$$cde^6f^3x^7+\frac{ce^7f^3x^8}{8}+x^6\left(\frac{be^5f^3}{6}+\frac{7cd^2e^5f^3}{2}\right)+x^5\left(bde^4f^3+7cd^3e^4f^3\right)+x^4\left(\frac{ae^3f^3}{4}+\frac{5bd^2e^3f^3}{2}+\frac{35cd^4e^3f^3}{4}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e*
*5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**3/
4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3 + 1
0*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*
d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*
f**3)
```

3.611 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

```
[Out] 1/4*a^2*f^3*(e*x+d)^4/e+1/3*a*b*f^3*(e*x+d)^6/e+1/8*(2*a*c+b^2)*f^3*(e*x+d)
^8/e+1/5*b*c*f^3*(e*x+d)^10/e+1/12*c^2*f^3*(e*x+d)^12/e
```

Rubi [A] time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1114, 631}

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*
f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^
12)/(12*e)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx &= \frac{f^3\,\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^2\,dx,x,d+ex\right)}{e}\\
&= \frac{f^3\,\text{Subst}\left(\int x\left(a+bx+cx^2\right)^2\,dx,x,(d+ex)^2\right)}{2e}\\
&= \frac{f^3\,\text{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right)dx,\right.}{2e}\\
&= \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{\left(b^2+2ac\right)f^3(d+ex)^8}{8e}+\frac{bc}{}
\end{aligned}$$

Mathematica [B] time = 0.07, size = 405, normalized size = 3.89

$$f^3\left(\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 1
4*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21
*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*
b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (
d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5
 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/
6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*
b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(
2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12)
```

fricas [B] time = 0.75, size = 715, normalized size = 6.88

$$\frac{1}{12}x^{12}f^3e^{11}c^2+x^{11}f^3e^{10}dc^2+\frac{11}{2}x^{10}f^3e^9d^2c^2+\frac{55}{3}x^9f^3e^8d^3c^2+\frac{165}{4}x^8f^3e^7d^4c^2+\frac{1}{5}x^{10}f^3e^9cb+66x^7f^3e^6d^5c^2+2x^9f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/12*x^12*f^3*e^11*c^2 + x^11*f^3*e^10*d*c^2 + 11/2*x^10*f^3*e^9*d^2*c^2 +
55/3*x^9*f^3*e^8*d^3*c^2 + 165/4*x^8*f^3*e^7*d^4*c^2 + 1/5*x^10*f^3*e^9*c*b
 + 66*x^7*f^3*e^6*d^5*c^2 + 2*x^9*f^3*e^8*d*c*b + 77*x^6*f^3*e^5*d^6*c^2 +
9*x^8*f^3*e^7*d^2*c*b + 66*x^5*f^3*e^4*d^7*c^2 + 24*x^7*f^3*e^6*d^3*c*b + 1
65/4*x^4*f^3*e^3*d^8*c^2 + 42*x^6*f^3*e^5*d^4*c*b + 1/8*x^8*f^3*e^7*b^2 + 1
/4*x^8*f^3*e^7*c*a + 55/3*x^3*f^3*e^2*d^9*c^2 + 252/5*x^5*f^3*e^4*d^5*c*b +
 x^7*f^3*e^6*d*b^2 + 2*x^7*f^3*e^6*d*c*a + 11/2*x^2*f^3*e*d^10*c^2 + 42*x^4
*f^3*e^3*d^6*c*b + 7/2*x^6*f^3*e^5*d^2*b^2 + 7*x^6*f^3*e^5*d^2*c*a + x*f^3*
d^11*c^2 + 24*x^3*f^3*e^2*d^7*c*b + 7*x^5*f^3*e^4*d^3*b^2 + 14*x^5*f^3*e^4*
d^3*c*a + 9*x^2*f^3*e*d^8*c*b + 35/4*x^4*f^3*e^3*d^4*b^2 + 35/2*x^4*f^3*e^3
*d^4*c*a + 1/3*x^6*f^3*e^5*b*a + 2*x*f^3*d^9*c*b + 7*x^3*f^3*e^2*d^5*b^2 +
14*x^3*f^3*e^2*d^5*c*a + 2*x^5*f^3*e^4*d*b*a + 7/2*x^2*f^3*e*d^6*b^2 + 7*x^
2*f^3*e*d^6*c*a + 5*x^4*f^3*e^3*d^2*b*a + x*f^3*d^7*b^2 + 2*x*f^3*d^7*c*a +
 20/3*x^3*f^3*e^2*d^3*b*a + 5*x^2*f^3*e*d^4*b*a + 1/4*x^4*f^3*e^3*a^2 + 2*x
*f^3*d^5*b*a + x^3*f^3*e^2*d*a^2 + 3/2*x^2*f^3*e*d^2*a^2 + x*f^3*d^3*a^2
```

giac [B] time = 0.43, size = 615, normalized size = 5.91

$$\frac{1}{2}\left(fx^2e+2\,dfx\right)c^2d^{10}f^2+\left(fx^2e+2\,dfx\right)bcd^8f^2+\frac{1}{2}\left(fx^2e+2\,dfx\right)b^2d^6f^2+\left(fx^2e+2\,dfx\right)acd^6f^2+\left(fx^2e+2\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^2*d^10*f^2 + (f*x^2*e + 2*d*f*x)*b*c*d^8*f^2 + 1/
2*(f*x^2*e + 2*d*f*x)*b^2*d^6*f^2 + (f*x^2*e + 2*d*f*x)*a*c*d^6*f^2 + (f*x^
2*e + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*a^2*d^2*f^2 + 1/120*(1
50*(f*x^2*e + 2*d*f*x)^2*c^2*d^8*f^4*e + 200*(f*x^2*e + 2*d*f*x)^3*c^2*d^6*
f^3*e^2 + 240*(f*x^2*e + 2*d*f*x)^2*b*c*d^6*f^4*e + 150*(f*x^2*e + 2*d*f*x)
^4*c^2*d^4*f^2*e^3 + 240*(f*x^2*e + 2*d*f*x)^3*b*c*d^4*f^3*e^2 + 90*(f*x^2*
e + 2*d*f*x)^2*b^2*d^4*f^4*e + 180*(f*x^2*e + 2*d*f*x)^2*a*c*d^4*f^4*e + 60
```

```
*(f*x^2*e + 2*d*f*x)^5*c^2*d^2*f*e^4 + 120*(f*x^2*e + 2*d*f*x)^4*b*c*d^2*f^
2*e^3 + 60*(f*x^2*e + 2*d*f*x)^3*b^2*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^
3*a*c*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^2*a*b*d^2*f^4*e + 10*(f*x^2*e +
 2*d*f*x)^6*c^2*e^5 + 24*(f*x^2*e + 2*d*f*x)^5*b*c*f*e^4 + 15*(f*x^2*e + 2*
d*f*x)^4*b^2*f^2*e^3 + 30*(f*x^2*e + 2*d*f*x)^4*a*c*f^2*e^3 + 40*(f*x^2*e +
 2*d*f*x)^3*a*b*f^3*e^2 + 30*(f*x^2*e + 2*d*f*x)^2*a^2*f^4*e)/f^3
```

maple [B] time = 0.00, size = 1413, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*f^3*c^2*x^12+d*f^3*e^10*c^2*x^11+1/10*(27*d^2*f^3*e^9*c^2+e^3*f^3
*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4))*x^10+1/9*(25*d^3*f^3*c^2*e^8
+3*d*f^3*e^2*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4)+e^3*f^3*(8*(6*c*d
^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4))*x^9+1/8*(8*d^4*f^3*c^2*
e^7+3*d^2*f^3*e*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4)+3*d*f^3*e^2*(8
*(6*c*d^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4)+e^3*f^3*(8*(4*c*d
^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+b*d^2+a)*c*e^4+(6*c*d^2*e^2+b*e^2)^2))*x^8+1
/7*(d^3*f^3*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4)+3*d^2*f^3*e*(8*(6*
c*d^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4)+3*d*f^3*e^2*(8*(4*c*d
^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+b*d^2+a)*c*e^4+(6*c*d^2*e^2+b*e^2)^2)+e^3*f^
3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)))*x^
7+1/6*(d^3*f^3*(8*(6*c*d^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4)+
3*d^2*f^3*e*(8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+b*d^2+a)*c*e^4+(6*c*d^2
*e^2+b*e^2)^2)+3*d*f^3*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)
*(6*c*d^2*e^2+b*e^2))+e^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d
^3*e+2*b*d*e)^2))*x^6+1/5*(d^3*f^3*(8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+
b*d^2+a)*c*e^4+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*f^3*e*(8*(c*d^4+b*d^2+a)*c*d*e^
3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*f^3*e^2*(2*(c*d^4+b*d^2+a)
*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e))*x^5+1/4*(d^3*f^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*
b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e
^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d*f^3*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+
e^3*f^3*(c*d^4+b*d^2+a)^2)*x^4+1/3*(d^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*f^3*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d
*e)+3*d*f^3*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2*d^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a)^2)*x^2+d^3*f^3*(c*d^4+b*d^2+a)^2
*x
```

maxima [B] time = 1.03, size = 439, normalized size = 4.22

$$\frac{1}{12}c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^
9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 + 72
*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a
*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2
+ 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^
3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*
c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*
(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2*d^10 +
```

```
18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x^2 + (c^2
*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

mupad [B] time = 1.47, size = 419, normalized size = 4.03

$$\frac{e^3\,f^3\,x^4\,\left(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2\,d^4+168\,b\,c\,d^6+165\,c^2\,d^8\right)}{4}+\frac{c^2\,e^{11}\,f^3\,x^{12}}{12}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 16
8*b*c*d^6))/4 + (c^2*e^11*f^3*x^12)/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2 +
(e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*(2*
a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*f^3*
x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6)
)/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 + 42*a*c
*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24*b*c*d^
2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2 + 252*b
*c*d^4))/5 + (c*e^9*f^3*x^10*(2*b + 55*c*d^2))/10 + c^2*d*e^10*f^3*x^11 + (
c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3
```

sympy [B] time = 0.21, size = 722, normalized size = 6.94

$$c^2de^{10}f^3x^{11}+\frac{c^2e^{11}f^3x^{12}}{12}+x^{10}\left(\frac{bce^9f^3}{5}+\frac{11c^2d^2e^9f^3}{2}\right)+x^9\left(2bcde^8f^3+\frac{55c^2d^3e^8f^3}{3}\right)+x^8\left(\frac{ace^7f^3}{4}+\frac{b^2e^7f^3}{8}+9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/5
 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**8*
f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**3 +
 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 +
24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7
*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*
c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3 + 7*
b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*f**3) +
x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f**3/2 + 3
5*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e**3*f**3/4
) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**
3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/
3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7
*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2) + x*(a**2
*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f**3 + 2*b*c*d**
9*f**3 + c**2*d**11*f**3)
```

3.612 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=159

$$\frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3b}{}$$

```
[Out] 1/4*a^3*f^3*(e*x+d)^4/e+1/2*a^2*b*f^3*(e*x+d)^6/e+3/8*a*(a*c+b^2)*f^3*(e*x+
d)^8/e+1/10*b*(6*a*c+b^2)*f^3*(e*x+d)^10/e+1/4*c*(a*c+b^2)*f^3*(e*x+d)^12/e
+3/14*b*c^2*f^3*(e*x+d)^14/e+1/16*c^3*f^3*(e*x+d)^16/e
```

Rubi [A] time = 0.32, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1114, 631}

$$\frac{a^2bf^3(d+ex)^6}{2e}+\frac{a^3f^3(d+ex)^4}{4e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a
*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c
*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) +
(c^3*f^3*(d + e*x)^16)/(16*e)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx &= \frac{f^3\,\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^3 \, dx,x,d+ex\right)}{e}\\
&= \frac{f^3\,\text{Subst}\left(\int x\left(a+bx+cx^2\right)^3 \, dx,x,(d+ex)^2\right)}{2e}\\
&= \frac{f^3\,\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4\right.\right.}{2e}\\
&= \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)f^3(d+ex)^8}{8e}+
\end{aligned}$$

Mathematica [B] time = 0.04, size = 801, normalized size = 5.04

$$f^3\left(\frac{1}{16}c^3e^{15}x^{16}+c^3de^{14}x^{15}+\frac{3}{14}c^2\left(35cd^2+b\right)e^{13}x^{14}+c^2d\left(35cd^2+3b\right)e^{12}x^{13}+\frac{1}{4}c\left(455c^2d^4+78bcd^2+b^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^
2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10
+ 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d
^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2
*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^
2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 +
42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8
 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a
*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)
*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4
 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a
*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 +
 ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^
3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11
 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35
*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 +
 (c^3*e^15*x^16)/16)
```

fricas [B] time = 0.76, size = 1635, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*x^16*f^3*e^15*c^3 + x^15*f^3*e^14*d*c^3 + 15/2*x^14*f^3*e^13*d^2*c^3 +
 35*x^13*f^3*e^12*d^3*c^3 + 455/4*x^12*f^3*e^11*d^4*c^3 + 3/14*x^14*f^3*e^1
3*c^2*b + 273*x^11*f^3*e^10*d^5*c^3 + 3*x^13*f^3*e^12*d*c^2*b + 1001/2*x^10
*f^3*e^9*d^6*c^3 + 39/2*x^12*f^3*e^11*d^2*c^2*b + 715*x^9*f^3*e^8*d^7*c^3 +
 78*x^11*f^3*e^10*d^3*c^2*b + 6435/8*x^8*f^3*e^7*d^8*c^3 + 429/2*x^10*f^3*e
^9*d^4*c^2*b + 1/4*x^12*f^3*e^11*c*b^2 + 1/4*x^12*f^3*e^11*c^2*a + 715*x^7*
f^3*e^6*d^9*c^3 + 429*x^9*f^3*e^8*d^5*c^2*b + 3*x^11*f^3*e^10*d*c*b^2 + 3*x
^11*f^3*e^10*d*c^2*a + 1001/2*x^6*f^3*e^5*d^10*c^3 + 1287/2*x^8*f^3*e^7*d^6
*c^2*b + 33/2*x^10*f^3*e^9*d^2*c*b^2 + 33/2*x^10*f^3*e^9*d^2*c^2*a + 273*x^
5*f^3*e^4*d^11*c^3 + 5148/7*x^7*f^3*e^6*d^7*c^2*b + 55*x^9*f^3*e^8*d^3*c*b^
2 + 55*x^9*f^3*e^8*d^3*c^2*a + 455/4*x^4*f^3*e^3*d^12*c^3 + 1287/2*x^6*f^3*
e^5*d^8*c^2*b + 495/4*x^8*f^3*e^7*d^4*c*b^2 + 1/10*x^10*f^3*e^9*b^3 + 495/4
*x^8*f^3*e^7*d^4*c^2*a + 3/5*x^10*f^3*e^9*c*b*a + 35*x^3*f^3*e^2*d^13*c^3 +
 429*x^5*f^3*e^4*d^9*c^2*b + 198*x^7*f^3*e^6*d^5*c*b^2 + x^9*f^3*e^8*d*b^3
+ 198*x^7*f^3*e^6*d^5*c^2*a + 6*x^9*f^3*e^8*d*c*b*a + 15/2*x^2*f^3*e*d^14*c
^3 + 429/2*x^4*f^3*e^3*d^10*c^2*b + 231*x^6*f^3*e^5*d^6*c*b^2 + 9/2*x^8*f^3
*e^7*d^2*b^3 + 231*x^6*f^3*e^5*d^6*c^2*a + 27*x^8*f^3*e^7*d^2*c*b*a + x*f^3
*d^15*c^3 + 78*x^3*f^3*e^2*d^11*c^2*b + 198*x^5*f^3*e^4*d^7*c*b^2 + 12*x^7*
f^3*e^6*d^3*b^3 + 198*x^5*f^3*e^4*d^7*c^2*a + 72*x^7*f^3*e^6*d^3*c*b*a + 39
/2*x^2*f^3*e*d^12*c^2*b + 495/4*x^4*f^3*e^3*d^8*c*b^2 + 21*x^6*f^3*e^5*d^4*
b^3 + 495/4*x^4*f^3*e^3*d^8*c^2*a + 126*x^6*f^3*e^5*d^4*c*b*a + 3/8*x^8*f^3
*e^7*b^2*a + 3/8*x^8*f^3*e^7*c*a^2 + 3*x*f^3*d^13*c^2*b + 55*x^3*f^3*e^2*d^
9*c*b^2 + 126/5*x^5*f^3*e^4*d^5*b^3 + 55*x^3*f^3*e^2*d^9*c^2*a + 756/5*x^5*
```

```
f^3*e^4*d^5*c*b*a + 3*x^7*f^3*e^6*d*b^2*a + 3*x^7*f^3*e^6*d*c*a^2 + 33/2*x^
2*f^3*e*d^10*c*b^2 + 21*x^4*f^3*e^3*d^6*b^3 + 33/2*x^2*f^3*e*d^10*c^2*a + 1
26*x^4*f^3*e^3*d^6*c*b*a + 21/2*x^6*f^3*e^5*d^2*b^2*a + 21/2*x^6*f^3*e^5*d^
2*c*a^2 + 3*x*f^3*d^11*c*b^2 + 12*x^3*f^3*e^2*d^7*b^3 + 3*x*f^3*d^11*c^2*a
+ 72*x^3*f^3*e^2*d^7*c*b*a + 21*x^5*f^3*e^4*d^3*b^2*a + 21*x^5*f^3*e^4*d^3*
c*a^2 + 9/2*x^2*f^3*e*d^8*b^3 + 27*x^2*f^3*e*d^8*c*b*a + 105/4*x^4*f^3*e^3*
d^4*b^2*a + 105/4*x^4*f^3*e^3*d^4*c*a^2 + 1/2*x^6*f^3*e^5*b*a^2 + x*f^3*d^9
*b^3 + 6*x*f^3*d^9*c*b*a + 21*x^3*f^3*e^2*d^5*b^2*a + 21*x^3*f^3*e^2*d^5*c*
a^2 + 3*x^5*f^3*e^4*d*b*a^2 + 21/2*x^2*f^3*e*d^6*b^2*a + 21/2*x^2*f^3*e*d^6
*c*a^2 + 15/2*x^4*f^3*e^3*d^2*b*a^2 + 3*x*f^3*d^7*b^2*a + 3*x*f^3*d^7*c*a^2
 + 10*x^3*f^3*e^2*d^3*b*a^2 + 15/2*x^2*f^3*e*d^4*b*a^2 + 1/4*x^4*f^3*e^3*a^
3 + 3*x*f^3*d^5*b*a^2 + x^3*f^3*e^2*d*a^3 + 3/2*x^2*f^3*e*d^2*a^3 + x*f^3*d
^3*a^3
```

giac [B] time = 0.51, size = 1360, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*b*c^2*d^12*f
^2 + 3/2*(f*x^2*e + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*c^2
*d^10*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b^3*d^8*f^2 + 3*(f*x^2*e + 2*d*f*x)*a*b
*c*d^8*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x
)*a^2*c*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(f*x^2*e + 2*
d*f*x)*a^3*d^2*f^2 + 1/560*(980*(f*x^2*e + 2*d*f*x)^2*c^3*d^12*f^6*e + 1960
*(f*x^2*e + 2*d*f*x)^3*c^3*d^10*f^5*e^2 + 2520*(f*x^2*e + 2*d*f*x)^2*b*c^2*
d^10*f^6*e + 2450*(f*x^2*e + 2*d*f*x)^4*c^3*d^8*f^4*e^3 + 4200*(f*x^2*e + 2
*d*f*x)^3*b*c^2*d^8*f^5*e^2 + 2100*(f*x^2*e + 2*d*f*x)^2*b^2*c*d^8*f^6*e +
2100*(f*x^2*e + 2*d*f*x)^2*a*c^2*d^8*f^6*e + 1960*(f*x^2*e + 2*d*f*x)^5*c^3
*d^6*f^3*e^4 + 4200*(f*x^2*e + 2*d*f*x)^4*b*c^2*d^6*f^4*e^3 + 2800*(f*x^2*e
 + 2*d*f*x)^3*b^2*c*d^6*f^5*e^2 + 2800*(f*x^2*e + 2*d*f*x)^3*a*c^2*d^6*f^5*
e^2 + 560*(f*x^2*e + 2*d*f*x)^2*b^3*d^6*f^6*e + 3360*(f*x^2*e + 2*d*f*x)^2*
a*b*c*d^6*f^6*e + 980*(f*x^2*e + 2*d*f*x)^6*c^3*d^4*f^2*e^5 + 2520*(f*x^2*e
 + 2*d*f*x)^5*b*c^2*d^4*f^3*e^4 + 2100*(f*x^2*e + 2*d*f*x)^4*b^2*c*d^4*f^4*
e^3 + 2100*(f*x^2*e + 2*d*f*x)^4*a*c^2*d^4*f^4*e^3 + 560*(f*x^2*e + 2*d*f*x
)^3*b^3*d^4*f^5*e^2 + 3360*(f*x^2*e + 2*d*f*x)^3*a*b*c*d^4*f^5*e^2 + 1260*(
f*x^2*e + 2*d*f*x)^2*a*b^2*d^4*f^6*e + 1260*(f*x^2*e + 2*d*f*x)^2*a^2*c*d^4
*f^6*e + 280*(f*x^2*e + 2*d*f*x)^7*c^3*d^2*f*e^6 + 840*(f*x^2*e + 2*d*f*x)^
6*b*c^2*d^2*f^2*e^5 + 840*(f*x^2*e + 2*d*f*x)^5*b^2*c*d^2*f^3*e^4 + 840*(f*
x^2*e + 2*d*f*x)^5*a*c^2*d^2*f^3*e^4 + 280*(f*x^2*e + 2*d*f*x)^4*b^3*d^2*f^
4*e^3 + 1680*(f*x^2*e + 2*d*f*x)^4*a*b*c*d^2*f^4*e^3 + 840*(f*x^2*e + 2*d*f
*x)^3*a*b^2*d^2*f^5*e^2 + 840*(f*x^2*e + 2*d*f*x)^3*a^2*c*d^2*f^5*e^2 + 840
*(f*x^2*e + 2*d*f*x)^2*a^2*b*d^2*f^6*e + 35*(f*x^2*e + 2*d*f*x)^8*c^3*e^7 +
 120*(f*x^2*e + 2*d*f*x)^7*b*c^2*f*e^6 + 140*(f*x^2*e + 2*d*f*x)^6*b^2*c*f^
2*e^5 + 140*(f*x^2*e + 2*d*f*x)^6*a*c^2*f^2*e^5 + 56*(f*x^2*e + 2*d*f*x)^5*
b^3*f^3*e^4 + 336*(f*x^2*e + 2*d*f*x)^5*a*b*c*f^3*e^4 + 210*(f*x^2*e + 2*d*
f*x)^4*a*b^2*f^4*e^3 + 210*(f*x^2*e + 2*d*f*x)^4*a^2*c*f^4*e^3 + 280*(f*x^2
*e + 2*d*f*x)^3*a^2*b*f^5*e^2 + 140*(f*x^2*e + 2*d*f*x)^2*a^3*f^6*e)/f^5
```

maple [B] time = 0.00, size = 7697, normalized size = 48.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.16, size = 920, normalized size = 5.79

$$\frac{1}{16}c^3e^{15}f^3x^{16}+c^3de^{14}f^3x^{15}+\frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}f^3x^{14}+\left(35\,c^3d^3+3\,bc^2d\right)e^{12}f^3x^{13}+\frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^
13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 + 7
8*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 +
 (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b
^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 + 429*b
*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2
145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12
*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13
86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6
*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 +
42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*
(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c
)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a
^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*c^3*d^13 + 78*b
*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3
+ 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*
d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a
*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7
 + a^3*d^3)*f^3*x
```

mupad [B] time = 1.65, size = 825, normalized size = 5.19

$$\frac{3\,e^7\,f^3\,x^8\,\left(a^2\,c+a\,b^2+72\,a\,b\,c\,d^2+330\,a\,c^2\,d^4+12\,b^3\,d^2+330\,b^2\,c\,d^4+1716\,b\,c^2\,d^6+2145\,c^3\,d^8\right)}{8}+\frac{e^5\,f^3\,x^6\,\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 +
 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b +
42*b^3*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 +
462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*f^3*x^10*(b^3 + 5
005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10
 + (c^3*e^15*f^3*x^16)/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^3*x^4*
(a^3 + 84*b^3*d^6 + 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c
*d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 +
 (c*e^11*f^3*x^12*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e^6*f^3*x^
7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d^4 + 1386*
b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^5*(5*a^2*b
+ 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 +
 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3*x^9*(b^3 + 7
15*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*a*b*c) + (3*c^
2*e^13*f^3*x^14*(b + 35*c*d^2))/14 + c^3*d*e^14*f^3*x^15 + d*e^2*f^3*x^3*(a
^3 + 12*b^3*d^6 + 35*c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
+ 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d^6) + (3*d^2*e*f^
3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 + c^2*d*e^12*f^3*x^1
3*(3*b + 35*c*d^2) + 3*c*d*e^10*f^3*x^11*(a*c + b^2 + 91*c^2*d^4 + 26*b*c*d
^2)
```

sympy [B] time = 0.39, size = 1654, normalized size = 10.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13*
f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*c*
*3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 + 39
*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d
*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*
d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2
+ b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**9*f**
3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*a*c**2*d
**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*b*c**2*d*
*5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**3/8 + 3*a*
b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 +
9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e*
*7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*
b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6*f**3 + 12
*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c**2*d**7*e**6*f*
*3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2 + 21*a**2*c*d**2
*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*
a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3
+ 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**
2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b**2*d**3*e**4*f**3 + 756
*a*b*c*d**5*e**4*f**3/5 + 198*a*c**2*d**7*e**4*f**3 + 126*b**3*d**5*e**4*f*
*3/5 + 198*b**2*c*d**7*e**4*f**3 + 429*b*c**2*d**9*e**4*f**3 + 273*c**3*d**
11*e**4*f**3) + x**4*(a**3*e**3*f**3/4 + 15*a**2*b*d**2*e**3*f**3/2 + 105*a
**2*c*d**4*e**3*f**3/4 + 105*a*b**2*d**4*e**3*f**3/4 + 126*a*b*c*d**6*e**3*
f**3 + 495*a*c**2*d**8*e**3*f**3/4 + 21*b**3*d**6*e**3*f**3 + 495*b**2*c*d*
*8*e**3*f**3/4 + 429*b*c**2*d**10*e**3*f**3/2 + 455*c**3*d**12*e**3*f**3/4)
 + x**3*(a**3*d*e**2*f**3 + 10*a**2*b*d**3*e**2*f**3 + 21*a**2*c*d**5*e**2*
f**3 + 21*a*b**2*d**5*e**2*f**3 + 72*a*b*c*d**7*e**2*f**3 + 55*a*c**2*d**9*
e**2*f**3 + 12*b**3*d**7*e**2*f**3 + 55*b**2*c*d**9*e**2*f**3 + 78*b*c**2*d
**11*e**2*f**3 + 35*c**3*d**13*e**2*f**3) + x**2*(3*a**3*d**2*e*f**3/2 + 15
*a**2*b*d**4*e*f**3/2 + 21*a**2*c*d**6*e*f**3/2 + 21*a*b**2*d**6*e*f**3/2 +
 27*a*b*c*d**8*e*f**3 + 33*a*c**2*d**10*e*f**3/2 + 9*b**3*d**8*e*f**3/2 + 3
3*b**2*c*d**10*e*f**3/2 + 39*b*c**2*d**12*e*f**3/2 + 15*c**3*d**14*e*f**3/2
) + x*(a**3*d**3*f**3 + 3*a**2*b*d**5*f**3 + 3*a**2*c*d**7*f**3 + 3*a*b**2*
d**7*f**3 + 6*a*b*c*d**9*f**3 + 3*a*c**2*d**11*f**3 + b**3*d**9*f**3 + 3*b*
*2*c*d**11*f**3 + 3*b*c**2*d**13*f**3 + c**3*d**15*f**3)
```

3.613 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal. Leaf size=193

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*
a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{x}{c} - \frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{ce}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2ce} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Su}}{}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] time = 0.14, size = 219, normalized size = 1.13

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

fricas [B] time = 0.86, size = 1231, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2)
)*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 -
4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4))
- (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d - sqrt
(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6
 - 4*a*c^7)*e^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*
c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3
- 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(((b^2*c^3 - 4*a*c
^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - b^3 +
```

```
 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2
- a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(
((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c
^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt((
(b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^
7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)
*e*x - 2*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(b^2*c^6 - 4*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + 2*
x)/c
```

giac [B] time = 0.43, size = 1194, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*b*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) +
 x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*
(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*
d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6 - 2*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e^5 + b*d^2*e^4 +
 a*e^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*
e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6
 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))))*e^(-4)/c + x/c
```

maple [C] time = 0.07, size = 158, normalized size = 0.82

$$\frac{x}{c}+\frac{\left(-\mathrm{RootOf}\left(\_Z^{4} c\, e^{4}+4\_Z^{3} c d\, e^{3}+c\, d^{4}+b\, d^{2}+\left(6 c\, d^{2} e\right.\right.\right.}{2 c e\left(2 c\, e^{3} \mathrm{RootOf}\left(\_Z^{4} c\, e^{4}+4\_Z^{3} c d\, e^{3}+c\, d^{4}+b\, d^{2}+\left(6 c\, d^{2} e^{2}+b\, e^{2}\right)\_Z^{2}+\left(4 c\, d^{3} e+2 d e b\right)\_Z+a\right)^{3}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/c*x+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*c*d
*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(c*e^4*_Z^4+4*c*d*
e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.32, size = 3988, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*
e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13
- 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)
))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b
^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 +
 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e
^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 -
 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^
2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^
4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*
a*b^2*c*e^12))/c)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e
^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((
8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4
*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c
 - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2
*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 -
 8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*
e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((
2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 -
 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*
b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^
2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2
*c^2*e^12 - 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
```

```
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + atan(((-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c
^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 -
 b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) +
 (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 +
2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2
*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - (
(8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^
4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^
2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2
- 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^
12))/c)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*
d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))
)^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c
^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c +
(2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 - b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^
11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c
^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^
14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^
3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2
+ b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12
- 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + x/c
```

sympy [A] time = 3.07, size = 178, normalized size = 0.92

$$\mathrm{RootSum}\left(t^4\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+a^3,\left(t\mapsto t\log\left(x+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3, L
ambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_
t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*
c*e - a*b**2*e)))) + x/c
```

3.614 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=81

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] 1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*arctanh((b+2*c*(e*x+d)^2)/(-4*a
*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]
*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce}$$
$$= \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+e$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e} + \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.95

$$\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

fricas [B] time = 0.71, size = 434, normalized size = 5.36

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right)}{4\left(b^2c-4\,ac^2\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2
)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a))/((b^2*c - 4*a*c^2)*e)]
```

giac [A] time = 0.40, size = 130, normalized size = 1.60

$$-\frac{b\arctan\left(\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\sqrt{-b^2+4\,ac}\,c} + \frac{e^{(-1)}\log\left(cd^4+2\left(x^2e+2\,dx\right)cd^2e+\left(x^2e+2\,dx\right)^2ce^2+bd^2+\left(x^2e+2\,d\right.\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(
-1)/(sqrt(-b^2 + 4*a*c)*c) + 1/4*e^(-1)*log(c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2
*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a)/c
```

maple [C] time = 0.00, size = 151, normalized size = 1.86

$$\frac{\left(\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3e^3+3\,\mathrm{RootOf}}{2e\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\,\mathrm{Ro}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c
*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.76, size = 278, normalized size = 3.43

$$\frac{4ace\,\ln\left(cd^4+4cd^3ex+6cd^2e^2x^2+bd^2+4cde^3x^3+2bdex+ce^4x^4+be^2x^2+a\right)}{16ac^2e^2-4b^2ce^2}-\frac{b^2e\,\ln\left(cd^4+4cd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^
2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b
^2*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^
2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*ata
n(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*
a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(
1/2))
```

sympy [B] time = 1.64, size = 280, normalized size = 3.46

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)+2a+2b^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)+\left(\frac{b\sqrt{}}{4ce}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x
**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))
 + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*
c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a
*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

3.615 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=164

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*
c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan((e*x+d)
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)
/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1142, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Sub}}{}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(c}{\sqrt{b+\sqrt{b^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.09, size = 175, normalized size = 1.07

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[
b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

fricas [B] time = 0.84, size = 703, normalized size = 4.29

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b^2c-4\,ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4\,ac^3\right)e^4}}+b}{\left(b^2c-4\,ac^2\right)e^2}}\,\log\left(\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)e^3\sqrt{-\frac{\left(b^2c-4\,ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4\,ac^3\right)e^4}}+b}{\left(b^2c-4\,ac^2\right)e^2}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)
) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-(
(b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*
c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqr
t(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c -
4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2
)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(
1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*
c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*lo
g(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*
c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c
^3)*e^4)) + e*x + d) + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b
^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4
)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x +
d)
```

giac [B] time = 0.45, size = 1285, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)
```

maple [C] time = 0.01, size = 140, normalized size = 0.85

$$\frac{\left(\mathrm{RootOf}\left(\_Z^4 c\, e^4+4\_Z^3 cd\, e^3+c\, d^4+b\, d^2+\left(6c\, d^2 e^2+b\right.\right.\right.}{2e\left(2c\, e^3\, \mathrm{RootOf}\left(\_Z^4 c\, e^4+4\_Z^3 cd\, e^3+c\, d^4+b\, d^2+\left(6c\, d^2 e^2+b\, e^2\right)\_Z^2+\left(4c\, d^3 e+2deb\right)\_Z+a\right)^3+6cd\, e^2\, \mathrm{Ro}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*
d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{(ex+d)^4 c+(ex+d)^2 b+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.74, size = 590, normalized size = 3.60

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3+\sqrt{-\left(4\,a\,c-b^2\right)^3}-4\,a\,b\,c}{8\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\left(x\,\left(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12}\right)+\frac{\left(x\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c^3\,d\,e^{1}\right.}{8\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\right.}\right.}{a\,c\,e^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*
a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) + (
(x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^1
3)*(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e
^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))*(-(b
^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8
*a*b^2*c^2*e^2)))^(1/2) - 2*atanh(((((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e
^12 - 2*b^2*c*e^12) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*
e^13 - 32*a*b*c^3*d*e^13)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b
^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*
e^11))/(a*c*e^10))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c*e^
2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)
```

sympy [A] time = 1.35, size = 104, normalized size = 0.63

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2+4b^3e^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3ac^2e^3-16}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t**3
*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

3.616 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] -arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1142, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{e}$$
$$= -\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)
```

fricas [A] time = 0.87, size = 272, normalized size = 6.33

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\,\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac-\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\,\left(2\,cd^3+bd\right)ex+a}\right)}{2\,\sqrt{b^2-4\,ac}\,e}, -\frac{\sqrt{-b^2+4\,a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
[Out] [1/2*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)
*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x
^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqr
t(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

giac [A] time = 0.45, size = 53, normalized size = 1.23

$$\frac{\arctan\left(\frac{2\,cd^2+2\,\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
[Out] arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/sqr
t(-b^2 + 4*a*c)
```

maple [C] time = 0.01, size = 129, normalized size = 3.00

$$\frac{\left(\mathrm{Ro}\right.}{2e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)^3+6cd\,e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^
2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 0.09, size = 61, normalized size = 1.42

$$\frac{\operatorname{atan}\left(\frac{2\,a\,c\,d^2+4\,a\,c\,d\,e\,x+2\,a\,c\,e^2\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{e\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((a*b + 2*a*c*d^2 + 2*a*c*e^2*x^2 + 4*a*c*d*e*x)/(a*(4*a*c - b^2)^(1/2)
))/(e*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.04, size = 168, normalized size = 3.91

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqrt(
-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b*
*2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)
```

3.617 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=94

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

```
[Out] ln(e*x+d)/a/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e+1/2*b*arctanh((b+2*c*(e
*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)}dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x}dx, x, (d+ex)^2\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{2ae}$$

$$= \frac{\log(d+ex)}{ae} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{4ae} - \frac{b\,\text{Subst}\left(\int \frac{}{a+b}\right.}{}$$

$$= \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4a}\right.}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,e} + \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+\right.}{4ae}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.36

$$\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4a\right.}{4ae\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)
```

fricas [A] time = 0.88, size = 468, normalized size = 4.98

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right)-\left(\right.}{4\left(ab^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2
*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (
b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*b^
2 - 4*a^2*c)*e)]
```

giac [**B**] time = 1.15, size = 274, normalized size = 2.91

$$-\frac{e^{(-1)}\log\left(\left|cx^4e^4+4\,cdx^3e^3+6\,cd^2x^2e^2+4\,cd^3xe+cd^4+bx^2e^2+2\,bdxe+bd^2+a\right|\right)}{4\,a}+\frac{e^{(-1)}\log\left(|xe+d|\right)}{a}-\frac{\Big(\frac{ab}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/a + e^(-1)*log(abs(x*e + d
))/a - 1/4*(a*b*c*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2
*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sq
rt(b^2 - 4*a*c) - a*b*c*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c)
```

maple [**C**] time = 0.01, size = 184, normalized size = 1.96

$$\frac{\ln\left(ex+d\right)}{ae}+\frac{\left(-c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)\right.}{2ae\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R-c*d^3-b*d)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf
(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+
2*b*d*e)*_Z+a))+ln(e*x+d)/a/e
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**B**] time = 2.50, size = 2173, normalized size = 23.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d
*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e - 8*a*c*e))/
(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*
((b^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e
^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b
^2*e - 8*a*c*e)^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 -
40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*(4*a*b^2*e^2 - 16*a
^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19)
)/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*
a*c - 6*b^2)) - (((b*(2*b^2*e - 8*a*c*e)^2*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^
19))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) - (b^3*(12
*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*
b^2*e - 8*a*c*e)*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 4
0*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*a*e*(4*a*c - b^2)^(1
/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^
3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b^2*c^2*
e^14) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*(((2*b^2*e - 8*a*c*e
)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b
^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(16*a^2*e^2*
(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*
e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2
*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*(4*a*b^2*e^2 - 16*a^2*c*
e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17
- 40*a*b*c^3*d^2*e^17))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e
^2))))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c
^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2
*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^
2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*a*e*(4*a*c - b^2)^(1/2)
*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^
17 - 40*a*b*c^3*d^2*e^17))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e -
 8*a*c*e)^2*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))
/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)))/(b^2*c^4*e^1
4*(25*a*c - 6*b^2)) + (16*a^3*x*(((3*b^3 - 8*a*b*c)*((b^2*(((2*b^2*e - 8*a*
c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^
2)) + 20*b*c^3*d*e^17))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2
*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2
*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2
) + (b^2*(2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a
^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*a*c - 6
*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^
2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 1
6*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 -
 16*a^2*c*e^2)) - (b^3*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(64*a^3*e^3
*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)^2*(24*b^3*c^2*d*e^18 - 80*a*
b*c^3*d*e^18))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2))
)/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b
^2*c^2*e^14)))/(2*a*e*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 6.04, size = 320, normalized size = 3.40

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)+2ab^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)-2ac+b^2+bc\cdots}{bce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x
**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*
e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)
) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*(4
*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-4*a*c
 + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*a*c +
b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e
**2)) + log(d/e + x)/(a*e)
```

3.618 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

```
[Out] -1/a/e/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2
)*(1-b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx=\frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{e}$$

$$=-\frac{1}{ae(d+ex)}+\frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{ae}$$

$$=-\frac{1}{ae(d+ex)}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x\right.}{2ae}$$

$$=-\frac{1}{ae(d+ex)}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{\sqrt{c}\left(1\right.}{}$$

Mathematica [A] time = 0.36, size = 206, normalized size = 1.06

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e)
```

fricas [B] time = 0.88, size = 1339, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2
 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sq
rt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt
(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a
^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2
*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log
(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*
a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b^2 - 4*a^4*c
)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3
```

```
*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^
3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*
e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 -
5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a
^4*c)*e^2))) + sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/
((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^
3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2
)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - 2)/(a*e^
2*x + a*d*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

maple [C] time = 0.01, size = 168, normalized size = 0.86

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4 c e^4 + 4\_Z^3 cd e^3 + c d^4 + b d^2 + \left(6c d^2 e^2 + b e\right.\right.\right.}{2ae\left(2c e^3 \mathrm{RootOf}\left(\_Z^4 c e^4 + 4\_Z^3 cd e^3 + c d^4 + b d^2 + \left(6c d^2 e^2 + b e^2\right)\_Z^2 + \left(4c d^3 e + 2deb\right)\_Z + a\right)^3 + 6cd e^2 \mathrm{R}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e
^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a/e/(e
*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.39, size = 3844, normalized size = 19.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*
```

```
c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3
*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^
(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*
c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12)
 + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a
^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 -
 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)
+ 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 -
 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^
6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b
^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4
*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) - (-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*
(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*
c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c
^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^
5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e
^11) + 2*a^3*c^4*e^10))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*
e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3
*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^
3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16
*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2
*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c
*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*
a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2
*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*
e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c
^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11
)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c
*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*
```

```
e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-
(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(
1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
2*a^3*b^2*c^3*d*e^11) - (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*
e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + (
(x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b
^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a
^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*
a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) + 2*a^3*c^4*e^10))*(-(b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - 1/(a*
e*(d + e*x))
```

sympy [A] time = 4.27, size = 211, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2e^4-128a^4b^2ce^4+16a^3b^4e^4\right)+t^2\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3, L
ambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e**3
- 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b*
*5*e + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e + a*e
**2*x)
```

3.619 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

```
[Out] -1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^
2/e-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/e/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2e}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,(d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,(d+ex\right.}{2a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+e\right.}{4a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex\right.}{4a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,e}-\frac{b\log(d+ex)}{a^2e}+
\end{aligned}$$

Mathematica [A] time = 0.13, size = 154, normalized size = 1.27

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)
```

fricas [B] time = 1.11, size = 810, normalized size = 6.69

$$\left[-\frac{2\,ab^2 - 8\,a^2c + \left(\left(b^2 - 2\,ac\right)e^2x^2 + 2\left(b^2 - 2\,ac\right)dex + \left(b^2 - 2\,ac\right)d^2\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2e^4x^4 + 8\,c^2de^3x^3 + 2\,c^2d^4 + 2\,(6}{ce^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3
*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2
*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*
c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*
a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3
*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e)]
```

giac [A] time = 0.40, size = 102, normalized size = 0.84

$$\frac{be^{(-1)}\log\left(c + \frac{b}{(xe+d)^2} + \frac{a}{(xe+d)^4}\right)}{4\,a^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\,\sqrt{-b^2 + 4\,ac}\,a^2} - \frac{e^{(-1)}}{2\,(xe + d)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^2 + 1/2*(b^2 - 2*a*c)
*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/(sqrt(-b^2 + 4*a*
c)*a^2) - 1/2*e^(-1)/((x*e + d)^2*a)
```

maple [C] time = 0.01, size = 213, normalized size = 1.76

$$-\frac{b\ln(ex + d)}{a^2e} - \frac{1}{2\,(ex + d)^2\,ae} + \frac{\left(\mathrm{RootOf}\left(\_Z^4ce^4 + 4\_Z^3cd\,e^3 + c\,d^4 + b\,d^2 + \left(6c\,d^2e^2 + b\,e^2\right)\_Z^2 + \left(4c\,d^3e + \right.\right.\right.}{2a^2e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4 + 4\_Z^3cd\,e^3 + c\,d^4 + b\,d^2 + \left(6c\,d^2e^2 + \right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c*d*e^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d
^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*
d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b
*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.86, size = 4950, normalized size = 40.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(4*a*c - b^2)^(3/2)*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((
(20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4
*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (6*b*c^4*e^17)
/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (c^5*e^16
)/a^3 - (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*
c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^
2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*
(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(
16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(4*a^2*e
*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^
3*e^19 - 12*a^3*b^3*c^2*e^19))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(
4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (((((((20*a^3*
c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e
^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c
- b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e
)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2
*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)) + (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e
 - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) + (6*b*c^4*e^17)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - (
(2*a*c - b^2)^3*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(64*a^9*e^3*(4*a
*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c -
b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c))))/(4*a^2*c^4*e^14 + b^4*c^2*e^14
 - 4*a*b^2*c^3*e^14) + (16*a^6*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3
*e - 8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*
a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a
^3*c*e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4
*d*e^16)/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (
2*c^5*d*e^15)/a^3 - ((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3
*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8
*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1
/2)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*
e - 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)
))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18 - 1
2*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e))/(16*a^7*e^2*(1
6*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
 24*a*b^2*c)) + (((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d
```

```
*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a
*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1/2
)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*e
- 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + ((((2*b^3*e -
8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b
*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4*d*e^
16)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^
18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^3)/(32*a^9*e^3*(4*a*c - b^2)^(3/2
)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^
2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^
2*e^14 - 4*a*b^2*c^3*e^14) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 -
9*a*b^2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/a^3 + (((4*a*b^2*c^3*e^15 - a^2*c^4
*e^15 + 6*a*b*c^4*d^2*e^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16
+ 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e
)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(
2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*
e^2)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c
^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b
^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*
a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) -
 ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*
d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*
(4*a*c - b^2)^(1/2))))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^2*(2*
b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b
*c^3*d^2*e^17))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))
/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b
^2*c^3*e^14)) + (2*a^3*(4*a*c - b^2)*(((2*b^3*e - 8*a*b*c*e)*((((4*a^2*b^3*
c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)
/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17
 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c -
 b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)
*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8
*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2))))/(2*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)) + (((4*a*b^2*c^3*e^15 - a^2*c^4*e^15 + 6*a*b*c^4*d^2*e
^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 +
 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 +
 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*
(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^3*(4*a^4*b^2*
c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(64*a^9*e^3*(4
*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)))*(2*a
*c - b^2))/(2*a^2*e*(4*a*c - b^2)^(1/2)) - (log(((c^5*e^16*x^2)/a^3 - ((b +
 a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2
- a*c + 6*b*c*d^2))/a^2 - ((b + a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b
^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/a +
 (2*c^3*e^18*x^2*(10*a*c + b^2))/a + (b*c^2*e^16*(b + a^2*e*(-(2*a*c - b^2)
^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*
c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (4*c^3*d*e^17*x
*(10*a*c + b^2))/a))/(4*a^2*e) + (6*b*c^4*e^17*x^2)/a^2 + (12*b*c^4*d*e^16*
x)/a^2))/(4*a^2*e) + (c^4*e^14*(b + c*d^2))/a^3 + (2*c^5*d*e^15*x)/a^3)*((c
^5*e^16*x^2)/a^3 - ((b - a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(
1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/a^2 - ((b - a^2*e*(-(2*a*c - b^
2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b
^2*c*d^2 - 2*a*b*c))/a + (2*c^3*e^18*x^2*(10*a*c + b^2))/a + (b*c^2*e^16*(b
 - a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2
```

```
 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x
))/a^2 + (4*c^3*d*e^17*x*(10*a*c + b^2))/a))/(4*a^2*e) + (6*b*c^4*e^17*x^2)
/a^2 + (12*b*c^4*d*e^16*x)/a^2))/(4*a^2*e) + (c^4*e^14*(b + c*d^2))/a^3 + (
2*c^5*d*e^15*x)/a^3))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) - (b*log(d + e*x))/(a^2*e) - 1/(2*a*e*(d^2 + e^2*x^2 + 2*d*e*x))
```

sympy [B] time = 146.46, size = 464, normalized size = 3.83

$$\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3ce\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right) + 2a^2b^2e\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right.}{2ac^2e^2 - b^2ce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)
))*log(2*d*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e) - sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*
c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2
*e) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))*log(2*d
*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2
- b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e + 4*a*
d*e**2*x + 2*a*e**3*x**2) - b*log(d/e + x)/(a**2*e)
```

3.620 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

```
[Out] -1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/e*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/e
*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{e}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{3ae}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}-\frac{\text{Subst}\left(\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4}\,dx,x,d+\right.}{3a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b}}\right.}{2a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+}{\sqrt{b-\sqrt{b^2-4}}}\right.}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] time = 0.21, size = 235, normalized size = 1.05

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e)
```

fricas [B] time = 0.93, size = 2044, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*
d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2)
)*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 +
a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^
11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^
4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*
a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a
^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e
^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4
 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt(
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4
*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4
*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 -
 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 +
3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a
^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c
)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*
c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2
- 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) + 3*sqrt(1/2)*(a^2*e^4*x^3
 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^
6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b
^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 +
 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e
^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10
*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 2*a)/(a^2*e^4*x^3 + 3
*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)
```

giac [B] time = 0.49, size = 1243, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
 - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
```

```
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/a^2 + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e + 3*b*
d^2 - a)*e^(-1)/((x*e + d)^3*a^2)
```

maple [C] time = 0.01, size = 188, normalized size = 0.84

$$-\frac{1}{3\,(ex+d)^3\,ae}+\frac{b}{(ex+d)\,a^2e}+\frac{\left(\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\right.\right.}{2a^2e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4
+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z
+a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.83, size = 5214, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3 + e^3*x^3 +
3*d*e^2*x^2 + 3*d^2*e*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*
a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c -
 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a
^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c
^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
```

```
b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*
((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a
^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 2
5*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)
))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4
*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + ((b^4*(
-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12
 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*
c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2
*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2
)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2
+ 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*
c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 -
 8*a^7*b^2*c^4*d*e^11)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b
*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^
2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^
12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(
32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b
^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1
/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*
c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - ((b^4*(-(4*a*c
- b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6
*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
+ 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^
5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2
- 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) +
 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/
2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
 + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7
*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12
 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b
^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*
c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
8*a^6*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) -
 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b
^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^
2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(
1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13
- 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*
c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^1
```

```
2) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + (
-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c
^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 +
 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10
*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5
*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4
*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3
*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i)/((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^
5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2
- 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*
b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^
(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13
 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*
c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2
*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^
12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - (-(
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5
*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 1
6*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b
^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a
^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d
*e^11 - 8*a^7*b^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*(-(b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 1
6*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*2i
```

sympy [A] time = 12.77, size = 347, normalized size = 1.55

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3}+\text{RootSum}\left(t^4\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e*
*2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum(_t**4*(256*a**7*c**
2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c**3
*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, L
ambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**
3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40
*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d - 3*a*b
**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))
```

3.621 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=270

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)t}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/
4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(
b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1120, 1166, 205}

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)t}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx, x, d+\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\text{Su}}{4\left(b\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.47, size = 263, normalized size = 0.97

$$\frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

fricas [B] time = 0.96, size = 2454, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*x + sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) +
```

```
b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))
*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*s
qrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a
*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sq
rt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 +
12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((
3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2
 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(-(
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*
c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^
2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*
d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*
e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*
c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 +
 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a
^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c -
 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^
2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6
*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*
e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c
^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + 4*a*d)/((b^2*c - 4*a*c^2)*e^5*x^4
+ 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c
- 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.52, size = 1304, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (
```

```
6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) + x - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) +
x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
 + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(
d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d
*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) - sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*l
og(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c)
+ 1/2*(b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + 2*a*x*e + 2*a*d)/(
(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*
e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 323, normalized size = 1.20

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^{4} c\, e^{4}+4\_Z^{3} c d\, e^{3}+c\, d^{4}+b\, d^{2}+\left(6 c\, d^{2} e^{2}\right.\right.\right.}{4\left(4 a c-b^{2}\right) e\left(2 c\, e^{3} \operatorname{RootOf}\left(\_Z^{4} c\, e^{4}+4\_Z^{3} c d\, e^{3}+c\, d^{4}+b\, d^{2}+\left(6 c\, d^{2} e^{2}+b\, e^{2}\right)\_Z^{2}+\left(4 c\, d^{3} e+2 d e b\right)\_Z+a\right)^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2*d*b*e/(4*a*c-b^2)*x^2-1/2*(3*b*d^2+2*a)/(4*
a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*
sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d
^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b e^{3} x^{3}+3\, b d e^{2} x^{2}+b d^{3}+\left(3\, b d^{2}+2\, a\right) e x+2\, a d}{2\left(\left(b^{2} c-4\, a c^{2}\right) e^{5} x^{4}+4\left(b^{2} c-4\, a c^{2}\right) d e^{4} x^{3}+\left(b^{3}-4\, a b c+6\left(b^{2} c-4\, a c^{2}\right) d^{2}\right) e^{3} x^{2}+2\left(2\left(b^{2} c-4\, a c^{2}\right) d^{3}+\left(b^{3}-4\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + b*d^3 + (3*b*d^2 + 2*a)*e*x + 2*a*d)/((b^2
*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^
2)*e) - 1/2*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*
c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
```

```
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

mupad [B] time = 4.76, size = 7327, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(-(((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^
13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c
^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^
14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c
^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^
2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614
4*a^5*b^2*c^6*e^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a
^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*
a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11
+ 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c
^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2
- 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i + ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3
*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*
c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b
^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^
14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
- 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*
a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^
2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^
12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^
9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^
3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c
^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(
1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c
^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/(((((64*b
^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b
^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b
*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^
3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12
 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 24
0*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*
b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d
```

e^11)/(8(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^1
2 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^
2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*
c^6*e^2)))^(1/2) - ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^
4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*
a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 -
96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a
*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c
^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*
e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4
*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e
^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*
a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b
^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 -
24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*
b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (4*a^2*b*c^2*e^10 + 3*a*b^3*c
e^10)/(4(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(-(b^9 + (-(4
*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(3
2*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/
2)*2i - ((2*a*d + b*d^3)/(2*e*(4*a*c - b^2)) + (x*(2*a + 3*b*d^2))/(2*(4*a*
c - b^2)) + (b*e^2*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*x^2)/(2*(4*a*c - b^2))
)/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e)
+ c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768
*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c
^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3
840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 -
32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*
b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a
^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (
x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3
*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*
c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^1
2 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*1i - (((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*
a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((
64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a
^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a
^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*

```
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*
b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^
6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b
^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^
2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 -
 6144*a^5*b^2*c^6*e^2)))^(1/2) + (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a
*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)))*1i)/((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b
^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^
2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 -
 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 3
84*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 1638
4*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 -
192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4
 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 2
4*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^
4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096
*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e
^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*
e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12)
)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (((-(4*a*c - b^2)^9)^(1/2) - b^9 +
768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^
6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2
+ 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12
 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024
*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 1638
4*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*
b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/
2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a
*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3
)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*
e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))
^(1/2) + (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(b^4*c*e^12 + 8*a^2*c^3*
e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) - (4*a^2*b*c^
2*e^10 + 3*a*b^3*c*e^10)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
))))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 51
2*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240
*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b
^2*c^6*e^2)))^(1/2)*2i
```

sympy [B] time = 8.97, size = 573, normalized size = 2.12

$$\frac{-2ad - bd^3 - 3bde^2x^2 - be^3x^3 + x}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d - b*d**3 - 3*b*d*e**2*x**2 - b*e**3*x**3 + x*(-2*a*e - 3*b*d**2*e))
/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*
e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**
2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*
b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2
 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**6*c**7*
e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a**4*b**4*c**5*e**4 - 327680*a*
*3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4 - 6144*a*b**10*c**2*e**4 + 25
6*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2
 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t
**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e*
*3 + 64*_t*a**2*c**2*e - 128*_t*a*b**2*c*e - 4*_t*b**4*e + 4*a*c*d + 3*b**2
*d)/(4*a*c*e + 3*b**2*e))))
```

3.622 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*arctanh(
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\right.}{\left(b^2-4ac\right)e}$$

$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 1.03

$$\frac{\frac{2a+b(d+ex)^2}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))
) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2))/(2*e)
```

fricas [B] time = 0.92, size = 1021, normalized size = 10.53

$$\left[\frac{\left(b^3-4\,abc\right)e^2x^2+2\left(b^3-4\,abc\right)dex+2\,ab^2-8\,a^2c+\left(b^3-4\,abc\right)d^2-\left(bce^4x^4+4\,bcde^3x^3+bcd^4+\right.}{2\left(\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)e^5x^4+4\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)de^4x^3+\left(b^5-8\,ab^3c+16\,a^2bc^2+6\left(b^4c-8\,ab^2c^2+\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*c
 + (b^3 - 4*a*b*c)*d^2 - (b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*
d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(b^2 -
4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/(
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*
```

```
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*
b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*
c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a
*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4
+ 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^
4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [A] time = 0.57, size = 171, normalized size = 1.76

$$\frac{b \arctan\left(\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bd^2+\left(x^2e+2\,dx\right)be+2\,a}{2\left(cd^4+2\left(x^2e+2\,dx\right)cd^2e+\left(x^2e+2\,dx\right)^2ce^2+bd^2+\left(x^2e+2\,dx\right)be+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/(
(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2 + (x^2*e + 2*d*x)*b*e + 2*a)
/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x
^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 276, normalized size = 2.85

$$\frac{}{2\left(4ac-b^2\right)e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e/(4*a*c-b^2)*x^2-b*d/(4*a*c-b^2)*x-1/2/e*(b*d^2+2*a)/(4*a*c-b^2))/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)+1/2/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d
*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int-\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*
e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a
)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b
*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 -
 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2)*e)
```

mupad [B] time = 1.77, size = 427, normalized size = 4.40

$$\frac{b\,\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)^4\,\left(x\,\left(\frac{b^3\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)+x^2\,\left(\frac{b^3\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{b^2\,c^2\,e^8}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)-\frac{b^3\,\left(16\,a^2\,c^3\,e^8-4\,a\,b^2\,c^2\,e^8+8\,a\,b\,c^3\,d^2\,e^8-2\,b^3\right.}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}}{2\,b^2\,c^2\,e^6}}{e\,\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*atan(((4*a*c - b^2)^4*(x*((b^3*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9))/(a*e
^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7)/(a*(4*a*c - b^2)^(7/2))) + x^2
*((b^3*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c - b^2)^(11/2)) -
(b^2*c^2*e^8)/(a*(4*a*c - b^2)^(7/2))) - (b^3*(16*a^2*c^3*e^8 - 4*a*b^2*c^2
*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(11/2
)) - (b^2*c^2*d^2*e^6)/(a*(4*a*c - b^2)^(7/2))))/(2*b^2*c^2*e^6)))/(e*(4*a*
c - b^2)^(3/2)) - ((2*a + b*d^2)/(2*e*(4*a*c - b^2)) + (b*e*x^2)/(2*(4*a*c
- b^2)) + (b*d*x)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c
*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [B] time = 4.88, size = 495, normalized size = 5.10

$$\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e}-\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(
4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - b*sqrt(-1/(4*a
*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) + (-2*a - b*d**2 - 2*b*d*e*x -
b*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e -
2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x*
*3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d*
*2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c*
*2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.623 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=254

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b \cdots\right.}{\sqrt{2}e\left(b^2-4ac\right)^{3/2} \cdots}$$

```
[Out] -1/2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1
/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*
b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/
2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1119, 1166, 205}

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b \cdots\right.}{\sqrt{2}e\left(b^2-4ac\right)^{3/2} \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x, d\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)\text{Subs}}{2\left(b^2\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-}}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-}}$$

Mathematica [A] time = 0.98, size = 247, normalized size = 0.97

$$-\frac{\frac{b(d+ex)+2c(d+ex)^3}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

fricas [B] time = 0.78, size = 2474, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*x - sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
```

```
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((a*b^
8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*s
qrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 +
 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*
c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4
)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt(1/2)*(
(a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/
2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))
 - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^
2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 -
64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^
4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt
(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3
*c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^
4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))) + 2*b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.53, size = 1312, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-
```

```
(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x - sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*
d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-
1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e
^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
- 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^
2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) -
 sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2
 - b)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^
(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 -
4*a*c) - 1/2*(2*c*x^3*e^3 + 6*c*d*x^2*e^2 + 6*c*d^2*x*e + 2*c*d^3 + b*x*e +
 b*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 +
 b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 319, normalized size = 1.26

$$\frac{\left(2\,\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2\right.\right.\right.}{4\left(4ac-b^2\right)e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (c*e^2/(4*a*c-b^2)*x^3+3*d*c*e/(4*a*c-b^2)*x^2+1/2*(6*c*d^2+b)/(4*a*c-b^2)*
x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((2*_R
^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2
+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,ce^3x^3+6\,cde^2x^2+2\,cd^3+\left(6\,cd^2+b\right)ex+bd}{2\left(\left(b^2c-4\,ac^2\right)e^5x^4+4\left(b^2c-4\,ac^2\right)de^4x^3+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^3x^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x + b*d)/((b^
2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6
*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b
*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d
^2)*e) + 1/2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4
*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (
```

```
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

mupad [B] time = 3.95, size = 7200, normalized size = 28.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c
^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4
*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14
+ 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (
8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e
^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2
)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) -
 (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b
^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5
*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11
)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^
13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13
 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a
^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2
) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2
 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^
6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c
^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e
^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*
b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*
a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((((-(4*a
*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3
)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*
e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))
^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 51
2*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192
*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
 (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*
c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096
*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e
^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12
 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b
```

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^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 40
96*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3
*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e
^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*((6
4*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 512*a*b^7*c^
3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c
^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7
*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))
/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4
*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^
5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a
^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6
*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 384
0*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^
2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (4*a*c^4*e^10 + 3*b^2*c^3*e^
10)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(((-(4*a*c - b^2
)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2
i - atan(-(((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*
e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^1
4 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*
e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2
+ 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 +
 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a
^6*b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2
*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a
*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 -
 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2
)*1i + ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13
 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 -
512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^
3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*
c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12
 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240
*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b
^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^4
*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c
```

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- b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*
b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 128
0*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*1i
)/((4*a*c^4*e^10 + 3*b^2*c^3*e^10)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) - ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^
6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3
*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2
 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2
+ 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*
a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*
c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*
c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e
^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 61
44*a^6*b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a
*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (
-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)
/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^
(1/2) + ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^1
3 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 -
 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*
a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a
^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2
*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^1
2 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^
4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^
4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c
 - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^
3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*
c^5*e^2)))^(1/2)*2i + ((x*(b + 6*c*d^2))/(2*(4*a*c - b^2)) + (b*d + 2*c*d^3
)/(2*e*(4*a*c - b^2)) + (c*e^2*x^3)/(4*a*c - b^2) + (3*c*d*e*x^2)/(4*a*c -
b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3
*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [B] time = 18.53, size = 578, normalized size = 2.28

$$\frac{bd + 2cd^3 + 6cde^2x^2 + 2ce^3x^3 + x}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d + 2*c*d**3 + 6*c*d*e**2*x**2 + 2*c*e**3*x**3 + x*(b*e + 6*c*d**2*e))/(
8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e
- 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*
```

```
d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b*
*3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 -
 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**7*c**6*e*
*4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**5*b**4*c**4*e**4 - 327680*a**4
*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 - 6144*a**2*b**10*c*e**4 + 256*
a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2 -
 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**2*c**3 + 24*a*b**2*c**2 +
 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a
**4*b**2*c**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 12
8*_t*a**2*b*c**2*e - 16*_t*a*b**3*c*e - 4*_t*b**5*e + 4*a*c**2*d + 3*b**2*c
*d)/(4*a*c**2*e + 3*b**2*c*e))))
```

3.624 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{-b-2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*arcta
nh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)
^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{c\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (\ldots\right.}{\left(b^2-4ac\right)e}$$
$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, \ldots\right.}{\left(b^2-4ac\ldots\right.}$$
$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.12, size = 98, normalized size = 1.00

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcT
an[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 - 4
*a*c)*e)
```

fricas [B] time = 0.92, size = 1042, normalized size = 10.63

$$\left[-\frac{2\left(b^2c-4\,ac^2\right)e^2x^2+4\left(b^2c-4\,ac^2\right)dex+b^3-4\,abc+2\left(b^2c-4\,ac^2\right)d^2+2\left(c^2e^4x^4+4\,c^2de^3x^3+c^2\ldots\right.}{2\left(\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)e^5x^4+4\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)de^4x^3+\left(b^5-8\,ab^3c+16\,a^2bc^2+6\left(b^4c-8\,ab^2c^2\ldots\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4
+ (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2
*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c -
 (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c
```

```
+ 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c
^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*e^4*x^4 + 4*c^
2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3
+ b*c*d)*e*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2
 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [A] time = 0.46, size = 172, normalized size = 1.76

$$-\frac{2\,c\arctan\left(\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{2\left(cd^4+2\left(x^2e+2\,dx\right)cd^2e+\left(x^2e+2\,dx\right)^2ce^2+bd^2+\left(x^2e+2\,dx\right)be+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1
)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e
 + b)/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2
 + (x^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 270, normalized size = 2.76

$$\frac{\cdots}{\left(4ac-b^2\right)e\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (c*e/(4*a*c-b^2)*x^2+2*c*d/(4*a*c-b^2)*x+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d
^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,c\int-\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2
)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
+ b)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^
3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)
```

mupad [B] time = 1.72, size = 417, normalized size = 4.26

$$\frac{\frac{2\,c\,d^2+b}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{c\,e\,x^2}{4\,a\,c-b^2}+\frac{2\,c\,d\,x}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}+\frac{2\,c\,\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)^4\left(x\left(\frac{8\,c^4\,d\,e^7}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((b + 2*c*d^2)/(2*e*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (2*c*d*x)/(4
*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e +
4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*atan(((4*a*c - b^2)^4*(x*((8
*c^4*d*e^7)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*(b^3*c^2*d*e^9 - 4*a*b*c^3*d
*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8)/(a*(4*a*c - b^2)^(7
/2)) - (4*b*c^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/(a*e^2*(4*a*c - b^2)^(11/2
))) + (4*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(7/2)) + (4*b*c^2*(8*a^2*c^3*e^8 - 2
*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)
^(11/2))))/(8*c^4*e^6)))/(e*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 4.68, size = 495, normalized size = 5.05

$$-\frac{c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(
-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*sqrt(-1/(4*
a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**
3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**
2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x +
2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e
- 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) +
x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*
d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*
c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.625 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=299

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{}\right.}{2}$$

```
[Out] 1/2*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^2
+c*e^4*(d/e+x)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/e*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1106, 1092, 1166, 205}

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*
(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^2}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-2\left(b^2e^{4}\cdots\right.}{a+be^2x^2\cdots}\cdots\right)}{2a\left(b^2-\cdots\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-\cdots}\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3\cdots}}$$

Mathematica [A] time = 0.89, size = 271, normalized size = 0.91

$$\frac{\frac{2(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x
)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 1
2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*
e)
```

fricas [B] time = 1.17, size = 3228, normalized size = 10.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*a
*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2
```

)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2 *(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 6 4*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4* c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b ^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 2 0*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4 *b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c ^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^ 6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2* c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2 *b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^ 3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4 *a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 1 8*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9* c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*l og((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^ 2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^ 2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 2 3*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64 *a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^ 2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^ 2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e ^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c) *d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^ 6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a ^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt (1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^ 7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18 *a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c ^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x ^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^ 2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b ^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^ 8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4 *c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7 *c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18* a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^ 3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4* c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +

```
48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)
```

giac [B] time = 0.46, size = 1357, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*l
og(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d
*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*
e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d
^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x - s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(
-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3
- 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a*b^2 - 4*a^2*c) + 1/2*(b*c*x^3*
e^3 + 3*b*c*d*x^2*e^2 + 3*b*c*d^2*x*e + b*c*d^3 + b^2*x*e - 2*a*c*x*e + b^2
*d - 2*a*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e +
 c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(a*b^2*e - 4*a^2*c*e))
```

maple [C] time = 0.02, size = 364, normalized size = 1.22

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^4 c\, e^4+4\_Z^3 c d\, e^3+c\, d^4+b\, d^2+\left(6 c\, d^2 e^2+b\, e\right.\right.\right.}{4\left(4 a c-b^2\right) a e\left(2 c\, e^3 \operatorname{RootOf}\left(\_Z^4 c\, e^4+4\_Z^3 c d\, e^3+c\, d^4+b\, d^2+\left(6 c\, d^2 e^2+b\, e^2\right)\_Z^2+\left(4 c\, d^3 e+2 d e b\right)\_Z+a\right)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*d
^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*sum((-_R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2+6*a
*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_
R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bce^3x^3 + 3\,bcde^2x^2 + bcd^3 + \left(3\,bcd^2 + b^2 - 2\,a}{2\left(\left(ab^2c - 4\,a^2c^2\right)e^5x^4 + 4\left(ab^2c - 4\,a^2c^2\right)de^4x^3 + \left(ab^3 - 4\,a^2bc + 6\left(ab^2c - 4\,a^2c^2\right)d^2\right)e^3x^2 + 2\left(2\left(ab^2c - 4\,a^2c^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - 2*a*c)*e*
x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^
2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 +
2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) - 1/2*int
egrate(-(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a
```

mupad [B] time = 4.85, size = 9056, normalized size = 30.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 2
40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8
*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^
6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 6
4*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64
*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 1
6384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*
e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^
8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*
e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288
*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c
*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 -
6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 7
2*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^
2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i - (-(b^11 + b^2*(
-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c
^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
```

^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) *(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3 *b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^ 4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^ 13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a ^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^ 4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840 *a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e ^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6* b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a ^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^ 2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^ 2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2* b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/ 2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^ 9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632* a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e ^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64* a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^ 2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840 *a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b ^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^ 7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4 *b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6* c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3* c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^1 1)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2* c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8* a^3*b^2*c))) + (-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 28 8*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(- (4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10* c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 2 88*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^ 2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d* e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d *e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3 *b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2 *c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7 *c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 2 40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8 *b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b* c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (5*b^3*c^4*e^10 - 36*a*b*c^5*e^10)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*2i + atan(((-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i - (-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15

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04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^
8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*
e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^1
2 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*
c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*
b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))
- (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 76
8*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^
2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2
 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1
/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*
c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 24
0*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*
b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c
^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^1
2))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (-(b^11 - b^2*(-(4*a*c - b^
2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^
4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2
+ 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6
*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5
*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^1
2 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^
7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^1
4 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/
2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4
 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^
9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2
+ 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*
a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 +
 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b
^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280
*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (
1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*
c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x
*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4
*c^2 - 8*a^3*b^2*c))) + (5*b^3*c^4*e^10 - 36*a*b*c^5*e^10)/(4*(a^2*b^6 - 64
*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^
9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b
^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4
096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^
3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*2i - ((b^2*d -
 2*a*c*d + b*c*d^3)/(2*a*e*(4*a*c - b^2)) + (x*(b^2 - 2*a*c + 3*b*c*d^2))/(
2*a*(4*a*c - b^2)) + (b*c*e^2*x^3)/(2*a*(4*a*c - b^2)) + (3*b*c*d*e*x^2)/(2
*a*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*
d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.626 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+e}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2\right.}$$

```
[Out] 1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+
1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)^(3/2)/e+ln(e*x+d)/a^2/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/e
```

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+e}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(
d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-b}{x(a+bx+cx}\right.}{2a\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{-b^2+4ac}{ax}\right.\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Sub}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Sub}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\log}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-}}{2a^2\left(b^2-4\right.}$$

Mathematica [A] time = 0.47, size = 235, normalized size = 1.45

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b\right.}{\left(b^2-4ac\right)}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e)
```

fricas [B] time = 1.21, size = 2476, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
```

```
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*x^4 + 4*(a^2*b^4
*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*x^2 + 2*(2*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e
), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2
*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + 2*
((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6
*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b
^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^
4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^
2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 +
4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e
*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*e^5*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d
^2)*e^3*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d^2)*e)]
```

giac [B] time = 1.25, size = 454, normalized size = 2.80

$$-\frac{\left(a^{2} b^{3} c e^{3} - 6 \, a^{3} b c^{2} e^{3}\right) \sqrt{b^{2} - 4 \, a c} \log\left(\left|b x^{2} e^{2} + 2 \, b d x e + \sqrt{b^{2} - 4 \, a c} x^{2} e^{2} + 2 \, \sqrt{b^{2} - 4 \, a c} d x e + b d^{2} + \sqrt{b^{2} - 4 \, a c}\right.\right.}{4 \, \left(a^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2
 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^
2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(
b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2
*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^4*b^4*
c*e^4 - 8*a^5*b^2*c^2*e^4 + 16*a^6*c^3*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4
+ 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d
*x*e + b*d^2 + a))/a^2 + e^(-1)*log(abs(x*e + d))/a^2 + 1/2*(a*b*c*x^2*e^2
+ 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*e^(-1)/((c*x^4*e^4 + 4*c*d*x
^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*
d^2 + a)*(b^2 - 4*a*c)*a^2)
```

maple [C] time = 0.03, size = 693, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*d/(4*a*c-b^2
)*x-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c*d^2+1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c
-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/a^2/(4*a*c-b^2)/e*sum((c*e^3*(4*a
*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*c
-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^
2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*
d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))+ln(e*
x+d)/a^2/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 11.35, size = 11072, normalized size = 68.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b
*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + log(d
 + e*x)/(a^2*e) - (log((((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b
^2)^3))^(1/2) - 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^
3))^(1/2) - 1)*((b*c^2*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c
- b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2
*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (2*b*c^2*e^16*(2*b^3 - 10*a*
c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a*c - b^2)) - (2*b*c^3*e^18*x^2*(10*
a*c - b^2))/(a*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c
```

```
 - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17
*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(
4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)
))/(4*a^2*e) + (b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^
2 - 4*a*c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*
a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) - ((a^2*e*(-(b^2*(
6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*(-(b^2*(6*a*
c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*(-(b^
2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 +
 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))
/a^2 - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a
*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*(4*a*c - b^2)) + (4*b*c^3
*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^
3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^
4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*
a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)))/(4*a^2*e) + (b^2*c^4*e^14*(b^2 - 4*a*
c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*a*c - b^
2)^3)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*
a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) +
(b*atan(((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2)
 - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))
*(x^2*((((((b*((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^
18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560
*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5
*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 19
2*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a
*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560
*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5
*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e
^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 9
6*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a
^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17
+ 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^
3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 +
1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 -
12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^
4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*
e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 +
 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)
^3*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056
*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)
*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c
^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 -
400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a
*b^2*c)*((((6*a*b^5*c^4*e^17 + 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18
 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^
6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e
 + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e
^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2
- 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 12
8*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5
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*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3
*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (b^3*c^5*e^16)/(a^3*b^6 - 64*a
^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*((b*((320*a^5*b*c^6*e^18 - 2*a
^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 -
184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(
a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*
a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a
^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 -
184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a
^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*
e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*
(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6
*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19
- 2688*a^6*b^3*c^5*e^19))/(32*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3
*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 40
0*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c))) + x*((((((b*((2*(320*a^5*b*c^6*
d*e^17 - 2*a^2*b^7*c^3*d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e
^17))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e -
128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 1
2*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2
688*a^6*b^3*c^5*d*e^18))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2
*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2
*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*
b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*
e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*
e^18 - 2688*a^6*b^3*c^5*d*e^18))/(4*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a
^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*
b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((2*(6*a*b^5*c^4*d*e^1
6 + 80*a^3*b*c^6*d*e^16 - 44*a^2*b^3*c^5*d*e^16))/(a^3*b^6 - 64*a^6*c^3 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*
d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*
b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 -
184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18)
)/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 -
256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*
a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c
^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^
(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^1
8 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e
^18))/(32*a^6*e^3*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*
a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b
^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((2*(6*a*b^5*c^4*d*e^16 + 80
*a^3*b*c^6*d*e^16 - 44*a^2*b^3*c^5*d*e^16))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*d*e^17
 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^
2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^
4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/((a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^
```

5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (2*b^3*c^5*d*e^15)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/(4*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/(16*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c))) + (((b*((4*a*b^6*c^3*e^15 - 33*a^2*b^4*c^4*e^15 + 68*a^3*b^2*c^5*e^15 - 44*a^2*b^3*c^5*d^2*e^15 + 6*a*b^5*c^4*d^2*e^15 + 80*a^3*b*c^6*d^2*e^15)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (((b*(6*a*c - b^2)*((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b^3*(6*a*c - b^2)^3*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*

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c^6*d^2*e^17))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^
4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2
 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((4*a*b^6*c^3*e^15
- 33*a^2*b^4*c^4*e^15 + 68*a^3*b^2*c^5*e^15 - 44*a^2*b^3*c^5*d^2*e^15 + 6*a
*b^5*c^4*d^2*e^15 + 80*a^3*b*c^6*d^2*e^15)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) - (((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*
a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b
^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b
^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*
e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e
^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17
 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*
d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 1
92*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^
2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(
2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2
)) - (b^4*c^4*e^14 - 4*a*b^2*c^5*e^14 + b^3*c^5*d^2*e^14)/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (b*(6*a*c - b^2)*((b*(6*a*c - b^2)*
((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5
*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*
b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*
a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17
 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^1
7 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*
d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2
*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*
a^2*e*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96
*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 +
192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184
*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^
17 + 2560*a^7*b*c^6*d^2*e^17))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a
^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 -
48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) +
(b^2*(6*a*c - b^2)^2*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4
*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 25
6*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1
056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e
^17))/(32*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4
*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*
b^2*c^2 - 72*a*b^4*c))))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4
*e^14))*(6*a*c - b^2))/(2*a^2*e*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.627 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=348

$$-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10\cdots\right.\right.}{2\sqrt{\cdots}}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)
/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e*x+d)*2^
(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*
c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.65, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10\cdots\right.\right.}{2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d
 + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 16*
a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex\right.}$$

Mathematica [A] time = 1.63, size = 339, normalized size = 0.97

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}\right.}{}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
```

```
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

fricas [B] time = 1.18, size = 4330, normalized size = 12.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c
+ 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*
c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b
^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5
- 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 25
00*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*
b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5
*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b
^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*
c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
+ 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c
^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^
7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b
^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 250
0*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
- 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*b
^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*
b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^
4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c
^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
```

```
10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4
*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^
2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^
4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500
*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^
9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b
*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*
a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^
2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 1
0*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*
a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2
)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
- 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4
*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2
500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*
a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^9
*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*
c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5
*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))))/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 +
 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c
 - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 -
 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2
*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)
```

giac [B] time = 0.79, size = 847, normalized size = 2.43

$$\frac{\left(2\left(3a^3b^2c-10a^4c^2\right)\sqrt{2ab+2\sqrt{b^2-4ac}\,a}\,\sqrt{b^2-4ac}\,\left|a^2b^2e^2-4a^3ce^2\right|e^2-\left(a^2b^2e^2-4a^3ce^2\right)^2\left(3b^3-13abc\right)\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt
(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 - (a^2*b^2*e^2 - 4*a^3*c*e
^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a) + (3*a^4*b^7 -
 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*
a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*a^3
*b*c*e^2 + sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*
e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a^5*
b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a))
 + 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*s
qrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 + (a^2*b^2*e^2 - 4*a^3*
c*e^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (3*a^4*b^
7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 -
 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*
a^3*b*c*e^2 - sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4
*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a
^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(
a)) - 1/2*(b^2*c*e^(-1)/(x*e + d) - 2*a*c^2*e^(-1)/(x*e + d) + b^3*e^(-1)/(
x*e + d)^3 - 3*a*b*c*e^(-1)/(x*e + d)^3)/((a^2*b^2 - 4*a^3*c)*(c + b/(x*e +
 d)^2 + a/(x*e + d)^4)) - e^(-1)/((x*e + d)*a^2)
```

maple [C] time = 0.03, size = 1304, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e^2/(4*a*c
-b^2)*x^3*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a*c-b^2)*x^2+3/2/a^2/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
*d*c*e/(4*a*c-b^2)*x^2*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2*d^2+3/2/a^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c+1/
2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2
+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c
-b^2)*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^3-1/4/a^2/(4*a*c-b^2)/e*su
m((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c
*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a^2/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 6.38, size = 10556, normalized size = 30.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 +
240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^
10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 -
3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 +
 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*
a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-
(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c
^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^
8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5
*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a
^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 98
3040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*
e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^
9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 15
72864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12
 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e
^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c
^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*
b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 4
58752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
+ 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^
4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
+ 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^
2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a
^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 +
144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 1
43360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^1
2) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*
a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240
*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*
b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^
6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*
a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2
*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6
*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4
*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^1
0*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 32768
0*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14)
 + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^
3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*
a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^14*b*c^8*e^12
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+ 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11) - (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e

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^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^
5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*
a^11*b^2*c^8*d*e^11) + 128000*a^10*c^9*e^10 + 504*a^6*b^8*c^5*e^10 - 8112*a
^7*b^6*c^6*e^10 + 48704*a^8*b^4*c^7*e^10 - 129280*a^9*b^2*c^8*e^10))*(-(9*b
^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*2i - atan(((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6
*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a
^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*
c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*
b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*
e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^1
2*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b
^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2
077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2
 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144
*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*
e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*
c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*
a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 +
61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^
6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*
b^13*c^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a
^11*b^7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) +
204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11
 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c
^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 409
6*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3
*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^1
2*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8
*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^1
1*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*
c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*
b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^1
0*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2
 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/
2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5
*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11
*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6
*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 384
0*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e
^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4
*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*
b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144
```

*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e
^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^
14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10
*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 12615
68*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 -
3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*
e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*
c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b
^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)
^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*
a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213
*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a
^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^
2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^
4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3
*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*
b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 12
80*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*
(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e
^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*
c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^1
0*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 -
327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3
*c^7*d*e^13) + 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^
11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a
^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 14
4*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11
- 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2
*c^8*d*e^11) - (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
+ 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c
*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 -
6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*
e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6
*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 +
9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656
*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c
- b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2
- 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1
/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*
a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240
*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*
b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 -
6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^
14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b
*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*
a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^
13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c

```
^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^
7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800
*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 301
12*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e
^11 - 458752*a^11*b^2*c^8*d*e^11) + 128000*a^10*c^9*e^10 + 504*a^6*b^8*c^5*
e^10 - 8112*a^7*b^6*c^6*e^10 + 48704*a^8*b^4*c^7*e^10 - 129280*a^9*b^2*c^8*
e^10))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 2
40*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^1
0*b^2*c^5*e^2)))^(1/2)*2i - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*
a*b*c*d))/(a*(a*b^2 - 4*a^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3))/(2*a*(
a*b^2 - 4*a^2*c)) - (2*x^3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2))/(a*(a*b^2 - 4*
a^2*c)) + (2*a*b^2 - 8*a^2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*
a*b*c*d^2)/(2*a*e*(a*b^2 - 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*
b^2*c*d^2*e - 11*a*b*c*e))/(2*a*(a*b^2 - 4*a^2*c)))/(a*d + x*(a*e + 3*b*d^2
*e + 5*c*d^4*e) + x^3*(b*e^3 + 10*c*d^2*e^3) + b*d^3 + c*d^5 + x^2*(10*c*d^
3*e^2 + 3*b*d*e^2) + c*e^5*x^5 + 5*c*d*e^4*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.628 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=213

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+ex)}{a^3e}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\cdots}{\cdots}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-
4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b^2*c+b^4
)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e-2*
b*ln(e*x+d)/a^3/e+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

Rubi [A] time = 0.39, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\,\mathrm{l}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d
+ e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subs}}{}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d\right.}$$

Mathematica [**A**] time = 0.52, size = 284, normalized size = 1.33

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}\right.}{\left(b^2-4ac\right)^{3/2}}}{2a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e)
```

fricas [**B**] time = 1.90, size = 4562, normalized size = 21.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
```

(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^
4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 -
8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c
 + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6
 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5
x^4 + 4(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 1
5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c +
 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)
d^5 + 2(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*
c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^
6*c^2)*d^2)*e), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a
^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^
2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2
*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x
 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 +
6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2
*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*
(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*
d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c +
6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b

```
^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3
 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16
*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
+ 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d
^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e
^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*
c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 +
 (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2
)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8
*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*
a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3
)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e)]
```

giac [A] time = 0.43, size = 224, normalized size = 1.05

$$\frac{\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2+4\,ac}} + \frac{be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{2\,a^3} + \frac{\left(\frac{b^3c-3\,abc^2}{a} + \frac{\left(b^4e-4\,ab^2ce+2\,a^2c^2 \cdots\right.}{(xe+d)^2a}\right.}{2\left(b^2-4\,ac\right)a^2\left(c+\frac{b}{(xe+d)^2} \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a
*c))*e^(-1)/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) + 1/2*b*e^(-1)*log(c +
 b/(x*e + d)^2 + a/(x*e + d)^4)/a^3 + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e -
 4*a*b^2*c*e + 2*a^2*c^2*e)*e^(-1)/((x*e + d)^2*a))*e^(-1)/((b^2 - 4*a*c)*a
^2*(c + b/(x*e + d)^2 + a/(x*e + d)^4)) - 1/2*e^(-1)/((x*e + d)^2*a^2)
```

maple [C] time = 0.04, size = 1014, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
```

```
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2
)*x^2*b^2-2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/a^2/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*
c-b^2)*x*b^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/
(4*a*c-b^2)*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)/e/(4*a*c-b^2)*b^3+1/a^3/(4*a*c-b^2)/e*sum((b*c*e^3*(4*a*c-b^2)*_R^3+3*
b*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2*
c-b^4)*_R+4*a*b*c^2*d^3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(2*_R^3*c*
e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^
4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*
d*e)*_Z+a))-1/2/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 12.32, size = 12436, normalized size = 58.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^4*(b*e^4 + 15*c*d^2*e^4) + a*d^2 + b*d^4 + c*d^6 + x*(2*a*d*e + 4*b*
d^3*e + 6*c*d^5*e) + x^2*(a*e^2 + 6*b*d^2*e^2 + 15*c*d^4*e^2) + x^3*(20*c*d
^3*e^3 + 4*b*d*e^3) + c*e^6*x^6 + 6*c*d*e^5*x^5) + (log((((b + a^3*e*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*
(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2
*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*
b^2*c^2*d^2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a
*b^2*c))/(a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*(-(b^4 + 6*a^2*c^2
- 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e
^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (
8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3
*e) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2
*c - 23*a*b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69
*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 +
69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x
^2*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^
3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e
^15*x*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3))*(((b - a^3*e*(-(b^4 + 6*a^2*c
^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*(-(b^4 + 6
*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b
^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^
2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(
```

a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*
c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 1
0*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (8*c^3*d*e^
17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3*e) - (4*c
^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*
b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 -
41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2
- 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x^2*(3*a*c
- b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^
2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a
c - b^2)^3)/(a^6(4*a*c - b^2)^3)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c
*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 +
48*a^5*b^2*c^2*e^2)) - (2*b*log(d + e*x))/(a^3*e) - (atan(((2*a^9*b^6*(4*a
*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c -
b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e
^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(276*a^5*b*c^7*d*
e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^1
6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8*
c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e
^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*
(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264
*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^
2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3
*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2
*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*
(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (
((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*
a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 6
4*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c
^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*
a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a
^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^
6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9
*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 +
48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2
)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12*
a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 -
46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/
(a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)
))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b
^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))
- (b*((((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^
17 + 30*a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5
*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a
^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^
3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2
*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*
e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^1
8 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 -

672*a^9*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((8*(276*a^5*b*c^7*d*e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/(a^9*e^3*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))) + x^2*((((4*(54*a^3*c^8*e^16 - 2*b^6*c^5*e^16 + 18*a*b^4*c^6*e^16 - 54*a^2*b^2*c^7*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7*e^17 - 6*a^2*b^7*c^4*e^17 + 65*a^3*b^5*c^5*e^17 - 233*a^4*b^3*c^6*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((((4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(2*a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64 *a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((4*(276*a^5*b*c^7*e^17 - 6*a^2*b^7*c^4*e^17 + 65*a^3*b^5*c^5*e^17 - 233*a^4*b^3*c^6*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(2*a^9*e^3*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))) + (((((4*(36*a^6*c^7*e^15 + 4*a^2*b^8*c^3*e^15 - 45*a^3*b^6*c^4*e^15 + 170*a^4*b^4*c^5*e^15 - 225*a^5*b^2*c^6*e^15 + 6*a^2*b^7*c^4*d^2*e^15 - 65*a^3*b^5*c^5*d^2*e^15 + 233*a^4*b^3*c^6*d^2*e^15 - 276*a^5*b*c^7*d^2*e^15))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (4*(2*b^7*c^4*e^14 - 20*a*b^5*c^5*e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^8*d^2*e^14 + 2*b^6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^2*e^14))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^1

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0*b*c^6*d^2*e^17))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2
+ 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^
2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12
*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a
^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^
7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*
a^10*b*c^6*d^2*e^17))/(2*a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e
^2 + 48*a^5*b^2*c^2*e^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c
))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^
3*b^2*c^3 + 72*a*b^6*c)) - (b*((((4*(36*a^6*c^7*e^15 + 4*a^2*b^8*c^3*e^15 -
 45*a^3*b^6*c^4*e^15 + 170*a^4*b^4*c^5*e^15 - 225*a^5*b^2*c^6*e^15 + 6*a^2*
b^7*c^4*d^2*e^15 - 65*a^3*b^5*c^5*d^2*e^15 + 233*a^4*b^3*c^6*d^2*e^15 - 276
*a^5*b*c^7*d^2*e^15))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
) - (((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 11
8*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*
c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b
^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) +
(2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*
e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3
*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17
- 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c
^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*
b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e
- 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48
*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(
3/2)) - (((((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^1
6 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^
4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272
*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^
8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^
17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2
*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64
*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 1
2*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a
^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*
b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c
^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^
17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d
^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b
^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e
^2 + 48*a^5*b^2*c^2*e^2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(a^7*b^8*c^2*e
^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*
a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 -
 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(2*a^9*e^3*(4*a*c - b
^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 -
 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*
(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))))/(36
*a^4*c^6*e^14 + b^8*c^2*e^14 - 12*a*b^6*c^3*e^14 + 48*a^2*b^4*c^4*e^14 - 72
*a^3*b^2*c^5*e^14))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(a^3*e*(4*a*c - b^2)^(3/
2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.629 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=408

$$\frac{b\left(5b^2-19ac\right)}{2a^3e\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}}{2\sqrt{2}\,a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a^3/(
-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d
)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-19*a*c+5*b^2
)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a
^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 3.68, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19\right.\right.}{2\sqrt{2}\,a^3e\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*c)
)/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*
(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]
*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4 -
29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subs}}{}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b\right.}$$

Mathematica [A] time = 2.98, size = 384, normalized size = 0.94

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \cdots}{12a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

fricas [B] time = 1.92, size = 5734, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^
4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*d^4 + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6
*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^
3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2
*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c
 + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 +
 (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^
2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a
^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d
^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
- 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7
*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 82
50*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2
4108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^1
0*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 5042
1*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410
*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*
b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c
^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^
2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^
6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (125
*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4
*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(
-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c
^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(
a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4
*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^
4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10
*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2
```

*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d
^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4
*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 24
15*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 -
12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 1232
5*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (
1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 96
04*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2
 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*
((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a
^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^
2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*
c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3
 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b
^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*
a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*
d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8
250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 -
24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 4341
0*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7
*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*
c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c
^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c
^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (12
5*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^
4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt
(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*
c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625
*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b
^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*
a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^
2 - 64*a^10*c^3)*e^2))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c -
4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a
^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 +
10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^
2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*
d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^
4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2
415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2

```
 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83
630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 123
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x +
(1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9
604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^
2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^
10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 7940
8*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^
5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^
9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((
a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))))/((a^3*b^2*c
- 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a
^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*
c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*
b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b
^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c
)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4
 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b
^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)
```

giac [B] time = 0.55, size = 1987, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((5*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^3*c*e^2 - 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*
a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 3
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*
b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2
- 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b
```

```
^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3
*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x
 - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*
e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^3*b^2 - 4*a^4*c) + 1/2*(b^
3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x^2*e^2 +
 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 + b^4*x*e
- 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d)/((c*x^
4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 +
 2*b*d*x*e + b*d^2 + a)*(a^3*b^2*e - 4*a^4*c*e)) + 1/3*(6*b*x^2*e^2 + 12*b*
d*x*e + 6*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^3)
```

maple [C] time = 0.04, size = 1518, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/a^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b^3*c*e
^2/(4*a*c-b^2)*x^3+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x^2-3/2/a^3/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*
c^2*d^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^2-1/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*
c-b^2)*x*c^2+2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4
*a*c-b^2)*x*b^4+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/a^3/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/a^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+
1/4/a^3/(4*a*c-b^2)/e*sum((b*c*e^2*(19*a*c-5*b^2)*_R^2+2*d*e*b*c*(19*a*c-5*
b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-14*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e
^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4
*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d
*e)*_Z+a))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 8.72, size = 12239, normalized size = 30.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9
*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^
2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^
7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 2197
44*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) -
 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c
*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2
- 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*
b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b
^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 16
5*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 -
 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^1
1*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14
+ 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^1
4 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*
c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^1
6*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13
+ 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c
^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^1
0*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200
*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12
- 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^1
2 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4
*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*
b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 -
488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4
*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*
a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*
e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 -
25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 357
67*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c
^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^
12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280
*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*
((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*
b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 21
```

5040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246
*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*
c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5
e^2)))^(1/2)(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a
^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 98
3040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*
e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^
9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 15
72864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12
+ 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*
e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18
*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^
10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 14
58688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*
e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*
c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458
688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^
9*d*e^11)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2
+ 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 614
4*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) -
80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c
^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a
^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4
*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b
^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 11
6928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(
4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*
c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 +
3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*
c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^
9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*
a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 -
6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^
5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 9175
04*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816
*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12
- 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c
^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^1
0*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280
*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 +
400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*
d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280
*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) - (-(25*b^15 - 25*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b
^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c
- b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 +
4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^
6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b
^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
- 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6

*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) + 476672*a^13*b*c^10*e^10 + 1800*a^9*b^9*c^6*e^10 - 29080*a^10*b^7*c^7*e^10 + 176032*a^11*b^5*c^8*e^10 - 473216*a^12*b^3*c^9*e^10))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225*b^3*c*d^2*e^3 - 62*a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2*b^2)) + (3*x^5*(5*b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2)) + (2*x^3*(15*b^4*d*e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*d*e^2 - 285*a*b*c^2*d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 + 45*b^3*c*d^5 + 28*a^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3 - 171*a*b*c^2*d^5))/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b*c^2*e^5))/(2*a*(4*a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84*a^2*c^2*d^2*e - 40*a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a*b*c^2*d^4*e))/(6*a*(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^4 + 10*a*b^3*d^2 + 15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^2*c*d^4 - 57*a*b*c^2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x^2*(10*b*d^3*e^2 + 21*c*d^5*e^2 + 3*a*d*e^2) + x^5*(b*e^5 + 21*c*d^2*e^5) + a*d^3 + b*d^5 + c*d^7 + x^3*(a*e^3 + 10*b*d^2*e^3 + 35*c*d^4*e^3) + x^4*(35*c*d^3*e^4 + 5*b*d*e^4) + x*(3*a*d^2*e + 5*b*d^4*e + 7*c*d^6*e) + c*e^7*x^7 + 7*c*d*e^6*x^6) + atan(((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2* c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i)/((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e

^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) - (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)

```
+ 476672*a^13*b*c^10*e^10 + 1800*a^9*b^9*c^6*e^10 - 29080*a^10*b^7*c^7*e^10
 + 176032*a^11*b^5*c^8*e^10 - 473216*a^12*b^3*c^9*e^10))*(-(25*b^15 + 25*b^
6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^
3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 -
49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10
*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.630 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=341

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+\right.}{4\sqrt{2}\,e\left(b^2-4\right.}$$

```
[Out] 1/4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-
1/8*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+
c*(e*x+d)^4)+3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1120, 1178, 1166, 205}

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+\right.}{4\sqrt{2}\,e\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2
 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*
a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx\right.}{4\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2 e\left(a+b(d\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2 e\left(a+b(d\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2 e\left(a+b(d\right.}$$

Mathematica [A] time = 4.43, size = 328, normalized size = 0.96

$$\frac{\frac{(d+ex)\left(4ac-7b^2-12bc(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(2\right.}{}}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2
 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3*b^
2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
```

```
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(8*e)
```

fricas [B] time = 1.57, size = 6633, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*e^7*x^7 + 168*b*c^2*d*e^6*x^6 + 2*(252*b*c^2*d^2 + 19*b^2*c
 - 4*a*c^2)*e^5*x^5 + 24*b*c^2*d^7 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2
)*d)*e^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2
)*d^2)*e^3*x^3 + 2*(19*b^2*c - 4*a*c^2)*d^5 + 2*(252*b*c^2*d^5 + 10*(19*b^2
*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*x^2 + 2*(5*b^3 + 16*a*b*c)*
d^3 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3
*(5*b^3 + 16*a*b*c)*d^2)*e*x - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2
)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*
e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*
c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c
+ 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)
))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b
^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*
x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((a*b^13 - 8*
a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a
^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^
4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8
 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c +
80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 16
0*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/(
(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c
^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e
^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4
*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*
a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 3
```

2*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^

```
7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*
c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^
2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280
*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4
*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*
b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*
c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^
5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^
6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*
c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6
*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*e^2))) + 6*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)
*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e
^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d
^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

giac [B] time = 0.75, size = 1688, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(d*e^(-1) + x + sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) +
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c
*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 -
4*a*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e
^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d
*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*
e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
```

```
c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(
d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(
2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*
c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^4 - 8*a*b^2*c
+ 16*a^2*c^2) - 1/8*(12*b*c^2*x^7*e^7 + 84*b*c^2*d*x^6*e^6 + 252*b*c^2*d^2*
x^5*e^5 + 420*b*c^2*d^3*x^4*e^4 + 420*b*c^2*d^4*x^3*e^3 + 252*b*c^2*d^5*x^2
*e^2 + 84*b*c^2*d^6*x*e + 12*b*c^2*d^7 + 19*b^2*c*x^5*e^5 - 4*a*c^2*x^5*e^5
 + 95*b^2*c*d*x^4*e^4 - 20*a*c^2*d*x^4*e^4 + 190*b^2*c*d^2*x^3*e^3 - 40*a*c
^2*d^2*x^3*e^3 + 190*b^2*c*d^3*x^2*e^2 - 40*a*c^2*d^3*x^2*e^2 + 95*b^2*c*d^
4*x*e - 20*a*c^2*d^4*x*e + 19*b^2*c*d^5 - 4*a*c^2*d^5 + 5*b^3*x^3*e^3 + 16*
a*b*c*x^3*e^3 + 15*b^3*d*x^2*e^2 + 48*a*b*c*d*x^2*e^2 + 15*b^3*d^2*x*e + 48
*a*b*c*d^2*x*e + 5*b^3*d^3 + 16*a*b*c*d^3 + 3*a*b^2*x*e + 12*a^2*c*x*e + 3*
a*b^2*d + 12*a^2*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d
^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e
+ 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 704, normalized size = 2.06

$$\frac{3\left(-4\,\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\right.\right.\right.}{16\left(16a^2c^2-8a\,b^2c+b^4\right)e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*
c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*
a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a*b*c*
d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+
3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d
^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.02, size = 12677, normalized size = 37.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 1
5360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/
(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13 - 28672*
a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13
- 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^
5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7
*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^
6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b
^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b
^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a
^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^1
2*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*
b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*
((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
- 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c
^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c
*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^
2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*
e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*
a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4
*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^
12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) -
 b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b
^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*
e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76
80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 + 936*b^8*c^3
*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 30
72*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786
432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((1024*b^
15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 34406
4*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*
e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e
^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^
14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 25
6*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920
*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024
*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576
*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c
^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*
```

c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*
b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c
^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5
*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12)
)/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))
((9((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
- 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*
c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*
c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e
^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7
*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i)/((13
5*b^5*c^3*e^10 + 1080*a*b^3*c^4*e^10 + 432*a^2*b*c^5*e^10)/(64*(b^12 + 4096
*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5
*b^2*c^5 - 24*a*b^10*c)) - (((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 307
2*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 7864
32*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^1
5*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064
*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e
^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12
+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6
144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^
14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^1
4 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*
a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*
a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*
a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^
3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c
^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b
^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^
7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*
c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^1
0*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 -
1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e
^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d
*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
+ (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a
*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*
b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b
^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*
a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*
a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491
20*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + ((18432*a^4*c^7*d*
e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^1
1)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*
b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*
b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*
c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175

040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8
*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3
840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14
 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81
920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 9
6*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1
/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*
a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*
b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2
 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2
 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e
^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^
15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*
c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2
 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*
a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86016
0*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262
1440*a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 +
72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^
5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*
e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53
760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1
966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2
)))^(1/2)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a
^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 614
40*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 4
0*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5
*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*
a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/
2)*2i + atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*
e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^
7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13
 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4
*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 2202009
6*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a
^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^
4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 -
560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5
 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^
2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 5376
0*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196
6080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2))
)^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2
*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440
*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*
a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b
^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^
8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)
 + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 115
20*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*
a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^
4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c

- b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 -
11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c
^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c
^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4
*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 +
936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128
*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c
^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^
2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6
*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^
2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
- ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e
^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4
*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13
)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4
*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*
a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3
*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b
^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(
1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^
7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e
^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 768
0*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860
160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2
621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) -
81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 -
1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 10
48576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b
^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7
*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*
a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*
b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 56
0*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 +
61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2
- 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*
a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 19660
80*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^
(1/2)*1i)/((135*b^5*c^3*e^10 + 1080*a*b^3*c^4*e^10 + 432*a^2*b*c^5*e^10)/(6
4*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*
c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (((786432*a^6*c^8*e^12 - 192*b^12*
c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c
^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*
c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)
) + ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d
*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a
^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^
13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 256
0*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a
^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)
^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*
b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20
*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7
680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 8
60160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2)

- 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + ((18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i - ((x^2*(15*b^3*d*e - 40*a*c^2*d^3*e + 190*b^2*c*d^3*e + 252*b*c^2*d^5*e + 48*a*b*c*d*e))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(5*b^3*e^2 - 40*a*c^2*d^2*e^2 + 190*b^2*c*d^2*e^2 + 420*b*c^2*d^4*e^2 + 16*a*b*c*e^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(84*b*c^2*

```
d^3*e^3 - 4*a*c^2*d*e^3 + 19*b^2*c*d*e^3))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^5*(19*b^2*c*e^4 - 4*a*c^2*e^4 + 252*b*c^2*d^2*e^4))/(8*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2 + 12*a^2*c + 15*b^3*d^2 - 20*a*c^2*d^4 +
95*b^2*c*d^4 + 84*b*c^2*d^6 + 48*a*b*c*d^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) + (5*b^3*d^3 - 4*a*c^2*d^5 + 19*b^2*c*d^5 + 12*b*c^2*d^7 + 3*a*b^2*d +
 12*a^2*c*d + 16*a*b*c*d^3)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2
*e^6*x^7)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*x^6)/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

3.631 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx$

Optimal. Leaf size=150

$$\frac{2a+b(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/4*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*b*(b
+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*arctanh(
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1142, 1114, 638, 614, 618, 206}

$$\frac{2a+b(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\right.}{4\left(b^2-4a\right.}$$
$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$
$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$
$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

Mathematica [A] time = 0.21, size = 146, normalized size = 0.97

$$\frac{\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2}-\frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 -
 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (1
2*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])
/(4*(b^2 - 4*a*c)^2*e)
```

fricas [B] time = 1.07, size = 3739, normalized size = 24.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^5
 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^3*
c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^
2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c
^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 +
27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^
2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a
*b^3*c - 20*a^2*b*c^2)*d)*e*x - 6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^7 + 2*(1
4*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)
*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c
^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3
*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^6 + 15*b^2*c
^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*c + 4*(2*b*c^
3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 +
b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e
^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3
*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))
/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
- 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e
^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +
 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*
b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*
e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*
c^2 - 4*a*b*c^3)*d*e^5*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c
^3)*d^2)*e^4*x^4 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^
3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b
^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 +
a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4
*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 12*(b*c^3*e^8*x^8
 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(
14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^
2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 +
 (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 +
 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^
2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a
*b*c^2)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
```

```
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.69, size = 365, normalized size = 2.43

$$-\frac{3\,bc\arctan\left(\frac{2\,cd^2+2\,\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2d^6+18\,\left(x^2e+2\,dx\right)bc^2d^4e+18\,\left(x^2e+2\,dx\right)^2bc^2d^2e^2+9\,b^2cd^4+\cdots}{4\left(cd^4+2\left(x^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(
-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*d^6
+ 18*(x^2*e + 2*d*x)*b*c^2*d^4*e + 18*(x^2*e + 2*d*x)^2*b*c^2*d^2*e^2 + 9*b
^2*c*d^4 + 6*(x^2*e + 2*d*x)^3*b*c^2*e^3 + 18*(x^2*e + 2*d*x)*b^2*c*d^2*e +
 9*(x^2*e + 2*d*x)^2*b^2*c*e^2 + 2*b^3*d^2 + 10*a*b*c*d^2 + 2*(x^2*e + 2*d*
x)*b^3*e + 10*(x^2*e + 2*d*x)*a*b*c*e + a*b^2 + 8*a^2*c)/((c*d^4 + 2*(x^2*e
 + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e +
 a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 544, normalized size = 3.63

$$\frac{\cdots}{2\left(16a^2c^2-8a\,b^2c+b^4\right)e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*
c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4
+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*
d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+
10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+
```

```
4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+
a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.85, size = 1182, normalized size = 7.88

$$-\frac{\frac{9\,x^4\,\left(b^2\,c\,e^3+10\,b\,c^2\,d^2\,e^3\right)}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{8\,a^2\,c+}{}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3+12\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((9*x^4*(b^2*c*e^3 + 10*b*c^2*d^2*e^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (a*b^2 + 8*a^2*c + 2*b^3*d^2 + 9*b^2*c*d^4 + 6*b*c^2*d^6 + 10*a*b*c*d^2
)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e + 27*b^2*c*d^2*e + 45*
b*c^2*d^4*e + 5*a*b*c*e))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*
b^2*c*e^2 + 10*b*c^2*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3 +
 9*b^2*c*d^2 + 9*b*c^2*d^4 + 5*a*b*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*
b*c^2*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*x^5)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*atan(((b^4*(4*a*c - b^2)^5 + 16*
a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*c^4*e^8)/
(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*
c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((18*b^2*c^4*d*e^7)/(a*(4*a*c -
b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*c^2*d*e^9 -
16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^3*c^2*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8
- 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*
c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (9*b^2*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
))))/(18*b^2*c^4*e^6)))/(e*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 14.45, size = 1671, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**4*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3
```

```
6*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)*
*5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) - 3*b*c*sqrt(-1/(4*a
*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)
**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*sqrt(
-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c + 6*
b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) + (-8*a**2*c - a*b**2 - 10*a*b*c*d**2 -
 2*b**3*d**2 - 9*b**2*c*d**4 - 6*b*c**2*d**6 - 36*b*c**2*d*e**5*x**5 - 6*b*
c**2*e**6*x**6 + x**4*(-9*b**2*c*e**4 - 90*b*c**2*d**2*e**4) + x**3*(-36*b*
*2*c*d*e**3 - 120*b*c**2*d**3*e**3) + x**2*(-10*a*b*c*e**2 - 2*b**3*e**2 -
54*b**2*c*d**2*e**2 - 90*b*c**2*d**4*e**2) + x*(-20*a*b*c*d*e - 4*b**3*d*e
- 36*b**2*c*d**3*e - 36*b*c**2*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e
+ 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b
**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2
*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4
*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e*
*9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 -
 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7
 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e*
*7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**
6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d*
*3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3
*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c
*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**
5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d
*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c
*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6
*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*
b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c
**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**
2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d*
*2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c
**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*
c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d
**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*
d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

3.632 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=363

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+\right.}{8\sqrt{2}\,ae\left(b^2\right.}$$

```
[Out] -1/4*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2
+1/8*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2)^2/e/(a
+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/
(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan((e*x+d)*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c-b*(-52*a*c
+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1119, 1178, 1166, 205}

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+\right.}{8\sqrt{2}\,ae\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e*x
)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c
]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\text{Subst}\left(\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,d\right.}{4\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

Mathematica [A] time = 4.86, size = 382, normalized size = 1.05

$$\frac{-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2(d+ex)\left(8abc+20ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{a\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\right.}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c
```

```
^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
(Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^
3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)
```

fricas [B] time = 1.52, size = 7701, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*x^6 +
2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^5 + 10*(7*(b^2
*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*x^4 + 2*(b^2*c^2 + 20*
a*c^3)*d^7 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2
+ 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*x^3 + 4*(b^3*c + 14*a*b*c^2)*d^5 + 2*(21
*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*
c + 36*a^2*c^2)*d)*e^2*x^2 + 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + 2*(7*(b
^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c +
 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*
b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^
2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2
)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10
 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 10
24*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b
^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^
5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1
280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15
000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000
*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c
+ 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5
 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2
*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*
a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2
- 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 3
5*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 1
60*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqr
t((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*
c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^1
0 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1
024*a^8*c^5)*e^2))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e
^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c
```

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- 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c
 - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)
*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*
b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a
^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2
- 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000
*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^
3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 27
20*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 +
40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a
^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^1
1*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21
824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3
*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^
6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + s
qrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 -
 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^
3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 +
 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3
*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 +
 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*
a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^
4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a
^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 128
0*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*
b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*
b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6
*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2
)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a
^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqr
```

t((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - 2*(a*b^3 - 16*a^2*b*c)*d)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*

```
a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)
*e)
```

giac [B] time = 0.87, size = 2295, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d^
2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d
^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*
d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^
2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2
*d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e
^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*x^7*e^7 + 20*a*
c^3*x^7*e^7 + 7*b^2*c^2*d*x^6*e^6 + 140*a*c^3*d*x^6*e^6 + 21*b^2*c^2*d^2*x^
5*e^5 + 420*a*c^3*d^2*x^5*e^5 + 35*b^2*c^2*d^3*x^4*e^4 + 700*a*c^3*d^3*x^4*
e^4 + 35*b^2*c^2*d^4*x^3*e^3 + 700*a*c^3*d^4*x^3*e^3 + 21*b^2*c^2*d^5*x^2*e
^2 + 420*a*c^3*d^5*x^2*e^2 + 7*b^2*c^2*d^6*x*e + 140*a*c^3*d^6*x*e + b^2*c^
2*d^7 + 20*a*c^3*d^7 + 2*b^3*c*x^5*e^5 + 28*a*b*c^2*x^5*e^5 + 10*b^3*c*d*x^
4*e^4 + 140*a*b*c^2*d*x^4*e^4 + 20*b^3*c*d^2*x^3*e^3 + 280*a*b*c^2*d^2*x^3*
e^3 + 20*b^3*c*d^3*x^2*e^2 + 280*a*b*c^2*d^3*x^2*e^2 + 10*b^3*c*d^4*x*e + 1
40*a*b*c^2*d^4*x*e + 2*b^3*c*d^5 + 28*a*b*c^2*d^5 + b^4*x^3*e^3 + 5*a*b^2*c
*x^3*e^3 + 36*a^2*c^2*x^3*e^3 + 3*b^4*d*x^2*e^2 + 15*a*b^2*c*d*x^2*e^2 + 10
```

```
8*a^2*c^2*d*x^2*e^2 + 3*b^4*d^2*x*e + 15*a*b^2*c*d^2*x*e + 108*a^2*c^2*d^2*
x*e + b^4*d^3 + 5*a*b^2*c*d^3 + 36*a^2*c^2*d^3 - a*b^3*x*e + 16*a^2*b*c*x*e
 - a*b^3*d + 16*a^2*b*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 +
4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8*a^2
*b^2*c*e + 16*a^3*c^2*e))
```

maple [C] time = 0.05, size = 885, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e^5*(2
0*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*c*d^2
+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(140*a*
c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/8*e^
2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^2+5*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21*b^2*
c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*d^4+1
0*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^2*d^4
+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(16*a^2
*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*s
um((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-
16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.43, size = 14584, normalized size = 40.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^5*(2*b^3*c*e^4 + 420*a*c^3*d^2*e^4 + 21*b^2*c^2*d^2*e^4 + 28*a*b*c^2*e^
4))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*b^4*d*e + 21*b^2*c^2*d^5
*e + 108*a^2*c^2*d*e + 420*a*c^3*d^5*e + 20*b^3*c*d^3*e + 280*a*b*c^2*d^3*e
 + 15*a*b^2*c*d*e))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(b^2*c^2*
d*e^5 + 20*a*c^3*d*e^5))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(20*a*
c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*b^4*d^
2 - a*b^3 + 140*a*c^3*d^6 + 10*b^3*c*d^4 + 108*a^2*c^2*d^2 + 7*b^2*c^2*d^6
+ 16*a^2*b*c + 15*a*b^2*c*d^2 + 140*a*b*c^2*d^4))/(8*a*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (x^3*(b^4*e^2 + 36*a^2*c^2*e^2 + 700*a*c^3*d^4*e^2 + 20*b^3*c
*d^2*e^2 + 35*b^2*c^2*d^4*e^2 + 5*a*b^2*c*e^2 + 280*a*b*c^2*d^2*e^2))/(8*a*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3 + 20*a*c^3*d^7 + 2*b^3*c*d^5 + 3
6*a^2*c^2*d^3 + b^2*c^2*d^7 - a*b^3*d + 16*a^2*b*c*d + 5*a*b^2*c*d^3 + 28*a
```

*b*c^2*d^5)/(8*a*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(140*a*c^3*d^3*e^3 + 7*b^2*c^2*d^3*e^3 + 2*b^3*c*d*e^3 + 28*a*b*c^2*d*e^3))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*

c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^1
4 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^1
4 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 9
6*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2)
- 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^
9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a
*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^
13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*
e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6
*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b
^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a
c(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 -
40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^
7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080
*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^
(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*
b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 -
256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b
*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a
^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*
c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 4
0*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7
*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*
a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(
1/2)*1i)/((8000*a^3*c^7*e^10 - 35*b^6*c^4*e^10 - 84*a*b^4*c^5*e^10 + 12720*
a^2*b^2*c^6*e^10)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256*
a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*
a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 550
5024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*
a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((
67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*
d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 3670016
0*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*
d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1
280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*
c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^
4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 +
256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 +
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10
160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c
^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/
(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^
16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^
10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^
11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c
^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a
^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20
*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7
680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 8
60160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2
- 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3
*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*
c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*
a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a
^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 14

72*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*2i + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13

+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4

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*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)
+ (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^
5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a
^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 +
 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7
*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4
*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*
b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*
c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b
^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1
i)/((8000*a^3*c^7*e^10 - 35*b^6*c^4*e^10 - 84*a*b^4*c^5*e^10 + 12720*a^2*b^
2*c^6*e^10)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256*a*b^13
*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^
9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a
^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^
8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((671088
64*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13
 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*
b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13
)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^
14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14
 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a
^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(
-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^
3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1
863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(
a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2
*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5
*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 +
 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^
6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*
a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262
1440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^1
1 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*
e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^
10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2
*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2
*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2
 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a
*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 -
40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^
7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080
*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^
(1/2) + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11
 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^
7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 -
 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^
2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c
^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*
```

```
b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c
^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*
a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 -
1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7
*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(
512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b
^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14
- 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 +
163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*
c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4
*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b
^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863
680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3
*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^
2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^
2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^
8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)^15
)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3488
0*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^
7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 10
48576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b
^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^
4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^
6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)
^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880
*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7
 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104
8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^
14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*
b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440
*a^12*b^2*c^9*e^2)))^(1/2)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.633 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{-b-2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+e\right.}$$

```
[Out] 1/4*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c*(
b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*arctanh
((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] time = 0.18, antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1142, 1107, 614, 618, 206}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]
])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,d\right.}{2\left(b^2-4ac\right)}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

Mathematica [A] time = 0.18, size = 147, normalized size = 0.97

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)
^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*
c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
4*(b^2 - 4*a*c)^2*e)
```

fricas [B] time = 1.13, size = 3708, normalized size = 24.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 1
8*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3
- 4*a*c^4)*d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d
)*e^3*x^3 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4
+ 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*
c^2 - 4*a*b*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*
```

```
(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c
^2 - 20*a^2*c^3)*d)*e*x + 12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2
 + b*c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*
d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d
^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c
^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2
*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (
b^2*c^2 + 2*a*c^3)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d
*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d
^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqr
t(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x
^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 1
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 +
 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d
^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2
 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*
c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2
*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b
^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 18*(b^3*c^2
- 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3 - 4*a*c^4)*
d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*x^3 -
 b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*(9*(b^2*c^3
 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2
*c^3)*d)*e*x - 24*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*c^3)*e
^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*d^6 + (70*c
^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d^5 + 10*b*c
^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^
3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2
)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2
*a*c^3)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
```

```
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.63, size = 365, normalized size = 2.40

$$\frac{6\,c^2\arctan\left(\frac{2\,cd^2+2\,\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3d^6+36\,\left(x^2e+2\,dx\right)c^3d^4e+36\,\left(x^2e+2\,dx\right)^2c^3d^2e^2+18\,bc^2d^4+12\,(x}{4\left(cd^4+2\,(x^2e+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-
1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*d^6 +
36*(x^2*e + 2*d*x)*c^3*d^4*e + 36*(x^2*e + 2*d*x)^2*c^3*d^2*e^2 + 18*b*c^2*
d^4 + 12*(x^2*e + 2*d*x)^3*c^3*e^3 + 36*(x^2*e + 2*d*x)*b*c^2*d^2*e + 18*(x
^2*e + 2*d*x)^2*b*c^2*e^2 + 4*b^2*c*d^2 + 20*a*c^2*d^2 + 4*(x^2*e + 2*d*x)*
b^2*c*e + 20*(x^2*e + 2*d*x)*a*c^2*e - b^3 + 10*a*b*c)/((c*d^4 + 2*(x^2*e +
 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a
)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 541, normalized size = 3.56

$$\frac{}{\left(16a^2c^2-8a\,b^2c+b^4\right)e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2*d*
e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c*d^
2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5*a*
c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*c^2
*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3
*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d
*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.80, size = 1157, normalized size = 7.61

$$\frac{\frac{-b^3+4\,b^2\,c\,d^2+18\,b\,c^2\,d^4+10\,a\,b\,c+}{4\,e\left(16\,a^2\,c^2-8\,a\,b^2\,c\right.}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3+12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((12*c^3*d^6 - b^3 + 20*a*c^2*d^2 + 4*b^2*c*d^2 + 18*b*c^2*d^4 + 10*a*b*c)/
(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(45*c^3*d^4*e + 5*a*c^2*e + b^2
*c*e + 27*b*c^2*d^2*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*x^4*(b*c^2*e^3
+ 10*c^3*d^2*e^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e^5*x^6)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*d*x*(5*a*c^2 + b^2*c + 9*c^3*d^4 + 9*b*c^2
*d^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(3*b*c^2*e^2 + 10*c^3*d^2*
e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*x^5)/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
 + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
 + 8*c^2*d*e^7*x^7) + (6*c^2*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x*((72*c^6*d*e^7)/(a*(4*a*c - b^2)^
(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*(b^5*c^2*d*e^9 - 8*a*b^3*
c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))) + x^2*((36*c^6*e^8)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*
c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (
36*c^6*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
6*b*c^4*(32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^
2*e^8 - 8*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6)))/(e*(4*a*c - b^2)^(5/
2))
```

sympy [B] time = 13.93, size = 1646, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 3
6*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + 3*c**2*sqrt(-1/(4*a*c -
 b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) -
 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**4*c**3*sqrt(-1/(4*
a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2 + 6*c**
3*d**2)/(6*c**3*e**2))/e + (10*a*b*c + 20*a*c**2*d**2 - b**3 + 4*b**2*c*d**
2 + 18*b*c**2*d**4 + 12*c**3*d**6 + 72*c**3*d*e**5*x**5 + 12*c**3*e**6*x**6
 + x**4*(18*b*c**2*e**4 + 180*c**3*d**2*e**4) + x**3*(72*b*c**2*d*e**3 + 24
0*c**3*d**3*e**3) + x**2*(20*a*c**2*e**2 + 4*b**2*c*e**2 + 108*b*c**2*d**2*
e**2 + 180*c**3*d**4*e**2) + x*(40*a*c**2*d*e + 8*b**2*c*d*e + 72*b*c**2*d*
```

```
*3*e + 72*c**3*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**
2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 1
28*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c
*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8
*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c
**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3
*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**
4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e*
*7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c*
*4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**
5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**
2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b*
*3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d
**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**
2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a
*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b*
*5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 76
8*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 +
1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b
**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b*
*5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512
*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 +
 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a
*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b
**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

3.634 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=437

$$\frac{\left(\frac{d}{e}+x\right)\left(3bce^2\left(b^2-8ac\right)\left(\frac{d}{e}+x\right)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}\right.}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{}}}$$

```
[Out] 1/4*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^2
+c*e^4*(d/e+x)^4)^2+1/8*(d/e+x)*((-7*a*c+b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+
b^2)*e^2*(d/e+x)^2)/a^2/(-4*a*c+b^2)^2/(a+b*e^2*(d/e+x)^2+c*e^4*(d/e+x)^4)+
3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
b^4-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(5/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4-10*a*b^2*c+56*a^2*c^2-b
*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 5.36, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1106, 1092, 1178, 1166, 205}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b\left(b^2-8\right.\right.}{8\sqrt{2}\,a^2e\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 -
 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a +
 b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^3}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-4(b^2\cdots}{(a+be^2x\cdots}\right.}{4a\left(b^2\cdots\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)(3\cdots\right.}{8a^2\left(b^2-4ac\right)^2 e\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)(3\cdots\right.}{8a^2\left(b^2-4ac\right)^2 e\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)(3\cdots\right.}{8a^2\left(b^2-4ac\right)^2 e\cdots}$$

Mathematica [A] time = 6.17, size = 463, normalized size = 1.06

$$\frac{28a^2c^2(d+ex)-25ab^2c(d+ex)-24abc^2(d+ex)^3+3b^4(d+ex)+3b^3c(d+ex)^3}{8a^2e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-8abc\cdots\right.}{8\sqrt{2}\,a^2e\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] (-(b^2*(d + e*x)) + 2*a*c*(d + e*x) - b*c*(d + e*x)^3)/(4*a*(-b^2 + 4*a*c)*
e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4*(d + e*x) - 25*a*b^2*c*(d
 + e*x) + 28*a^2*c^2*(d + e*x) + 3*b^3*c*(d + e*x)^3 - 24*a*b*c^2*(d + e*x)
^3)/(8*a^2*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqr
t[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (3*
Sqrt[c]*(-b^4 + 10*a*b^2*c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

fricas [B] time = 1.94, size = 8554, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 42*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6
+ 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^
5*x^5 + 10*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2
*c^3)*d)*e^4*x^4 + 6*(b^3*c^2 - 8*a*b*c^3)*d^7 + 2*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28
*a^2*c^3)*d^2)*e^3*x^3 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + 2*(6
3*(b^3*c^2 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3
+ 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + 2*(3*b^5 - 20*a*b^3*c -
 4*a^2*b*c^2)*d^3 + 2*(21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*
c + 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 2
0*a*b^3*c - 4*a^2*b*c^2)*d^2)*e*x + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e
^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^
5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x
^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c
^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b
^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*
a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt(
(b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^
10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b
^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
- 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*
c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)
*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^
6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*
b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 6758
```

```
4*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4
*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a
^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) -
 (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*
c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9
 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b
^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2
*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a
^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6
*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c
^5)*e^2))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^
8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*
(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^
4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*
a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b
^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)
*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a
^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 +
(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^
5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^
3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b
^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219
*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c
 + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)
*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 128
0*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4
189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 -
 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d -
27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9
*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*
a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*
b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) - (b^14 - 32*a*b^12*c +
464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5
 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2
*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 16
0*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqr
t((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((
a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14
*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^
2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) - 3*sqrt(1/2
)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14
*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*
c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(
a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c
^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a
^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6
- 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
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b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)
*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a
^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c
^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 +
 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-
(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a
^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c
^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3
*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 6
40*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^
10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208
*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a
^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^
15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^
4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b
^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10
*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2
*c^4 - 1024*a^15*c^5)*e^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885
*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 -
 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*
c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8
*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 2
19*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8
*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^
5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1
280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b
^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*
e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)
*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b
^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*
c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16
*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b
^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189
*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c
+ 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2
*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4
)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*
a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^
6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(2
1*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^
4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*
b^2*c^6 + 38416*a^4*c^7)*d - 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 42
4*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5
+ 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a
^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c +
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 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e
^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^
4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt
(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 -
(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2
*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a
^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 -
20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*
a^10*c^5)*e^2))) + 2*(5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 1
6*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c
^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 1
0*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c +
 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2
*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 +
 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3
*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)
```

giac [B] time = 0.56, size = 2487, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^3*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^
2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) -
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b
^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x -
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e
^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^
3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d
*e + 16*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
```

```
c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)
*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
+ sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 -
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*
a*b*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*
b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))))/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2) + 1/8*(3*b^3*c^2*x^7*e^7 - 24*a
*b*c^3*x^7*e^7 + 21*b^3*c^2*d*x^6*e^6 - 168*a*b*c^3*d*x^6*e^6 + 63*b^3*c^2*
d^2*x^5*e^5 - 504*a*b*c^3*d^2*x^5*e^5 + 105*b^3*c^2*d^3*x^4*e^4 - 840*a*b*c
^3*d^3*x^4*e^4 + 105*b^3*c^2*d^4*x^3*e^3 - 840*a*b*c^3*d^4*x^3*e^3 + 63*b^3
*c^2*d^5*x^2*e^2 - 504*a*b*c^3*d^5*x^2*e^2 + 21*b^3*c^2*d^6*x*e - 168*a*b*c
^3*d^6*x*e + 3*b^3*c^2*d^7 - 24*a*b*c^3*d^7 + 6*b^4*c*x^5*e^5 - 49*a*b^2*c^
2*x^5*e^5 + 28*a^2*c^3*x^5*e^5 + 30*b^4*c*d*x^4*e^4 - 245*a*b^2*c^2*d*x^4*e
^4 + 140*a^2*c^3*d*x^4*e^4 + 60*b^4*c*d^2*x^3*e^3 - 490*a*b^2*c^2*d^2*x^3*e
^3 + 280*a^2*c^3*d^2*x^3*e^3 + 60*b^4*c*d^3*x^2*e^2 - 490*a*b^2*c^2*d^3*x^2
*e^2 + 280*a^2*c^3*d^3*x^2*e^2 + 30*b^4*c*d^4*x*e - 245*a*b^2*c^2*d^4*x*e +
 140*a^2*c^3*d^4*x*e + 6*b^4*c*d^5 - 49*a*b^2*c^2*d^5 + 28*a^2*c^3*d^5 + 3*
b^5*x^3*e^3 - 20*a*b^3*c*x^3*e^3 - 4*a^2*b*c^2*x^3*e^3 + 9*b^5*d*x^2*e^2 -
60*a*b^3*c*d*x^2*e^2 - 12*a^2*b*c^2*d*x^2*e^2 + 9*b^5*d^2*x*e - 60*a*b^3*c*
d^2*x*e - 12*a^2*b*c^2*d^2*x*e + 3*b^5*d^3 - 20*a*b^3*c*d^3 - 4*a^2*b*c^2*d
^3 + 5*a*b^4*x*e - 37*a^2*b^2*c*x*e + 44*a^3*c^2*x*e + 5*a*b^4*d - 37*a^2*b
^2*c*d + 44*a^3*c^2*d)/((a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e)*(c*x^4*e
^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*
b*d*x*e + b*d^2 + a)^2)
```

maple [C] time = 0.05, size = 1010, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*e
^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2+6
3*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a
^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2
*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c-
3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*b^3*c
^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+60*a*b^3
*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6+21*b^3*c
^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a
*b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*
c^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37*
a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2+2*b*c*d*e*(-8*a
*c+b^2)*_R-8*a*b*c^2*d^2+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(2*_R^3*c*e^3+
6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*
```

```
e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)
*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6 +
 (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^5*x
^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3
)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*
c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c
^3)*d^2)*e^3*x^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b^3*c^2
 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(3*b^5
 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2
)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2
 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2)*d^2)*e*x + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b
*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 -
8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 +
10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2
+ 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*
d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e) - 3/8*integrate(-((b
^3*c - 8*a*b*c^2)*e^2*x^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2 + 2*(b^3*c - 8*a*b
*c^2)*d*e*x + (b^3*c - 8*a*b*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)
```

mupad [B] time = 7.80, size = 16086, normalized size = 36.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((3*b^5*d^3 + 44*a^3*c^2*d + 6*b^4*c*d^5 + 28*a^2*c^3*d^5 + 3*b^3*c^2*d^7 +
 5*a*b^4*d - 4*a^2*b*c^2*d^3 - 49*a*b^2*c^2*d^5 - 37*a^2*b^2*c*d - 20*a*b^3
*c*d^3 - 24*a*b*c^3*d^7)/(8*a^2*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(3
*b^5*e^2 - 4*a^2*b*c^2*e^2 + 60*b^4*c*d^2*e^2 + 280*a^2*c^3*d^2*e^2 + 105*b
^3*c^2*d^4*e^2 - 20*a*b^3*c*e^2 - 840*a*b*c^3*d^4*e^2 - 490*a*b^2*c^2*d^2*e
^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(6*b^4*c*e^4 + 28*a^2*c^
3*e^4 - 49*a*b^2*c^2*e^4 + 63*b^3*c^2*d^2*e^4 - 504*a*b*c^3*d^2*e^4))/(8*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5*d*e + 280*a^2*c^3*d^3*e + 6
3*b^3*c^2*d^5*e + 60*b^4*c*d^3*e - 12*a^2*b*c^2*d*e - 504*a*b*c^3*d^5*e - 4
90*a*b^2*c^2*d^3*e - 60*a*b^3*c*d*e))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
```

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) + (5*x^4*(28*a^2*c^3*d*e^3 + 21*b^3*c^2*d^3*e^3 + 6*b^4*c*d*e^3 - 49*a*b^
2*c^2*d*e^3 - 168*a*b*c^3*d^3*e^3))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (21*x^6*(b^3*c^2*d*e^5 - 8*a*b*c^3*d*e^5))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x*(5*a*b^4 + 44*a^3*c^2 + 9*b^5*d^2 - 37*a^2*b^2*c + 30*b^4*c*d
^4 + 140*a^2*c^3*d^4 + 21*b^3*c^2*d^6 - 12*a^2*b*c^2*d^2 - 245*a*b^2*c^2*d^
4 - 60*a*b^3*c*d^2 - 168*a*b*c^3*d^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (3*x^7*(b^3*c^2*e^6 - 8*a*b*c^3*e^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 +
 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^
2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c
^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d
*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 +
 b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^
2*d*e^7*x^7) - atan((((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*
a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 +
1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4
096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^
8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b
^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^
5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^
6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5
120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 3
27680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6
*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) -
1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c
^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30105
60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^
2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 5376
0*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1
966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e
^2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^
12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360
*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/
(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7
*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c
 - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^
3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936
*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8
*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*
a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26
21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*
(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^
15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069
824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^
7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^
10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949
120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*1i + ((3612672*a^6
*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*
c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 33914
88*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240
```

*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (
((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c
^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 3670
0160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*
c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^
2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a
^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b
^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*
b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9
(b^19 + b^4(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c
^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*
a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^
16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b
^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*
a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (22020096*a^9*c^9*e^
12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*
e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b
^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 2
4*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144
*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*
c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^
5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8
 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^1
8*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4
*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^
6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) +
(x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4
*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6
*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^1
5)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440
*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*
c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a
^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3
*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*
c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^1
4*b^2*c^9*e^2)))^(1/2)*1i)/((567*b^7*c^5*e^10 - 10368*a*b^5*c^6*e^10 - 1693
44*a^3*b*c^8*e^10 + 67824*a^2*b^3*c^7*e^10)/(256*(a^4*b^12 + 4096*a^10*c^6
 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6
144*a^9*b^2*c^5)) + ((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a
*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1
410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 40
96*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8
*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^
15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5
*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6
 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 51
20*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 32
7680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^
4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 301056
0*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b
^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2

- 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760
*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 19
66080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^
2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^1
2*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*
a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(
512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*
b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3
+ 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*
a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1
048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*
b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a
^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 262
1440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a
^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(
-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^1
5*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 10698
24*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7
*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^1
0*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 29491
20*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) - ((3612672*a^6*c^9
*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*
d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a
^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6
*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67
108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d
*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160
*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*
d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 -
1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b
*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c
^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8
+ 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^
19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 -
8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*
b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512
*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c
^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*
c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13
*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (22020096*a^9*c^9*e^12 -
768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12
+ 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c
^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^
5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9
*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9
+ 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^
9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 4
9*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*
e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2
- 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^
7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(
14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5

*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*2i - atan((((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*(((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a

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^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*1i + ((9*(b^4*(-(4*a*c - b^2
)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c
^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 234393
6*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^
8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160
*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2
621440*a^14*b^2*c^9*e^2)))^(1/2)*((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*
e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c
^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(
a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c
^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^12 - 768*
a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 20
27520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^
12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^1
0*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*
c^5)) + ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^
5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^1
3 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a
^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^
6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(
262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 409
60*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(
32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2
*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1
069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(
1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720
*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048
*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2
949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*
a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a
^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6
 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b
^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2
- 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2
 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^
8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*
c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^
12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b
^2*c^3)))*1i)/((567*b^7*c^5*e^10 - 10368*a*b^5*c^6*e^10 - 169344*a^3*b*c^8*
e^10 + 67824*a^2*b^3*c^7*e^10)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10
*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c
^5)) + ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*
a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5
- 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 +
720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258
048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2
+ 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*(((22020096*
a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4
*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 2339
6352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^
10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*
c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2
```

*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 917504
0*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d
*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*
a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a
^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5
*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a
^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^
2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*
a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316
864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b
^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a
^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^
12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a
^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1
/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^
2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 -
1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^
(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 72
0*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 25804
8*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 +
2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (3612672*a^6
*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*
c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 33914
88*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240
*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (
x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*
c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*
c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2
) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440
*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*
c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a
^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3
*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*
c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^1
4*b^2*c^9*e^2)))^(1/2)*((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 403
2*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11
+ 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 +
 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*
a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c
^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*
b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603
008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*
a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + ((
67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3
*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 367001
60*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^
8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2
- 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9
*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7
*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^
8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b
^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2
+ 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6
*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(51
2*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*

```
c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10
*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^1
3*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)
^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^
3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936
*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8
*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*
a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26
21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))))
)*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b
^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 106
9824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/
2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a
^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a
^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 294
9120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.635 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=255

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-\right.}{}$$

```
[Out] 1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^
2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*
c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*ar
ctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e+ln(e*x
+d)/a^3/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

Rubi [A] time = 0.49, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+e\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(
d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2(b^2-4a\ldots}{x(a+bx+\ldots}\ldots\right)}{4a\left(b\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16\ldots}{4a^2\left(b^2-4ac\right)^2\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16\ldots}{4a^2\left(b^2-4ac\right)^2\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16\ldots}{4a^2\left(b^2-4ac\right)^2\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16\ldots}{4a^2\left(b^2-4ac\right)^2\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16\ldots}{4a^2\left(b^2-4ac\right)^2\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16\ldots}{4a^2\left(b^2-4ac\right)^2\ldots}$$

Mathematica [A] time = 3.95, size = 391, normalized size = 1.53

$$\frac{\frac{a^2\left(2ac-b^2-bc(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{a\left(16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^4+2b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b\ldots\right.}{\left(b^2-4ac\right)^5\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e)
```

fricas [B] time = 3.21, size = 9908, normalized size = 38.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 +
 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c
^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3
+ 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11
*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^
2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*
c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c
^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^
2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*
a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a
^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a
^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c -
 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6
0*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 -
 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*
c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e
*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 +
 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
```

```
4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*
c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 4
8*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*
a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*
a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3
 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^
2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^
6*c^5)*d^2)*e^7*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
- 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6
*b*c^4)*d)*e^6*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*
c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*
a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c
^4)*d^2)*e^5*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 6
4*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b
*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128
*a^7*c^4)*d)*e^4*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*
b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6
 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*
(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^
2)*e^3*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c
^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^
5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4
)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*
a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*
c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5
*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c +
48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e), 1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3
+ 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*
e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(
a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^
3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28
*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (
4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*
```

a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 1
0*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3
 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64
*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b
*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6
*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b
^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 - 10*a*b^3*c^3 +
30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7
 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 +
30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*
d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*
a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*
b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b
^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c
^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*
b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^
3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8
*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 3
0*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c
- 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2
+ 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^
2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*
c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^
3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)
*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 +
 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^
4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 6
4*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6
 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 +
 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c
^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d
^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*
e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6

```
4*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 +
(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x
^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)
*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5
)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*
d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4 +
 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^2)*e^7*x
^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^3
 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d)*e^6*x^
5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4
 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^4 + 30
*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^5 +
10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^3 + (a^3*
b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d)*e^4*
x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3 + 14*(a^3*b
^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6 + 15*(a^3*b^7*c
- 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*(a^3*b^8 - 10*a^4
*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^2)*e^3*x^2 + 4*(2
*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^5 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x + (a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*
a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*
c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6
*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 -
64*a^7*b*c^3)*d^2)*e)]
```

giac [B] time = 1.72, size = 1012, normalized size = 3.97

$$-\frac{\left(a^3b^7ce^3 - 14\,a^4b^5c^2e^3 + 70\,a^5b^3c^3e^3 - 120\,a^6bc^4e^3\right)\sqrt{b^2-4\,ac}\,\log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*
c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a
)) - (a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*c
^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^6*b^8*c*e^4 - 16*a^7*b^6*c^2*e^4 + 96*a^8*b^4*c^3*e^4 - 256*a^9*b^2*
c^4*e^4 + 256*a^10*c^5*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3
+ 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a
))/a^3 + e^(-1)*log(abs(x*e + d))/a^3 + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3
*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 -
12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^
```

```
6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c
^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e
^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 7
0*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d
*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^
2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3
*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4
*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^
2*b^3*c*d*e - a^3*b*c^2*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^
2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2
 - 4*a*c)^2*a^3)
```

maple [C] time = 0.08, size = 4477, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)
*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*
c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c
^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d
*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3
*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-3/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21/a/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^3+3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^3*c^2-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^
4)*x*b^2*c^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/a/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/
e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^3*d^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2
-8*a*b^2*c+b^4)*b^3*c^2*d^6-29/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b
^4)*b^2*c^2*d^4+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c*d^4
-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c*d^2-29/4/a/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-7/2/a/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e
^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2*b*c^2-1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*
c+b^4)*x*b^5+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
```

```
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2+1/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4+16/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+24/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2*c^3*d^2+6*a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)
*c^2+3/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4+4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+16/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8
*a*b^2*c+b^4)*x*c^3+4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-2
1/4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+ln(e*x+d)/a^3/e-21/a/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*b*c^3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/a^2/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2*b^3*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-105/2/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3
*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*d^2+15/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^3*d^2-70/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^3*b+10/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^3*b^3-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3*b^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4
-105/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/a^2
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c^2*d^4-87/2/a/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2*d^2+6/a^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c*d^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 17.98, size = 19440, normalized size = 76.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
```

^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + log(d + e*x)/(a^3*e) - (log((((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e) + (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6))*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e) - (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*(4*a*c - b^2)^4) + (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4) + (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*atan((x*((((((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*

b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 2496
0*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^
18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*
a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c
^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*
a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 -
10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^
2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e +
2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*
c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9
*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 -
294912*a^12*b^3*c^8*d*e^18))/(4*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 -
4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c
^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)
))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 25
60*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 8
0*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2
*c^4*e^2)) + (b*((2*(8960*a^7*b*c^9*d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^
3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6*d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 1102
4*a^6*b^3*c^8*d*e^16))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*
b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*
(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 +
276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17
- 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240
*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) -
((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560
*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*
d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9
*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 2949
12*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c
*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(
a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c
^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e +
320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)
)/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^
2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2*c^2 -
10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*
b^2*c)^3*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*
c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^
10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18
))/(32*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10
*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2
c^5)))(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6
*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c
^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^
5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d
*e^15))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*
a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(8960*a^7*b*c^9
*d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6
*d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 11024*a^6*b^3*c^8*d*e^16))/(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^1
3*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b
^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a
^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163
840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 -

3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*
e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^
10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a
^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b
^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*
b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^
3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5
*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120
*a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a
^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 409
6*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e
^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^
13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*
b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b
^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 +
3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*
a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(16
3840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 -
3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d
*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b
^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*
a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*
b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11
*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2))
- (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b
^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*
a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840
*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18
+ 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/(4*a^3*e*(4*a*
c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*
a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*
e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e -
2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4
*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328
*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18
- 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*
c^8*d*e^18))/(16*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2
- 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7
*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 -
1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*
c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*b^10 - 64
00*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*
b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(
15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b
^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(
4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e
^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900
*a^4*b^2*c^6*e^14) + (x^2*((((((b*((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e
^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 +
3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 -
24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6
144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*
a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19
- 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 2
4960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19
- 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4
*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*

e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9
*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*
a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c
)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 256
0*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e
^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^
19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c
^8*e^19))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 -
 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b
^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1
280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048
*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e -
40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 6
40*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b*((8
960*a^7*b*c^9*e^17 - 6*a^2*b^11*c^4*e^17 + 137*a^3*b^9*c^5*e^17 - 1217*a^4*
b^7*c^6*e^17 + 5256*a^5*b^5*c^7*e^17 - 11024*a^6*b^3*c^8*e^17)/(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3
*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18
 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6
 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 -
 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 128
0*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^1
9 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 +
 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^1
9 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a
^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^
4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a
^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*
c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*
b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^
5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2
 - 10*a*b^2*c)^3*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b
^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10
*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(64*a
^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240
*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*
(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*
a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 385
0*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5*e^16 - 21
*a*b^7*c^6*e^16 + 147*a^2*b^5*c^7*e^16 - 343*a^3*b^3*c^8*e^16)/(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((8960*a^7*b*c^9*e^17 - 6*a^2*b^11*c^4*
e^17 + 137*a^3*b^9*c^5*e^17 - 1217*a^4*b^7*c^6*e^17 + 5256*a^5*b^5*c^7*e^17
 - 11024*a^6*b^3*c^8*e^17)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*
a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (
((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a
^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*
b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 -
 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2
048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e
 - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b
^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10
*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4
*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 -
 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 2
4*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 614
4*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3

*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(32*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) - (((b*((4*a^2*b^12*c^3*e^15 - 93*a^3*b^10*c^4*e^15 + 854*a^4*b^8*c^5*e^15 - 3889*a^5*b^6*c^6*e^15 + 8808*a^6*b^4*c^7*e^15 - 7952*a^7*b^2*c^8*e^15 + 6*a^2*b^11*c^4*d^2*e^15 - 137*a^3*b^9*c^5*d^2*e^15 + 1217*a^4*b^7*c^6*d^2*e^15 - 5256*a^5*b^5*c^7*d^2*e^15 + 11024*a^6*b^3*c^8*d^2*e^15 - 8960*a^7*b*c^9*d^2*e^15)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13

*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 9
7280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3
*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*
c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5
120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^
8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10
*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2
*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*
c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2))
)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (((b*((4
*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952*
a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552*
a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 27
6*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e
^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 409
6*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10
*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c
^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*
c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5
*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*
c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*
b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^1
7 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^
13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^
2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6
*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
+ 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))
/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10
*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2
*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^
9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*
a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 32
8*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^
2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 29491
2*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(8*a^3*e*(4*a*c - b^
2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^
6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a
^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^
4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e
 - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*
e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*
b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)
^3*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 51
20*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 +
16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e
^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*
b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e
^17 - 163840*a^13*b*c^9*d^2*e^17))/(64*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*
c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 655
36*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3
840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(1
5/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c
- b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2*e^14 - 20*a*b^8*c
^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^1
4)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b
^2*c^4 - 120*a*b^8*c)) - (3*b*((((4*a^2*b^12*c^3*e^15 - 93*a^3*b^10*c^4*e^1
5 + 854*a^4*b^8*c^5*e^15 - 3889*a^5*b^6*c^6*e^15 + 8808*a^6*b^4*c^7*e^15 -

7952*a^7*b^2*c^8*e^15 + 6*a^2*b^11*c^4*d^2*e^15 - 137*a^3*b^9*c^5*d^2*e^15
+ 1217*a^4*b^7*c^6*d^2*e^15 - 5256*a^5*b^5*c^7*d^2*e^15 + 11024*a^6*b^3*c^8
*d^2*e^15 - 8960*a^7*b*c^9*d^2*e^15)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1
0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^
2*c^5) - (((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4
*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*
e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*
d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*
b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a
^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^
3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e +
320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)
*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120
*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16
384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^1
7 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^
7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^1
7 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80
*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*
c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280
*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^
5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*
a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*
a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e
- 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^
4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e
^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) -
(b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^14 - 616*a^3*b^4*c^7
*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^2*b^5*c^7*d^2*e^14
- 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6*b^12 + 4096*a^12*c
^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4
- 6144*a^11*b^2*c^5) + (b*((b*((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^1
6 + 1052*a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16
- 32768*a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^
16 - 36*a^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6
*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^
10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*
c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*
e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*
c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9
*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a
^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328
*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2
*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912
*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 -
4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^
3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 2
40*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5))
)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b*(b^4
+ 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e -
1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^
17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17
+ 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^
17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c
^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 22
7328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c
^9*d^2*e^17))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e
^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a
^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2

```
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2
 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*
b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^
8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^
11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^
6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^1
7 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*
b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^1
7))/(32*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4
*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*
e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9
*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*c^3 + 40*
a^2*b^2*c^2 - 11*a*b^4*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^
6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b
^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 614
40*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15
/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a
^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 - 64
00*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*
b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.636 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=484

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3e\left(b^2-4ac\right)^2(d+ex)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\,a^3e}$$

```
[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)+1/4*(b^2-2*a
*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+1/
8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(-4*a*c+b
^2)^2/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+b*
(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+(-124*a^2*b*
c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.23, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan}{8\sqrt{2}\,a^3e\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (
b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 -
 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4
 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[
b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5
*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
```

```
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4}{8a^2(}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d\right.}$$

Mathematica [A] time = 6.24, size = 560, normalized size = 1.16

$$-\frac{1}{a^3e(d+ex)} + \frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e*
x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(
d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d +
 e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (
3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 3
7*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c
- 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] +
60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

fricas [B] time = 2.50, size = 10260, normalized size = 21.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^
2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d
)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4
*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c
^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*
x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a
^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^
3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5
*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a
^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1
024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351
310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/
((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^
18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*
c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(
4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7
+ 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b
^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 8
10000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*
b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 +
 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625
*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^
4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
 + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)
*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c
^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684
672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894
```

*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a
^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2
*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c
^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a
^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*
a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2)))
+ 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3
*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^
8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
c^3 + 126(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2
*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3
+ 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^
3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^
4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6
*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*
e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16
*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 +
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b
^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^1
0 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 -
1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 3
51310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6
)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*
a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^
6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27
*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^
7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a
*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 -
810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^
9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5
+ 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((6
25*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*
a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^
5)*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11
*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 56
84672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 38
94*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 +
(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b
^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8
*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
+ 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 16
0*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))
) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*
e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a

```
^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c
 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b
^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^
3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 +
35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*
c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d
)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6
 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c +
16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9
+ 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c
 + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2
*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b
^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4
 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c
^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 128
0*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*
b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-
27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*
c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825
*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8
 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*
a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c
^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt(
(625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 59188
6*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b
^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*
c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^
11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 -
5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c +
3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5
- (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11
*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b
^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 506
25*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c
^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c +
160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2
))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2
)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32
*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^
5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6
+ 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)
*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d
^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c
```

```
+ 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4
*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a
^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7
*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c
^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2
 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6
*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1
280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^
9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log
(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^
4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 778
25*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c
^8 - 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 200
6*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6
*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqr
t((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591
886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15
*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^1
9*c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*
b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6
- 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c
+ 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^
5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^
11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2
*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 5
0625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4
*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e
^2))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 +
 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 +
126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^
3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^
6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7
+ 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 1
6*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3
*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*
c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*
b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)*d)*e)
```

giac [B] time = 1.22, size = 1412, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
[Out] 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt
```

```
(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*e^2 - 8*a^4*b
^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 - (a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^
2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*
c)*a) + (5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3
+ 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2
*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^3*b
^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 + sqrt((a^3*b^5*e^2 - 8*a^4*b^3
*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^
2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4 - 8*a^5*b^2*
c*e^4 + 16*a^6*c^2*e^4))))*e^(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2
*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 1
6*a^5*c^2*e^2)*abs(a)) + 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^
2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*
abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 + (a^3*b^4*e^2 - 8*
a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(
2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^
9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a
^12*b*c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-
1)/((x*e + d)*sqrt((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 - sqrt
((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*
a^5*b^2*c*e^4 + 16*a^6*c^2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/
(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^2*e^4))))*e^(-3)/((a^7*b^6*c - 12
*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*
e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*abs(a)) - 1/8*(7*b^4*c^2*e^(-1)/(x*
e + d) - 47*a*b^2*c^3*e^(-1)/(x*e + d) + 52*a^2*c^4*e^(-1)/(x*e + d) + 14*b
^5*c*e^(-1)/(x*e + d)^3 - 99*a*b^3*c^2*e^(-1)/(x*e + d)^3 + 136*a^2*b*c^3*e
^(-1)/(x*e + d)^3 + 7*b^6*e^(-1)/(x*e + d)^5 - 43*a*b^4*c*e^(-1)/(x*e + d)^
5 + 25*a^2*b^2*c^2*e^(-1)/(x*e + d)^5 + 68*a^3*c^3*e^(-1)/(x*e + d)^5 + 9*a
*b^5*e^(-1)/(x*e + d)^7 - 66*a^2*b^3*c*e^(-1)/(x*e + d)^7 + 108*a^3*b*c^2*e
^(-1)/(x*e + d)^7)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b/(x*e + d)^2
 + a/(x*e + d)^4)^2) - e^(-1)/((x*e + d)*a^3)
```

maple [C] time = 0.07, size = 6821, normalized size = 14.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 14.38, size = 18112, normalized size = 37.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
[Out] - ((x^4*(15*b^6*e^3 + 324*a^3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*
e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*
```

a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*
e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108
*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^
6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 +
25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 +
50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/
(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 +
140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3
*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^
3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 36
4*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e
+ 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*
a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*
d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91
*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b
*c^3*d^3*e^2 - 1554*a*b^2*c^3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*
b^2*c)) + (3*x^7*(60*a^2*c^4*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))
/(a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b
^6*d^4 - 64*a^3*b^2*c + 25*a*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180
*a^2*c^4*d^8 + 15*b^4*c^2*d^8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227
*a*b^3*c^2*d^6 + 392*a^2*b*c^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4
- 91*a*b^4*c*d^4)/(8*a*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b
^2*d^2*e^3 + 84*c^2*d^6*e^3 + 2*a*b*e^3 + 20*a*c*d^2*e^3 + 70*b*c*d^4*e^3)
+ x^7*(36*c^2*d^2*e^7 + 2*b*c*e^7) + x*(a^2*e + 5*b^2*d^4*e + 9*c^2*d^8*e +
6*a*b*d^2*e + 10*a*c*d^4*e + 14*b*c*d^6*e) + x^4*(5*b^2*d*e^4 + 126*c^2*d^
5*e^4 + 10*a*c*d*e^4 + 70*b*c*d^3*e^4) + a^2*d + x^2*(10*b^2*d^3*e^2 + 36*c
^2*d^7*e^2 + 6*a*b*d*e^2 + 20*a*c*d^3*e^2 + 42*b*c*d^5*e^2) + x^6*(84*c^2*d
^3*e^6 + 14*b*c*d*e^6) + x^5*(b^2*e^5 + 126*c^2*d^4*e^5 + 2*a*c*e^5 + 42*b*
c*d^2*e^5) + b^2*d^5 + c^2*d^9 + c^2*e^9*x^9 + 2*a*b*d^3 + 2*a*c*d^5 + 2*b*
c*d^7 + 9*c^2*d*e^8*x^8) - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 10
48576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*
b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*
a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26
21440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 10
48576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*
b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*
a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26
21440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 10
48576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*
b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*
a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26

21440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144
*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4
*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 12
4017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 141733920
7680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a
^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*
b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13
 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145
925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722
688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 283467841536
0*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384
*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*
c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 267
3082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*
a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b
^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^1
1*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12
 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b
^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^
12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869
815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 174731349
1968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*
a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^1
1 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 1687
8108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967
040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 154384780492
8*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648
*a^19*b^2*c^13*d*e^11)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998
60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576
*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*
c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*
b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440
*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998
60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576
*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*
c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*
b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440
*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998
60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576
*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*
c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*
b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440
*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15
*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14
 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 1240171
80672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*
a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b

^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^1
3*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 23
0686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 2214592512
0*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a
^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^2
3*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25
*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e
^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 26730823
68*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*
b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^
9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^1
2 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 23
0400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c
^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 +
89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 86981581
2096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*
a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*
c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 1
91038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 168781086
72*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a
^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^1
7*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19
*b^2*c^13*d*e^11)*1i)/((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23
*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 27
68240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672
*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*
b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^
11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e
^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 2306867
20*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^1
9*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b
^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7

*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918

430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 320834057 0112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22 *c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 220 7803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a ^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8 *c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12 *e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11 *b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14* c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9* d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d* e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e ^11) + 191102976000*a^17*c^14*e^10 + 2851200*a^9*b^16*c^6*e^10 - 92568960*a ^10*b^14*c^7*e^10 + 1312630272*a^11*b^12*c^8*e^10 - 10611136512*a^12*b^10*c ^9*e^10 + 53445353472*a^13*b^8*c^10*e^10 - 171591892992*a^14*b^6*c^11*e^10 + 342580396032*a^15*b^4*c^12*e^10 - 388363714560*a^16*b^2*c^13*e^10))*(-(9* (25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794* a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1 1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19 *c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^ 2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^ 7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*2i - a tan(((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1 0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266 40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^ 8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4 *a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8 *b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^ 12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a ^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1 /2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1 0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266 40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^ 8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4 *a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8 *b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^ 12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a ^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1 /2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1 0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266 40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^ 8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4 *a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8 *b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^ 12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a ^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1 /2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 115343 36*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17 *c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^1 4 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 283 4678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 30236569 76384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15* b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d

*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13
+ 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1
417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 377
9571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 118
5410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^2
1*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 +
21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 4967733
65760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*
a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*
b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12
+ 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^
12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c
^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12
- 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 11
01055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^
9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*
d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11
+ 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 8
69815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 174
7313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i +
(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 1
7794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^
5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5
*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a
*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18
*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^
4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b
^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(
(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 1
7794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^
5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5
*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a
*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18
*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^
4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b
^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(
(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 1
7794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^
5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5
*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a
*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18
*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^
4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b
^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(
x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^
16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*
e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 4
96068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 28346784
15360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384
*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*
c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13
+ 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124
017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 141733
9207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 37795712
20480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 11854109
73696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3

*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 2163
0025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760
*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*
b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c
^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 986
1120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^
16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^
12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 15
43847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055
131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^2
2*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^1
1 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89
374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815
812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 17473134
91968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i)/((-(9*
(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*
a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1
1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8
- 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19
*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^
2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2
- 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^
7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*
(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*
a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1
1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8
- 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19
*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^
2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2
- 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^
7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*
(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*
a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1
1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8
- 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19
*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^
2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2
- 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^
7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(10
99511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^
21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14
- 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068
722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360
*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25
*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d
*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 27
68240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 12401718
0672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 14173392076
80*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480
*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696
*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12
- 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 216300257
28*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18
*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c
^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e
^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*

a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^
6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 -
333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847
804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 110105513164
8*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3
*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2
207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 8937485
1072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 86981581209
6*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968
*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) - (-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627
776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e
^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 221459
25120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a
^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^
7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^1
2*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 +
11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640
*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^2
0*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*
b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^
5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*
c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 22039
7568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*
b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^
8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12
 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x
*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^2
0*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 -
 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 33322696
7040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a
^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b
^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11

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+ 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 220780339
2*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^1
4*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b
^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^
4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) + 191102976000*a^17*c^1
4*e^10 + 2851200*a^9*b^16*c^6*e^10 - 92568960*a^10*b^14*c^7*e^10 + 13126302
72*a^11*b^12*c^8*e^10 - 10611136512*a^12*b^10*c^9*e^10 + 53445353472*a^13*b
^8*c^10*e^10 - 171591892992*a^14*b^6*c^11*e^10 + 342580396032*a^15*b^4*c^12
*e^10 - 388363714560*a^16*b^2*c^13*e^10))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7
680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2
+ 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8
*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.637 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=325

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3e\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b\right.}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)^2+1/4*(b^2-2*a*
c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+1
/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*c+b
^2)^2/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+30*a^2*b^2*c
^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*
c+b^2)^(5/2)/e-3*b*ln(e*x+d)/a^4/e+3/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^4/
e
```

Rubi [A] time = 0.58, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b
^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2
- 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*
ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2
)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x
)^4])/(4*a^4*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4}{4a^2\left(b^2\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4}{4a^2\left(b^2\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+\right.}$$

Mathematica [A] time = 6.18, size = 491, normalized size = 1.51

$$-\frac{3b\log(d+ex)}{a^4e} - \frac{1}{2a^3e(d+ex)^2} + \frac{-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{-46a^2bc^2-28a^2c^3(d+ex)^2+29}{4a^3e\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2)
+ (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*
(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4
*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b
^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2
*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^4*c - 30
*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*
```

a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)

fricas [B] time = 5.16, size = 15165, normalized size = 46.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7
+ 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56
*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^
7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*
b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b
^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a
^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3
 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*
a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 +
 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7
9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b
*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x
^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(
b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 +
30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^
3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^
4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*
c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10
*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4
*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2
*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3
*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8
+ a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*
```

a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*
x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*
c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^
3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3
*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 +
3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*
e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 +
(2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 -
12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*
b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c
+ 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*
c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^
3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d
^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2
- 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48
*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c
^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c
^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^
3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 +
2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*
a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b

^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 - 24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e), -1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60 *a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^

5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c

```
 - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a
^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 1
2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b
^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 -
24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b
^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2
 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6
*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*
b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3
 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7
*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 -
 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b
^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 -
 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^
2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*
a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*
e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10
+ 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4
*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 +
2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e)]
```

giac [A] time = 0.64, size = 377, normalized size = 1.16

$$\frac{3\left(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2\right)\sqrt{-b^2+4\,ac}}+\frac{3\,be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{4\,a^4}-\frac{e^{(-1)}}{2\,(xe+d)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(x*e
+ d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sq
rt(-b^2 + 4*a*c)) + 3/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^4
 - 1/2*e^(-1)/((x*e + d)^2*a^3) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*
c^4 + 2*(5*b^6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)*e^(-
1)/(x*e + d)^2 + (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*
b*c^3*e^2)*e^(-2)/(x*e + d)^4 + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2
*c^2*e^3 - 6*a^4*c^3*e^3)*e^(-3)/(x*e + d)^6)*e^(-1)/((b^2 - 4*a*c)^2*a^4*(
c + b/(x*e + d)^2 + a/(x*e + d)^4)^2)
```

maple [C] time = 0.09, size = 5575, normalized size = 17.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 22.45, size = 21465, normalized size = 66.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] (log(((27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b
^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2
)^6) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2
/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2
*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*
b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*(-
(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5
))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 +
 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3
*(4*a*c - b^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2
*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2
 - 20*a*c*d*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^
2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^
8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4
*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c
^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c
^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6))*((27*c
^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 +
10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2)^6) - ((3*
b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(
4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6
- 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2
 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
(6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^
4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*(4*a*c - b
^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30
*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2
*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d
*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4
*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4
*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)
^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*
b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c^5*e^16*x^2
*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x
*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6)))*(6*b^11*e + 960*
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a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6
144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2
+ 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - ((x
^4*(6*b^6*e^3 + 100*a^3*c^3*e^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 +
4200*a^2*c^4*d^4*e^3 + 420*b^4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^
2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a
^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2
*e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c
*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261
*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3
 - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 1
2*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^
3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 +
 b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) +
 (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e +
168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 21
6*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*
c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
 + (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d
^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435
*a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b
^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6
- 7*a*b^2*c^3*d*e^6))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 3
2*a^4*c^2 + 6*b^6*d^4 - 16*a^3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3
*c^3*d^4 + 60*a^2*c^4*d^8 + 6*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^
2*d^2 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^
2*c^2*d^4 - 36*a*b^4*c*d^4)/(4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^
4*(15*b^2*d^2*e^4 + 210*c^2*d^6*e^4 + 2*a*b*e^4 + 30*a*c*d^2*e^4 + 140*b*c*
d^4*e^4) + x^8*(45*c^2*d^2*e^8 + 2*b*c*e^8) + x^5*(6*b^2*d*e^5 + 252*c^2*d^
5*e^5 + 12*a*c*d*e^5 + 112*b*c*d^3*e^5) + x^3*(20*b^2*d^3*e^3 + 120*c^2*d^7
*e^3 + 8*a*b*d*e^3 + 40*a*c*d^3*e^3 + 112*b*c*d^5*e^3) + x^7*(120*c^2*d^3*e
^7 + 16*b*c*d*e^7) + x*(6*b^2*d^5*e + 10*c^2*d^9*e + 2*a^2*d*e + 8*a*b*d^3*
e + 12*a*c*d^5*e + 16*b*c*d^7*e) + x^6*(b^2*e^6 + 210*c^2*d^4*e^6 + 2*a*c*e
^6 + 56*b*c*d^2*e^6) + x^2*(a^2*e^2 + 15*b^2*d^4*e^2 + 45*c^2*d^8*e^2 + 12*
a*b*d^2*e^2 + 30*a*c*d^4*e^2 + 56*b*c*d^6*e^2) + a^2*d^2 + b^2*d^6 + c^2*d^
10 + c^2*e^10*x^10 + 2*a*b*d^4 + 2*a*c*d^6 + 2*b*c*d^8 + 10*c^2*d*e^9*x^9)
- (3*b*log(d + e*x))/(a^4*e) - (3*atan((x^2*((((27000*a^6*c^11*e^16 + 27*b^
12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*b^8*c^7*e^16 - 20601*a^3*b^6*c
^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*b^2*c^10*e^16)/(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((129600*a^9*b*c^10*e^17 + 54*a^3*b^13*
c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17 - 57204*a^6*b^7*
c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^17)/(a^9*b^12 + 4
096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840
*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^18 + 6*a^6*b^14*
c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*
e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2
*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 -
1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 9
60*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e
- 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^
10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 9728
0*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))
/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2
*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c
^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*
c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3
```

*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((3*((153600*a^13*c^10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(8*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(32*a^8*e^2*(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((153600*a^13*c^10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(8*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^

6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 40
96*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*
a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^
3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4
*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 -
 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((129600*a^9*b*c^10*e^1
7 + 54*a^3*b^13*c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17
- 57204*a^6*b^7*c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^1
7)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^1
2*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^
18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 +
792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 -
 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240
*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)
- ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e
- 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c
^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9
*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^1
5*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 +
 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^
12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
+ 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e -
3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)
)/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^
2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 +
30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*e^19 - 12*a^9*
b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^
12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 2949
12*a^15*b^3*c^8*e^19))/(64*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 33
5*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 -
 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*
b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c
^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*
b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61
440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(1
5/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*
a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*
a^5*b^2*c^7*e^14) + (x*((((2*(27000*a^6*c^11*d*e^15 + 27*b^12*c^5*d*e^15 -
567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3*b^6*c^8*d*e^15
+ 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15))/(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(129600*a^9*b*c^10*d*e^16 + 54*a^3*b^1
3*c^4*d*e^16 - 1233*a^4*b^11*c^5*d*e^16 + 11583*a^5*b^9*c^6*d*e^16 - 57204*
a^6*b^7*c^7*d*e^16 + 156276*a^7*b^5*c^8*d*e^16 - 223200*a^8*b^3*c^9*d*e^16)
)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12
*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(153600*a^13*c^10*
d*e^17 + 6*a^6*b^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5
*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*
b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 2
4*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 -
6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e +
7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d
*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*
c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328
*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096
*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^

2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*
a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))
*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e -
120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 -
80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^
2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
5120*a^8*b^2*c^4*e^2)) - (3*((3*((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*
c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^
8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199
680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*
a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) -
((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e -
120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*
c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a
^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18
- 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^
5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4
e^2)(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*
a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 +
30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 384
0*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(1
63840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18
- 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c
^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(4*a^
4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e
^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^
9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*
c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b
^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3
+ 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b
^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a
^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840
*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^
18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(16*a^8*e^2*
(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640
*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 +
4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^
2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6
- 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5
*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b
^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^
9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 -
199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c +
240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^
5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4
*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b
^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 249
60*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*
e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 8
0*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2
*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1
280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c
^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6
- 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e -
3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e

)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*
e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b
^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(
4*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8
*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)
*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*
b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*
c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*
b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^
6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((2*(129
600*a^9*b*c^10*d*e^16 + 54*a^3*b^13*c^4*d*e^16 - 1233*a^4*b^11*c^5*d*e^16 +
 11583*a^5*b^9*c^6*d*e^16 - 57204*a^6*b^7*c^7*d*e^16 + 156276*a^7*b^5*c^8*d
*e^16 - 223200*a^8*b^3*c^9*d*e^16))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*
b^2*c^5) - (((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*c^3*d*e^17 - 108*a^7*
b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*
a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^
17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a
^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2
*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144
*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10
*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 -
97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^
8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6
*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3
*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*
a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 -
2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b
^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3
 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c
^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^
12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18
- 294912*a^15*b^3*c^8*d*e^18))/(32*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12
+ 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3
840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*
c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a
^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6
100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 6553
6*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 38
40*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15
/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c -
 b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14
 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14
- 32400*a^5*b^2*c^7*e^14) - (((((36*a^3*b^14*c^3*e^15 - 14400*a^10*c^10*e^1
5 - 837*a^4*b^12*c^4*e^15 + 8046*a^5*b^10*c^5*e^15 - 40941*a^6*b^8*c^6*e^15
 + 116532*a^7*b^6*c^7*e^15 - 177588*a^8*b^4*c^8*e^15 + 119520*a^9*b^2*c^9*e
^15 + 54*a^3*b^13*c^4*d^2*e^15 - 1233*a^4*b^11*c^5*d^2*e^15 + 11583*a^5*b^9
*c^6*d^2*e^15 - 57204*a^6*b^7*c^7*d^2*e^15 + 156276*a^7*b^5*c^8*d^2*e^15 -
223200*a^8*b^3*c^9*d^2*e^15 + 129600*a^9*b*c^10*d^2*e^15)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9
*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*
e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3
*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b
^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 2
2272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2
*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^

2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (27*b^13*c^4*e^14 - 594*a*b^11*c^5*e^14 + 43200*a^6*b*c^10*e^14 + 5319*a^2*b^9*c^6*e^14 - 24732*a^3*b^7*c^7*e^14 + 62748*a^4*b^5*c^8*e^14 - 82080*a^5*b^3*c^9*e^14 + 27000*a^6*c^11*d^2*e^14 + 27*b^12*c^5*d^2*e^14 + 4779*a^2*b^8*c^7*d^2*e^14 - 20601*a^3*b^6*c^8*d^2*e^14 + 47790*a^4*b^4*c^9*d^2*e^14 - 56700*a^5*b^2*c^10*d^2*e^14 - 567*a*b^10*c^6*d^2*e^14)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + (3*((3*((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 22272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(8*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^1

7 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2
*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^
12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2
*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(32*a^8
e^2(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2
+ 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b
^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b
^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) +
 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2)
 + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2
)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*
a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(10800*a^6*c^8*e^14 + 27*b^1
2*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^
5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*
b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2
*c^5 + 120*a*b^10*c)) - (b*((3*((36*a^3*b^14*c^3*e^15 - 14400*a^10*c^10*e^1
5 - 837*a^4*b^12*c^4*e^15 + 8046*a^5*b^10*c^5*e^15 - 40941*a^6*b^8*c^6*e^15
 + 116532*a^7*b^6*c^7*e^15 - 177588*a^8*b^4*c^8*e^15 + 119520*a^9*b^2*c^9*e
^15 + 54*a^3*b^13*c^4*d^2*e^15 - 1233*a^4*b^11*c^5*d^2*e^15 + 11583*a^5*b^9
*c^6*d^2*e^15 - 57204*a^6*b^7*c^7*d^2*e^15 + 156276*a^7*b^5*c^8*d^2*e^15 -
223200*a^8*b^3*c^9*d^2*e^15 + 129600*a^9*b*c^10*d^2*e^15)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9
*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*
e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3
*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b
^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 2
2272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2
*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^
2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e
 + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*
c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960
*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24
576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17
- 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9
*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17
+ 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^1
0*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^
7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/
(4*a^4*e*(4*a*c - b^2)^(5/2)) - (((3*((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*
c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c
^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*
b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^
7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16
- 22272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*
b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8
*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^1
1*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b
^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 +
960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 -
 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^
17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*
b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^

```
17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*
b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560
*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^1
0*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*
a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*
(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*
(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 1
20*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e
^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*
e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*
d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960
*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*
d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(8*a
^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*
e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a
^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6
*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2
*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c
^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b
^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) + (27*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^
12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*
b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b
^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17
 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*
b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^1
7))/(64*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*
a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^
(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c -
b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*
(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2
*(4*a*c - b^2)^(13/2)*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c
^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^
6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 +
3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(b
^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*a^4*e*(4*a*c - b^2)^(5/2
))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

3.638 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

```
[Out] f^4*x/c-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1122, 1166, 205}

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4\,\mathrm{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{f^4x}{c} - \frac{f^4\,\mathrm{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce}$$
$$= \frac{f^4x}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce} - \frac{\left(\left(b+\cdots\right.\right.}{\cdots}$$
$$= \frac{f^4x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{b\cdots}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4\cdots}}}$$

Mathematica [A] time = 0.05, size = 222, normalized size = 1.10

$$\frac{f^4\left(-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)\right)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

fricas [B] time = 0.89, size = 1346, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*
d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*
sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^
6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + s
qrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^
16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)
*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/
2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*
f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*
```

```
a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4
))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(
-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4
*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a
*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 -
 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*
a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c
)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b
^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*
f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*
(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))))/c
```

giac [B] time = 0.54, size = 1245, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f^4*x/c + 1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 + b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-
1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e
^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
- 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^
2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 +
 b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^
2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*
d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 + b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(
-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5
+ b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c
*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)*e^(-4)/c))))*e^(-4)/c
```

maple [C] time = 0.00, size = 164, normalized size = 0.81

$$\frac{f^4 x}{c}+\frac{f^4 \left(-\mathrm{RootOf}\left(\_Z^4 c\, e^4+4\_Z^3 c d\, e^3+c\, d^4+b\, d^2+\left(6 c\, d^2 \right.\right.\right.}{2 c e \left(2 c\, e^3 \,\mathrm{RootOf}\left(\_Z^4 c\, e^4+4\_Z^3 c d\, e^3+c\, d^4+b\, d^2+\left(6 c\, d^2 e^2+b\, e^2\right)\_Z^2+\left(4 c\, d^3 e+2 d e b\right)\_Z+a\right)^3+6 c \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] f^4*x/c+1/2*f^4/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+
4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+
a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 1.34, size = 4605, normalized size = 22.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c +
 ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*
a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8
+ b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c
*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c
^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e
^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^
2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i + ((2*b^
4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^
3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^
13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1
/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a
*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3
*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 -
 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/(((2*b^4*d*e^11*f^8
 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^12*f^4 -
 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x
*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*
f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
 a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e
^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f
^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 +
 b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*
e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^
12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^1
```

```
4 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(
1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-
(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*
f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 -
8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 + b^2*f^8*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*2i + atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*
f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^
13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(
b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f
^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8
*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 +
2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i
+ ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((1
6*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*
c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^
2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e
^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f
^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 +
 b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^
12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/(((2*b^4*d*
e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^
12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/
c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))
*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3
*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2
 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a
*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c
^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^
2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^
2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3
*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b
^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^
8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*
e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8
))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*
a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^
3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 - b^2
*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^
2)))^(1/2)*2i + (f^4*x)/c
```

sympy [A] time = 3.62, size = 219, normalized size = 1.08

$$\mathrm{RootSum}\left(t^4\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\left(48a^2bc^2e^2f^8-28ab^3ce^2f^8+4b^5e^2f^8\right)+a^3f^{16},\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**2*
f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3
*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b**4*
e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f**12)
))) + f**4*x/c
```

3.639 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=87

$$\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] 1/4*f^3*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*f^3*arctanh((b+2*c*(e*x+d)^
2)/(-4*a*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^3\,\mathrm{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{f^3\,\mathrm{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2e}$$
$$= \frac{f^3\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4ce} - \frac{\left(bf^3\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+\right.}{4ce}$$
$$= \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{\left(bf^3\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{2ce}$$
$$= \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e} + \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.92

$$\frac{f^3\left(\log\left(a+b(d+ex)^2+c(d+ex)^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

fricas [A] time = 0.94, size = 446, normalized size = 5.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,bf^3\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right.}{4\left(b^2c-4\,ac^2\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*
d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x +
 b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c -
 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*e^2*x^2 + 4*c*d*
e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*f^3*lo
g(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*
c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]
```

giac [B] time = 0.43, size = 162, normalized size = 1.86

$$-\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{2\sqrt{-b^2+4\,ac}\,c}+\frac{f^3e^{(-1)}\log\left(cd^4f^2+2\left(fx^2e+2\,dfx\right)cd^2fe+bd^2f^2+\left(fx^2e+2\,d\right.\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))*e^(-1)/(sqrt(-b^2 + 4*a*c)*c) + 1/4*f^3*e^(-1)*log(c*d^4*f^2 +
 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2
+ (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)/c
```

maple [C] time = 0.00, size = 154, normalized size = 1.77

$$\frac{f^3\left(\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)^3 e^3+3\,\mathrm{RootC}}{2e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4 c\,e^4+4\_Z^3 cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)^3+6cd\,e^2\,\mathrm{Rc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z
^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(efx+df\right)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 0.44, size = 287, normalized size = 3.30

$$\frac{4\,a\,c\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}-\frac{b\,f^3\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*
c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2)
- (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*
e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*
a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^
4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e
^2 - 4*b^2*c*e^2)
```

sympy [B] time = 1.95, size = 332, normalized size = 3.82

$$\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)+bd}{be^2f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d
*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
 + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqr
t(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2
+ (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*
e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)
) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

3.640 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}$$

```
[Out] -1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-
4*a*c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*f^2*arctan
((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2
))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1142, 1130, 205}

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^2\,\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e} + \frac{\left(\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.10, size = 178, normalized size = 1.05

$$\frac{f^2\left(\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

fricas [B] time = 0.89, size = 799, normalized size = 4.70

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+\left(b^2c-4\,ac^2\right)\sqrt{\frac{f^8}{\left(b^2c^2-4\,ac^3\right)e^4}}\,e^2}{\left(b^2c-4\,ac^2\right)e^2}}\,\log\left(ef^6x+df^6+\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{f^8}{\left(b^2c^2-4\,ac^3\right)e^4}}\,e^3\sqrt{-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3
)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*
c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b^2*c
- 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2)
)) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*
a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*
(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b
^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)
*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(
1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4
 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a
*c^2)*e^2))) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^
2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 -
sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(
b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c
- 4*a*c^2)*e^2)))
```

giac [B] time = 0.47, size = 1325, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e +
(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^
2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 -
2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3
+ 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*
c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^
(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^
4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)
```

maple [C] time = 0.00, size = 143, normalized size = 0.84

$$\frac{f^2 \left(\mathrm{RootOf}\left(\_Z^4 c e^4 + 4\_Z^3 cd e^3 + c d^4 + b d^2 + \left(6c d^2 e^2 +\right.\right.\right.}{2e\left(2c e^3 \mathrm{RootOf}\left(\_Z^4 c e^4 + 4\_Z^3 cd e^3 + c d^4 + b d^2 + \left(6c d^2 e^2 + b e^2\right)\_Z^2 + \left(4c d^3 e + 2deb\right)\_Z + a\right)^3 + 6cd e^2 \mathrm{Ro}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d
^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^
4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(efx + df\right)^2}{(ex+d)^4 c + (ex+d)^2 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.79, size = 683, normalized size = 4.02

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3\,f^4+f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)+\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}-32\right.}{8\,\left(16\,a^2\,c^3\,e^2-\right.}\right.}{a\,c\,e^{10}\,f^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^4
*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4 - 2
*b^2*c*e^12*f^4) + ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^1
3 - 32*a*b*c^3*d*e^13)*(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^
4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11*f^4
 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*(-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)
^(1/2) - 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(
1/2) - 2*atanh((((f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*f^4)/(8*
(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4
 - 2*b^2*c*e^12*f^4) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d
*e^13 - 32*a*b*c^3*d*e^13)*(f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*
c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11
*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*((f^4*(-(4*a*c - b^2)^3)^(1/2)
- b^3*f^4 + 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2))
)^(1/2)
```

sympy [A] time = 1.59, size = 124, normalized size = 0.73

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}c^{3}e^{4}-128ab^{2}c^{2}e^{4}+16b^{4}ce^{4}\right)+t^{2}\left(-16abce^{2}f^{4}+4b^{3}e^{2}f^{4}\right)+af^{8},\left(t\mapsto t\log\left(x+\frac{64t^{3}ac^{2}\cdots}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*lo
g(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f**6
)/(e*f**6))))
```

3.641 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=44

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[Out] -f*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1142, 1107, 618, 206}

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= -\frac{f\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{e}$$
$$= -\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.07

$$\frac{f\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e)
```

fricas [A] time = 0.89, size = 274, normalized size = 6.23

$$\left[\frac{f\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac-\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right)}{2\,\sqrt{b^2-4\,ac}\,e}, -\frac{\sqrt{-b^2+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(s
qrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x
 + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

giac [A] time = 0.40, size = 62, normalized size = 1.41

$$\frac{f\arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*
f))*e^(-1)/sqrt(-b^2 + 4*a*c)
```

maple [C] time = 0.00, size = 130, normalized size = 2.95

$$\frac{f\,(\mathrm{R}}{2e\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)^3+6cd\,e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx+df}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.62, size = 477, normalized size = 10.84

$$\frac{f\,\mathrm{atan}\left(\frac{\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2-\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8\right)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)1\mathrm{i}}{2e\sqrt{b^2-4ac}}+\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2+\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{1\cdots}\right.}{2e\sqrt{b^2-4ac}}\right.}{2e\sqrt{b^2-4ac}}}{\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2-\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8\right)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)}{2e\sqrt{b^2-4ac}}-\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2+\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{1\cdots}\right.}{2e\sqrt{b^2-4ac}}\right.}{2e\sqrt{b^2-4ac}}}\right.}{e\sqrt{b^2-4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (f*atan(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^2
*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2))
+ 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c)^(1/2)) + (f*(4*c^2*d^2*e^7*f + 4*
c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*
b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2
- 4*a*c)^(1/2)))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8
+ 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c
)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c)^(1/2)) - (f*(4*c^2*d^2*e^7*
f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2
 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b
^2 - 4*a*c)^(1/2))))*1i)/(e*(b^2 - 4*a*c)^(1/2))
```

sympy [B] time = 1.19, size = 189, normalized size = 4.30

$$-\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4acf\sqrt{-\frac{1}{4ac-b^2}}+b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}+\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4acf\sqrt{-\frac{1}{4ac-b^2}}-b^2f\sqrt{\cdots}}{2ce^2f\cdots}\right.}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))/(
2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(4*a
*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*
f))/(2*e)
```

3.642 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=103

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

```
[Out] ln(e*x+d)/a/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e/f+1/2*b*arctanh((b+2*
c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/f/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*
e*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
```

```
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2aef} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d+ex)^2 \right.}{2aef}$$

$$= \frac{\log(d+ex)}{aef} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4aef} - \frac{b\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef} + \frac{b\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,ef} + \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d\right.}{4aef}$$

Mathematica [A] time = 0.07, size = 131, normalized size = 1.27

$$\frac{4\sqrt{b^2-4ac}\,\log(d+ex) - \left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4a\right.}{4aef\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

fricas [A] time = 0.92, size = 474, normalized size = 4.60

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right) - \left(\right.}{4\left(ab^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) -
 (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*
b^2 - 4*a^2*c)*e*f)]
```

giac [B] time = 1.19, size = 285, normalized size = 2.77

$$-\frac{e^{(-1)}\log\left(\left|cx^4e^4+4\,cdx^3e^3+6\,cd^2x^2e^2+4\,cd^3xe+cd^4+bx^2e^2+2\,bdxe+bd^2+a\right|\right)}{4\,af}+\frac{e^{(-1)}\log\left(|xe+d|\right)}{af}-\frac{\Big(ab \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a*f) + e^(-1)*log(abs(x*e
 + d))/(a*f) - 1/4*(a*b*c*f*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 -
4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2
+ 2*a))/sqrt(b^2 - 4*a*c) - a*b*c*f*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sq
rt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*
a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c*f^2)
```

maple [C] time = 0.01, size = 190, normalized size = 1.84

$$\frac{\ln\left(ex+d\right)}{aef}+\frac{\left(-c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\_Z+a\right)\cdots}{2aef\left(2c\,e^3\,\mathrm{RootOf}\left(\_Z^4c\,e^4+4\_Z^3cd\,e^3+c\,d^4+b\,d^2+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2deb\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f/a/e*sum((-_R^3*c*e^3-3*_R^2*c*d*e^2-c*d^3+(-3*c*d^2-b)*_R*e-b*d)/(2*_
R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=Root
Of(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*
e+2*b*d*e)*_Z+a))+ln(e*x+d)/a/e/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.46, size = 2520, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e*f) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b
*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e*f - 8*a*c*
e*f))/(2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) - (b*atan((16*a^3*f^3*x*(4*a
*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^
3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f
^2)) + (20*b*c^3*d*e^17)/f))/(16*a^2*e^2*f^2*(4*a*c - b^2)) - ((2*b^2*e*f -
 8*a*c*e*f)^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*
d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*e^17)/f))
/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*
(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a^2*e^2*f^3*(4*a*b^2*e^2*f^2
 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - (((b*(
2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20
*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*
e^17)/f))/(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)
) - (b^3*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(16*a^3*e^3*f^4*(4*a*c
 - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(6*b^3*c^2*d*e^18*f - 20*a*b*
c^3*d*e^18*f))/(4*a*e*f^2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b
^2)^(1/2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/
2)*(25*a*c - 6*b^2))))/(b^2*c^2*e^14) + (2*f^3*(3*b^3 - 8*a*b*c)*(4*a*c - b
^2)^(3/2)*((b^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f + ((2*b^2*e*f -
8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17
*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c - b^
2)) - ((2*b^2*e*f - 8*a*c*e*f)^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f
 + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20
*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(4*(4*a*b^
2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^
2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a^2*e^2*f^3*(4
*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(b^2*c^4*e^14*(25*a*c -
 6*b^2)) + (16*a^3*f^3*x^2*(4*a*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((
10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*
e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c
- b^2)) - (((10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f
- 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)))*(2*b^2*e*f
- 8*a*c*e*f)^2)/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a^2*e^2*f^3*(4*a*
b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2
)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*((10*b*c^3*e^18)/f + ((2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2
- 16*a^2*c*e^2*f^2)))*(2*b^2*e*f - 8*a*c*e*f))/(4*a*e*f*(4*a*b^2*e^2*f^2 -
16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(6*b^3*c^2*e^19*f - 20*a*b*c^
3*e^19*f))/(32*a^3*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f
)^2*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a*e*f^2*(4*a*b^2*e^2*f^2 - 1
6*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(2
5*a*c - 6*b^2))))/(b^2*c^2*e^14) - (2*f^3*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2
 - 14*a*b^2*c)*((b*(2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^
2*e^16))/f + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e
^17*f - 20*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/
(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(
2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(32*a^3
*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(2*a*b^2*c^2*e
^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a*e*f^2*(4*a*b^2*
e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(b^2*c^4*e^14*(25*a*c
- 6*b^2))))/(2*a*e*f*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 7.16, size = 348, normalized size = 3.38

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)-2}{bce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e
 + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) -
 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2
)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c +
 b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-8*a**
2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*
a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) -
 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

3.643 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

```
[Out] -1/a/e/f^2/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Rubi [A] time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1142, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx,x,d+ex\right)}{ef^2}$$

$$= -\frac{1}{aef^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{aef^2}$$

$$= -\frac{1}{aef^2(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\right.}{2aef^2}$$

$$= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,ef^2} - \ldots$$

Mathematica [A] time = 0.34, size = 209, normalized size = 1.02

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2aef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e*f^2)
```

fricas [B] time = 0.94, size = 1477, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*
a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2
*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3
*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*
c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^
2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c
```

```
+ 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*
c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8))
+ b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x +
 a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^
2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((
a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sq
rt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
- 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) + sqr
t(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((
a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*
c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4
*a^2*b*c^2)*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^
4*c)*e^2*f^4))) - 2)/(a*e^2*f^2*x + a*d*e*f^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

maple [C] time = 0.01, size = 174, normalized size = 0.85

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^{4} c\, e^{4}+4\_Z^{3} c d\, e^{3}+c\, d^{4}+b\, d^{2}+\left(6 c\, d^{2} e^{2}+b\right.\right.\right.}{2 a e\, f^{2}\left(2 c\, e^{3} \operatorname{RootOf}\left(\_Z^{4} c\, e^{4}+4\_Z^{3} c d\, e^{3}+c\, d^{4}+b\, d^{2}+\left(6 c\, d^{2} e^{2}+b\, e^{2}\right)\_Z^{2}+\left(4 c\, d^{3} e+2 d e b\right)\_Z+a\right)^{3}+6 c d\, e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c
*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a/
e/f^2/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 3.87, size = 4339, normalized size = 21.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)

[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6)
 - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*
e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e
^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
+ 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6
- 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
+ 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6
- 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^1
4*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))
)^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2
) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2
)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^1
4*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2
*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^
4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^
12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^
6) - (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4
*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) -
((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^1
3*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f
^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12
*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 1
6*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2
*a^3*b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - ata
n(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b
^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((
x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*
f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f
^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*
a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a
^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*
a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a
^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^1
```

```
0) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/
2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4
*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*
(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^1
0 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^
13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8
*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^
8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b
^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6) -
(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*
c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(
8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^1
0 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 +
16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f^8
+ 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5
*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*
b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^
4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - 1/(a*e*(
d*f^2 + e*f^2*x))
```

sympy [A] time = 4.90, size = 258, normalized size = 1.26

$$\mathrm{RootSum}\left(t^4\left(256a^5c^2e^4f^8-128a^4b^2ce^4f^8+16a^3b^4e^4f^8\right)+t^2\left(48a^2bc^2e^2f^4-28ab^3ce^2f^4+4b^5e^2f^4\right)+c^3,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a**
3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f**4
 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**
3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*
_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d
 - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x
)
```

3.644 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

```
[Out] -1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)/a^2/e/f^3+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+
d)^4)/a^2/e/f^3-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/
2))/a^2/e/f^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^2*
e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2aef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,(d+ex$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,(d\right.}{2a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(\right.}{4a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\log\left(a+b(d+ex)^2+c(d\right.}{4a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,ef^3} - \frac{b\log(d+}{a^2ef^3}$$

Mathematica [A] time = 0.14, size = 157, normalized size = 1.18

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

fricas [B] time = 1.22, size = 828, normalized size = 6.23

$$\left[-\frac{2\,ab^2 - 8\,a^2c + \left(\left(b^2 - 2\,ac\right)e^2x^2 + 2\left(b^2 - 2\,ac\right)dex + \left(b^2 - 2\,ac\right)d^2\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2e^4x^4 + 8\,c^2de^3x^3 + 2\,c^2d^4 + 2\,(6}{ce^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2
 - 4*a^3*c)*d^2*e*f^3), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2
+ 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x
 + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*
(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2*e*f^3)]
```

giac [B] time = 1.11, size = 348, normalized size = 2.62

$$\frac{be^{(-1)}\log\left(\left|cx^4e^4 + 4\,cdx^3e^3 + 6\,cd^2x^2e^2 + 4\,cd^3xe + cd^4 + bx^2e^2 + 2\,bdxe + bd^2 + a\right|\right)}{4\,a^2f^3} - \frac{be^{(-1)}\log\left(\left|xe + d\right|\right)}{a^2f^3} - \frac{}{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f^3) - b*e^(-1)*log(
abs(x*e + d))/(a^2*f^3) - 1/2*e^(-1)/((x*e + d)^2*a*f^3) + 1/4*((a^2*b^2*c*
f^3*e^3 - 2*a^3*c^2*f^3*e^3)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2
*a))/sqrt(b^2 - 4*a*c) - (a^2*b^2*c*f^3*e^3 - 2*a^3*c^2*f^3*e^3)*log(abs(-b
*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*
e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^4*c*
f^6)
```

maple [C] time = 0.01, size = 222, normalized size = 1.67

$$-\frac{b\ln\left(ex+d\right)}{a^{2}e\,f^{3}}-\frac{1}{2\left(ex+d\right)^{2}ae\,f^{3}}+\frac{\left(\mathrm{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2\right.\right.\right.}{2a^{2}e\,f^{3}\left(2c\,e^{3}\,\mathrm{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^3/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c*d*e^2+b*c*d^3-a*c*d+b^2*d+(3*b*c
*d^2-a*c+b^2)*_R*e)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*
e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e
^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)
/a^2/e/f^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 6.98, size = 5947, normalized size = 44.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*f^9*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3*e*f^3 - 8*a*
b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f^9
) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8
*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^
3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*f^6))*(2*b^3*e*f
^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (2*c^5*d*
e^15)/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6)
)/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3
*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*
(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^
9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))
/(4*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18*f
^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)
^2)/(16*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
)/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*
a*b^3*c)*((((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f
^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 -
 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c -
 b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a
^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))/(4*a^5*
e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + ((((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^1
7*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*
(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f
```

^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*
f^6))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e
^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*a*c - b^2)^3)/(32*a^9*e^3*f^18*(4*a
*c - b^2)^(3/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b
^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e
^14) + (16*a^6*f^9*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*((((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*b^3*e*f^3 - 8*a*b
*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (c^5*e^16)/(a^3*f^9
) - ((2*a*c - b^2)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f
^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^
3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b
^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12
*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12
*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*e*f^3
*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19
*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2)^2)/(32*a^7*e^2*f^15*(4*a*c - b^
2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3*e*f^3 - 8*a*b*
c*e*f^3)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9
))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c
^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12*(4*a*c -
b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 -
4*a^2*b^2*e^2*f^6)) + ((12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(
2*a*c - b^2)^3)/(64*a^9*e^3*f^18*(4*a*c - b^2)^(3/2)) + (((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4
*a*b^2*c^3*e^14) + (2*a^3*f^9*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^
2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/(a^3*f^9) + (((((4*a^2*b^3*c^2*e^16*f^6 +
20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^
6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*
a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3 - a^2*c^4*e^15*f^3 +
6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a
^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16*
f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^
16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9
- 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*
a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b
^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)*(4*a^4*b^2*c^2*e^
17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(8*a^5*e
f^12(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*
e*f^3*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)^2
*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e
^17*f^9))/(32*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*
f^6))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14
- 4*a*b^2*c^3*e^14)) + (2*a^3*f^9*(4*a*c - b^2)*(((2*b^3*e*f^3 - 8*a*b*c*e*
f^3)*((((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^1
6*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f
^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^

```
2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b
^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*
a*c - b^2)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3
*c^2*d^2*e^17*f^9))/(8*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4
*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (((((4*a^
2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2
*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^
2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/
(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*
e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3
- a^2*c^4*e^15*f^3 + 6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*a*c - b^2))/(4*a
^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^3*(4*a^4*b^2*c^2*e^17*f^9 -
40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(64*a^9*e^3*f^18*
(4*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 -
6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)))*(2
*a*c - b^2))/(2*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - 1/(2*a*e*(d^2*f^3 + e^2*f^
3*x^2 + 2*d*e*f^3*x)) - (b*log(d + e*x))/(a^2*e*f^3) - (log(((c^5*e^16*x^2)
/(a^3*f^9) - ((b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))
^(1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/(a^2*f^6) - ((b + a^2*e*f^3*(
-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 +
10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/(a*f^3) + (2*c^3*e^18*x^2*(10*a*c + b^
2))/(a*f^3) + (b*c^2*e^16*(b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*
a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d
*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f^3) + (4*c^3*d*e^17*x*(10*a*c
+ b^2))/(a*f^3)))/(4*a^2*e*f^3) + (6*b*c^4*e^17*x^2)/(a^2*f^6) + (12*b*c^4*
d*e^16*x)/(a^2*f^6)))/(4*a^2*e*f^3) + (c^4*e^14*(b + c*d^2))/(a^3*f^9) + (2
*c^5*d*e^15*x)/(a^3*f^9))*((c^5*e^16*x^2)/(a^3*f^9) - ((b - a^2*e*f^3*(-(2*
a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2 - a*c +
6*b*c*d^2))/(a^2*f^6) - ((b - a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a
*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c
))/(a*f^3) + (2*c^3*e^18*x^2*(10*a*c + b^2))/(a*f^3) + (b*c^2*e^16*(b - a^2
*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d
^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e
*x))/(a^2*f^3) + (4*c^3*d*e^17*x*(10*a*c + b^2))/(a*f^3)))/(4*a^2*e*f^3) +
(6*b*c^4*e^17*x^2)/(a^2*f^6) + (12*b*c^4*d*e^16*x)/(a^2*f^6)))/(4*a^2*e*f^3
) + (c^4*e^14*(b + c*d^2))/(a^3*f^9) + (2*c^5*d*e^15*x)/(a^3*f^9)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))
```

sympy [B] time = 156.63, size = 532, normalized size = 4.00

$$\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right) + 2a^2b^2ef^3\left(\frac{b}{4a^2ef^3} - \cdots\right.}{2ac^2e^2 - b^2ce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a
*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/(4*a**2*e*f**3) - sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 2*a**2*b
**2*e*f**3*(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*
e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*
a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*
e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f*
*3*(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*
d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e*f
```

```
**3 + 4*a*d*e**2*f**3*x + 2*a*e**3*f**3*x**2) - b*log(d/e + x)/(a**2*e*f**3
)
```

3.645 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

```
[Out] -1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2)
)/a^2/e/f^4*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^
(1/2))/a^2/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[c]
*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{3aef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4} \cdots\right.}{3a^2ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subs}\cdots}{\cdots}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\cdots}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-\cdots}}}$$

Mathematica [A] time = 0.10, size = 238, normalized size = 1.01

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e*f^4)
```

fricas [B] time = 0.95, size = 2212, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 -
4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/
((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x
+ 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c
 + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)
*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/
((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^
2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^
2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^
6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17
*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a
^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2
*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^
2*f^4*x + a^2*d^3*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^
4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*lo
g(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*
c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a
^11*c)*e^4*f^16)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16))
- b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) + 3*sqrt(1
/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*
e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5
*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a
*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)
*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16))
+ (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sq
rt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c
- 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 2*a)/(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)
```

giac [B] time = 0.57, size = 1249, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqr
```

```
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
 - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^2*f^4) + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e
+ 3*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^2*f^4)
```

maple [C] time = 0.01, size = 197, normalized size = 0.83

$$-\frac{1}{3\left(ex+d\right)^{3} ae f^{4}}+\frac{b}{\left(ex+d\right) a^{2}e f^{4}}+\frac{\left(\mathrm{RootOf}\left(\_Z^{4}c e^{4}+4\_Z\right.\right.}{2a^{2}e f^{4}\left(2c e^{3}\,\mathrm{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^4/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3
+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c
*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e
)*_Z+a))-1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 3.17, size = 5771, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f^
4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 +
16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8
 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^
3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^
14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3
*c^2*d*e^13*f^20) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^
9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8
*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12
 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 2
0*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2
*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e
^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2
*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a
^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^
20) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12
*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e
^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c
^4*d*e^11*f^12)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b
^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
- 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^
3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a
^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14
*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) - 16*a^10*
c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*
a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*
a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12
) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(
1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))
^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^
2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)
))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*
b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) + 16*a^10*c^4*e^12*f^16 + 4
*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12
 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^
12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e
^10*f^8))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
```

^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i)/((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e^10*f^8))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/

```
(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*2i
```

sympy [A] time = 14.01, size = 411, normalized size = 1.74

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}+\text{RootSum}\left(t^4\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d*
*2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootSum
(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*b**
4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*
f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t*log(
x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8
*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*c**3*
e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t*b**8*e*
f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2
*c**4*e + b**4*c**3*e))))
```

3.646 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=279

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4
)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(
-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1120, 1166, 205}

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b
^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{f^4\,\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx, x, \ldots\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f\ldots\right.}{4\ldots}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f^4\ldots}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\ldots}}$$

Mathematica [A] time = 0.46, size = 266, normalized size = 0.95

$$\frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

fricas [B] time = 0.73, size = 2578, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(b
*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
```

```
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt
(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 +
 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f
^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*
f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^
2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2
 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*
e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64
*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)))
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 +
sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*sqrt(f^16/((b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48
*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2
*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b
^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x
^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*
c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c
)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e
^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12
*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*
x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^
8 - 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3
+ 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))))/((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.60, size = 1370, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*f^4*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
```

```
^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1) + x + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d
^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^
4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^2
- 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*
b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*b*f^4*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1)
 + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(
-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 -
6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*
c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c) + 1/2*(b*f^
4*x^3*e^3 + 3*b*d*f^4*x^2*e^2 + 3*b*d^2*f^4*x*e + b*d^3*f^4 + 2*a*f^4*x*e +
 2*a*d*f^4)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c
*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 695, normalized size = 2.49

$$-\frac{b\,e^{2} f^{4} x^{3}}{2\left(c\,e^{4} x^{4}+4 c d\,e^{3} x^{3}+6 c\,d^{2} e^{2} x^{2}+4 c\,d^{3} e x+b\,e^{2} x^{2}+c\,d^{4}+2 b d e x+b\,d^{2}+a\right)\left(4 a c-b^{2}\right)}-\frac{}{2\left(c\,e^{4} x^{4}+4 c d\,e^{3} x^{3}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*e^2/(4*a*c-b^2)*x^3-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*e/(4*a
*c-b^2)*x^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*d^2-f^4/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*
c-b^2)*x*a-1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*d^3/e*b-f^4/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c
-b^2)*d/e*a+1/4*f^4/(4*a*c-b^2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2
*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=Ro
otOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^
3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} f^{4} \int -\frac{be^{2} x^{2}+2\,bdex+bd^{2}-2\,a}{\left(b^{2} c-4\,ac^{2}\right) e^{4} x^{4}+4\left(b^{2} c-4\,ac^{2}\right) de^{3} x^{3}+\left(b^{2} c-4\,ac^{2}\right) d^{4}+\left(b^{3}-4\,abc+6\left(b^{2} c-4\,ac^{2}\right) d^{2}\right) e^{2} x^{2}+ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^2
)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b
*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 + 2*a*d
)*f^4)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)
```

mupad [B] time = 5.09, size = 8025, normalized size = 28.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^
4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c
^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5
*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4
*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6
*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 +
 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^
2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^
4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a
^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2
 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i - ((128*a^3*c^4*
d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4
*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a
*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*
a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
(x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^
3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*
f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c
^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2
)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 -
96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^
2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a
^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2
*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*
c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
```

10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((2048*a^4*c^5*e^12*f^4 + 384*a^2
*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 10
24*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16
384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^
2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*
c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b
^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6
e^2)))^(1/2))(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^
8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^
7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 38
40*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^
8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c
^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
c)))(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^
2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 2
4*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^
4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)
^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2) - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(
b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) -
 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*
e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3
*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (3*a*
b^3*c*e^10*f^12 + 4*a^2*b*c^2*e^10*f^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c))))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*
c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*
a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^
2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*2i - ((x*(2*a*f^4
+ 3*b*d^2*f^4))/(2*(4*a*c - b^2)) + (b*d^3*f^4 + 2*a*d*f^4)/(2*e*(4*a*c - b
^2)) + (b*e^2*f^4*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*f^4*x^2)/(2*(4*a*c - b^
2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*
e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*
b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 102
4*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 163
84*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2
*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9
)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^
3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e

```
^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 +
 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e
^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*
a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 -
 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2
*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9
)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^
3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2)*1i - ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^
2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048
*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6
144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1
024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*
b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*
a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*
c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b
^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*
(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 -
1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)
 - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768
*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2
+ 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6
*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2
*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24
*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4
*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) -
b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32
*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 -
 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2
) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/
(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 +
 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2
*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24
*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4
*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6
*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 15
36*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 +
16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^1
4 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 7
68*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^
```

```
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-
(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 -
 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^
2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614
4*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2
)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3
*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*
b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^
6*e^2)))^(1/2) - (3*a*b^3*c*e^10*f^12 + 4*a^2*b*c^2*e^10*f^12)/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*((f^8*(-(4*a*c - b^2)^9)^(1/2) -
 b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(3
2*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/
2)*2i
```

sympy [B] time = 9.82, size = 641, normalized size = 2.30

$$\frac{-2adf^4 - bd^3f^4 - 3bde^2f^4x^2 - be^3f^4x^3 + x\left(\right.}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(-
2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e +
8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2
*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e
**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*
c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + Roo
tSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a
**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4
 - 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e
**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e**2*f**8 +
16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 + 9*a*b**4*f
**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t**3*a**
2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**3 + 64
*_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f**8 + 4*a*c*d*
f**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12))))
```

3.647 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*f^3*
arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
 4*a*c)^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f^3\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx,x,(d+ex)^2\right)}{2e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(bf^3\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(bf^3\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{\left(b^2-4ac\right)}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.13, size = 103, normalized size = 1.00

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x
)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2)))/(2*e)
```

fricas [B] time = 0.89, size = 1077, normalized size = 10.46

$$\left[\frac{\left(b^3-4\,abc\right)e^2f^3x^2+2\left(b^3-4\,abc\right)def^3x+\left(2\,ab^2-8\,a^2c+\left(b^3-4\,abc\right)d^2\right)f^3-\left(bce^4f^3x^4+4\,bcde^3f^3x^3+\left(6\,bc\right.\right.}{2\left(\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)e^5x^4+4\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)de^4x^3+\left(b^5-8\,ab^3c+16\,a^2bc^2+6\left(b^4c-8\,ab^2c^2+\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2
- 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x
^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d
^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 +
 b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^
2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*
x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4
```

```
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^
2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*
f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c
)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e
^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [B] time = 0.52, size = 211, normalized size = 2.05

$$\frac{bf^3 \arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bd^2f^5+\left(fx^2e+2\,dfx\right)bf^4e+2}{2\left(cd^4f^2+2\left(fx^2e+2\,dfx\right)cd^2fe+bd^2f^2+\left(fx^2e+2\,dfx\right)^2ce^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a
*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2*f^5 + (f*x^2
*e + 2*d*f*x)*b*f^4*e + 2*a*f^5)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*
f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e +
 a*f^2)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 500, normalized size = 4.85

$$-\frac{be\,f^3x^2}{2\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a\right)\left(4ac-b^2\right)}-\frac{}{\left(c\,e^4x^4+4cd\,e^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*e/(4*a*c-b^2)*x^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*d/(4*a*c-b^2)*
x-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*b*d^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*a
+1/2*f^3/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d
^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-bf^3\int -\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2
)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*
d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3*x +
 (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*
x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a
*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

mupad [B] time = 1.90, size = 460, normalized size = 4.47

$$\frac{b\,f^3\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)+x^2\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,e^8\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)-\frac{b^3\,f^6\,\left(16\,a^2\,c^3\,e^8-4\,a\,b^2\,c^2\,e^8+8\,a\,b\cdots\right.}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{1\cdots}}\cdots\right)}{2\,b^2\,c^2\,e^6\,f^6}\cdots\right)}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*f^3*atan(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^
9))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)^(
7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c -
 b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(16*a
^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a
*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(7/2)))
)/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2))/(2*e
*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a*c - b^2
)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e
) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [B] time = 5.29, size = 556, normalized size = 5.40

$$\frac{bf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3cf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}-\frac{bf^3\sqrt{-\frac{1}{\left(4ac-b\cdots\right.}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f**
3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3) -
 b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c
*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 +
 (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqrt(-1/
(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b
*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 - 2*b*d*e
*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a
*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b*
*2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3
 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d
*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.648 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\,f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,f^2\left(\sqrt{b^2-4ac}\right.}{\sqrt{2}\,e\left(b^2-4a\right.}$$

```
[Out] -1/2*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)+1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1119, 1166, 205}

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\,f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,f^2\left(\sqrt{b^2-4ac}\right.}{\sqrt{2}\,e\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^
2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{f^2\,\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)f^2\right)\text{S}}{2(}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\,\text{ta}}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-}}$$

Mathematica [A] time = 0.94, size = 250, normalized size = 0.95

$$-\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

fricas [B] time = 0.87, size = 2600, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] -1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(2
*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
```

```
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2
*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4
 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*
a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/
((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c -
4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*
c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e
^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((
3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^
3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
- 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^
3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c +
4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a
*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c
)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt
(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2
*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 2
56*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))))/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.67, size = 1378, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
```

```
c))^2*c*f^2*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d*e^(-1) + x + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*
d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2*e^2 - 4*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b
*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^
(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2
*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2*e^2 - 4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c) - 1/
2*(2*c*f^2*x^3*e^3 + 6*c*d*f^2*x^2*e^2 + 6*c*d^2*f^2*x*e + 2*c*d^3*f^2 + b*
f^2*x*e + b*d*f^2)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 693, normalized size = 2.63

$$\frac{c\,e^2 f^2 x^3}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a\right)\left(4ac-b^2\right)}+\frac{}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*c*e^2/(4*a*c-b^2)*x^3+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)
*x^2+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c*d^2+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2
)*x*b+f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*d^3/e*c+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*d/e*b+1/4*f^2/(4*a*c-b^2)/e*sum((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf
(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+
2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,f^2\int -\frac{2\,ce^2x^2+4\,cdex+2\,cd^2-b}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^
2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(
2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3 + b*
d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)
```

mupad [B] time = 4.40, size = 7835, normalized size = 29.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*e
*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*c
- b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d
^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((f^4*(-(4*a*c - b^2)^9)^(1/2))
/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4
 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 -
 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*
b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*
c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*
c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^
2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^
4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*
e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4
*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7
*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b
*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*
c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e
^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (((f^4*(-(4*a*c - b^2)^9)^(1
/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4
*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*
b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*
b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^
7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
```

^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6 + (3*b
^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (((f
^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b
^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^
10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^
2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8
*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^
5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14
 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c -
 b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 -
16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*
b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^
2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*
a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((
f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*
b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b
^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e
^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b
^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*
b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 128
0*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((
8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b
^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^1
4 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c
 - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 -
 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 2
40*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6
*b^2*c^5*e^2))^(1/2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a
*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b
^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4
*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))*((
f^4*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^4 + 768*a^4*b*c^4*f^4 + 96*a^2*b^5*c^2
*f^4 - 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^1
0*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2
 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i + atan(((-((b^9*f^4)/32 + (f^4*(-(4*a*c
 - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^
3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2
*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))
^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*
f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^
2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a
^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b
^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b
^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7
*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))
/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^
(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b

```
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (-((b^9*f^4)/32 + (f^4*(
-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3
*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*
b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^
5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4
*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7
*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 +
 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 -
128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 204
8*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x
*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4
*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b
^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^
4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2
 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/
2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2
- 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)
/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 -
5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6
 + (3*b^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
 + (-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 -
 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 2
4*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^
4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c -
 b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 7
68*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b
*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*
f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*
c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*
c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 -
 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f
^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4
 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) - (-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f
^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2
 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^
5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a
*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13
 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a
^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((
b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*
b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b
```

```
^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e
^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^
12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11
*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))))*(-(b^9*f^4 + f^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^4 -
96*a^2*b^5*c^2*f^4 + 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^
2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a
^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i
```

sympy [B] time = 19.76, size = 646, normalized size = 2.46

$$\frac{bdf^2 + 2cd^3f^2 + 6cde^2f^2x^2 + 2ce^3f^2x^3 + x}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d*f**2 + 2*c*d**3*f**2 + 6*c*d*e**2*f**2*x**2 + 2*c*e**3*f**2*x**3 + x*(
b*e*f**2 + 6*c*d**2*e*f**2))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*
a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b
**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**
3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*
d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootS
um(_t**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**
5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 -
 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**
2*f**4 + 8192*a**3*b**3*c**3*e**2*f**4 - 1536*a**2*b**5*c**2*e**2*f**4 + 16
*b**9*e**2*f**4) + 16*a**2*c**3*f**8 + 24*a*b**2*c**2*f**8 + 9*b**4*c*f**8,
 Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c
**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*
b*c**2*e*f**4 - 16*_t*a*b**3*c*e*f**4 - 4*_t*b**5*e*f**4 + 4*a*c**2*d*f**6
+ 3*b**2*c*d*f**6)/(4*a*c**2*e*f**6 + 3*b**2*c*e*f**6))))
```

3.649 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] -1/2*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*f*a
rctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
4*a*c)^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,(d+ex)^2\right)}{2e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{(cf)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{\left(b^2-4ac\right)e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2cf)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,d\right.}{\left(b^2-4a\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.12, size = 99, normalized size = 1.01

$$-\frac{f\left(\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\right)}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*A
rcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/((b^2
 - 4*a*c)*e)
```

fricas [B] time = 0.66, size = 1066, normalized size = 10.88

$$\left[-\frac{2\left(b^2c-4\,ac^2\right)e^2fx^2+4\left(b^2c-4\,ac^2\right)defx+2\left(c^2e^4fx^4+4\,c^2de^3fx^3+\left(6\,c^2d^2+bc\right)e^2fx^2+2\left(2\,c^2d^3+bcd\right.\right.}{2\left(\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)e^5x^4+4\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)de^4x^3+\left(b^5-8\,ab^3c+16\,a^2bc^2+6\left(b^4c-8\,ab^2c^2\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^2
*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3
 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2
*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*
c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c
*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (b^3 - 4*a*b*c +
2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
```

```
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*
x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 +
(6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*
d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^
2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*
c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [B] time = 0.44, size = 211, normalized size = 2.15

$$-\frac{2\,cf\arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2f^3+2\left(fx^2e+2\,dfx\right)cf^2e}{2\left(cd^4f^2+2\left(fx^2e+2\,dfx\right)cd^2fe+bd^2f^2+\left(fx^2e+2\,dfx\right)^2ce^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*
a*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^2*f^3 + 2*(
f*x^2*e + 2*d*f*x)*c*f^2*e + b*f^3)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d
^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*
e + a*f^2)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 484, normalized size = 4.94

$$\frac{cef\,x^2}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a\right)\left(4ac-b^2\right)}+\frac{}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2)*x+f/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)/(4*a*c-b^2)/e*c*d^2+1/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*b+f*c/(4*a*c
-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,cf\int-\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)
*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d
```

```
^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2
)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x + (2
*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
 (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)
```

mupad [B] time = 1.91, size = 442, normalized size = 4.51

$$\frac{\frac{f\left(2cd^2+b\right)}{2e\left(4ac-b^2\right)}+\frac{2cdfx}{4ac-b^2}+\frac{cefx^2}{4ac-b^2}}{a+x^2\left(6cd^2e^2+be^2\right)+bd^2+cd^4+x\left(4ced^3+2bed\right)+ce^4x^4+4cde^3x^3}+\frac{2cf\,\mathrm{atan}\left(\frac{\left(4ac-b^2\right)^4\left(x\left(\frac{8c^4de^7f^2}{a\left(4ac-b^2\right)^{7/}}\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*f
*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*
b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*atan(((4*a*c - b^2
)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*f^2*(b^3*c^2*d
*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8*f^
2)/(a*(4*a*c - b^2)^(7/2)) - (4*b*c^2*f^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/
(a*e^2*(4*a*c - b^2)^(11/2))) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)^(7/2))
 + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*
c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(11/2))))/(8*c^4*e^6*f^2)))/(e*(4*a*c -
b^2)^(3/2))
```

sympy [B] time = 5.07, size = 525, normalized size = 5.36

$$-\frac{cf\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3f\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2f\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4cf\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e}+\frac{cf\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\;\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt(
-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4*c
*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
 c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sqrt(-
1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*
f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
(b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b**2*e
 + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**
4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**
4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**
2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2
*c*d**3*e**2))
```

3.650 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d}{2aef\left(b^2-4ac\right)\left(a+b(d+e\right.}$$

```
[Out] 1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4
)+1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(3/2)/e/f+ln(e*x+d)/a^2/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/
e/f
```

Rubi [A] time = 0.30, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d}{2aef\left(b^2-4ac\right)\left(a+b(d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a
 + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \cdots\right.}{2\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex\cdots}{a^2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex\cdots}{a^2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex\cdots}{a^2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\cdots\right.}{2a^2\left(\cdots\right.}$$

Mathematica [**A**] time = 0.44, size = 238, normalized size = 1.37

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6ab\cdots\right.}{\left(b^2-\cdots\right.}}{4a^2ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e*f)
```

fricas [**B**] time = 1.72, size = 2486, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b
^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2
*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*d^2)*e*f), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*
b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2
)*d^2 + 2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 +
(b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2
*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)
*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*
x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*
e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^
3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
+ 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 1
6*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x
^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f)]
```

giac [B] time = 1.43, size = 476, normalized size = 2.74

$$-\frac{\left(a^2b^3cfe^3 - 6\,a^3bc^2fe^3\right)\sqrt{b^2-4\,ac}\,\log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-4\,ac}\,dxe + bd^2 + \sqrt{b^2-4\,ac}\right.\right.}{4\left(a^4b^4cf\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2
*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e +
b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^
3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2
*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(
a^4*b^4*c*f^2*e^4 - 8*a^5*b^2*c^2*f^2*e^4 + 16*a^6*c^3*f^2*e^4) - 1/4*e^(-1
)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f) + e^(-1)*log(abs(x*e + d))/(
a^2*f) + 1/2*(a*b*c*x^2*e^2 + 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*
e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
+ b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*a^2*f)
```

maple [C] time = 0.03, size = 714, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*d/(4*a*c
-b^2)*x-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c*d^2+1/f/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*
c-b^2)*c-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/f/a^2/(4*a*c-b^2)/e*sum
(((4*a*c-b^2)*_R^3*c*e^3+3*(4*a*c-b^2)*_R^2*c*d*e^2+4*a*c^2*d^3-b^2*c*d^3+5
*a*b*c*d-b^3*d+(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*c-b^3)*_R*e)/(2*_R^3*c*e^3+6
*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e
^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*
_Z+a))+ln(e*x+d)/a^2/e/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 11.69, size = 13434, normalized size = 77.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a*f + x^2*(b*e^2*f + 6*c*d^2*e^2*
f) + x*(4*c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c*d*
e^3*f*x^3) - (log((((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c -
b^2)^3))^(1/2) - 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c
- b^2)^3))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10
*a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2)
/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2
```

 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2*
b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*
a*c - b^2))/(a*f*(4*a*c - b^2)))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c
^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17
x^2(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*
c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^5*e^16*x^2)/(a
^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(
4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3))*((b^3*c^5
*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a
^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/
(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a
*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3
*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(
a^2*f) - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*f*
(4*a*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) + (4
*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2)))))/(4*a^2*e*f) - (b*c^3*
e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2
)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b
*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (
b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^
5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a
^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f
^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + log(d + e*x)/(a^2*e
*f) + (b*atan((x^2*((((b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*
e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*
c*f^3 + 48*a^5*b^2*c^2*f^3) - (((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^
18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 -
64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128
*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^
3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5
*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12
*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f
^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3
*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 25
6*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a
^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^
4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^
3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f -
 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^1
9*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3
*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3
 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e
^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(
4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*
e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a
^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3
 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 -
64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2
 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(
2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a
^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4
*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*
c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560
*a^7*b*c^6*e^19*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64
*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3
 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 40
0*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b
^2*c)*((((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*e^17*f - 44*a^2*b^3*c^5*e^17*f)
/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (

((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f
^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f
^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e
*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 +
1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19
f^3))/(2(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2
f^3)(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f -
24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2
*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^
2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^
2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^3*c^5*e^16)/(a^3*
b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*
((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f
^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f
^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e
*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 +
1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19
f^3))/(2(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2
f^3)(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b
*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4
*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*
c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2
*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 +
48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2
*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b
^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2
*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*
f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^
6*e^19*f^3))/(32*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3
- 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e
^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a
c - b^2)^3(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16*a^6*
b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^
7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2)))
/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14) + (x*((((((b*(6*a
c - b^2)((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3
*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^
3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*
e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*
a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^
18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*
a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^
2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c -
b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^
2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*
f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b
^3*c^5*d*e^18*f^3))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^
3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5
*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f
- 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^
2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^
2)) + (b*((2*(6*a*b^5*c^4*d*e^16*f - 44*a^2*b^3*c^5*d*e^16*f + 80*a^3*b*c^6
*d*e^16*f))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c
^2*f^3) + (((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^
3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c
^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3
*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12

*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(32*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((2*(6*a*b^5*c^4*d*e^16*f - 44*a^2*b^3*c^5*d*e^16*f + 80*a^3*b*c^6*d*e^16*f))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (2*b^3*c^5*d*e^15)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (b*(6*a*c - b^2)*((b*(6*a*c - b^2)*((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(16*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2)))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14) + (((b*((4*a*b^6*c^3*e^15*f - 33*a^2*b^4*c^4*e^15*f + 68*a^3*b^2*c^5*e^15*f + 6*a*b^5*c^4*d^2*e^15*f + 80*a^3*b*c^6*d^2*e^15*f - 44*a^2*b^3*c^5*d^2*e^15*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3

+ 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f
- 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a
^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^
3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b
^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*
a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 2
56*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b
^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*
b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*
e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*(6*a*c - b
^2)*((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^
16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*
c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f
^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)
+ ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a
^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 -
256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2
*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 -
2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b
^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 1
92*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^
(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f
- 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*
a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f
^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*
b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(8*a^2*e*f*(4*a*c -
b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^
2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f
- 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^
2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(4*a^4*b^8*c^
2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b
^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3
- 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*
b^3*c^5*d^2*e^17*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 6
4*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(16*a^6*b^6*f^3*(4
*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^
3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2))*(3*b^6 - 4
0*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b
^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14)*(6*b^6 - 400*a^3*c^3
+ 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16
*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 1
92*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9
/2))*((((4*a*b^6*c^3*e^15*f - 33*a^2*b^4*c^4*e^15*f + 68*a^3*b^2*c^5*e^15*f
+ 6*a*b^5*c^4*d^2*e^15*f + 80*a^3*b*c^6*d^2*e^15*f - 44*a^2*b^3*c^5*d^2*e^
15*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3
) - (((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e
^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7
*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*
f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)
+ ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*
a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 -
256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^
2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 -
2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*
b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 +
192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*
e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5
*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f

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 - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^
2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^
2)) - (b^4*c^4*e^14 - 4*a*b^2*c^5*e^14 + b^3*c^5*d^2*e^14)/(a^3*b^6*f^3 - 6
4*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*(6*a*c - b^
2)*((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^1
6*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c
^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^
2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) +
 ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^
4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 2
56*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*
e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2
688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^
4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 19
2*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(
3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f
- 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a
^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^
3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b
^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(8*a^2*e*f*(4*a*c - b
^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2
*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b
^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a
*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4
*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12
*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4
*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(32*a^4*e^2*f^2*(4*a*c - b^
2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)
*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^
3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2*e^14 - 12*a*b^4*c^3
*e^14 + 36*a^2*b^2*c^4*e^14)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*
b^4*c)))*(6*a*c - b^2))/(2*a^2*e*f*(4*a*c - b^2)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.651 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$

Optimal. Leaf size=360

$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-\right.\right.}{2}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/f^2/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d
)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e
*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c
+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.60, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 - 2*a*c + b*
c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d
 + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c]*
(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Sub}}{}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b\right.}$$

Mathematica [A] time = 1.47, size = 342, normalized size = 0.95

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}\right.}{}}{4a^2ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
```

```
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

fricas [B] time = 1.20, size = 4520, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*
a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*
a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c
+ 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x +
((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4
*c)*d)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3
- 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 19
71*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b
^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4
- 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1054
9*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*
b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^
2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3
*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*
a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sq
rt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5
625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8
*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)
*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 +
14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2
*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
```

```
^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c
 - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*
c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^
3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
+ 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 1
2*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6
*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c -
385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c
^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^
11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
 - 5200*a^5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3
)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))) + sqrt
(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^
5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*
x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*
x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a
^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*
c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a
^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*
c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 +
420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*
f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)
*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d -
 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4
*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c
 + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^9*c +
 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)
*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^
4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)
) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a
^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))))/((a^2*b^2*c - 4*a^3*c^2
)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*
b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*
c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*
(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^
2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*
d)*e*f^2)
```

giac [B] time = 1.02, size = 999, normalized size = 2.78

$$-\frac{\frac{b^2ce^{(-1)}}{(fxe+df)f}-\frac{2\,ac^2e^{(-1)}}{(fxe+df)f}+\frac{b^3fe^{(-1)}}{(fxe+df)^3}-\frac{3\,abcfe^{(-1)}}{(fxe+df)^3}}{2\left(a^2b^2-4\,a^3c\right)\left(c+\frac{bf^2}{(fxe+df)^2}+\frac{af^4}{(fxe+df)^4}\right)}-\frac{e^{(-1)}}{(fxe+df)a^2f}+\frac{\left(\left(3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3\right)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/2*(b^2*c*e^(-1)/((f*x*e + d*f)*f) - 2*a*c^2*e^(-1)/((f*x*e + d*f)*f) + b
^3*f*e^(-1)/(f*x*e + d*f)^3 - 3*a*b*c*f*e^(-1)/(f*x*e + d*f)^3)/((a^2*b^2 -
 4*a^3*c)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e + d*f)^4)) - e^(-1)/((f
*x*e + d*f)*a^2*f) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*
a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 + 2*(3*a^3*b^2*c - 1
0*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^
2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)*e^2 - (a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)^
2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2
)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2 + sqrt(
(a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2)^2 - 4*(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*
e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*e^4))))*e^(-3
)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*f^6*abs(a^2*b^2*f^4*e^2 - 4*a^
3*c*f^4*e^2)*abs(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 8
0*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 - 2*(3*a^3*b^2*c -
 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(
a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)*e^2 - (a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2
)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1
/2)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2 - sqr
t((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2)^2 - 4*(a^3*b^2*f^8*e^4 - 4*a^4*c*f^
8*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*e^4))))*e^(
-3)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*f^6*abs(a^2*b^2*f^4*e^2 - 4*
a^3*c*f^4*e^2)*abs(a))
```

maple [C] time = 0.03, size = 1346, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e^
2/(4*a*c-b^2)*x^3*b^2-3/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a*c-b^2)*x^2+3/2/f^2/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)*x^2*b^2-3/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*x*c^2*d^2+3/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/f^2/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)/(4*a*c-b^2)*x*b*c+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/f^2/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6
```

```
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-
b^2)*b^2*c-3/2/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b*c+1/2/f^2/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)*d/e/(4*a*c-b^2)*b^3-1/4/f^2/a^2/(4*a*c-b^2)/e*sum(((10*a*c-3*b^2)*_R^2*
c*e^2+10*a*c^2*d^2-3*b^2*c*d^2+2*(10*a*c-3*b^2)*_R*c*d*e+13*a*b*c-3*b^3)/(2
*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=Ro
otOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^
3*e+2*b*d*e)*_Z+a))-1/f^2/a^2/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 7.29, size = 12008, normalized size = 33.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*
c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^
10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c
^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e
^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3
*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^
10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a
^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800
*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^
2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^
2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*
a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4
*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10
- 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*
a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3
*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*
f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10)
- 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c
^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 +
 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^
12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 +
30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*
c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8
*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^1
1*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
```

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40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 +
 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4
 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2
*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a
^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*
b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24
*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104
8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
)*1i)/((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c
^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^1
0*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^
2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*
e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^1
0*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^
6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*
a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2
*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
+ 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2
*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a
^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*
e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 -
 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a
^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*
d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f
^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) +
 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^
4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 -
1261568*a^13*b^3*c^7*e^12*f^8 + 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^1
2*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 3
0112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c
^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
```

144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*
c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11
*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6) - (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^
9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a
^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*
a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 409
6*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3
840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4
)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2
077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b
^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144
*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*
c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6
*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c
^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10
 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576
*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2
*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f
^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1
572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^1
1*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8
 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^
14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^
6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^
6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^
6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^
10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^1
1*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6) +
128000*a^10*c^9*e^10*f^4 + 504*a^6*b^8*c^5*e^10*f^4 - 8112*a^7*b^6*c^6*e^10
*f^4 + 48704*a^8*b^4*c^7*e^10*f^4 - 129280*a^9*b^2*c^8*e^10*f^4))*(-(9*b^13
 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 128
0*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^
4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*2i - atan(((-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4
+ 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^
4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^
2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^
6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^
3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*
a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*
b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a
^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8
*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 2
4*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b

^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14
*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 10
48576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^1
3*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e
^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^1
0 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^
9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^1
2*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 8519
68*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^
12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a
^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^
12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a
^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*
d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^
6)*1i + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 +
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*
c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^
10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c
^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e
^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3
*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^
10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a
^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800
*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^
2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^
2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*
a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4
*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10
- 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*
a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3
*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*
f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10)
+ 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c
^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 -
 1261568*a^13*b^3*c^7*e^12*f^8 + 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^
12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 +
30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*
c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8
*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^1
1*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i)/((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 +
 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4
 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2
f^4)))^(1/2)((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a
^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b

```
^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*
b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24
*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104
8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
) - (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2
*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a
^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*
e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b
^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f
^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2
*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c
*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 2
40*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^
4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10
*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^1
4*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 15
72864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16
*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e
^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10
 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 19
2*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e
^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 126
1568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c
^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 3011
2*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*
e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144
*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5
*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^
6 - 458752*a^11*b^2*c^8*d*e^11*f^6) + 128000*a^10*c^9*e^10*f^4 + 504*a^6*b^
8*c^5*e^10*f^4 - 8112*a^7*b^6*c^6*e^10*f^4 + 48704*a^8*b^4*c^7*e^10*f^4 - 1
29280*a^9*b^2*c^8*e^10*f^4))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2
6880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 -
 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 5
1*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e
^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*
c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*2i
 - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*a*b*c*d))/(a*(a*b^2 - 4*a
^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3))/(2*a*(a*b^2 - 4*a^2*c)) - (2*x^
```

```
3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2))/(a*(a*b^2 - 4*a^2*c)) + (2*a*b^2 - 8*a^
2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*a*b*c*d^2)/(2*a*e*(a*b^2
- 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*b^2*c*d^2*e - 11*a*b*c*e)
)/(2*a*(a*b^2 - 4*a^2*c)))/(x^2*(10*c*d^3*e^2*f^2 + 3*b*d*e^2*f^2) + x*(a*e
*f^2 + 3*b*d^2*e*f^2 + 5*c*d^4*e*f^2) + x^3*(b*e^3*f^2 + 10*c*d^2*e^3*f^2)
+ b*d^3*f^2 + c*d^5*f^2 + a*d*f^2 + c*e^5*f^2*x^5 + 5*c*d*e^4*f^2*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.652 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=228

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log(d+ex)}{a^3ef^3}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2\cdots}{\sqrt{\cdots}}\right.}{a^3ef^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/f^3/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/
a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b
^2*c+b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3
/2)/e/f^3-2*b*ln(e*x+d)/a^3/e/f^3+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
/f^3
```

Rubi [A] time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\,\mathrm{lo}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2*a*c + b*c
*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(
a^3*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Sul}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Sul}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\right.}$$

Mathematica [A] time = 0.52, size = 287, normalized size = 1.26

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+\right.}{\left(b^2-4ac\right)^{3/2}}}{2a^3ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e*f^3)
```

fricas [B] time = 4.14, size = 4604, normalized size = 20.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^
4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 -
8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c
+ 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3
*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c
+ 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b
^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (
a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e*f^3), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^
2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^
3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b
^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)
*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*
b^3*c + 28*a^3*b*c^2)*d)*e*x + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6
 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^
```

```
4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 -
6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sq
rt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2
*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 1
6*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2
*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(
5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6
- 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3*x^6 + 6*(a^3
*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*f^3*
x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2
+ 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*
c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a
^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^
2*c + 16*a^6*c^2)*d^2)*e*f^3)]
```

giac [B] time = 1.30, size = 687, normalized size = 3.01

$$\frac{\left(a^3b^4cf^3e^3 - 6\,a^4b^2c^2f^3e^3 + 6\,a^5c^3f^3e^3\right)\sqrt{b^2-4\,ac}\,\log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-4\,ac}\,dxe + b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] 1/2*((a^3*b^4*c*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b
^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*s
qrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^4*c
*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b^2 - 4*a*c)*log
(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*
c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^4*c*f^6*e^4 - 8*a^
7*b^2*c^2*f^6*e^4 + 16*a^8*c^3*f^6*e^4) + 1/2*b*e^(-1)*log(abs(c*x^4*e^4 +
4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x
*e + b*d^2 + a))/(a^3*f^3) - 2*b*e^(-1)*log(abs(x*e + d))/(a^3*f^3) - 1/2*(
2*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^
4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*d
*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^2*
```

```
b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a^2*
b*c*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*(x*e + d)^2*
a^3*f^3)
```

maple [C] time = 0.03, size = 1047, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4
*a*c-b^2)*x^2*b^2-2/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/f^3/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*c*d/(4*a*c-b^2)*x*b^2-1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2
/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2*c*d^2-3/2/f^3/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a
*c-b^2)*b*c+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^3+1/f^3/a^3/(4*a*c-b^2
)/e*sum(((4*a*c-b^2)*_R^3*b*c*e^3+3*(4*a*c-b^2)*_R^2*b*c*d*e^2+4*a*b*c^2*d^
3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d+(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2
*c^2+5*a*b^2*c-b^4)*_R*e)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3
+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/f^3/a^2/e/(e*x+d)^2-2*b
*ln(e*x+d)/a^3/e/f^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 13.52, size = 14830, normalized size = 65.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^3*e*f^
3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2*f^3 +
6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6*f^3 + c
*e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (log((((b + a^3*e*f^3*(-(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*f^3*(-(
```

```
b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^
2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a
*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2
+ 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*f^3*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*
b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*
c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*
f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a
^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)
^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c
- b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(
4*a*c - b^2)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9
*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*
d^2 - 4*a*b*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^
3)/(a^6*f^9*(4*a*c - b^2)^3))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2
*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2
*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b
^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^
2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c
))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*f^3*(-(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3
*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(
a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c
- b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*
b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)^2) + (4*b*
c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2) +
 (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2
)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b
^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b
*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^3)/(a^6*f^9
*(4*a*c - b^2)^3)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48
*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*
c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (2*b*log(d + e*x))/(a^3*e*f^3) - (at
an(((2*a^9*b^6*f^9*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*f^9*(4*a*c - b^2)^(9/
2) - 24*a^10*b^4*c*f^9*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*f^9*(4*a*c - b
^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*
e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c
*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(276*a^5*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4
*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 - 233*a^4*b^3*c^6*d*e^16*f^3))/(a^6
*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(
480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6
+ 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 6
4*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12
*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6
*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^
8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*
c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*
c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 -
 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6
*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f
^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e
*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 -
12*a^4*b^4*c*e^2*f^6)) - (((((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 4
6*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e
^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^
```

2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*
a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b
^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3
 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 +
3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^
18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b
^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e
^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6
)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^
4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e
*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e
^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*
b^3*c^5*d*e^18*f^9))/(a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3
*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3
*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*
c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 -
 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((8*(480
*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 3
0*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a
^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*
b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*
e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b
^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3
*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3
*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6
*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c
^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^
7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)
)/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4
*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5
*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 -
64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^
3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (((8*(276*a^5
*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 -
233*a^4*b^3*c^6*d*e^16*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f
^9 + 48*a^8*b^2*c^2*f^9) - (((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 4
6*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e
^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^
2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*
a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 4
8*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2
*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^
3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b
*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 26
4*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/(a^9*e^3*f^9*(4*a*c
 - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2
*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*
(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3
+ 72*a*b^6*c))) + x^2*((((4*(54*a^3*c^8*e^16 - 2*b^6*c^5*e^16 + 18*a*b^4*c^
6*e^16 - 54*a^2*b^2*c^7*e^16))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c
*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(6*a^2*b^7*c^4*e^17*f^3 - 65*a^3*b^5*c^5*
e^17*f^3 + 233*a^4*b^3*c^6*e^17*f^3 - 276*a^5*b*c^7*e^17*f^3))/(a^6*b^6*f^9
 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(480*a^8*
c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c

```
^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12
*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 6
4*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*
b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 64
0*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 +
 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2
*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b
*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^
6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*
c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 -
 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (((
((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 +
 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9
*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^
5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*
f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*
e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^
7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 4
8*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*
c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7
*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a
^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 -
672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(a^3*e*f^3*(4*a*c - b^
2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*
f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^
4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2
)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 -
 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^
7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 +
640*a^10*b*c^6*e^19*f^9))/(2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*
a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*
a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 -
 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^
4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*(((((
(4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 +
30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*
c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5
*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f
^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e
^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7
*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48
*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c
))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*
e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^
6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 6
72*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(a^3*e*f^3*(4*a*c - b^2
)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f
^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4
*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^
2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) + (((4*(6*a^2*b^7*c^4*e^17*f^3 - 65*a^3*
b^5*c^5*e^17*f^3 + 233*a^4*b^3*c^6*e^17*f^3 - 276*a^5*b*c^7*e^17*f^3))/(a^6
*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(
480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a
^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*
f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e
*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 -
 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*
f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4
```

*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5
*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 -
64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^
3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c
^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*
a*b^2*c)^3*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*
c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(2*a^9*
e^3*f^9*(4*a*c - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^
9 + 48*a^8*b^2*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c
))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 38
2*a^3*b^2*c^3 + 72*a*b^6*c))) + (((((4*(36*a^6*c^7*e^15*f^3 + 4*a^2*b^8*c^3
*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e^15*f^3 - 225*a^5*b^
2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7*c^4*d^2*e^15*f^3 -
65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15*f^3))/(a^6*b^6*f^9 -
64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((4*(2*a^4*b^9*
c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7
*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*
b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16
*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^
7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a
^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c
^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10
*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*
f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b
^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e
^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6
)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f
^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12
*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 +
48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^
2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (4*(2*b^7*c^4*e^14 - 20*a*b^5*c^5*
e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^8*d^2*e^14 + 2*b^
6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^2*e^14))/(a^6*b^6
*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((((4*(2*
a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 -
208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^
6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*
d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9
- 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^
3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a
^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 +
640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d
^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))
/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a
^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c
*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)
) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b
*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e
^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c
^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9
+ 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3*
(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a
^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*
f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(
4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^
5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^
9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^1
7*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b
^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^

```
17*f^9))/(2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*
a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 +
 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*c^3 + 36*a
^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 2
88*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((4*(36*a^6*c^7*e^15
*f^3 + 4*a^2*b^8*c^3*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e
^15*f^3 - 225*a^5*b^2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7
*c^4*d^2*e^15*f^3 - 65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15*
f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9
) - (((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^
4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b
*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*
a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64
*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*
a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^1
7*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5
*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a
^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^
2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2
*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 -
12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3
+ 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*
b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2
*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (((((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b
^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480
*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 -
6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*
d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^
2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2
*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^
4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3
*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d
^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9
 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2
*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 -
6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2
*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f
^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f
^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2
*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9
- 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^
9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^
6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(
b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/
(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*
b^4*c*e^2*f^6)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(a^7*b^8*c^2*e^17*f^9 -
12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^
9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c
^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f
^9))/(2*a^9*e^3*f^9*(4*a*c - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*
a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2
- 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*
b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))))/(36*a^4*c^6*e^14 + b^8*c^2*e^14
- 12*a*b^6*c^3*e^14 + 48*a^2*b^4*c^4*e^14 - 72*a^3*b^2*c^5*e^14))*(b^4 + 6*
a^2*c^2 - 6*a*b^2*c))/(a^3*e*f^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.653 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=423

$$\frac{b\left(5b^2-19ac\right)}{2a^3ef^4\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2ef^4\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)}{2\sqrt{2}\,a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4a}}}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/f^4/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a
^3/(-4*a*c+b^2)/e/f^4/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/
e/f^4/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-
19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a
*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] time = 3.55, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19\right.\right.}{2\sqrt{2}\,a^3ef^4\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 19*
a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^
2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)
) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) - (S
qrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^4}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \cdots$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b^2-2ac+\cdots}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\cdots}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)\cdots}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)\cdots}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)\cdots}$$

Mathematica [A] time = 3.02, size = 387, normalized size = 0.91

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \cdots}{12a^3ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

<u>fricas</u> [B] time = 1.37, size = 5954, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^
4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*d^4 + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6
*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x
^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5
*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a
^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c
)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*
b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2
)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x
+ ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a
^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10
*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 31
5*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4
 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*
e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c
^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*
b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - (125*b^14 - 2425*a*b^12*
c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^
5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b
^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a
^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 6
4*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3
*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a
^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*
c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b
^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4
```

```
*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*
a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*
f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*
(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^
3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a
^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^
10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^
5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*
b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 -
 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 +
43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5
*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*
b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*
a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2
401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^
4*f^16)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c
^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^
7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3
)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^
6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 -
12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*
b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 +
(a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(
a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^
2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2
)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*
d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6
 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a
^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)
*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*
c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^
7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 123
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x +
(1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9
604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^
2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c +
 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + (125*b^14 - 2425*a*b^12*c + 18
940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*
c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c +
 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*
c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^
3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3
)*e^2*f^8))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^
2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a
^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*
a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d
^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2
)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2
```

```
 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^
2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a
^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 3
9525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^
5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^
3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3
- 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a
^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^
2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^1
1 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4
- 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16))
 + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160
932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e
*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1
386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))))/((a^3*b^2*c - 4*a^4*c^2)*e^8
*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c +
 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)
*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*
b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a
^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a
^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*
b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*
d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)
```

giac [B] time = 0.71, size = 2002, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/4*((5*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^3*c*e^2 - 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*
a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 3
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*
b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
```

```
- sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2
- 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b
^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3
*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x
 - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*
e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^3*b^2*f^4 - 4*a^4*c*f^4) +
 1/2*(b^3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x
^2*e^2 + 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 +
b^4*x*e - 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d
)/((a^3*b^2*f^4*e - 4*a^4*c*f^4*e)*(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2
*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)) + 1/3*(6*b
*x^2*e^2 + 12*b*d*x*e + 6*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^3*f^4)
```

maple [C] time = 0.03, size = 1569, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 3/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/f^4/a^3/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
*b^3*c*e^2/(4*a*c-b^2)*x^3+9/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x
^2-3/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/f^4/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)/(4*a*c-b^2)*x*b*c^2*d^2-3/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^
2-1/f^4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*x*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^4+3/2/f^4/a^2/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-
1/f^4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-
b^2)*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+1/4/f^4/a^3/(4*a*c-
b^2)/e*sum(((19*a*c-5*b^2)*_R^2*b*c*e^2+19*a*b*c^2*d^2-5*b^3*c*d^2+2*(19*a*
```

```
c-5*b^2)*_R*b*c*d*e-14*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c
*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/3/
f^4/a^2/e/(e*x+d)^3+2/f^4/a^3*b/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 10.45, size = 13781, normalized size = 32.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2
*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^
2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4
096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8
+ 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2
*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^
8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 614
4*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c
^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20
- 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a
^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*
d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^2
0 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 9830
40*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^
19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^
16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 153600
0*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^
2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^
12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*
a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*
c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f
^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816
*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13
*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*
c^9*d*e^11*f^12)*1i + (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*
a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 21
```

9744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
 - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240
*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^
8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15
- 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*
f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c
^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*
(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^
9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*
f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 104
8576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^1
1*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*
e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f
^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*
b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^
12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 -
 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*
b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^1
2*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2
401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^
16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e
^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^1
2 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1
871872*a^15*b^2*c^9*d*e^11*f^12)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9
)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*
a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c
 - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
+ 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c
^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^1
1*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1
/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*
a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c -
 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*
f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2
*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 40
96*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 +
 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*
f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20
 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^
19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*
e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20
 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680

*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) - (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) + 1800*a^9*b^9*c^6*e^10*f^8 - 29080*a^10*b^7*c^7*e^10*f^8 + 176032*a^11*b^5*c^8*e^10*f^8 - 473216*a^12*b^3*c^9*e^10*f^8 + 476672*a^13*b*c^10*e^10*f^8))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225*b^3*c*d^2*e^3 - 62*a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2*b^2)) + (3*x^5*(5*b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2))

+ (2*x^3*(15*b^4*d*e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*d*e^2 - 285*a*b*c^2*d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 + 45*b^3*c*d^5 + 28*a^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3 - 171*a*b*c^2*d^5))/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b*c^2*e^5))/(2*a*(4*a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84*a^2*c^2*d^2*e - 40*a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a*b*c^2*d^4*e))/(6*a*(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^4 + 10*a*b^3*d^2 + 15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^2*c*d^4 - 57*a*b*c^2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x*(3*a*d^2*e*f^4 + 5*b*d^4*e*f^4 + 7*c*d^6*e*f^4) + x^4*(35*c*d^3*e^4*f^4 + 5*b*d*e^4*f^4) + x^2*(10*b*d^3*e^2*f^4 + 21*c*d^5*e^2*f^4 + 3*a*d*e^2*f^4) + x^5*(b*e^5*f^4 + 21*c*d^2*e^5*f^4) + x^3*(a*e^3*f^4 + 10*b*d^2*e^3*f^4 + 35*c*d^4*e^3*f^4) + a*d^3*f^4 + b*d^5*f^4 + c*d^7*f^4 + c*e^7*f^4*x^7 + 7*c*d*e^6*f^4*x^6) + atan(((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12)*1i + (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2

*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*
f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c
^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*
(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^
9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*
f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 104
8576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^1
1*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*
e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f
^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*
b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^
12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 -
 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*
b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^1
2*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2
401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^
16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e
^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^1
2 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1
871872*a^15*b^2*c^9*d*e^11*f^12)*1i)/((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9
)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*
a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c
 - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
- 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c
^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^1
1*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1
/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*
a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c +
 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*
f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2
*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 40
96*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 +
 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*
f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20
 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^
19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*
e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20
 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680
*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*
b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f
^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480
*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4
*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12
*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a
^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c
^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f

```
^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a
^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^
8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8
*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) - (-(25*b^15 + 25*b^6*(-(4
*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*
c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3
*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8
+ 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f
^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*
e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*
b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744
*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 6
15*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9
*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 -
6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25
*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767
*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6
 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*
c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^
8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*
c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^2
0 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 104857
6*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^
2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*
f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 -
1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^
14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*
f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 286
7200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a
^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f
^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 14586
88*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b
^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*
f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 4
88096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280
*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) + 1800*a^9*b^
9*c^6*e^10*f^8 - 29080*a^10*b^7*c^7*e^10*f^8 + 176032*a^11*b^5*c^8*e^10*f^8
 - 473216*a^12*b^3*c^9*e^10*f^8 + 476672*a^13*b*c^10*e^10*f^8))*(-(25*b^15
+ 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.654 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=353

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}\,f^4\left(-2b\sqrt{b^2-4a\ldots\right.}{4\sqrt{2}\,e\left(b^2-\ldots\right.}$$

```
[Out] 1/4*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4
)^2-1/8*f^4*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e
*x+d)^2+c*(e*x+d)^4)+3/8*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5
/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*f^4*arctan((e*x+d)*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1120, 1178, 1166, 205}

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}\,f^4\left(-2b\sqrt{b^2-4a\ldots\right.}{4\sqrt{2}\,e\left(b^2-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))
/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*
b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]
*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4\,\text{Subst}\left(\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\right.}{4\left(b^2-4ac\right)}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

Mathematica [A] time = 4.45, size = 331, normalized size = 0.94

$$\frac{f^4\left(\frac{(d+ex)\left(4ac-7b^2-12bc(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\cdots}{\cdots}\right.}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/
((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*
```

```
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

fricas [B] time = 1.24, size = 6770, normalized size = 19.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] -1/16*(24*b*c^2*e^7*f^4*x^7 + 168*b*c^2*d*e^6*f^4*x^6 + 2*(252*b*c^2*d^2 +
19*b^2*c - 4*a*c^2)*e^5*f^4*x^5 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d
)*e^4*f^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^
2)*d^2)*e^3*f^4*x^3 + 2*(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5
*b^3 + 16*a*b*c)*d)*e^2*f^4*x^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*
d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + 2*(12*b*c^2*
d^7 + (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*
c)*d)*f^4 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
+ 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*
f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*
c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^1
0 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1
024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x +
27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*
a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 8
0*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 1
2288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640
*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40
*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 6
40*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^
8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3
+ 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 +
```

```
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)
*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-
((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b
^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2
*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4
 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640
*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*
a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)
*d*f^12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)
*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*
a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 2
0*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a
^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b
^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*
c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^
2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)
*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b
^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 -
 (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 2
0*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a
^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5
*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*a*b^6
*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3
*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*
a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*
b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^
3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^
4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*
b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5
)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*
c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
```

```
d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5
 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*
x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5
 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^
2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10
 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 10
24*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*
b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^
12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^
8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b
^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3
*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^
5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a
^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*
c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 1
60*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))))
/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 3
0*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*
(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^
2)*d^2)*e)
```

giac [B] time = 0.80, size = 1844, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(d
*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*f^4*e^2 - 8*
(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*
d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
```

```
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4
 - b^2*f^4 - 4*a*c*f^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*
e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^4 - 8*a*b
^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*f^4*x^7*e^7 + 84*b*c^2*d*f^4*x^6*e^6 + 2
52*b*c^2*d^2*f^4*x^5*e^5 + 420*b*c^2*d^3*f^4*x^4*e^4 + 420*b*c^2*d^4*f^4*x^
3*e^3 + 252*b*c^2*d^5*f^4*x^2*e^2 + 84*b*c^2*d^6*f^4*x*e + 12*b*c^2*d^7*f^4
 + 19*b^2*c*f^4*x^5*e^5 - 4*a*c^2*f^4*x^5*e^5 + 95*b^2*c*d*f^4*x^4*e^4 - 20
*a*c^2*d*f^4*x^4*e^4 + 190*b^2*c*d^2*f^4*x^3*e^3 - 40*a*c^2*d^2*f^4*x^3*e^3
 + 190*b^2*c*d^3*f^4*x^2*e^2 - 40*a*c^2*d^3*f^4*x^2*e^2 + 95*b^2*c*d^4*f^4*
x*e - 20*a*c^2*d^4*f^4*x*e + 19*b^2*c*d^5*f^4 - 4*a*c^2*d^5*f^4 + 5*b^3*f^4
*x^3*e^3 + 16*a*b*c*f^4*x^3*e^3 + 15*b^3*d*f^4*x^2*e^2 + 48*a*b*c*d*f^4*x^2
*e^2 + 15*b^3*d^2*f^4*x*e + 48*a*b*c*d^2*f^4*x*e + 5*b^3*d^3*f^4 + 16*a*b*c
*d^3*f^4 + 3*a*b^2*f^4*x*e + 12*a^2*c*f^4*x*e + 3*a*b^2*d*f^4 + 12*a^2*c*d*
f^4)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 +
b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 3432, normalized size = 9.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c*d^2-2*f^4
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b*c-63/2*f^4/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+5*f^4/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c^2-95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2*b^2*c-6*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x*a*b*c*d^2-2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e*a*b*c-
105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+5/2*f
^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*a-95/8*f^4/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-6*f^4/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d
*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b*c+3/16*f^4/(16*a^2*c^2-8*a*b^2*c+b^4)
```

```
/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4
+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z
+a))-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3-21/2*f^
4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2*d^6+5/2*f^4/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c^2*d^4-95/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x*b^2*c*d^4-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*d^7/e*b*c^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*a
*c^2-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*b^2*c-3/2*f^4
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d/e*a^2*c-3/8*f^4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*d/e*a*b^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^5*a-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^5*b^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^7-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3-15/8*f^
4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*d^2-3/2*f^4/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*c-3/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*x*a*b^2-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e
*b^3-105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*c^2*d^4+
5*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*c^2*d^2-21/2*f^4/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*c^2*d*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-63/2*f^4/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b*d^2
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
[Out] Timed out
```

mupad [**B**] time = 7.52, size = 13840, normalized size = 39.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8
- 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 102
4*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*
e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76
80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c
^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 229376
0*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*
e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8
*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c
)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 +
20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))
/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*
(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*
a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*
b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4
*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a
^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144
0*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*
f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5
*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*
c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f
^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*
(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*
e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*
e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^
8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936
*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*
f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*
a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 +
 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96
*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + (-(9*(b^15*f^8 + f^8*(-
(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160
*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6
*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 -
 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a
^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196608
0*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(
1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^
9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 917504
0*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d
*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 -
2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 8192
0*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*
a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c
 - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b
^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c
^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^1
2*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*
b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) +
 (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*
f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^
10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6
```

```
*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8
+ f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^
8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 6
1440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*
b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^
6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^
5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*
b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)))*1i)/((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 819
20*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*
b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^
8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*
b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*
b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a
^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13
 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^
4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 220200
96*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*
a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a
^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7
*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^
4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c
^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c
^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4
*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 1
92*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12
*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 +
 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 614
4*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/
2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11
520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b
^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 +
720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258
048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2
949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7
*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^
2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a
^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 +
 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) - (-(9*(b^1
5*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*
c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f
^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a
^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3
*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^
6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^
2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 167
77216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d
*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128
*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^
```

11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c
^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256
*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8
+ f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^
8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 6
1440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2
*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f
^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2
- 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(
-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a
^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b
^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1
048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*
b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^
7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440
*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11
*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12
 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6
144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^
12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
256*a^3*b^2*c^3 - 16*a*b^6*c))) + (135*b^5*c^3*e^10*f^12 + 1080*a*b^3*c^4*e
^10*f^12 + 432*a^2*b*c^5*e^10*f^12)/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*
c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)
)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 -
560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*
a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^
2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680
*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 8601
60*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26
21440*a^10*b^2*c^9*e^2)))^(1/2)*2i - ((x^3*(5*b^3*e^2*f^4 + 16*a*b*c*e^2*f^
4 - 40*a*c^2*d^2*e^2*f^4 + 190*b^2*c*d^2*e^2*f^4 + 420*b*c^2*d^4*e^2*f^4))/
(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(19*b^2*c*d*e^3*f^4 - 4*a*c^2*d
*e^3*f^4 + 84*b*c^2*d^3*e^3*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5
*(19*b^2*c*e^4*f^4 - 4*a*c^2*e^4*f^4 + 252*b*c^2*d^2*e^4*f^4))/(8*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2*f^4 + 12*a^2*c*f^4 + 15*b^3*d^2*f^4 -
20*a*c^2*d^4*f^4 + 95*b^2*c*d^4*f^4 + 84*b*c^2*d^6*f^4 + 48*a*b*c*d^2*f^4))
/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(15*b^3*d*e*f^4 - 40*a*c^2*d^3*e
*f^4 + 190*b^2*c*d^3*e*f^4 + 252*b*c^2*d^5*e*f^4 + 48*a*b*c*d*e*f^4))/(8*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*b^3*d^3*f^4 - 4*a*c^2*d^5*f^4 + 19*b^2*c
*d^5*f^4 + 12*b*c^2*d^7*f^4 + 3*a*b^2*d*f^4 + 12*a^2*c*d*f^4 + 16*a*b*c*d^3
*f^4)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*e^6*f^4*x^7)/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*f^4*x^6)/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
 + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
 + 8*c^2*d*e^7*x^7) + atan((((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 +
 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*
a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*
c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*
a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*
a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491
20*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*

e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^1
1*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22
020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*
a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131
072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 1638
40*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2
*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*
a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7
*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))
/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^1
6*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^1
0*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*
b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4
- 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e
^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15
*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 +
11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a
*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2
+ 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 2
58048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 +
2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c
^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*
a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144
*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8
+ 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + ((9*
(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b
^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c
^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 10485
76*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14
*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^
8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^1
0*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 -
16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c
^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 2936
0128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280
*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(12
8*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b
^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-
(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2
*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8
- 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11
*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^
2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e
^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c
^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*
a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^1
2*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
)*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560
*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5
*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^
4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*
a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26214

40*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^
11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^
12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*
e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2
- 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i)/(((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) -
 b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f
^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 -
 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c
*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^
2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*
e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*
b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344
064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*
d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b
^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
- 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3
*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*
e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f
^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11
520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b
^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 +
720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258
048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2
949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^
8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^
4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4
)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4
*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(
1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9
*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6
*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*
b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*
c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^
6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (
18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f
^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*
c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)
) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8
))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
) - ((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 5
60*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a
^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2
 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*
a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86016
0*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262
1440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*
d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a
^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^1
3 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^
2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
+ (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 204
80*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(1
6*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9
(f^8(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*
b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*
c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048
576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^1

```
4*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b
^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^
10*b^2*c^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4
- 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2
*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a
^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*
b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f
^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 +
1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^
20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 -
 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 +
 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2
 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*
c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/
(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*
b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) + (135*b^5*c^3*e^10*f^12 + 1080*a
*b^3*c^4*e^10*f^12 + 432*a^2*b*c^5*e^10*f^12)/(64*(b^12 + 4096*a^6*c^6 + 24
0*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24
*a*b^10*c))))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c
^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^
8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(
a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e
^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e
^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8
*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.655 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=159

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2\cdots}{\sqrt{\cdots}}\right)}{e\left(b^2-4ac\right)\cdots}$$

```
[Out] 1/4*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*
b*f^3*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*
f^3*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1142, 1114, 638, 614, 618, 206}

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2\cdots}{\sqrt{\cdots}}\right)}{e\left(b^2-4ac\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^3\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\left(3bf^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\right.}{4\left(b^2-4ac\right)}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d+e\right.}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d+e\right.}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d+e\right.}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.94

$$\frac{f^3\left(\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2} - \frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((
b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2
 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

fricas [B] time = 1.00, size = 3843, normalized size = 24.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5
*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*f^3*x
^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*f^3*
x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(
b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*
(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*f^3*x + (6*
(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4
*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*c^3*e^8*
f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f^3*x^6 +
4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2
 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3
*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c
 + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5
+ a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 + 2*b^2*c^2*d^6
 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*sqrt(b^2 - 4*a*c
)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^
2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2
+ 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 +
c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^
4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c
^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x +
 ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2
 - 4*a*b*c^3)*d*e^5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*
c^3)*d^2)*e^4*f^3*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b
^2*c^2)*d)*e^3*f^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*
a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 -
4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^
2)*d)*e*f^3*x + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3
*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*
f^3 - 12*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2
*c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*
d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 1
0*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b
^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*
d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3
*d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f
^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
```

```
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 1
4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(
14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 1
28*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*
b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2
*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.78, size = 447, normalized size = 2.81

$$-\frac{3\,bcf^3\arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2d^6f^7+18\left(fx^2e+2\,dfx\right)bc^2d^4f^6e+9\,b^2cd^4f^7+18\left(fx^2e+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))*e^(-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1
/4*(6*b*c^2*d^6*f^7 + 18*(f*x^2*e + 2*d*f*x)*b*c^2*d^4*f^6*e + 9*b^2*c*d^4*
f^7 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*d^2*f^5*e^2 + 18*(f*x^2*e + 2*d*f*x)*b
^2*c*d^2*f^6*e + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 6*(f*x^2*e + 2*d*f*x)^3
*b*c^2*f^4*e^3 + 9*(f*x^2*e + 2*d*f*x)^2*b^2*c*f^5*e^2 + 2*(f*x^2*e + 2*d*f
*x)*b^3*f^6*e + 10*(f*x^2*e + 2*d*f*x)*a*b*c*f^6*e + a*b^2*f^7 + 8*a^2*c*f^
7)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2
*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e
 + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 2181, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -3/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*f^3/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2*e^4*b*c^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-45/2*f^3/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*b*c^2*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2-9/4*f^3/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*
```

```
c*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-30*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^3-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2*b^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^3-45/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*c^2*d^4-27/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c*d^
2-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c-1/2*f^3/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*b^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-9*f^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*b
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*b^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*x*c-5*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*
c-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*d*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*f^3/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b*c^2*d^6-9/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c
^2-8*a*b^2*c+b^4)/e*b^2*c*d^4-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*a*b*c*d^2-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b^3*d^
2-2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a^2*c-1/4*f^3/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a*b^2+3/2*f^3*b*c/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [**B**] time = 4.02, size = 1267, normalized size = 7.97

$$-\frac{\frac{8a^2 c f^3+a b^2 f^3+10 a b c d^2 f^3+2 b^3 d^2 f^3+9 b^2 c d^4 f^3+6 b\cdots}{4 e\left(16 a^2 c^2-8 a b^2 c+b^4\right)}\cdots}{x^2\left(6 b^2 d^2 e^2+30 b c d^4 e^2+2 a b e^2+28 c^2 d^6 e^2+12 a c d^2 e^2\right)+x^6\left(28 c^2 d^2 e^6+2 b c e^6\right)+x\left(4 e b^2 d^3+1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^6
*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e
*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3))/(
```

```
4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c^2*d^
2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*d^2*f^
3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*b*c
^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*f^3*x^5
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*
a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^
6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e)
 + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*
(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4
 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*
a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*atan(((b^4*(4*a*c - b^2
)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*
c^4*e^8*f^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^
3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^
2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((9*b^3*c^2*f^6
*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (18*b^2*c^4*d*e^7*f^6)/(a*
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^2*c^4*d^2*e^6*f
^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*f^6
*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8 - 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8
 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(18*b^2*c^4*e^6*f^6)))/(e*(4*a*c - b^
2)^(5/2))
```

sympy [B] time = 14.58, size = 1794, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**
4*f**3*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*f**3*sqrt(-1/(4*a*c
- b**2)**5) - 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*f**
3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b*c**
2*e**2*f**3))/(2*e) - 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x
**2 + (192*a**3*b*c**4*f**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3
*f**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**
2)**5) - 3*b**7*c*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**
2*d**2*f**3)/(6*b*c**2*e**2*f**3))/(2*e) + (-8*a**2*c*f**3 - a*b**2*f**3 -
10*a*b*c*d**2*f**3 - 2*b**3*d**2*f**3 - 9*b**2*c*d**4*f**3 - 6*b*c**2*d**6*
f**3 - 36*b*c**2*d*e**5*f**3*x**5 - 6*b*c**2*e**6*f**3*x**6 + x**4*(-9*b**2
*c*e**4*f**3 - 90*b*c**2*d**2*e**4*f**3) + x**3*(-36*b**2*c*d*e**3*f**3 - 1
20*b*c**2*d**3*e**3*f**3) + x**2*(-10*a*b*c*e**2*f**3 - 2*b**3*e**2*f**3 -
54*b**2*c*d**2*e**2*f**3 - 90*b*c**2*d**4*e**2*f**3) + x*(-20*a*b*c*d*e*f**
3 - 4*b**3*d*e*f**3 - 36*b**2*c*d**3*e*f**3 - 36*b*c**2*d**5*e*f**3))/(64*a
**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4
*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a*
*2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6
*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*
d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9)
+ x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8
) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*
e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7)
 + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**
2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d*
*3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**
2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b*
*2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4
*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**
```

```
2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a
*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**
2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a*
*2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*
a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b*
*2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**
2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a*
*2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16
*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*
b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**
2*d**7*e**2))
```

3.656 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=375

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}\,f^2\left(\frac{b\left(b^2-52ac\right.}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\,ae\left(\right.}$$

```
[Out] -1/4*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2+1/8*f^2*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2
)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*f^2*
arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+2
0*a*c-b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1119, 1178, 1166, 205}

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}\,f^2\left(\frac{b\left(b^2-52ac\right.}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\,ae\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*
(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b
^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2\,\text{Subst}\left(\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right.}\right.}{4\left(b^2-4ac\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

Mathematica [A] time = 4.48, size = 385, normalized size = 1.03

$$\frac{f^2\left(-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2(d+ex)\left(8abc+20ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{a\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\text{ta}}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 2
0*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]
*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

fricas [B] time = 1.46, size = 7838, normalized size = 20.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*f^
2*x^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*f^2*x^5
+ 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*f^2*x^4 +
2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c +
 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c
+ 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*e^2*f^2*x^2 + 2*(7*
(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c
 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*x + 2*((b^2*c^2 + 20*a*c^3)*
d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 - (a*
b^3 - 16*a^2*b*c)*d)*f^2 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b
^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 +
3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2
*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^
3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4
*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2
- 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^
8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a
^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2
)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*
b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*
b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2
*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a
^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4
- (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^
7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^
4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c
^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-
((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20
*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^
8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c
```

```
+ 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^
4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8
*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6
 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^
4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*
a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5
 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b
^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*
b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x +
 ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*
b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*
sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^1
0 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1
024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*
b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c
^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^
3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3
 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^
3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*
b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*
b*c^5)*e*f^4 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*
c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*
c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)
)*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 -
20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024
*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6
*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + sqrt(1/2)*((a*b^4*c^2
- 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6
*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*
b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3
 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*
b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a
*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a
*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*
c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f
^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^
7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b
^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4
- 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 6
```

40*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c

```
^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8
+ a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2)*d^2)*e)
```

giac [B] time = 0.79, size = 2527, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e - 40*(d*e^(-1) + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d
^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(d*e^(-1) + x + sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1)
 + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2
*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) - sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(
d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c
*d*f^2*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a
*b*c*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(
d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((
d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2
*c*f^2*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 +
20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
- b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e
- 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*f^2*x^7*e^7 + 20*a*c^3*f^2*x^7*e^7 +
7*b^2*c^2*d*f^2*x^6*e^6 + 140*a*c^3*d*f^2*x^6*e^6 + 21*b^2*c^2*d^2*f^2*x^5*
e^5 + 420*a*c^3*d^2*f^2*x^5*e^5 + 35*b^2*c^2*d^3*f^2*x^4*e^4 + 700*a*c^3*d^
```

```
3*f^2*x^4*e^4 + 35*b^2*c^2*d^4*f^2*x^3*e^3 + 700*a*c^3*d^4*f^2*x^3*e^3 + 21
*b^2*c^2*d^5*f^2*x^2*e^2 + 420*a*c^3*d^5*f^2*x^2*e^2 + 7*b^2*c^2*d^6*f^2*x*
e + 140*a*c^3*d^6*f^2*x*e + b^2*c^2*d^7*f^2 + 20*a*c^3*d^7*f^2 + 2*b^3*c*f^
2*x^5*e^5 + 28*a*b*c^2*f^2*x^5*e^5 + 10*b^3*c*d*f^2*x^4*e^4 + 140*a*b*c^2*d
*f^2*x^4*e^4 + 20*b^3*c*d^2*f^2*x^3*e^3 + 280*a*b*c^2*d^2*f^2*x^3*e^3 + 20*
b^3*c*d^3*f^2*x^2*e^2 + 280*a*b*c^2*d^3*f^2*x^2*e^2 + 10*b^3*c*d^4*f^2*x*e
+ 140*a*b*c^2*d^4*f^2*x*e + 2*b^3*c*d^5*f^2 + 28*a*b*c^2*d^5*f^2 + b^4*f^2*
x^3*e^3 + 5*a*b^2*c*f^2*x^3*e^3 + 36*a^2*c^2*f^2*x^3*e^3 + 3*b^4*d*f^2*x^2*
e^2 + 15*a*b^2*c*d*f^2*x^2*e^2 + 108*a^2*c^2*d*f^2*x^2*e^2 + 3*b^4*d^2*f^2*
x*e + 15*a*b^2*c*d^2*f^2*x*e + 108*a^2*c^2*d^2*f^2*x*e + b^4*d^3*f^2 + 5*a*
b^2*c*d^3*f^2 + 36*a^2*c^2*d^3*f^2 - a*b^3*f^2*x*e + 16*a^2*b*c*f^2*x*e - a
*b^3*d*f^2 + 16*a^2*b*c*d*f^2)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^
2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8
*a^2*b^2*c*e + 16*a^3*c^2*e))
```

maple [C] time = 0.05, size = 4751, normalized size = 12.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 7/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6*b^2+21/8*
f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5*b^2*d^2+35/8*f^
2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^2+5/4*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^3+35/8*f^2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^2*c^2*d^4+3/8*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^4*d^2+105/2*f^2/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x*b*c^2*d^4+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*b^2*c*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*b*c^2+5/
8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e*b^2*c+35/2*f^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*c^3*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+2*f^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*a*x*b*c+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*a*d^3/e*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^3/e
*b^4+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+17
5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^3*d^4+5/8*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c+105/2*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*c^3*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*
e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
```

```
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*a*x^3*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/a*x^3*b^4+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x*c^2*d
^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+35/2*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3*d^6+5/2*f^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*d^7/e*c^3-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*d/e*b^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^7/
e*b^2*c^2+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^5/e*b^3*c+2
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*d/e*b*c+35/2*f^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+35*f^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*c^2*d^2+35*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^2*b^2*c+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4
)/a*x^5*b^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7
*b^2+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x^2*c^2+3/8*f
^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^4+7/8*f^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^2*c^2*d^6+5/4*f^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^3*c*d^4+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c
^2-8*a*b^2*c+b^4)/a*x^3*b^3*c*d^2+21/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2
-8*a*b^2*c+b^4)/a*x^2*b^2*c^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*
b^2*c+b^4)/a*x^2*b^3*c-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*
b^3+1/16*f^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*sum(((20*a*c+b^2)*_R^2*c*e^2+20
*a*c^2*d^2+b^2*c*d^2+2*(20*a*c+b^2)*_R*c*d*e-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*
_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^
4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_
Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 7.94, size = 16025, normalized size = 42.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3
*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13
 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^
8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262
144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*
a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*
(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))
*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 +
1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43
776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55
*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 +
1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6
*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a
^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621
440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^1
1*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2
 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c
^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^
2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 +
 b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*
c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^
5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4
 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c
^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2
 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2
 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c
^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672
*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d
*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 -
24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 614
4*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^
2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^
3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*
c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*
a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a
^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*
1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*
b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13
- 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8
*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^
8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(2621
44*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a
^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(
a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*
(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1
140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 437
76*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*
```

```
a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1
048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*
b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^
9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 26214
40*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11
*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2
- 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^
8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 +
b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c
^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5
*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 -
 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^
10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2
- 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^
9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*
a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*
e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2
*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3
*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c
 - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a
^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^
6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4
*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*
b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*
c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b
^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1
i)/(((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b
^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 -
 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*
b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8
*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(26214
4*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^
4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a
^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(
-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 11
40*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 4377
6*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a
*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 10
48576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b
^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*
c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 -
 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8
*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b
^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^
2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*
f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 -
25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a
*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e
```

^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*
a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*
c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*
b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^
3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6
*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*
a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b
^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c
^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^
6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) -
((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*
c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 367
00160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*
c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^
7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^
7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b
^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^
17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a
^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^
5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^1
5*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104857
6*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*
c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8
*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^
12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*
e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 550
5024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^1
2*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 12
80*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b^2*f
^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^
4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4
- 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a
*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^
2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 5376
0*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 196
6080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2
)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8
*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*
f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3
*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*
b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*
e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*
c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^
11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5
*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c
- b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*
c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e
^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^
7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (800
0*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 12720*a^2*b^2*c^6*e^10*f^6 - 84*
a*b^4*c^5*e^10*f^6)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4
*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(b^
17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a
^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^
5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^1

5*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104857
6*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*
c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8
*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^
12*b^2*c^9*e^2)))^(1/2)*2i + atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2
*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13
 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*
b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^
6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5
120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 3
27680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4
*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/
2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4
 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 +
 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)
^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 72
0*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 25804
8*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 29
49120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^
9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 +
 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c
^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a
^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b
^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b
^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(
a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2
*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5
*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 -
 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^
11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(51
2*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6
*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b
^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(
a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*
(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1
140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 437
76*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*
a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1
048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*
b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^
9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 26214
40*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*
b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13
+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b
^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6
 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 51
20*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 32
7680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2
) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4
+ 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 +
1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720
*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048

*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 294
9120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9
*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 +
2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^
2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*
a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^
8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^
9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^
3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a
^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*
e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*
e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*
c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 -
16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^1
1*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512
*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*
c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^
6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a
^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(
-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 11
40*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 4377
6*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a
*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 10
48576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b
^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2)*1i)/(((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b
^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 +
 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^
5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6
- 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6
144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 512
0*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327
680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b
^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2)
 - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 +
 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1
863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(
1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*
a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*
a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949
120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*
c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2
949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2
*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*
a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a
^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8
*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9
*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3
*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^
3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e
^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e
^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c
^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 1
6*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11
*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*
(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c
^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6

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*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 114
0*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776
*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*
b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104
8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^
14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*
b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440
*a^12*b^2*c^9*e^2)))^(1/2) - ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*
c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 917
5040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^
7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*
a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^
3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*
a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c
^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 348
80*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 18636
80*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*
b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*
b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*
a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*
e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 29491
20*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^1
2*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*
b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b
^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c
^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4
*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7
*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^
20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 -
 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 +
 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e
^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^
10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4
 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2
*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 +
 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3
*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^
8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^1
7*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^
2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5
*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15
*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576
*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c
^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*
c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^1
2*b^2*c^9*e^2)))^(1/2) + (8000*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 127
20*a^2*b^2*c^6*e^10*f^6 - 84*a*b^4*c^5*e^10*f^6)/(256*(a^2*b^12 + 4096*a^8*
c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
 - 6144*a^7*b^2*c^5))))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 348
80*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 18636
80*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*
b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*
```

```
b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*
a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*2i + ((x^5*(2*b^3*c*e^
4*f^2 + 21*b^2*c^2*d^2*e^4*f^2 + 28*a*b*c^2*e^4*f^2 + 420*a*c^3*d^2*e^4*f^2
))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4*e^2*f^2 + 36*a^2*c^2*e^
2*f^2 + 35*b^2*c^2*d^4*e^2*f^2 + 5*a*b^2*c*e^2*f^2 + 700*a*c^3*d^4*e^2*f^2
+ 20*b^3*c*d^2*e^2*f^2 + 280*a*b*c^2*d^2*e^2*f^2))/(8*a*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)) + (x*(3*b^4*d^2*f^2 - a*b^3*f^2 + 140*a*c^3*d^6*f^2 + 10*b^3*c
*d^4*f^2 + 108*a^2*c^2*d^2*f^2 + 7*b^2*c^2*d^6*f^2 + 16*a^2*b*c*f^2 + 15*a*
b^2*c*d^2*f^2 + 140*a*b*c^2*d^4*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (x^2*(3*b^4*d*e*f^2 + 108*a^2*c^2*d*e*f^2 + 420*a*c^3*d^5*e*f^2 + 20*b^3*
c*d^3*e*f^2 + 21*b^2*c^2*d^5*e*f^2 + 15*a*b^2*c*d*e*f^2 + 280*a*b*c^2*d^3*e
*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3*f^2 + 20*a*c^3*d^7*f
^2 + 2*b^3*c*d^5*f^2 + 36*a^2*c^2*d^3*f^2 + b^2*c^2*d^7*f^2 - a*b^3*d*f^2 +
 16*a^2*b*c*d*f^2 + 5*a*b^2*c*d^3*f^2 + 28*a*b*c^2*d^5*f^2)/(8*a*e*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(20*a*c^3*d*e^5*f^2 + b^2*c^2*d*e^5*f^2))/
(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(7*b^2*c^2*d^3*e^3*f^2 + 2*b^
3*c*d*e^3*f^2 + 140*a*c^3*d^3*e^3*f^2 + 28*a*b*c^2*d*e^3*f^2))/(8*a*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (f^2*x^7*(20*a*c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.657 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+\right.}$$

```
[Out] -1/4*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c
*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*f*a
rctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1142, 1107, 614, 618, 206}

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df + efx}{\left(a + b(d+ex)^2 + c(d+ex)^4\right)^3}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3cf)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\right.}{2\left(b^2-4ac\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

Mathematica [A] time = 0.18, size = 148, normalized size = 0.97

$$\frac{f\left(\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)
^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
))/(4*(b^2 - 4*a*c)^2*e)
```

fricas [B] time = 1.03, size = 3748, normalized size = 24.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^5
 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(10
*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^
```

```
3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x + 12*(c^4*e^8*f*x^8 + 8*
c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^
3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4
+ 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c
^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*
(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^
4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)
*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*
c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*
c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 +
 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/
((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^
5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +
10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e
^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 3
2*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2
 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c
^3 - 4*a*c^4)*d*e^5*f*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4
)*d^2)*e^4*f*x^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)
*d)*e^3*f*x^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*
d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^
5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x
 - 24*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6
 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*
c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^
3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a
*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 +
2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2
*a*c^3)*d^4 + a^2*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e
*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (12*(b^2*c^3 - 4*a*c^
4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4
*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^
7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4
```

```
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(
b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*
x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^
6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
+ 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.69, size = 445, normalized size = 2.91

$$\frac{6\,c^2 f \arctan\left(\frac{2\,cd^2 f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3d^6f^5+36\left(fx^2e+2\,dfx\right)c^3d^4f^4e+18\,bc^2d^4f^5+36\left(fx^2e+2\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4
*a*c)*f))*e^(-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*
(12*c^3*d^6*f^5 + 36*(f*x^2*e + 2*d*f*x)*c^3*d^4*f^4*e + 18*b*c^2*d^4*f^5 +
 36*(f*x^2*e + 2*d*f*x)^2*c^3*d^2*f^3*e^2 + 36*(f*x^2*e + 2*d*f*x)*b*c^2*d^
2*f^4*e + 4*b^2*c*d^2*f^5 + 20*a*c^2*d^2*f^5 + 12*(f*x^2*e + 2*d*f*x)^3*c^3
*f^2*e^3 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*f^3*e^2 + 4*(f*x^2*e + 2*d*f*x)*b
^2*c*f^4*e + 20*(f*x^2*e + 2*d*f*x)*a*c^2*f^4*e - b^3*f^5 + 10*a*b*c*f^5)/(
(c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f
*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e + 1
6*a^2*c^2*e))
```

maple [C] time = 0.05, size = 2132, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*f/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*e^4*c^3*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+45*f/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^3/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2+9/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^3/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b+60*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+45*f/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*c^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*d^4+27*f/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*d^2+5*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e/(16*a^
```

```
2*c^2-8*a*b^2*c+b^4)*x^2*a+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2*b^2+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x+18*f/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*c^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b+10*f/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d/
(16*a^2*c^2-8*a*b^2*c+b^4)*x*a+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d/(16*a^2*c^2-8*a*b^2*c
+b^4)*x*b^2+3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*c^3*d^6+9/2*f/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b*c^2*d^4+5*f/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16
*a^2*c^2-8*a*b^2*c+b^4)/e*a*c^2*d^2+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*b^2*c*d^2+5/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a*b*c-1/
4*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b^3+3*f*c^2/(16*a^2*c^2-8*a
*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^
3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*
c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.99, size = 1199, normalized size = 7.84

$$\frac{\frac{x^2\left(e\,f\,b^2\,c+27\,e\,f\,b\,c^2\,d^2+45\,e\,f\,c^3\,d^4+5\,a\,e\,f\,c^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3+12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4 +
 16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2*c
*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(b^4 +
16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
+ (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2
+ 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c
^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b
*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 4
0*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^
2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8
*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + (6*c^2*f*atan
(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^
```

```
2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*c^4*e^
10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((72*
c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72
*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2
*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (36*c^6*d^2*e^6*f^
2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(
32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^2*e^8 - 8
*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6*f^2)))/(e*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 13.97, size = 1707, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*f
*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**
5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1/(4
*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c**2*
f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqrt(-1/
(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5) + 36*a
*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a*c - b*
*2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*c*f + 20
*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*c**3*d**6
*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**2*e**4*f +
 180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**3*e**3*f) +
 x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**2*f + 180*c*
*3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c**2*d**3*e*f
+ 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d
**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*
a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d*
*4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b*
*5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3
*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*
e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d
**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7
+ 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*
d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c
*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b
*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*
c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2
*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b
*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b*
*3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*
c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a
**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 179
2*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3
*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*
c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a*
*3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 51
2*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b*
*3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5
*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

3.658 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=270

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2\right.}{}$$

```
[Out] 1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4
)^2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*
a*c+b^2)^2/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4
)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e/f+
ln(e*x+d)/a^3/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e/f
```

Rubi [A] time = 0.50, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - L
og[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\ldots}{4a^2\left(b^2-\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\ldots}{4a^2\left(b^2-\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\ldots}{4a^2\left(b^2-\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\ldots}{4a^2\left(b^2-\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\ldots}{4a^2\left(b^2-\ldots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\ldots}{4a^2\left(b^2-\ldots\right.}$$

Mathematica [A] time = 3.93, size = 394, normalized size = 1.46

$$\frac{\frac{a^2\left(2ac-b^2-bc(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{a\left(16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^4+2b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4a\ldots}\right.}{\left(b^2-4\ldots\right.}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)
```

fricas [B] time = 4.92, size = 9926, normalized size = 36.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 +
 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c
^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3
+ 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11
*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^
2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*
c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c
^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^
2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*
a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a
^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a
^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c -
 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6
0*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 -
 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*
c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e
*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 +
 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
```

4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*
c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 4
8*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*
a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*
a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3
 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^
2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3
*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 6
4*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 -
64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*
a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64
*a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
- 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2
*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*
a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c
^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128
*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*
c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64
*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3
 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c
^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a
^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7
*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^
4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 -
 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f), 1/4*(2*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3
+ 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 +
 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 1
1*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*

a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)* d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^ 2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 13 2*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3 *b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c ^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^ 2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^ 3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^ 5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2 *b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3* b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(1 4*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)* e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3* (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3 *c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)* sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^ 2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64* a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) *d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14* (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12* a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4 *c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^ 2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2* c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3* b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6 *c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b ^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^ 4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a ^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c ^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c ^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3) *d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3* (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^ 6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2* b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^ 3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4* ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^ 2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a *b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48* a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48 *a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a ^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4 *c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^

```
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a
^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^
3*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 -
64*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 -
 64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32
*a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*
c^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 6
4*a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*
b^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
 - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^
2*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*
c^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64
*a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*
c^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 12
8*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 6
4*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^
3 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (
a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^
7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a
^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f)]
```

giac [B] time = 1.70, size = 1044, normalized size = 3.87

$$-\frac{\left(a^3b^7cfe^3 - 14\,a^4b^5c^2fe^3 + 70\,a^5b^3c^3fe^3 - 120\,a^6bc^4fe^3\right)\sqrt{b^2-4\,ac}\,\log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3 - 120*
a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2
 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d
^2 + 2*a)) - (a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3
 - 120*a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e +
sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 -
4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*f^2*e^4 - 16*a^7*b^6*c^2*f^2*e^4 + 96*a^8*b^
4*c^3*f^2*e^4 - 256*a^9*b^2*c^4*f^2*e^4 + 256*a^10*c^5*f^2*e^4) - 1/4*e^(-1
)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f) + e^(-1)*log(abs(x*e + d))/(
a^3*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b
```

```
^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*
d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c +
24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d
^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^
3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c
*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4
*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2
+ 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4
*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2
*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2*d*e)*x)
*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3*f)
```

maple [C] time = 0.08, size = 4606, normalized size = 17.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2-1/f/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^
2-8*a*b^2*c+b^4)*b*c^2*d^2+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*b^5+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^5+1/2
/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2+ln(e*x+d)/a^3/e/
f-7/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/f/a^2
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+6/f*a/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+3/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-
8*a*b^2*c+b^4)*b^4+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3+4/f/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-21/4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2
-8*a*b^2*c+b^4)*b^2*c-1/2/f/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16
*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(4
8*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+1
6*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(
2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=R
ootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d
^3*e+2*b*d*e)*_Z+a))+10/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^3*b^3-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^3*b^2+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-
105/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/f/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
```

```
b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c^2*d^4-87/2/f/a/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2*d^2+6/f/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c*d^2-21/f/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^
3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3*c^2*d*e^4
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-105/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^4*b*d^2+15/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b^3*d^2-70/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3*b+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^3+24/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3*d^2+1/f/a^2/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4-3/f/a/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2
-8*a*b^2*c+b^4)*x*b*c^3+3/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^
4)*x*b^3*c^2-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2
+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/f/a/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/f/a/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*
a^2*c^2-8*a*b^2*c+b^4)*b*c^3*d^6+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a
*b^2*c+b^4)*b^3*c^2*d^6-29/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4
)*b^2*c^2*d^4+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c*d^4
-3/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c*d^2-29/4/f/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 18.49, size = 22621, normalized size = 83.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3
*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 -
21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d
^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^3*f + 40*b*c*d
^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^6*e^2*f + 2*a*b
*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^3*e*f + 8*c^2*d^
7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^4*(b^2*e^4*f + 70
*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(56*c^2*d^3*e^5*f +
12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*e^6*f) + b^2*d^4*f
+ c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^4*f + 2*b*c*d^6*f + 8*c
^2*d*e^7*f*x^7) - (log((((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)
/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*
c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^
16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^
2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*f*(-(b^2*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b +
3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*
a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^
2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a
^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) + (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71
*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))
/(a^4*f^2*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2
*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^
6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4))
)/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6
) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c
 - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c -
 b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6))*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 1
0*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*f*(-(b^2*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)
*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b
^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*f*(-(
b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2)
 - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*
e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*
a*b^2*c))/(a^2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2
*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) - (b*c^3*e^15*(7*a*c - b^2
)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*
b^2*c^2*d^2))/(a^4*f^2*(4*a*c - b^2)^4) + (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*
c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) + (2*b*c^4*d
*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a
*c - b^2)^4)))/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4
*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^
2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^
15*x*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -
 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b
^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a
^4*b^8*c*e^2*f^2)) + log(d + e*x)/(a^3*e*f) - (b*atan((x*(((((b*((2*(5120*
```

```
a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*
f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b
^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c
^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f
- 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^
3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*
b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*
e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8
*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^
7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3
 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^
10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/
(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^1
0*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 256
0*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*
b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^1
8*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 2273
28*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(4*a^3*e*f*(4
*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*
c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*
b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*
a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*
b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*
a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^
2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e
^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((2*(6*a^2*
b^11*c^4*d*e^16*f - 137*a^3*b^9*c^5*d*e^16*f + 1217*a^4*b^7*c^6*d*e^16*f -
5256*a^5*b^5*c^7*d*e^16*f + 11024*a^6*b^3*c^8*d*e^16*f - 8960*a^7*b*c^9*d*e
^16*f))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8
*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5
*f^3) - (((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a
^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^
17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b
^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 128
0*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^
10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 25
60*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6
*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^
18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227
328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10
*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^
3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3
+ 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^
6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f -
2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^
2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2
+ 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2
*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*
(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13*
b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3
 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10
*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*
d*e^18*f^3))/(32*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 18
0*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13
/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*
```

b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d*e^15))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(6*a^2*b^11*c^4*d*e^16*f - 137*a^3*b^9*c^5*d*e^16*f + 1217*a^4*b^7*c^6*d*e^16*f - 5256*a^5*b^5*c^7*d*e^16*f + 11024*a^6*b^3*c^8*d*e^16*f - 8960*a^7*b*c^9*d*e^16*f))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((b*((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d

*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(16*a^6*e^2*f^2*(4*a*c - b^2)^
5*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 25
60*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(
a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3
- 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(
b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6
*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2
*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^
6*f^3*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 384
0*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b
^2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c
^5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*
b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) + (x^2*((((((b*
((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18
*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3
*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3
 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^
10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f +
 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b
^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*
b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^
3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^
13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5
*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80
*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 +
 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*
a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^
2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e
*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40
*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*
a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^1
9*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 16384
0*a^13*b*c^9*e^19*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2
- 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2
 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^
12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3
 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*
c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f
 - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5
*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80
*a^4*b^8*c*e^2*f^2)) + (b*((8960*a^7*b*c^9*e^17*f - 6*a^2*b^11*c^4*e^17*f +
 137*a^3*b^9*c^5*e^17*f - 1217*a^4*b^7*c^6*e^17*f + 5256*a^5*b^5*c^7*e^17*f
 - 11024*a^6*b^3*c^8*e^17*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^1
0*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^
3 - 6144*a^11*b^2*c^5*f^3) + (((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e
^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b
^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^
6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 -
1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2
*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f +
 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7
*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^
3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^1
2*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 -
4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 +
 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -

40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(64*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5*e^16 - 21*a*b^7*c^6*e^16 + 147*a^2*b^5*c^7*e^16 - 343*a^3*b^3*c^8*e^16)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (((8960*a^7*b*c^9*e^17*f - 6*a^2*b^11*c^4*e^17*f + 137*a^3*b^9*c^5*e^17*f - 1217*a^4*b^7*c^6*e^17*f + 5256*a^5*b^5*c^7*e^17*f - 11024*a^6*b^3*c^8*e^17*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((b*((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^1

9*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e
^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*
c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*
a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4
*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3
*e*f*(4*a*c - b^2)^(5/2)) - (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*
e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*
a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*
c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97
280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*
c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(32*a^6*e^2*f^2*(4*a*c - b^2)^5
*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 256
0*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a
^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 -
 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b
^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*
(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*
c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6
f^3(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840
*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^
2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^
5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b
^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) - (((b*((4*a^2*b
^12*c^3*e^15*f - 93*a^3*b^10*c^4*e^15*f + 854*a^4*b^8*c^5*e^15*f - 3889*a^5
*b^6*c^6*e^15*f + 8808*a^6*b^4*c^7*e^15*f - 7952*a^7*b^2*c^8*e^15*f - 8960*
a^7*b*c^9*d^2*e^15*f + 6*a^2*b^11*c^4*d^2*e^15*f - 137*a^3*b^9*c^5*d^2*e^15
*f + 1217*a^4*b^7*c^6*d^2*e^15*f - 5256*a^5*b^5*c^7*d^2*e^15*f + 11024*a^6*
b^3*c^8*d^2*e^15*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 +
 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*
a^11*b^2*c^5*f^3) - (((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2
+ 1052*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^
6*e^16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 512
0*a^10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d
^2*e^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2
+ 3456*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*
f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^
9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e
*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a
^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3
*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*
a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^
17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 32
8*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^
9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*
d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 -
4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 +
 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -
 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b
^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a
^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^
2)^(5/2)) - (((b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 10
52*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^
16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^
10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e
^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 34
56*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3

+ 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(64*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*f^3*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*f^3*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) - (3*b*((((4*a^2*b^12*c^3*e^15*f - 93*a^3*b^10*c^4*e^15*f + 854*a^4*b^8*c^5*e^15*f - 3889*a^5*b^6*c^6*e^15*f + 8808*a^6*b^4*c^7*e^15*f - 7952*a^7*b^2*c^8*e^15*f - 8960*a^7*b*c^9*d^2*e^15*f + 6*a^2*b^11*c^4*d^2*e^15*f - 137*a^3*b^9*c^5*d^2*e^15*f + 1217*a^4*b^7*c^6*d^2*e^15*f - 5256*a^5*b^5*c^7*d^2*e^15*f + 11024*a^6*b^3*c^8*d^2*e^15*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 12

16*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*
c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 2
40*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^
11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1
280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^
2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^
10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17
*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^
6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4
*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^1
7*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3
))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2
- 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^
2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*
f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)
))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*
e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096
*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 512
0*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*
e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4
0*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6
*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4
*b^8*c*e^2*f^2)) - (b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^1
4 - 616*a^3*b^4*c^7*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^
2*b^5*c^7*d^2*e^14 - 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6
*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1
280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (b*(
(b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^
4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768
*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e
^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276*
a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7
*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^
6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 38
40*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*
e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4
0*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^
9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*
f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^
13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2
*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3
 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294
912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f
^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*
e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*
b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4
*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f
*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 20
48*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*
c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f
^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^
6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3
- 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b
^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d
^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^1
7*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(
4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*
a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6
*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1

```
280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4
 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 +
30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2
*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7
*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 -
 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*
c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3
 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*
b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6
*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*
e^17*f^3))/(32*a^6*e^2*f^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c
^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*
b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 -
 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10
*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2
- 11*a*b^4*c)*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*f^3*(4
*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840*a^11*b
^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^2)^(15/
2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*f^3*(
4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(b^10*c^2*e^14 - 20*a*b^8*
c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^
14)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*
b^2*c^4 - 120*a*b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*e*f*(4*a*c
 - b^2)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.659 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=499

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3ef^2\left(b^2-4ac\right)^2(d+ex)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+\right.}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}}$$

```
[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^2/(e*x+d)+1/4*(b^2
-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)
^4)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(
-4*a*c+b^2)^2/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5
*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^2/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/
2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b
^2)+(-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^
2/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)}{8\sqrt{2}\,a^3ef^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x))
 + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*
(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c)
 + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*
a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b
^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
```

```
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - $$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + $$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2 ef^2(d+ex)} + \frac{b^2-2ac+}{4a\left(b^2-4ac\right)ef^2(d+ex)(}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2 ef^2(d+ex)} + \frac{b^2-2ac+}{4a\left(b^2-4ac\right)ef^2(d+ex)(}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2 ef^2(d+ex)} + \frac{b^2-2ac+}{4a\left(b^2-4ac\right)ef^2(d+ex)(}$$

Mathematica [A] time = 6.21, size = 575, normalized size = 1.15

$$-\frac{1}{a^3ef^2(d+ex)} + \frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{4a^2ef^2\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
+ e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2
*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*
c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2
- 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*(-5*b
^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt
[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

fricas [B] time = 2.63, size = 10518, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")

[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^
2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d
)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4
*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a
^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a
^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 4
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 +
 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
+ 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 -
 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b
*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5
*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 +
 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c
^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*sqr
t(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^
4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^1
2 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^
4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 1
60*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4
*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 12
80*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 77825*a
*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 -
 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6
*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2
sqrt(1/2)((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b
^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 -
323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*b^1
0*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*
a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2
 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (125*b
^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^

4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c

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 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 -
6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*
c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2
*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4
 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4
*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 -
 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a
^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)
*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a
^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a
^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^
2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27
720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*
c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((6
25*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*
a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^
5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 7
7825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2
*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a
^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d
 + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*
a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*
c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250
*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3
12300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b
^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) +
(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623
534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b
^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*
b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^
10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4
- 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^
2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^
6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*
f^4))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x
^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^
5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4
)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (1
26*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*
f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 +
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 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e
*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 +
 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b
^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt
((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 5918
86*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*
b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19
*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4
*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4
- 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*
b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 57103
0*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)
*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 149
60*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b
^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12
250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4
- 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^1
6*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8))
 + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1
623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^
7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a
^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7
*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c
^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8
*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e
^2*f^4))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^
4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b
^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 +
2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a
^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*
c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 +
6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)
```

giac [B] time = 1.42, size = 1658, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] -1/8*(7*b^4*c^2*e^(-1)/((f*x*e + d*f)*f) - 47*a*b^2*c^3*e^(-1)/((f*x*e + d*
f)*f) + 52*a^2*c^4*e^(-1)/((f*x*e + d*f)*f) + 14*b^5*c*f*e^(-1)/(f*x*e + d*
f)^3 - 99*a*b^3*c^2*f*e^(-1)/(f*x*e + d*f)^3 + 136*a^2*b*c^3*f*e^(-1)/(f*x*
e + d*f)^3 + 7*b^6*f^3*e^(-1)/(f*x*e + d*f)^5 - 43*a*b^4*c*f^3*e^(-1)/(f*x*
e + d*f)^5 + 25*a^2*b^2*c^2*f^3*e^(-1)/(f*x*e + d*f)^5 + 68*a^3*c^3*f^3*e^(
-1)/(f*x*e + d*f)^5 + 9*a*b^5*f^5*e^(-1)/(f*x*e + d*f)^7 - 66*a^2*b^3*c*f^5
*e^(-1)/(f*x*e + d*f)^7 + 108*a^3*b*c^2*f^5*e^(-1)/(f*x*e + d*f)^7)/((a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e +
d*f)^4)^2) - e^(-1)/((f*x*e + d*f)*a^3*f) + 3/64*((5*a^6*b^13 - 112*a^7*b^1
1*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11
*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 + 2
*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b
+ 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*
b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^
4*e^2 + 16*a^5*c^2*f^4*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*
b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)*e^(-1)/((f*x*e + d*f)*f*sqrt
((a^3*b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 + sqrt((a^3*
b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^
8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^
2 + 16*a^5*c^3)))/(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e
^4))))*e^(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*
sqrt(b^2 - 4*a*c)*f^6*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^
2*f^4*e^2)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2
- 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*
c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 - 2*(5*a^4*b^6*c - 57*a^5*
b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*
c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*
e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)
*a))*arctan(2*sqrt(1/2)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^3*b^5*f^4*e^2 - 8*a
^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 - sqrt((a^3*b^5*f^4*e^2 - 8*a^4*b^3
*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8
*e^4 + 16*a^6*c^2*f^8*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*
b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4))))*e^(-3)/((a^7*b^6
*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*f^6*a
bs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*abs(a))
```

maple [C] time = 0.07, size = 7019, normalized size = 14.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 15.40, size = 20580, normalized size = 41.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*
c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61
26640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160
*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f
^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^1
2*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4
 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^1
6*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^
14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6
 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 1
6878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 33
3226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1
543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6
 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10
*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*
f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*
a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4
)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10
*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 -
 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684
160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^
2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*
b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f
^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*
a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*
c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*
e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*
e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c
^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^
7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^
25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 109951162777
6*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^1
6*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a
^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 12401718
0672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1
417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*
f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^1
2*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^1
2*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^
8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^
8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f
^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^1
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2*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a

^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 118541097369 6*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^ 22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18* c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^1 4*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15 *b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 154384780492 8*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 11010 55131648*a^19*b^2*c^13*d*e^11*f^6)*1i)/((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b ^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15 *c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a ^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e ^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b ^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^ 4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^ 15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 22078 03392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 8937485 1072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 8698158 12096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 17473 13491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17 794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5 *b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5* c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a* b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720* a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2 *f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080 *a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9 *e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b ^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^ 6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 22 5*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a *c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^2 0*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^1 0*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2 *f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120 *a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^ 4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14 *f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f ^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14 *f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e ^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c ^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26* b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^2 3*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^1 9*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19 *b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 4960687226 88*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 283 4678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f ^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12* f^8 - 10911744*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 12 2607894528*a^17*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1 438679826432*a^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 -

3949222428672*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^

10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) + 191102976000*a^17*c^14*e^10*f^4 + 2851200*a^9*b^16*c^6*e^10*f^4 - 92568960*a^10*b^14*c^7*e^10*f^4 + 1312630272*a^11*b^12*c^8*e^10*f^4 - 10611136512*a^12*b^10*c^9*e^10*f^4 + 53445353472*a^13*b^8*c^10*e^10*f^4 - 171591892992*a^14*b^6*c^11*e^10*f^4 + 342580396032*a^15*b^4*c^12*e^10*f^4 - 388363714560*a^16*b^2*c^13*e^10*f^4))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*2i - atan(((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 12

4017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6)*1i + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^

6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11
*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*
b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*
a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627
776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a
^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640
*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017
180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 -
1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^1
3*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c
^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a^13*b^21*c^3*e
^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*b^17*c^5*e^12*
f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*b^13*c^7*e^12*
f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^19*b^9*c^9*e^12
*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a^21*b^5*c^11*e
^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973696*a^23*b*c^13*
e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*
f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6
+ 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f
^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^
11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^1
1*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*
b^2*c^13*d*e^11*f^6)*1i)/((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2)
+ 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860
*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*
b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*
c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/
2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 10485
76*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^
4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^1
3*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2
*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(27
1790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10
*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^1
6*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12
*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8
*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*
b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 +
25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c
^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19
905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680
*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a
^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2
)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e
^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*
a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*
e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*
a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/
2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299
860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a
^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 10
48576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2
*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*
a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*
e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*
(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686
720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925

120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068
722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 283
4678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10
+ 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^
10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f
^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f
^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^
13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c
^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^2
3*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 30236569
76384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744
*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^1
5*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^1
7*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a
^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672
*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973
696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*
b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^1
8*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b
^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^
15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804
928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 110
1055131648*a^19*b^2*c^13*d*e^11*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^
2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*
c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 -
43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^
3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^
2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^
14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4
+ 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^1
5*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))
^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 +
9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 220780
3392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851
072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 86981581
2096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 174731
3491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-
(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 177
94*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*
b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c
^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b
^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c -
b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a
^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*
f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*
a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*
e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10
*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*
f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*
a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4
)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*
f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^

10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*
f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^
14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^
11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b
*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23
*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19
*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*
b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 49606872268
8*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834
678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^
10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f
^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2
673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122
607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 14
38679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 +
3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8
 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6
- 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 19103
8464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878
108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 33
3226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6
 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e
^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) + 191102976000*a^17*c^14*
e^10*f^4 + 2851200*a^9*b^16*c^6*e^10*f^4 - 92568960*a^10*b^14*c^7*e^10*f^4
+ 1312630272*a^11*b^12*c^8*e^10*f^4 - 10611136512*a^12*b^10*c^9*e^10*f^4 +
53445353472*a^13*b^8*c^10*e^10*f^4 - 171591892992*a^14*b^6*c^11*e^10*f^4 +
342580396032*a^15*b^4*c^12*e^10*f^4 - 388363714560*a^16*b^2*c^13*e^10*f^4))
(-(9(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 +
17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a
^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^
5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*
a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c
 - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 72
0*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e
^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 19660
80*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c
^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*2i - ((x^4*(15*b^6*e^3 + 324*a^
3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*e^3 + 12600*a^2*c^4*d^4*e^3
+ 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*a*b^3*c^2*d^2*e^3 + 5880*a^2
*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^
3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5 + 50
40*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108*a*b^2*c^3*d^2*e^5))/(8*a*(a
^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^6*d^3 + 90*b^5*c*d^5 + 648*a
^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 + 25*a*b^5*d - 681*a*b^3*c^2*
d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 + 50*a^2*b^2*c^2*d^3 - 194*a^2
*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/(4*a*(a^2*b^4 + 16*a^4*c^2 -
 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 + 140*b^4*c^2*d^3*e^4 + 30*b^
5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3*d*e^4 - 1036*a*b^2*c^3*d^3*
e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^8*(60*a^2*c^4*e^7 +
 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*
c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^3*c^3*d^2*e + 5040*a^2*c^4*d
^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 364*a^3*b*c^2*e + 450*b^5*c*d^
4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e + 5880*a^2*b*c^3*d^4*e - 310
8*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*d*e^2 + 150*b^5*c*d^3*e^2 +
2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2*
c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b*c^3*d^3*e^2 - 1554*a*b^2*c^
3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^7*(60*a^2*c^4
*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))/(a*(a^2*b^4 + 16*a^4*c^2 -

```
8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b^6*d^4 - 64*a^3*b^2*c + 25*a
*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180*a^2*c^4*d^8 + 15*b^4*c^2*d^
8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227*a*b^3*c^2*d^6 + 392*a^2*b*c
^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4 - 91*a*b^4*c*d^4)/(8*a*e*(a
^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b^2*d^2*e^3*f^2 + 84*c^2*d^6*
e^3*f^2 + 2*a*b*e^3*f^2 + 20*a*c*d^2*e^3*f^2 + 70*b*c*d^4*e^3*f^2) + x^6*(8
4*c^2*d^3*e^6*f^2 + 14*b*c*d*e^6*f^2) + x^2*(10*b^2*d^3*e^2*f^2 + 36*c^2*d^
7*e^2*f^2 + 6*a*b*d*e^2*f^2 + 20*a*c*d^3*e^2*f^2 + 42*b*c*d^5*e^2*f^2) + x^
4*(5*b^2*d*e^4*f^2 + 126*c^2*d^5*e^4*f^2 + 10*a*c*d*e^4*f^2 + 70*b*c*d^3*e^
4*f^2) + x^7*(36*c^2*d^2*e^7*f^2 + 2*b*c*e^7*f^2) + x^5*(b^2*e^5*f^2 + 126*
c^2*d^4*e^5*f^2 + 2*a*c*e^5*f^2 + 42*b*c*d^2*e^5*f^2) + x*(a^2*e*f^2 + 5*b^
2*d^4*e*f^2 + 9*c^2*d^8*e*f^2 + 6*a*b*d^2*e*f^2 + 10*a*c*d^4*e*f^2 + 14*b*c
*d^6*e*f^2) + a^2*d*f^2 + b^2*d^5*f^2 + c^2*d^9*f^2 + c^2*e^9*f^2*x^9 + 2*a
*b*d^3*f^2 + 2*a*c*d^5*f^2 + 2*b*c*d^7*f^2 + 9*c^2*d*e^8*f^2*x^8)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.660 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=343

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3ef^3\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6a\right.}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2+1/4*(b^2-
2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d
)^4)^2+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(
-4*a*c+b^2)^2/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+
30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2)
)/a^4/(-4*a*c+b^2)^(5/2)/e/f^3-3*b*ln(e*x+d)/a^4/e/f^3+3/4*b*ln(a+b*(e*x+d)
^2+c*(e*x+d)^4)/a^4/e/f^3
```

Rubi [A] time = 0.59, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d}{\sqrt{b^2-}}\right.}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2)
+ (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b
*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 -
 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d +
 e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx,x,d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx,x,(d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} -$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} +$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} +$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac}{4a\left(b^2-4ac\right)ef^3(d+ex)^2}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac}{4a\left(b^2-4ac\right)ef^3(d+ex)^2}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac}{4a\left(b^2-4ac\right)ef^3(d+ex)^2}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-2ac}{4a\left(b^2-4ac\right)ef^3(d+ex)^2}$$

Mathematica [A] time = 6.16, size = 509, normalized size = 1.48

$$-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{1}{2a^3ef^3(d+ex)^2}+\frac{-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2}{4a^2ef^3\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{-46a^2bc^2-28a^2c^3(d+e}{4a^3ef^3\,(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c
^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*
b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*f^3
*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3
*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] -
8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[
b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3) + (3*(-b
^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a
```

```
*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2
 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
```

fricas [B] time = 14.76, size = 15231, normalized size = 44.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7
+ 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56
*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^
7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*
b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b
^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a
^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3
 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*
a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 +
 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7
9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b
*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x
^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(
b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 +
30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^
3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^
4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*
c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10
*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4
*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2
*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3
*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8
+ a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
```

b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*
a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*
x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*
c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^
3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3
*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 +
3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*
e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 +
(2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 -
12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*
b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c
+ 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*
c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^
3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d
^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2
- 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48
*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c
^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c
^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^
3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 +
2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*
a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3

+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c
^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 +
48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*
c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b
^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64
*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^
4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 +
45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c
 - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7
*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*
a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a
^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^
3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 -
 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*
c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*e^11*f^3*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d*e^10*f^3*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*
b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*
c^3 - 64*a^7*b*c^4)*d)*e^8*f^3*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c
^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3
*c^3 - 64*a^7*b*c^4)*d^2)*e^7*f^3*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^
3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*
a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*f^3*x^5 + (2*a^5*b^7 - 24*a^6*b^5*
c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48
*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b
^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 +
32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^
5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c
^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^
6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c +
 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*f^3*x^3 + (a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 -
64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^
2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64
*a^8*b*c^3)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*
b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3
 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b
^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64
*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)
*e^2*f^3*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*
d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 +
 (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d
^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*
b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3), -1/4*(6*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8 + 48*(a*b^6*
c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*
c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a
^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6*c^2 -
 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*
b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*

b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 6

4*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*

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c^4 - 64*a^7*c^5)*e^11*f^3*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6
*b^2*c^4 - 64*a^7*c^5)*d*e^10*f^3*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*
a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*
b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*f^3*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b
^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6
*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*f^3*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^
4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 +
 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^
4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*f^3*x^5 + (2*a^5*b^7 - 24*a^6*
b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a
^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^6*c^2 - 1
2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b
^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 2
4*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5
*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*f^3*x^3 + (a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6
*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^
3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^
7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2
- 64*a^8*b*c^3)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*
c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a
^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2
- 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*d)*e^2*f^3*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c
^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d
^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^
4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3)]
```

giac [B] time = 1.63, size = 1735, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] 3/4*((a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 -
 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*
x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e
 + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*
c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 - 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5
*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a)))/(a^8*b^8*c*f^6*e^4 - 16*a^9*b^6*c^2*f^6*e^4 + 96*a^10*b^4*c^3*f^6*e^4
 - 256*a^11*b^2*c^4*f^6*e^4 + 256*a^12*c^5*f^6*e^4) - 1/4*(6*b^4*c^2*x^8*e^
8 - 42*a*b^2*c^3*x^8*e^8 + 60*a^2*c^4*x^8*e^8 + 48*b^4*c^2*d*x^7*e^7 - 336*
a*b^2*c^3*d*x^7*e^7 + 480*a^2*c^4*d*x^7*e^7 + 168*b^4*c^2*d^2*x^6*e^6 - 117
6*a*b^2*c^3*d^2*x^6*e^6 + 1680*a^2*c^4*d^2*x^6*e^6 + 336*b^4*c^2*d^3*x^5*e^
5 - 2352*a*b^2*c^3*d^3*x^5*e^5 + 3360*a^2*c^4*d^3*x^5*e^5 + 420*b^4*c^2*d^4
*x^4*e^4 - 2940*a*b^2*c^3*d^4*x^4*e^4 + 4200*a^2*c^4*d^4*x^4*e^4 + 336*b^4*
c^2*d^5*x^3*e^3 - 2352*a*b^2*c^3*d^5*x^3*e^3 + 3360*a^2*c^4*d^5*x^3*e^3 + 1
68*b^4*c^2*d^6*x^2*e^2 - 1176*a*b^2*c^3*d^6*x^2*e^2 + 1680*a^2*c^4*d^6*x^2*
e^2 + 48*b^4*c^2*d^7*x*e - 336*a*b^2*c^3*d^7*x*e + 480*a^2*c^4*d^7*x*e + 6*
```

```
b^4*c^2*d^8 - 42*a*b^2*c^3*d^8 + 60*a^2*c^4*d^8 + 12*b^5*c*x^6*e^6 - 87*a*b
^3*c^2*x^6*e^6 + 138*a^2*b*c^3*x^6*e^6 + 72*b^5*c*d*x^5*e^5 - 522*a*b^3*c^2
*d*x^5*e^5 + 828*a^2*b*c^3*d*x^5*e^5 + 180*b^5*c*d^2*x^4*e^4 - 1305*a*b^3*c
^2*d^2*x^4*e^4 + 2070*a^2*b*c^3*d^2*x^4*e^4 + 240*b^5*c*d^3*x^3*e^3 - 1740*
a*b^3*c^2*d^3*x^3*e^3 + 2760*a^2*b*c^3*d^3*x^3*e^3 + 180*b^5*c*d^4*x^2*e^2
- 1305*a*b^3*c^2*d^4*x^2*e^2 + 2070*a^2*b*c^3*d^4*x^2*e^2 + 72*b^5*c*d^5*x*
e - 522*a*b^3*c^2*d^5*x*e + 828*a^2*b*c^3*d^5*x*e + 12*b^5*c*d^6 - 87*a*b^3
*c^2*d^6 + 138*a^2*b*c^3*d^6 + 6*b^6*x^4*e^4 - 36*a*b^4*c*x^4*e^4 + 14*a^2*
b^2*c^2*x^4*e^4 + 100*a^3*c^3*x^4*e^4 + 24*b^6*d*x^3*e^3 - 144*a*b^4*c*d*x^
3*e^3 + 56*a^2*b^2*c^2*d*x^3*e^3 + 400*a^3*c^3*d*x^3*e^3 + 36*b^6*d^2*x^2*e
^2 - 216*a*b^4*c*d^2*x^2*e^2 + 84*a^2*b^2*c^2*d^2*x^2*e^2 + 600*a^3*c^3*d^2
*x^2*e^2 + 24*b^6*d^3*x*e - 144*a*b^4*c*d^3*x*e + 56*a^2*b^2*c^2*d^3*x*e +
400*a^3*c^3*d^3*x*e + 6*b^6*d^4 - 36*a*b^4*c*d^4 + 14*a^2*b^2*c^2*d^4 + 100
*a^3*c^3*d^4 + 9*a*b^5*x^2*e^2 - 68*a^2*b^3*c*x^2*e^2 + 122*a^3*b*c^2*x^2*e
^2 + 18*a*b^5*d*x*e - 136*a^2*b^3*c*d*x*e + 244*a^3*b*c^2*d*x*e + 9*a*b^5*d
^2 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a
^4*c^2)/((a^3*b^4*f^3*e - 8*a^4*b^2*c*f^3*e + 16*a^5*c^2*f^3*e)*(c*x^5*e^5
+ 5*c*d*x^4*e^4 + 10*c*d^2*x^3*e^3 + 10*c*d^3*x^2*e^2 + 5*c*d^4*x*e + c*d^5
 + b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + a*x*e + a*d)^2) + 3/4*
b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e
+ c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^4*f^3) - 3*b*e^(-1)*log(ab
s(x*e + d))/(a^4*f^3)
```

maple [C] time = 0.08, size = 5737, normalized size = 16.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 24.91, size = 25334, normalized size = 73.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] (log(((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c -
b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^
2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^
4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3
*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*
e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c
*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30
*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*f
^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c
```

```
 - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*
e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 10
0*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^
3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*
c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*
a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (
18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3
 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b
^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3
*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*
e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6))*((27*c^
5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6) - ((
3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8
*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*
c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b
^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e
*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a
*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^
2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3
*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*f^3*(-(b^6 -
20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^
(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c
*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 -
30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^3*d*e^17*x*(
b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2))
)/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e
^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*
c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b^4 + 10*a^2*
c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b
^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x*(b^4
+ 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6)))*(6*b^11*e*f^3 - 12
0*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b
^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c
^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*
b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - ((x^4*(6*b^6*e^3 + 100*a^3*c^3*e
^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 + 4200*a^2*c^4*d^4*e^3 + 420*b^
4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^2*d^2*e^3 + 2070*a^2*b*c^3*d^
2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) +
 (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5 + 560*a^2*c^4*d^
2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2*e^5))/(4*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c*d^5 + 200*a^3*c^3*d^3 + 240*
a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261*a*b^3*c^2*d^5 + 414*a^2*b*c^
3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3 - 68*a^2*b^3*c*d + 122*a^3*b
*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^5
*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 12*b^5*c*d*e^4 - 87*a*b^3*c^2*
d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^3*e^4))/(2*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 + b^4*c^2*e^7 - 7*a*b^2*c^3*e^
7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^2*(36*b^6*d^2*e + 9*a*b^5
*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e + 168*b^4*c^2*d^6*e - 68*a^2*b^
3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 216*a*b^4*c*d^2*e - 1305*a*b^3*
c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*c^3*d^6*e + 84*a^2*b^2*c^2*d^
2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^3*(6*b^6*d*e^2 + 100*a^
3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d^5*e^2 + 84*b^4*c^2*d^5*e^2 -
 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435*a*b^3*c^2*d^3*e^2 + 690*a^2*
b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c
) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6 - 7*a*b^2*c^3*d*e^6))/(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 32*a^4*c^2 + 6*b^6*d^4 - 16*a^
3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3*c^3*d^4 + 60*a^2*c^4*d^8 + 6
```

*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 - 87*a*b^3*c^2*d^6 + 13
8*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^2*c^2*d^4 - 36*a*b^4*c*d^4)/(
4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^4*(15*b^2*d^2*e^4*f^3 + 210*c
^2*d^6*e^4*f^3 + 2*a*b*e^4*f^3 + 30*a*c*d^2*e^4*f^3 + 140*b*c*d^4*e^4*f^3)
+ x^7*(120*c^2*d^3*e^7*f^3 + 16*b*c*d*e^7*f^3) + x*(6*b^2*d^5*e*f^3 + 10*c^
2*d^9*e*f^3 + 2*a^2*d*e*f^3 + 8*a*b*d^3*e*f^3 + 12*a*c*d^5*e*f^3 + 16*b*c*d
^7*e*f^3) + x^3*(20*b^2*d^3*e^3*f^3 + 120*c^2*d^7*e^3*f^3 + 8*a*b*d*e^3*f^3
 + 40*a*c*d^3*e^3*f^3 + 112*b*c*d^5*e^3*f^3) + x^2*(a^2*e^2*f^3 + 15*b^2*d^
4*e^2*f^3 + 45*c^2*d^8*e^2*f^3 + 12*a*b*d^2*e^2*f^3 + 30*a*c*d^4*e^2*f^3 +
56*b*c*d^6*e^2*f^3) + x^5*(6*b^2*d*e^5*f^3 + 252*c^2*d^5*e^5*f^3 + 12*a*c*d
*e^5*f^3 + 112*b*c*d^3*e^5*f^3) + x^8*(45*c^2*d^2*e^8*f^3 + 2*b*c*e^8*f^3)
+ x^6*(b^2*e^6*f^3 + 210*c^2*d^4*e^6*f^3 + 2*a*c*e^6*f^3 + 56*b*c*d^2*e^6*f
^3) + a^2*d^2*f^3 + b^2*d^6*f^3 + c^2*d^10*f^3 + c^2*e^10*f^3*x^10 + 2*a*b*
d^4*f^3 + 2*a*c*d^6*f^3 + 2*b*c*d^8*f^3 + 10*c^2*d*e^9*f^3*x^9) - (3*b*log(
d + e*x))/(a^4*e*f^3) + (3*atan((x^2*((((((54*a^3*b^13*c^4*e^17*f^3 - 1233*
a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57204*a^6*b^7*c^7*e^17
*f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9*e^17*f^3 + 129600*a
^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9
+ 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 61
44*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*f
^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c^
6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 - 1
99680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^
10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4
*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a
^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*
b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 384
0*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6
*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 1
63840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 +
 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*
f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1
0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*
f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5
*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^
3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c
^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b
^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 +
960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(
2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25
60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6))
- (27000*a^6*c^11*e^16 + 27*b^12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*
b^8*c^7*e^16 - 20601*a^3*b^6*c^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*
b^2*c^10*e^16)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240
*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^
14*b^2*c^5*f^9) + (3*((3*((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*
f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c
^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 -
199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b
^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^
4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*
a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4
*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 38
40*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^
6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 -
163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6
+ 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2
*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^
10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4

*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^
3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3
)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c
^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 2
27328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*
c^9*e^19*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*
a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120
*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*
f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3
840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*
b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3
*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6
144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680
*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9
+ 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^
7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^
9 - 163840*a^16*b*c^9*e^19*f^9))/(32*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^1
0*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c
^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9
+ 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^1
2*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 1
0*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*
a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7
675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((3*((54*a^3*b^13*
c^4*e^17*f^3 - 1233*a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57
204*a^6*b^7*c^7*e^17*f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9
*e^17*f^3 + 129600*a^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 -
24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a
^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6
*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*
f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*
b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*
c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*
b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c
^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^1
3*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9
+ 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*
b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 40
96*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5
120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2
*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^
4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10
*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - (((3*((153600*a^13*c^10*e^18
*f^6 + 6*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c
^5*e^18*f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 1006
08*a^11*b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 40
96*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6
*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3
- 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a
^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*
a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e
^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 2949

12*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*
f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2
*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 409
6*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*
c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^
3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b
^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*
e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f
^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*
e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 9728
0*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^
8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*
(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560
*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^
9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9
- 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*
(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*
e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^
2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e
^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) + (27*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^
10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^1
9*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912
*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(64*a^12*e^3*f^9*(4*a
c - b^2)^(15/2)(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 2
40*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*
a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^
3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 96
0*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 12
0*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(
4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*
b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15
/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*
(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*
c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c
^6*e^14 - 32400*a^5*b^2*c^7*e^14) - (x*((((2*(27000*a^6*c^11*d*e^15 + 27*b^
12*c^5*d*e^15 - 567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3
*b^6*c^8*d*e^15 + 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15))/(a
^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9
 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) -
 (((2*(129600*a^9*b*c^10*d*e^16*f^3 + 54*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4
*b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e
^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3 - 223200*a^8*b^3*c^9*d*e^16*f^3))/(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)
- (((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b
^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f
^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 19968
0*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1
0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*
f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^
5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b
^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 +
328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b
^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e
^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*
c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8
*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9
- 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*

a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f
^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3
+ 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 +
640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f
^6 - 80*a^5*b^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b
*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*
c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2
*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8
*c*e^2*f^6)) - (3*((3*((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e
^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a
^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8
*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f^9 + 4096*a^15*c^
6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 +
3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^1
5*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18
*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 2273
28*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/((4*a^4*b^10*
e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3
*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 +
4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*
b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^
3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - (
3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^1
5*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18
*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 2273
28*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/(4*a^4*e*f^3*
(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^
6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^
5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 2
40*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*
a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^
4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a
*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^
2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*
a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^
18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97
280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*
b^3*c^8*d*e^18*f^9))/(16*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^10*e^2*f^6 -
4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 +
5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15
*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^
9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 10*a^4*c^4 +
120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6
*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*
c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((2*(153600*a^13*c^10*d*
e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^
8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e
^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))
/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*
f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9
) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*
b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^
9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a
^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c

^8*d*e^18*f^9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c
^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b
^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*
a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^1
4*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e
f^3(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^
4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*
b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^
9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a
^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c
^8*d*e^18*f^9))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096
*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 512
0*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6
*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 +
3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*
c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e
*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f
^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*
e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - (3*((2*(129600*a^9*b*c^10*d*e^16*f^3 + 5
4*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4*b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^
6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3
 - 223200*a^8*b^3*c^9*d*e^16*f^3))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a
^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b
^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((2*(153600*a^13*c^10*d*e^17*f^6 + 6
*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d
*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 10
0608*a^11*b^4*c^8*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f
^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a
^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11
*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 -
 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18
*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^1
1*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*
d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^
9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 -
 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6
)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2
*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^
9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^
10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*
c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) +
 (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*
d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 38
40*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^
7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e
^18*f^9))/(32*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 1
80*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(1
3/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*
b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2
)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4
*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^1
5*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(
15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 +
 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3

*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14) + (((((36*
a^3*b^14*c^3*e^15*f^3 - 14400*a^10*c^10*e^15*f^3 - 837*a^4*b^12*c^4*e^15*f^
3 + 8046*a^5*b^10*c^5*e^15*f^3 - 40941*a^6*b^8*c^6*e^15*f^3 + 116532*a^7*b^
6*c^7*e^15*f^3 - 177588*a^8*b^4*c^8*e^15*f^3 + 119520*a^9*b^2*c^9*e^15*f^3
+ 129600*a^9*b*c^10*d^2*e^15*f^3 + 54*a^3*b^13*c^4*d^2*e^15*f^3 - 1233*a^4*
b^11*c^5*d^2*e^15*f^3 + 11583*a^5*b^9*c^6*d^2*e^15*f^3 - 57204*a^6*b^7*c^7*
d^2*e^15*f^3 + 156276*a^7*b^5*c^8*d^2*e^15*f^3 - 223200*a^8*b^3*c^9*d^2*e^1
5*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^
8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c
^5*f^9) - (((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^
8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16*
f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600*
a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^1
6*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792
*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b
^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*f^9 + 409
6*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*
c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 -
120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^
3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^
11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*
f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^1
6*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e
^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9
- 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 22732
8*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*
b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^
4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*
f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*
a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11
*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 -
3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 -
4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6
+ 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*
b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c
^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e
^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*
c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - (27*b^13*c^4*e^14 - 594*a*b^11*c^5*e
^14 + 43200*a^6*b*c^10*e^14 + 5319*a^2*b^9*c^6*e^14 - 24732*a^3*b^7*c^7*e^1
4 + 62748*a^4*b^5*c^8*e^14 - 82080*a^5*b^3*c^9*e^14 + 27000*a^6*c^11*d^2*e^
14 + 27*b^12*c^5*d^2*e^14 + 4779*a^2*b^8*c^7*d^2*e^14 - 20601*a^3*b^6*c^8*d
^2*e^14 + 47790*a^4*b^4*c^9*d^2*e^14 - 56700*a^5*b^2*c^10*d^2*e^14 - 567*a*
b^10*c^6*d^2*e^14)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 +
240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 614
4*a^14*b^2*c^5*f^9) + (3*((3*((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*
e^16*f^6 + 3156*a^8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*
a^10*b^7*c^6*e^16*f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*
e^16*f^6 + 153600*a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^
6*b^14*c^3*d^2*e^16*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*
d^2*e^16*f^6 + 792*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f
^6 + 100608*a^11*b^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)
+ ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c
^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^
2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*
a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^
17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*
a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11

*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(32*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((3*((36*a^3*b^14*c^3*e^15*f^3 - 14400*a^10*c^10*e^15*f^3 - 837*a^4*b^12*c^4*e^15*f^3 + 8046*a^5*b^10*c^5*e^15*f^3 - 40941*a^6*b^8*c^6*e^15*f^3 + 116532*a^7*b^6*c^7*e^15*f^3 - 177588*a^8*b^4*c^8*e^15*f^3 + 119520*a^9*b^2*c^9*e^15*f^3 + 129600*a^9*b*c^10*d^2*e^15*f^3 + 54*a^3*b^13*c^4*d^2*e^15*f^3 - 1233*a^4*b^11*c^5*d^2*e^15*f^3 + 11583*a^5*b^9*c^6*d^2*e^15*f^3 - 57204*a^6*b^7*c^7*d^2*e^15*f^3 + 156276*a^7*b^5*c^8*d^2*e^15*f^3 - 223200*a^8*b^3*c^9*d^2*e^15*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16*f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600*a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^16*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*

f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*
a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^1
1*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3
- 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^
9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*
c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 +
16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*
c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*
e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^
9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(
2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25
60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9))
)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^
2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*
e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3
*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*a^3
*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - ((
(3*((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^8*b^11*c
^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16*f^6 - 10
9824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600*a^13*c^1
0*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^16*f^6 -
108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792*a^9*b^8
*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b^4*c^8*d
^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b
^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^
3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*
c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15
360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^
8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9
- 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*
a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b
^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*b^10*e^2
*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^
2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 40
96*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6
*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c
^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(
b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c
*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*
f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*
e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*
a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^
17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 32
8*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12
*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c
^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(8*a^4*e*f^3*(4*a*c -
b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2
*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e
^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b
^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*
c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a
^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*
a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^
7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) + (27
*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^14*c^2*e^17*f

```
^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8
*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 +
 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15
*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2
*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f
^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/
(64*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 2
4*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^1
3*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4
*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2)
 + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b
^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^
3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) -
98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/
2)*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*
b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*
b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 -
7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(b^6 - 20*a^3*c^3 + 3
0*a^2*b^2*c^2 - 10*a*b^4*c))/(2*a^4*e*f^3*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.661 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=340

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{}}{e^2\sqrt{}}$$

```
[Out] -d*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^2/(a+b*(e*x+d)^3
+c*(e*x+d)^6)^(1/2)+1/2*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*(e*x+d)^3/(
b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)
^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/e^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1389, 1790, 1348, 429, 1385, 510}

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{}}{e^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*e
^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
```

```
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1389

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{\text{Subst}\left(\int \frac{-d+x}{\sqrt{a+bx^3+cx^6}}\,dx,x,d+ex\right)}{e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d}{\sqrt{a+bx^3+cx^6}}+\frac{x}{\sqrt{a+bx^3+cx^6}}\right)dx,x,d+ex\right)}{e^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx,x,d+ex\right)}{e^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx,x,d+ex\right)}{e^2}$$

$$= \frac{\left(\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx,\right.}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

$$= -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(}{b+\sqrt{}}\right.}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

Mathematica [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
[Out] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{ce^6x^6 + 6\,cde^5x^5 + 15\,cd^2e^4x^4 + cd^6 + \left(20\,cd^3 + b\right)e^3x^3 + 3\left(5\,cd^4 + bd\right)e^2x^2 + bd^3 + 3\left(2\,cd^5 + bd^2\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (20*
c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d^2
)*e*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(ex+d)^6c + (ex+d)^3b + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + (ex+d)^3\,b + (ex+d)^6\,c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
[Out] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(ex+d)^6c + (ex+d)^3b + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a + b\,(d + e\,x)^3 + c\,(d + e\,x)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
[Out] int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bd^3 + 3bd^2ex + 3bde^2x^2 + be^3x^3 + cd^6 + 6cd^5ex + 15cd^4e^2x^2 + 20cd^3e^3x^3 + 15cd^2e^4x^4 + 6cde^5x^5 + ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3 +
 c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d
**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

3.662 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}} dx$

Optimal. Leaf size=398

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}}{e^3}$$

```
[Out] 1/3*arctanh(1/2*(b+2*c*(e*x+d)^3)/c^(1/2)/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
)/e^3/c^(1/2)+d^2*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*
c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^3
/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)-d*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*
c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(
1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/e^3/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1389, 1790, 1348, 429, 1385, 510, 1352, 621, 206}

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*
Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^
3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*c*(d + e*x)^3)/
(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1348

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1389

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{\text{Subst}\left(\int \frac{(-d+x)^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{d^2}{\sqrt{a+bx^3+cx^6}} - \frac{2dx}{\sqrt{a+bx^3+cx^6}} + \frac{x^2}{\sqrt{a+bx^3+cx^6}}\right) dx, x, d+ex\right)}{e^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3} - \frac{(2d)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+\right.}{e^3}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, (d+ex)^3\right)}{3e^3} - \frac{\left(2d\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+e}{b+\sqrt{b^2-}}\right.}{e^3\sqrt{a}}$$
$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\right.}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$
$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\right.}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
[Out] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

fricas [F] time = 11.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{ce^6x^6 + 6\,cde^5x^5 + 15\,cd^2e^4x^4 + cd^6 + \left(20\,cd^3+b\right)e^3x^3 + 3\left(5\,cd^4+bd\right)e^2x^2 + bd^3 + 3\left(2\,cd^5+bd^2\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (2
0*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d
^2)*e*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(ex+d)^6c + (ex+d)^3b + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + (ex + d)^3 b + (ex + d)^6 c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(e*x+d)^3*b+(e*x+d)^6*c)^(1/2),x)
```

```
[Out] int(x^2/(a+(e*x+d)^3*b+(e*x+d)^6*c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(ex + d)^6 c + (ex + d)^3 b + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a + b\,(d + e\,x)^3 + c\,(d + e\,x)^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

```
[Out] int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bd^3 + 3bd^2ex + 3bde^2x^2 + be^3x^3 + cd^6 + 6cd^5ex + 15cd^4e^2x^2 + 20cd^3e^3x^3 + 15cd^2e^4x^4 + 6cde^5x^5 + ce^6x^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**
3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*
c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

3.663 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx$

Optimal. Leaf size=34

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[Out] 1/21*(2+3*x)^7+1/42*(2+3*x)^14+1/63*(2+3*x)^21
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1390, 14}

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1390

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx &= \frac{1}{3}\text{Subst}\left(\int x^6 \left(1+x^7+x^{14}\right) dx, x, 2+3x\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(x^6+x^{13}+x^{20}\right) dx, x, 2+3x\right)\\ &= \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

fricas [B] time = 0.75, size = 104, normalized size = 3.06

$$\frac{1162261467}{7}x^{21}+2324522934x^{20}+15496819560x^{19}+65431015920x^{18}+196293047760x^{17}+444930908256x^{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

giac [A] time = 0.35, size = 28, normalized size = 0.82

$$\frac{1}{63}\,(3\,x+2)^{21}+\frac{1}{42}\,(3\,x+2)^{14}+\frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")
```

```
[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

maple [B] time = 0.00, size = 105, normalized size = 3.09

$$\frac{1162261467}{7}x^{21}+2324522934x^{20}+15496819560x^{19}+65431015920x^{18}+196293047760x^{17}+444930908256x^{16}+79$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14),x)
```

```
[Out] 1162261467/7*x^21+2324522934*x^20+15496819560*x^19+65431015920*x^18+1962930
47760*x^17+444930908256*x^16+790988281344*x^15+15819767221203/14*x^14+13183
14865122*x^13+1269491970942*x^12+1015602174288*x^11+677082445416*x^10+37617
4427616*x^9+173635132896*x^8+66158154783*x^7+20588764518*x^6+5149786572*x^5
+1010576952*x^4+149902032*x^3+15808800*x^2+1056832*x
```

maxima [B] time = 0.54, size = 104, normalized size = 3.06

$$\frac{1162261467}{7}\,x^{21}+2324522934\,x^{20}+15496819560\,x^{19}+65431015920\,x^{18}+196293047760\,x^{17}+444930908256\,x^{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

mupad [B] time = 1.58, size = 29, normalized size = 0.85

$$\frac{(3\,x+2)^7\,\left(3\,(3\,x+2)^7+2\,(3\,x+2)^{14}+6\right)}{126}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1),x)
```

```
[Out] ((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126
```

sympy [B] time = 0.10, size = 107, normalized size = 3.15

$$\frac{1162261467x^{21}}{7}+2324522934x^{20}+15496819560x^{19}+65431015920x^{18}+196293047760x^{17}+444930908256x^{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] 1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x**
18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 1581976
7221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217428
8*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 + 6615
8154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 + 14990
2032*x**3 + 15808800*x**2 + 1056832*x
```

3.664 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[Out] 1/21*(2+3*x)^7+1/21*(2+3*x)^14+1/21*(2+3*x)^21+1/42*(2+3*x)^28+1/105*(2+3*x
)^35
```

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1390, 1352, 611}

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 +
 3*x)^35/105
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1390

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx &= \frac{1}{3}\text{Subst}\left(\int x^6 \left(1+x^7+x^{14}\right)^2 \, dx, x, 2+3x\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+x+x^2\right)^2 \, dx, x, (2+3x)^7\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+2x+3x^2+2x^3+x^4\right) dx, x, (2+3x)^7\right)\\
&= \frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21}+\frac{1}{42}(2+3x)^{28}+\frac{1}{105}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 188, normalized size = 3.36

$$\frac{16677181699666569x^{35}}{35}+1111812113311046x^{34}+1260053728419251 88x^{33}+92403940084078 4712x^{32}+4928210$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4 +
 (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7 +
197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*x^1
0)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 940692639189
29616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^15 + 8
72775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660147522
40080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20 + (2
6506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 306451507651
2846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711771792*x^2
5)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 + (262545832
6972530284475*x^28)/14 + 67899784121041365504*x^29 + (101849676181562048256
*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 + 12600537284
1925188*x^33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35
```

fricas [B] time = 0.66, size = 174, normalized size = 3.11

$$\frac{16677181699666569}{35}x^{35}+11118121133111046x^{34}+126005372841925188x^{33}+924039400840784712x^{32}+4928$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

giac [A] time = 0.42, size = 46, normalized size = 0.82

$$\frac{1}{105}\,(3\,x+2)^{35}+\frac{1}{42}\,(3\,x+2)^{28}+\frac{1}{21}\,(3\,x+2)^{21}+\frac{1}{21}\,(3\,x+2)^{14}+\frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

```
[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2)
^14 + 1/21*(3*x + 2)^7
```

maple [B] time = 0.00, size = 175, normalized size = 3.12

$$\frac{16677181699666569}{35}x^{35}+11118121133111046x^{34}+126005372841925188x^{33}+924039400840784712x^{32}+4928$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14)^2,x)
```

```
[Out] 17451466816*x+16677181699666569/35*x^35+67899784121041365504*x^29+262545832
6972530284475/14*x^28+875152864622814086340*x^26+2298383223254096766840*x^2
```

```
4+3614565944605222108800*x^22+3534290697929473864098*x^20+21945771660147522
40080*x^18+8727757740674554985 28*x^16+11118121133111046*x^34+12600537284192
5188*x^33+924039400840784712*x^32+4928210137817518464*x^31+1018496761815620
48256/5*x^30+4375763967252854465 64*x^27+889942562270387136*x^9+118214875016
20716192*x^11+9406926391892961632 4*x^13+46551709104168101529 6*x^15+14631040
32160519033200*x^17+2945285062308448290360*x^19+265069490388589180368 81/7*x
^21+3064515076512846852480*x^23+7584660010542711771792/5*x^25+8740667957868
0*x^4+443569828128*x^2+6077684727888102*x^6+197897276851452864*x^8+17344958
593049772048/5*x^10+354540694805720481 24*x^12+22169975754827019438 9*x^14+72
99544818384*x^3+4057390785756924/5*x^5+37727143432895007*x^7
```

maxima [B] time = 0.77, size = 174, normalized size = 3.11

$$\frac{16677181699666569}{35}\,x^{35}+11118121133111046\,x^{34}+126005372841925188\,x^{33}+924039400840784712\,x^{32}+49282$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 262545832697253028 4475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

mupad [B] time = 1.60, size = 46, normalized size = 0.82

$$\frac{(3\,x+2)^7}{21}+\frac{(3\,x+2)^{14}}{21}+\frac{(3\,x+2)^{21}}{21}+\frac{(3\,x+2)^{28}}{42}+\frac{(3\,x+2)^{35}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)
```

```
[Out] (3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*x
 + 2)^35/105
```

sympy [B] time = 0.15, size = 187, normalized size = 3.34

$$\frac{16677181699666569x^{35}}{35}+11118121133111046x^{34}+126005372841925188x^{33}+924039400840784712x^{32}+4928210$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*x
**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 1018496761815
62048256*x**30/5 + 67899784121041365504*x**29 + 2625458326972530284475*x**2
8/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 75846600
10542711771792*x**25/5 + 2298383223254096766840*x**24 + 3064515076512846852
480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x**21/7
+ 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 21945771660
```

```
14752240080*x**18 + 1463104032160519033200*x**17 + 872775774067455498528*x*
*16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 + 940692639
18929616324*x**13 + 35454069480572048124*x**12 + 11821487501620716192*x**11
 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 197897276851452
864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405739078575692
4*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 1
7451466816*x
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```